CAFSAC RESEARCH DOCUMENT 79/45

Response of Cohort Analysis to Input Parameters.

By D. Rivard and W. G. Doubleday

May 1979

CONTENTS

As indicated by Pope (1972), cohort analysis is used to reconstruct the age-composition of the stock ($N_{i,t}$) and to estimate the instantaneous rates of fishing mortality ($F_{i,t}$) for a specified number of years ($t=t_0,\ldots,t_f$) and a predetermined number of ages ($i=b,\ldots,m$). For the purposes of the following discussion, the $N_{i,t}$ and the $F_{i,t}$ will be refered to as the calculated quantities or the output variables. The method of cohort analysis uses historical information on catch-at-age ($C_{i,t}, i=b,\ldots,m;\ t=t_0,\ldots,t_f$), an estimate of the instantaneous rate of natural mortality (M), as well as an estimate of the instantaneous fishing mortalities for the last age-groups ($F_{m,t}$, $t = t_0,\ldots, t_f$) and for the last year of historical catch ($F_{i,t_f}, i=b,\ldots,m$). In this paper, the $C_{i,t}$, M, $F_{m,t}$ and F_{i,t_f} are refered to as the input parameters.

This paper investigates the effect of small perturbations of input parameters on the output variables. In technical terms, such an approach is called a "sensitivity analysis". Sensitivity analyses are often used to identify important parameters and to decide on the "relative worth of improving various parts of a data base" (Miller, 1974). In order to gain insight on the subject, three cases will be discussed: cod in 3Pn-4RS, plaice in division 4T (Schweigert, 1978) and herring in 4WX (Stobo, Gray and Metuzals, 1978). From these three numerical examples, which cover a wide range of values for somatic growth, we will study the response of output variables, as calculated by the method of cohort analysis, to changes in the initial values of input parameters.

Relative sensitivity coefficients.

Sensitivities are calculated as the ratio of the relative change of a calculated quantity to a small relative change in the parameters (see Rivard and Doubleday, 1979). Therefore, a negative value indicates that an increase (decrease) of the parameter value will give rise to a decrease (increase) of the output variable. On the other hand, a positive value indicates that an increase (decrease) of the parameter value will give rise to an increase (decrease) of the calculated quantity.

A value of zero for the relative sensitivity coefficients indicates that the calculated quantity is not influenced by a change of the initial parameter value. A value of ± 1 for the relative sensitivity indicates that a 1% change in the parameter value is accompanied by a 1% change in the output variable. This correspondence is approximately true for small changes of the parameter value. For larger changes of the parameter value, the exactitude of this correspondence depends upon the degree of nonlinearity of the model. For cohort analysis, linearity holds for surprisingly large perturbations of input parameters (Table 1). For perturbations of more than 50% of initial parameter value, the calculated population numbers can be approximated reasonably well by a linear projection of the relative sensitivity coefficients. Even though the model is not linear with respect to natural and fishing mortalities, the linear approximation applies reasonably well. Note finally that the model is linear with respect to catch values. In conclusion, we can say that the linear projection of relative sensitivities gives a reasonable image of the response of the cohort analysis to changes of initial parameters (catch-at-age, natural mortality, initial fishing mortalities).

A value between -1 and +1 for the relative sensitivity coefficients implies that the relative change in the output variable is smaller than the relative change in the parameter. A value less than -1 or a value greater than +1 indicates that the relative change in the output variable is larger than the relative change in the parameter. For example, a value of .5 indicates that a 1% change in the parameter value gives a .5% change in the output variable. Here again, this correspondence is approximately true for small changes of the parameter value (but note that Table 1 indicates that perturbations of more than 50% of initial parameter value still gives a good approximation of all parameters).

Parameter	% change of pop. nb. calculated from	% change in parameters .5	1	2	5	10	20	50	100
A. First year, first age-group									
M	actual perturbation	.29	.59	1.18	2.98	6.08	12.7	35.9	90.1
	sensitivities*	.29	.59	1.17	2.93	5.85	11.7	29.3	58.5
Initial F	actual perturbation	-.07	-.14	-.28	-.67	-1.28	-2.35	-4.68	-6.96
	sensitivities*	-.07	-.14	-.28	-.70	-1.41	-2.82	-7.05	-14.10
Catch (systematic)	actual perturbation	.5	1	2	5	10	20	50	100
	sensitivities*	.5	1	2	5	10	20	50	100
Individual catches	actual perturbation	.09	.18	0.35	0.89	1.77	3.54	8.86	17.7
	sensitivities*	.09	.18	0.35	0.89	1.77	3.54	8.86	17.7
B. Last year, first age-group									
M	actual perturbation	.05	.10	.19	.48	.97	1.94	4.89	9.92
	sensitivities*	.05	.10	.19	.48	.96	1.92	4.80	9.60
Initial F	actual perturbation	-.49	-.98	-1.94	-4.71	-8.99	-16.48	-33.0	-49.4
	sensitivities*	-.49	-.99	-1.98	-4.94	-9.88	-19.80	-49.4	-98.8
Catch (system.)	actual perturbation	.5	1	2	5	10	20	50	100
	sensitivities*	.5	1	2	5	10	20	50	100
Individual catches	actual perturbation	.5	1	2	5	10	20	50	100
	sensitivities*	.5	1	2	5	10	20	50	100

*calculated by assuming linear extension of the relative sensitivity coefficients.

TABLE 1 . Effect of individual parameter changes on recruitment as calculated by cohort analysis for the first year and the last year of the output table. This effect is expressed as a % change of the calculated recruitment.

Sensitivity of calculated recruitment to parameters.

In this section, we analyse the effect of changes of input parameters on the calculated recruitment (i.e. calculated population numbers of the first age-group in each year).

A. Sensitivity to catch data.

For our purposes, we will identify two major sources of error in catch-at-age data:

- systematic error: this is a persistent error which applies simultaneously to all catch values, during each year and for all ages (for example, the permanent misreporting of catches for all age-groups).

- specific error: these are errors related to individual catch values (discarding of early age-groups, random error due to sampling procedure, etc.).

The importance of both types of error for the estimation of recruitment can be assessed by the analysis of the relative sensitivity coefficients.

Systematic error. A persistent error in all catch-at-age figures has been simulated by a small relative perturbation of all entries of the catch matrix. In that case, the relative sensitivity coefficients of recruitment with respect to the catch values take the value of 1.00. This indicates that a 10% change of catch values will yield a 10% change of the estimated recruitment in each year. Since the model is linear with respect to the catches, this conclusion can be extended to any perturbation, large or small, of initial catches. If misreporting is persistent, the trends which are observed in recruitment will still be a good indicator of the relative changes in the year-class size. But if all catches are under-reported, all numbers of recruits, as well as population numbers, are underestimated.

If misreporting is not persistent but is suddenly reduced or absent, the trends in recruitment will no longer be in perfect correlation when plotted against an indicator of the year-class size. However, Gray (1978) demonstrated that the correlation is still high (92%, in his numerical example) and that the cohort analysis shifts fairly quickly to allow reasonable predictions. When we consider only one year of correct data, the recruitment figures are corrected to some degree for (m-b) years back in the cohort table. This readjustment is rapid since each age-group in the final year contributes to modify the estimation of recruitment in the (m-b) years preceding the change in reporting. In the next section, we discuss in detail the response of the calculated number of recruits to changes of individual catch values.

Specific error. The effect of changes of individual catch values on the calculated recruitment is particularly important for the assessment of future recruitment. The sensitivities are calculated for the recruitment in each year with respect to the individual catches (Table 2). Note that the relative sensitivities add to 1.00, since the model is linear with respect to individual catches. In consequence, the total effect (on recruitment) of perturbing two or more catch values can be deduced from the summation of the corresponding sensitivities.

For cod in 3Pn-4RS, the catches in the last year (1978, all ages) take a major role in the determination of recruitment figures. For instance, recruitment in recent years (77,78) is particularly sensitive to catches of age 4 and 5, in 1978. Table 2 also indicates that the recruitment figures are progressively less influenced by misreporting of catches of older age-groups, in the last year. This is illustrated by Figure 1, in which we can see that recent recruitments (76,77 and 78) are largely determined from our information on the latest catch-at-age data. In short, misreporting and poor estimation (related to sampling error) of catch in young age-groups will influence substantially our estimation of recruitment in recent years. From Table 2, we can also draw some conclusions regarding systematic misreporting of catch for early ages. Table 2 indicates that this form of misreporting has a marked effect on the estimation of recruitment in the latest year, but is of little importance for the estimation of recruitment in the early years (in our case, 73-75). This is the case because recruitment in early years is a linear function of the catch-at-age information. Therefore, the effect of perturbing a single catch value on the total number of recruits is attenuated by the presence of other catches which are seen to enter the model in a linear fashion. As a consequence of this, Table 2 indicates that a 10% misreporting in catch at age 4, in 1973, will change our 1973-recruitment by less than 2%. In summary, misreporting of catches in early ages (4 and 5, in the present case) is likely to influence sensibly the recruitment figures in recent years (77,78); but it is likely to become a negligeable source of error for all other years if the situation persists. In fact, Table 2 indicates that a 10% misreporting of catch at ages 4 and 5 will give an overall response of the calculated recruitment in 1973-76 which is within 2%, on the average, of the recruitment calculated by assuming complete reporting for young ages. On the other hand, a 10% misreporting of catches at ages 4 and 5 would change the recruitment figures in 77 and 78 by more than 10%.

For plaice in division 4T, similar trends can be observed (Table 2B). Misreporting of catch in young age-groups influences substantially the estimation of recruitment for the last few years. But recruitment in early years (1964-67) show a rather flat response to changes in individual catch values. Table 2 indicates that a 10% misreporting in any catch-at-age data during those years will change our recruitment estimates by less than 2% (less than 1% in most cases). This conclusion can be extended to the estimation of recruitment in every other year with the following exception. As we more towards 1977, recruitment becomes more and more dependent upon catch-at-age values of the last year (1977). The dependence of recruitment to the last year of catch data is clearly demonstrated in Table 2B. For recruitment of recent years (71-77), the errors related to the last year of catch data have an important effect on our estimates. On the other hand, errors on catch-at-age data in all other years have a limited effect on the estimation of recruitment.

For herring in 4WX, misreporting of catch in younger age-groups would also influence the estimation of recruitment for the last years (Table 2C). From 1965 through 1967, recruitment is not dependent upon the last year of catch data. During the 1965-72 period, the sensitivity of recruitment to individual catches is rather low (at the most, a 3.5% error in recruitment for a 10% error in individual catches). Thereafter recruitment becomes progressively more sensitive to catch values of the last year (see Figure 1C).

COD IN 3Pn-4RS

Recruitment in year	is influenced by catch at age 4	5	6	7	8	9	total
73	.192	.136	.209	.150	.135	.177	1.00
74	.031	.114	.240	.287	.327	--	1.00
75	.028	.175	.215	.582	--	--	1.00
76	.016	.098	.886	--	--	--	1.00
77	.032	.968	--	--	--	--	1.00
78	1.000	--	--	--	--	--	1.00

TABLE 2A . Effect of small perturbations of individual catch values on the calculated recruitment (i.e. population numbers for the first age-groups). The values given in the table are sensitivities of the calculated recruitment in each year with respect to the catch-at-age data which correspond to the year-class. Sensitivities add to 1.0 since the model is linear with respect to catch values.

TABLE 2B. PLAICE IN DIVISION 4T

Recruitment in year	is influenced by catch at age 3	4	5	6	7	8	9	10	11	12	13	14	15	16
1964	.01	.01	.08	.12	.17	.10	.08	.13	.09	.06	.04	.05	.03	.05
1965	.01	.02	.07	.13	.15	.08	.09	.12	.09	.06	.05	.05	.08	--
1966	.01	.01	.07	.12	.09	.09	.09	.12	.10	.08	.07	.15	--	--
1967	.01	.02	.06	.04	.10	.08	.08	.13	.11	.12	.25	--	--	--
1968	.01	.02	.01	.04	.09	.08	.09	.14	.08	.44	--	--	--	--
1969	.01	.02	.01	.05	.11	.11	.12	.11	.47	--	--	--	--	--
1970	.01	.02	.01	.04	.11	.11	.10	.60	--	--	--	--	--	--
1971	.01	.01	.01	.04	.11	.13	.68	--	--	--	--	--	--	--
1972	.01	.01	.01	.03	.12	.83	--	--	--	--	--	--	--	--
1973	.01	.01	.01	.05	.93	--	--	--	--	--	--	--	--	--
1974	.01	.02	.01	.96	--	--	--	--	--	--	--	--	--	--
1975	.01	.03	.96	--	--	--	--	--	--	--	--	--	--	--
1976	.02	.98	--	--	--	--	--	--	--	--	--	--	--	--
1977	1.00	--	--	--	--	--	--	--	--	--	--	--	--	--

TABLE 2C. HERRING IN 4WX

Recruitment in year	is influenced by catch at age 2	3	4	5	6	7	8	9	10
1965	.09	.14	.15	.22	.11	.13	.07	.05	.05
1966	.03	.06	.07	.21	.19	.19	.12	.06	.07
1967	.04	.09	.16	.32	.15	.12	.05	.04	.04
1968	.35	.22	.20	.10	.08	.03	.01	.01	.01
1969	.13	.13	.28	.25	.10	.04	.02	.03	.03
1970	.15	.30	.31	.09	.05	.04	.02	.05	--
1971	.19	.11	.21	.12	.14	.07	.16	--	--
1972	.14	.15	.20	.15	.11	.26	--	--	--
1973	.04	.08	.19	.18	.51	--	--	--	--
1974	.10	.17	.17	.56	--	--	--	--	--
1975	.16	.13	.72	--	--	--	--	--	--
1976	.11	.89	--	--	--	--	--	--	--
1977	1.00	--	--	--	--	--	--	--	--

B. Sensitivity to natural mortality.

As a whole, the sensitivity analysis reveals that an increase of natural mortality will generate higher estimates of recruitment in all years.

For cod in 3Pn-4RS, the sensitivity of recruitment with respect to natural mortality is described in Figure 1A. For the latest years (76-78), recruitment is the least influenced by perturbations in the initial value of natural mortality. The influence of M on recruitment is maximum for the earliest years (73-75). This general trend is related to the way in which natural mortality enters the model. For the latest year, recruitment appears as a simple function of $C_{4,78}$, $F_{4,78}$ and M. But as we go back in time, recruitment becomes a linear function of the catches, as prescribed by the standard equations of cohort analysis. Each of those linear terms is a function of M and any perturbation of natural mortality will influence each term independently. The total response of recruitment to M is calculated from the summation of the individual terms. If the catch matrix is relatively homogeneous for consecutive cohorts, which is the situation for cod, recruitment figures will tend to be more sensitive to natural mortality as we go back in time.

For plaice in division 4T, the sensitivities of recruitment with respect to natural mortality show the same general trends (Figure 2A): recruitment tend to be more sensitive to natural mortality as we go back in time. For the early years, the response of recruitment to M is even more pronounced than the response which has been recorded for cod (Fig. 2A). In fact, a 10% error in M will generate a 10% change or more in our estimation of recruitment for the first years (1964-71).

For herring in 4WX, the errors related to M are the major source of error in the estimation of the 1965-1972 recruitment. In the latest years (1975-1977), recruitment becomes less sensitive to natural mortality while misreporting of the 1977 catches and errors in the 1977 F's are the major sources of error (Figure 1C).

C. Sensitivity to F values in the last year.

As a whole, the relative sensitivity coefficients indicate that an increase (decrease) of the F values in the last year (which values are provided as input data) will generate a decrease (increase) of the estimated recruitment in the different years. This effect is opposite to the effect which is observed when natural mortality and catch data are perturbed: in these two cases, we saw that an increase (decrease) of the initial parameter value generates an increase (decrease) of the calculated recruitment.

For cod in 3Pn-4RS, the sensitivities of recruitment with respect to the initial fishing mortalities is described in Figure 1A. The 1973-75 recruitments are the least influenced by the perturbations of the 1978 fishing mortalities. After this period, the sensitivity of recruitment to the 1978 fishing mortalities increases up to 1978. In fact, a 50% error in the 1978 fishing mortality for age 4 would change the calculated recruitment by 50%, approximately. On the other hand, a 50% error in the 1978 fishing mortality for age 10 would change the recruitment number in 1973 by less than 7%, approximately. This trend is related to the way in which fishing mortality figures in the last year enter the predictive equation for recruitment. In fact, the F values are used solely to estimate population numbers at age in the last year. For previous years, the cohort analysis uses the catch matrix and an estimate of natural mortality to reconstruct population numbers. The initial error in F is progressively damped out as more terms are inserted in the cohort analysis. Recruitment in the early years benefits from the fact that more information is available for its evaluation.

For plaice in 4T, the initial error in F is also progressively damped out as more terms are inserted in the reconstruction of consecutive cohorts. Figure 1B indicates that the sensitivity of recruitment to the 1977 fishing mortalities increases gradually from an absolute value of .04 (i.e. no practical influence) in 1964, to a high of 1.0 in 1977.

For herring in 4WX, recruitment figures before 1972 show a low sensitivity to the 1977 F's. But the 1977 values of the instantaneous fishing mortalities take a major role in the estimation of recruitment in the last few years (Figure 1C). Within the last year (1977), the fishing mortalities of the youngest age-groups are the most important for the evaluation of the latest recruitment.

D. <u>Sensitivity to F values for the last age-groups.</u>

Recruitment numbers in the last (m-b+1) years are not modified by changes of the initial F values for the last age-groups. This is so because recruitment in the last (m-b+1) years is not a function of any of the fishing mortalities which apply to the last age-groups. For earlier years, if present, the relative sensitivity coefficients indicate that an increase (decrease) of the F values in the last age-groups will generate a decrease (increase) of estimated recruitment.

For cod in 3Pn-4RS, we have less than (m-b+1) years. Therefore, the input values for F in the last age-groups do not influence our estimation of recruitment numbers. The same is true for plaice in division 4T.

For herring in 4WX, the 1965-67 recruitments are dependent upon the initial F's for the last age-groups. Figure 1C indicates that recruitment is not sensitive to the initial F's for the last age-groups. In fact, a 20% error in the initial F's for the last age-group in different years will still produce recruitment estimates which are within 1% of the true value.

E. Overall sensitivity of recruitment to parameters.

In order to assess the effect of simultaneous perturbations of initial parameter values on the calculation of recruitment, we defined the following index, which can be calculated for each year:

$$I^2_{year} = \sum_{\theta} (XR_{year;\theta})^2 \quad ,$$

where $XR_{year;\theta}$= relative sensitivity of recruitment to parameter θ in a given year.

This index provides information on the overall sensitivity of the calculated recruitment in consecutive years with respect to initial parameter values. The square root of this index provides an approximation of the overall relative change which can be registered for the calculated recruitment when small perturbations are applied simultaneously to each parameter. The quantity I_{year} is not a measure of precision for recruitment estimates but an indicator of the overall sensitivity of recruitment to parameters.

We calculated the value of I_{year} for our three examples. As a whole, recruitment figures, as calculated by cohort analysis, are more sensitive to initial parameter values for the latest years.

For herring in 4WX, the calculated recruitment is the least sensitive for the 1967-72 period (Fig. 1D). During that period, recruitment is particularly sensitive to errors in the initial value of natural mortality (Fig. 1C). If natural mortality is not in fact constant from year to year, as we assumed in constructing the cohorts, the trends observed in the calculated recruitment may not be in perfect correlation with actual recruitment numbers. For the 1972-1977 period, recruitment becomes more sensitive to catch-at-age in the last year and to terminal F's (Fig. 1D). If we have no reason to believe that a change in the reporting practice occurred in 1977, then the errors in the 1977 F-values become a major source of error for the estimation of the 1972-77 recruitment. In summary, while the estimation of recruitment for 1965-72 is particularly sensitive to M, it becomes mainly dependent upon the 1977 F's and the 1977 catch data during the 1972-77 period. This shifting may be sufficient in itself to create artificial trends in recruitment figures when the full 1965-77 period is considered. The same general conclusions can be retained after the analysis of the sensitivities for plaice in 4T (Fig. 1B) and cod in 3Pn-4RS (Fig. 1A). Spurious trends in calculated recruitment should be a major concern when the initial value of mortality is chosen arbitrarily.

In order to permit objective comparison of the overall sensitivity of recruitment estimates for different species or for different stocks, we define an index

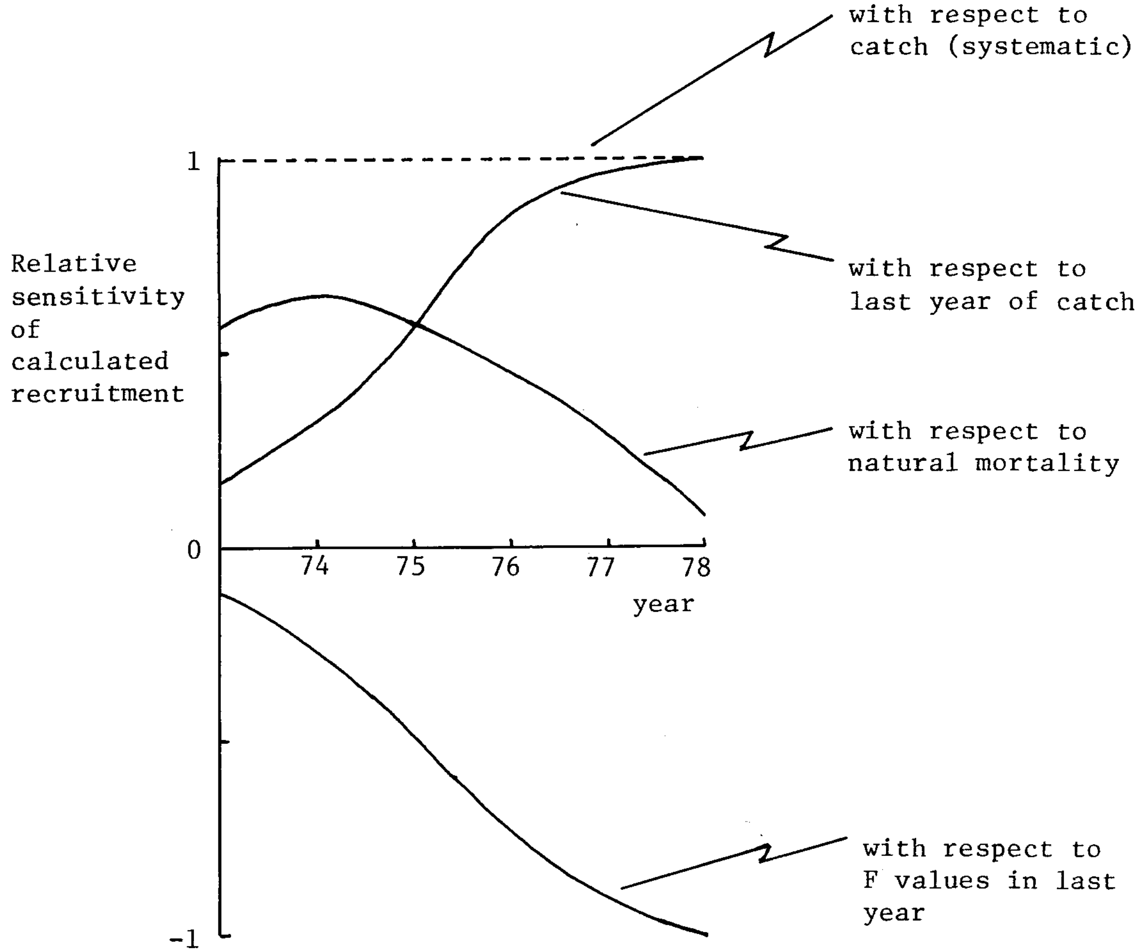
$$I_{sp} = \frac{1}{\text{nb of years}} \sum I_{year}$$

where the summation is taken over the years. The following Table summarizes the values obtained for cod, plaice and herring when I_{sp} is calculated:

Species	I_{sp}
Cod in 3Pn-4Rs	1.10
Plaice in 4T	1.28
Herring in 4WX	0.92

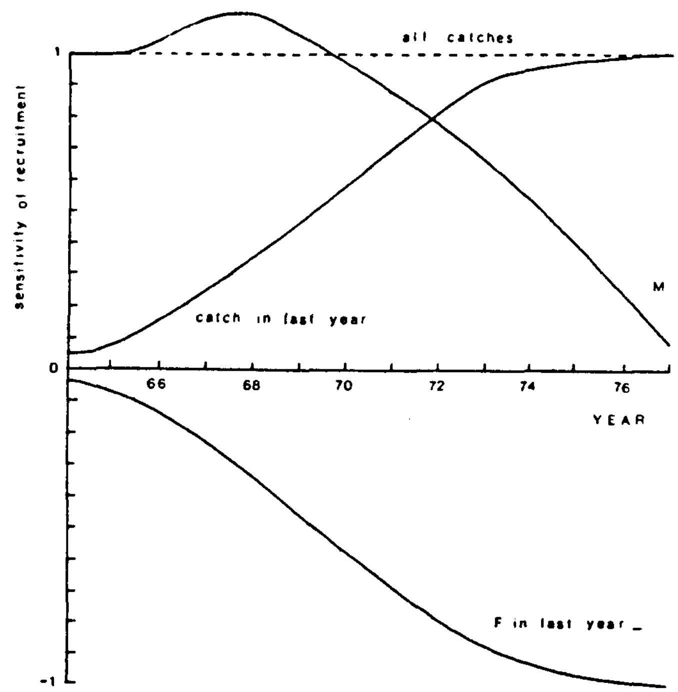
These results indicate that the calculation of recruitment for herring is the least sensitive to input parameters. For plaice in 4T, recruitment figures appear to be quite sensitive to input parameters. An examination of Figure 1B indicates that for plaice, the estimation of recruitment in recent years is sensitive to catches in the last year and to terminal F's.

In summary, the index I_{sp} gives insight on the overall importance of input parameters for the evaluation of recruitment in different species. The definition of this index constitute a first step towards the evaluation of the uncertainties associated with recruitment estimates.



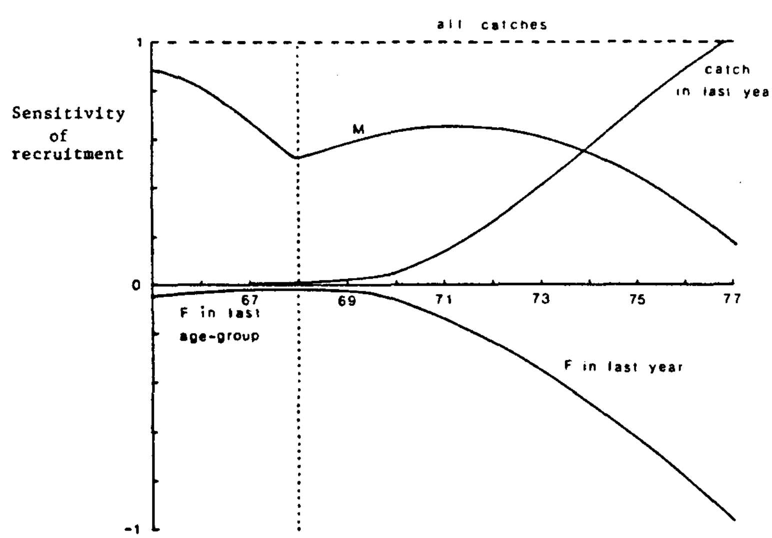
COD IN 3Pn-4RS

FIGURE 1A . Relative sensitivity of recruitment in different years with respect to input parameters.



PLAICE IN DIVISION 4T

Figure 1B . Relative sensitivity of recruitment in consecutive years with respect to input parameters.



HERRING IN 4WX

FIGURE 1C. Relative sensitivity of recruitment in consecutive years with respect to input parameters.

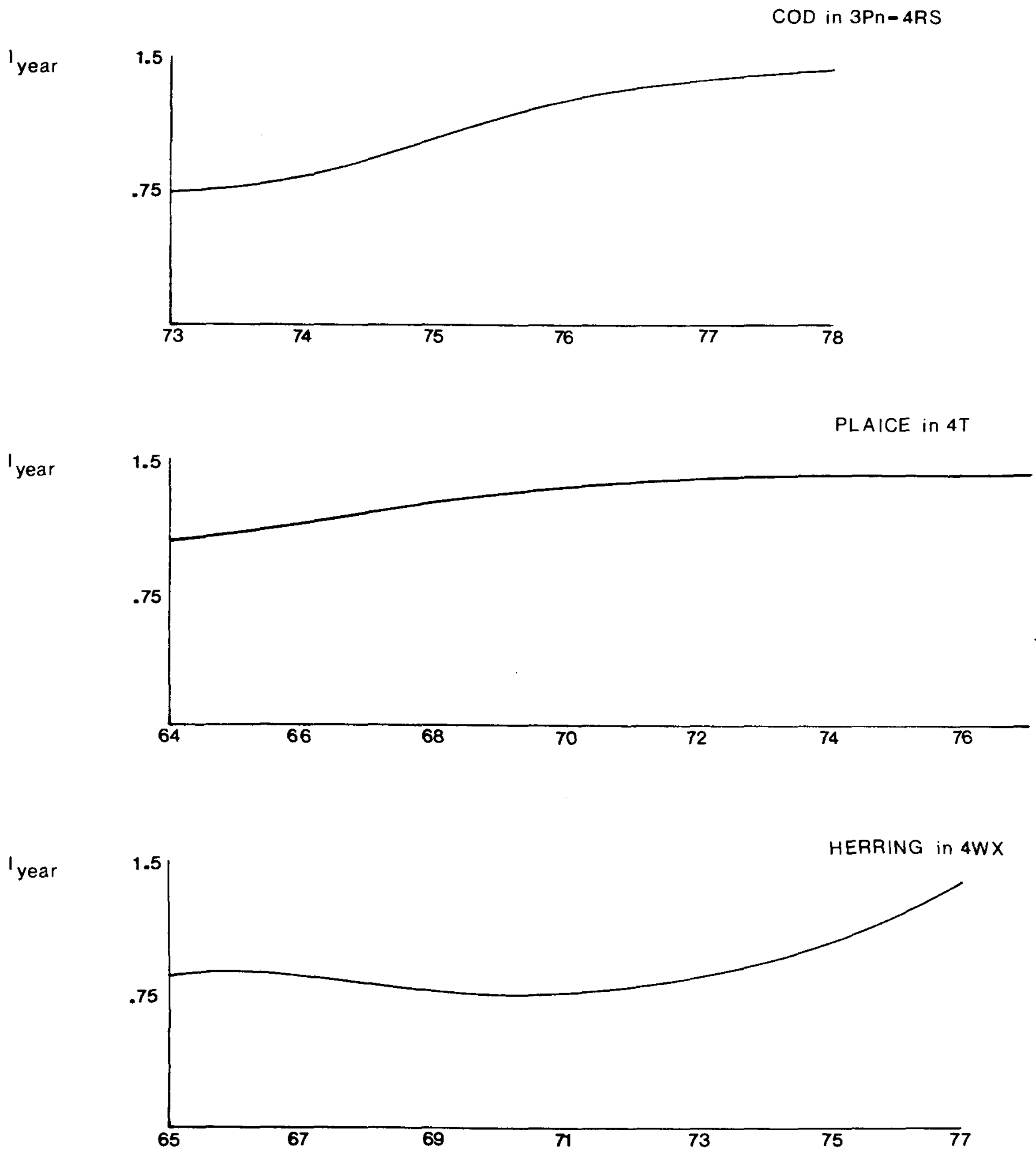


Figure 1D. Overall sensitivity of recruitment in consecutive years with respect to parameters.

Sensitivity of estimated fishing mortalities to parameters.

In this section, we analyse the effect of changes of input parameters on the age-specific fishing mortalities as calculated from the cohort analysis.

A. Sensitivity to catch data.

Systematic error. When a small relative perturbation is applied to all entries of the catch matrix, no change is registered in the estimated fishing mortalities. In fact, the relative sensitivity coefficients take the value of zero, which value indicates that any systematic perturbation of the catch data will not influence our estimation of the output variable. Therefore, if all catches are systematically misreported over the years, this practice will not modify our estimation of the instantaneous fishing mortalities.

Specific error. The effect of changes of individual catch values on the calculation of fishing mortalities has been studied by Gray (1978). In this paper, Dr Gray indicates that, when a continuous period of under-reporting practice is followed by one year of correct data, the fishing mortalities in the years preceding the change are underestimated. The underestimation is more pronounced for the latest years. Also the author notes that there is less error in estimating the F's of older ages.

For cod in 3Pn-4RS, we arrive at a similar conclusion (see Table 3A and Figure 2C). As a whole, younger age-groups in the latest years appear to be the most sensitive. In fact, the relative sensitivities approach -1, which value indicates that a 10% misreporting in the last year will generate an error of 10%, approximately, in the evaluation of the fishing mortalities for younger fish in the latest years. The negative sign indicates that under-reporting of catch in the last year generates an overestimation of F; on the other hand, full reporting of catch in the last year will generate an underestimation of F if catches have been under-reported in the preceding years. For the earliest years (1973-75), the sensitivities of F to catches of the last years are reduced considerably. In our example, the relative sensitivities suggest that a 10% misreporting in the last year will generate, for 1973, F values which are within 1.5% of the true F's.

For plaice in division 4T, the effect of changes in reporting is clearly the same as in cod (compare Tables 3B and 3A). As indicated in Figure 3C, instantaneous fishing mortalities are underestimated in the years preceding the change if a period of misreporting is followed by one year of full reporting. The underestimation becomes less and less important as we go back in the catch past-history. From 1964 through 1966 (i.e. the first 3 years of catch data), our results indicate that a 20% misreporting in the last year (1977) will generate F values which are within 1% of the true F's. Note finally that the error is far more important for the F's of younger age-groups.

For herring in 4WX (Table 3C), a change in reporting for the last year has no effect on the estimation of fishing mortalities from 1965 through 1968. From 1969 through 1972, the effect of a change in reporting on the estimation of the F's is minimal. But a change in the reporting practice for 1977 would have a marked effect on the estimation of the F's for 1973-76.

COD IN 3Pn-4RS

	Years before change in reporting					
Age	5	4	3	2	1	0
4	-.20	-.33	-.59	-.89	-.98	0.00
5	-.14	-.24	-.36	-.66	-.95	0.00
6	-.15	-.19	-.32	-.45	-.86	0.00
7	-.08	-.22	-.28	-.47	-.74	0.00
8	-.09	-.12	-.35	-.44	-.76	0.00
9	-.11	-.14	-.23	-.54	-.77	0.00
10	-.06	-.17	-.21	-.47	-.82	0.00
11	-.00	-.07	-.29	-.43	-.83	0.00
12	-.00	-.00	-.16	-.51	-.82	0.00
13	-.00	-.00	-.00	-.42	-.84	0.00
14	-.00	-.00	-.00	-.00	-.80	0.00
15	-.00	-.00	-.00	-.00	-.00	0.00

TABLE 3A . Effect of a change in reporting for the last year on the estimation of instantaneous fishing mortalities. The values given in the table are the relative sensitivities of the calculated fishing mortalities with respect to the catch data in the last year (last column of the catch matrix).

PLAICE IN DIVISION 4T

AGE	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
							YEARS							
3	¯0.05	¯0.08	¯0.15	¯0.25	¯0.44	¯0.47	¯0.60	¯0.69	¯0.83	¯0.93	¯0.97	¯0.97	¯0.99	0.00
4	¯0.03	¯0.05	¯0.08	¯0.15	¯0.25	¯0.45	¯0.48	¯0.61	¯0.69	¯0.83	¯0.94	¯0.98	¯0.99	0.00
5	¯0.05	¯0.03	¯0.05	¯0.08	¯0.16	¯0.26	¯0.46	¯0.49	¯0.62	¯0.70	¯0.84	¯0.95	¯0.99	0.00
6	¯0.04	¯0.06	¯0.04	¯0.06	¯0.09	¯0.17	¯0.28	¯0.47	¯0.50	¯0.64	¯0.72	¯0.86	¯0.97	0.00
7	¯0.05	¯0.04	¯0.07	¯0.05	¯0.07	¯0.11	¯0.20	¯0.30	¯0.50	¯0.55	¯0.70	¯0.79	¯0.94	0.00
8	¯0.01	¯0.06	¯0.05	¯0.09	¯0.06	¯0.09	¯0.13	¯0.23	¯0.34	¯0.56	¯0.62	¯0.80	¯0.92	0.00
9	0.00	¯0.02	¯0.08	¯0.06	¯0.11	¯0.07	¯0.10	¯0.16	¯0.26	¯0.38	¯0.62	¯0.73	¯0.93	0.00
10	0.00	0.00	¯0.02	¯0.09	¯0.08	¯0.13	¯0.09	¯0.13	¯0.20	¯0.32	¯0.46	¯0.75	¯0.90	0.00
11	0.00	0.00	0.00	¯0.02	¯0.11	¯0.09	¯0.16	¯0.12	¯0.19	¯0.28	¯0.43	¯0.59	¯0.92	0.00
12	0.00	0.00	0.00	0.00	¯0.03	¯0.13	¯0.11	¯0.20	¯0.16	¯0.26	¯0.38	¯0.58	¯0.83	0.00
13	0.00	0.00	0.00	0.00	0.00	¯0.04	¯0.16	¯0.13	¯0.25	¯0.20	¯0.35	¯0.53	¯0.82	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00	¯0.05	¯0.19	¯0.16	¯0.30	¯0.27	¯0.51	¯0.80	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.06	¯0.23	¯0.19	¯0.38	¯0.41	¯0.81	0.00
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.08	¯0.29	¯0.24	¯0.53	¯0.73	0.00
17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.15	¯0.42	¯0.41	¯0.80	0.00
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.24	¯0.57	¯0.78	0.00
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.26	¯0.78	0.00
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.55	0.00
21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3B

Herring in 4WX

AGE	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
							YEARS						
2	0.00	0.00	0.00	0.00	¯0.04	¯0.05	¯0.18	¯0.28	¯0.52	¯0.59	¯0.78	¯0.94	0.00
3	0.00	0.00	0.00	0.00	0.00	¯0.04	¯0.07	¯0.21	¯0.34	¯0.55	¯0.69	¯0.92	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	¯0.06	¯0.14	¯0.27	¯0.43	¯0.65	¯0.88	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.11	¯0.25	¯0.37	¯0.61	¯0.86	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.21	¯0.38	¯0.56	¯0.85	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.35	¯0.56	¯0.84	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.49	¯0.83	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	¯0.76	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE 3C

B. Sensitivity to natural mortality.

As a whole, the sensitivity analysis reveals that an increase of M generates a decrease in the estimate of the age-specific fishing mortalities in the different years.

For cod in 3Pn-4RS, the sensitivity of estimated fishing mortalities with respect to natural mortality is described in Figure 2B. As we go back in time, the estimated fishing mortalities become more sensitive to M. But within each year, the sensitivity of the age-specific fishing mortalities stays relatively constant for the different ages. Since they are entered as input data, the fishing mortalities for the last year and for the last age-groups are not influenced by changes of the initial value of natural mortality (relative sensitivities are zero). As we reconstruct the cohorts (dashed lines in Figure 2A), we see that the accuracy of M becomes an important consideration for the determination of instantaneous fishing mortalities.

For plaice in division 4T, the estimated fishing mortalities are even more sensitive to M. Except for the last years (1975-77) for which the sensitivities are rather constant in all ages, Figure 3B shows within each year a progressive decline in the absolute value of the sensitivities from the youngest to the oldest age-groups.

For herring in 4WX, similar trends can be observed (Figure 4B). Within each of the first year (1965-69), the estimated F's show a low sensitivity with respect to natural mortality for the oldest age-groups but higher sensitivity as we move towards the youngest age-groups. But the sensitivity of F in the youngest age-groups decreases in the latest years (1975-77). When we follow consecutive cohorts (dashed lines in Figure 4B), the sensitivity of F values with respect to M increases sharply as we reconstruct the cohorts.

C. Sensitivity to initial F values.

As a whole, positive perturbations of initial F values generate higher estimates of the instantaneous fishing mortalities in the different years. In other words, an overestimation of initial F values will lead to an overestimation of age-specific fishing mortalities.

For cod in 3Pn-4RS, Figure 2A indicates that the estimated fishing mortalities become less sensitive to initial F's as we go back in time. In other works, the initial error in the F values for the last year is rapidly damped out as we reconstruct the cohorts (dashed lines in Figure 2A). In general, the sensitivity analysis indicates that fishing mortalities readjust within 3 years to minimize the effect of this initial error in F (solid lines). In fact, a 10% error of the initial F values in 1978 will generate, during the 1973-75 period, F values which are within 2% of the true F's. There is an exception for the youngest and the oldest age-groups, as it is pictured in Figure 2A. Sensitivities are increasing for the older age-groups since we have assumed, for each year of the simulation, an initial error in the F values of the last age-groups. But Figure 2A indicates that the error in F values for the last age-groups also drops rapidly as we reconstruct the cohorts.

For plaice in 4T, we observe similar trends (Figure 3A). Youngest age-groups in the first years (1964-66) are the least influenced by initial F values. A look at Figure 3A also indicates that the 1964-66 cohorts are the least influenced by errors in estimating initial F values. The sensitivities are still high for the oldest ages but decrease sharply within 3 or 4 years and become insignificant in younger ages.

For herring in 4WX, the youngest age-groups (say ages 2-6) in the earliest years (1965-72) are the least influenced by errors in the initial F values (Figure 4A). Sensitivities are higher (between .5 and 1.0 in absolute value) for each age-group in the latest years (75-77) and for the last two age-groups (ages 9 and 10) in all years.

D. <u>Sensitivity of terminal F, as estimated from regression with effort.</u>

It is common practice to follow a cohort analysis with a regression of weighted average F for fully recruited ages on fishing effort. The regression is used to "predict" terminal F and hence estimate the initial stock size for a projection. Let us define the weighted average F for year i as

$$\overline{F}_i = \sum_j W_j F_{i,j}$$

The summation is taken over fully recruited ages, with weights proportional to estimated population at age. If fishing effort in year i is represented by E_i, then

$$\hat{\overline{F}}_T = \sum_i \frac{\overline{F}_i}{n} + \frac{(E_T - \overline{E}) \sum (E_i - \overline{E}) \overline{F}_i}{\sum (E_i - \overline{E})^2}$$

where $\hat{\overline{F}}_T$ and E_T are the weighted average terminal F and terminal effort, respectively. Thus, the sensitivity of $\hat{\overline{F}}_T$ to a parameter θ is calculated as

$$X\overline{F}_{T,\theta} = \frac{1}{n} \sum_i X\overline{F}_{i,\theta} + \frac{(E_T - \overline{E}) \sum (E_i - \overline{E}) X\overline{F}_{i,\theta}}{\sum (E_i - \overline{E})^2} \quad ,$$

where

$$\bar{XF}_{i,\theta} = \sum_j W_j \, XF_{i,j;\theta}$$

The quantities $XF_{i,j;\theta}$ are the sensitivities of $F_{i,j}$ with respect to a parameter θ . This formula allows the sensitivity of estimated terminal F to be calculated as a linear combination of the sensitivities of F's, as derived from the cohort analysis. From this formula, it appears that sensitivity will be high when extreme values of E_i are associated with extreme values of $\bar{XF}_{i,\theta}$. This situation is most likely to occur for the latest years, since the sensitivities of F are close to 1 for all ages in the latest years.

In conclusion, if a long period of stable fishing effort is followed by a large increase or decrease on the most recent year, the regression of F on effort may have little prediction value for terminal F.

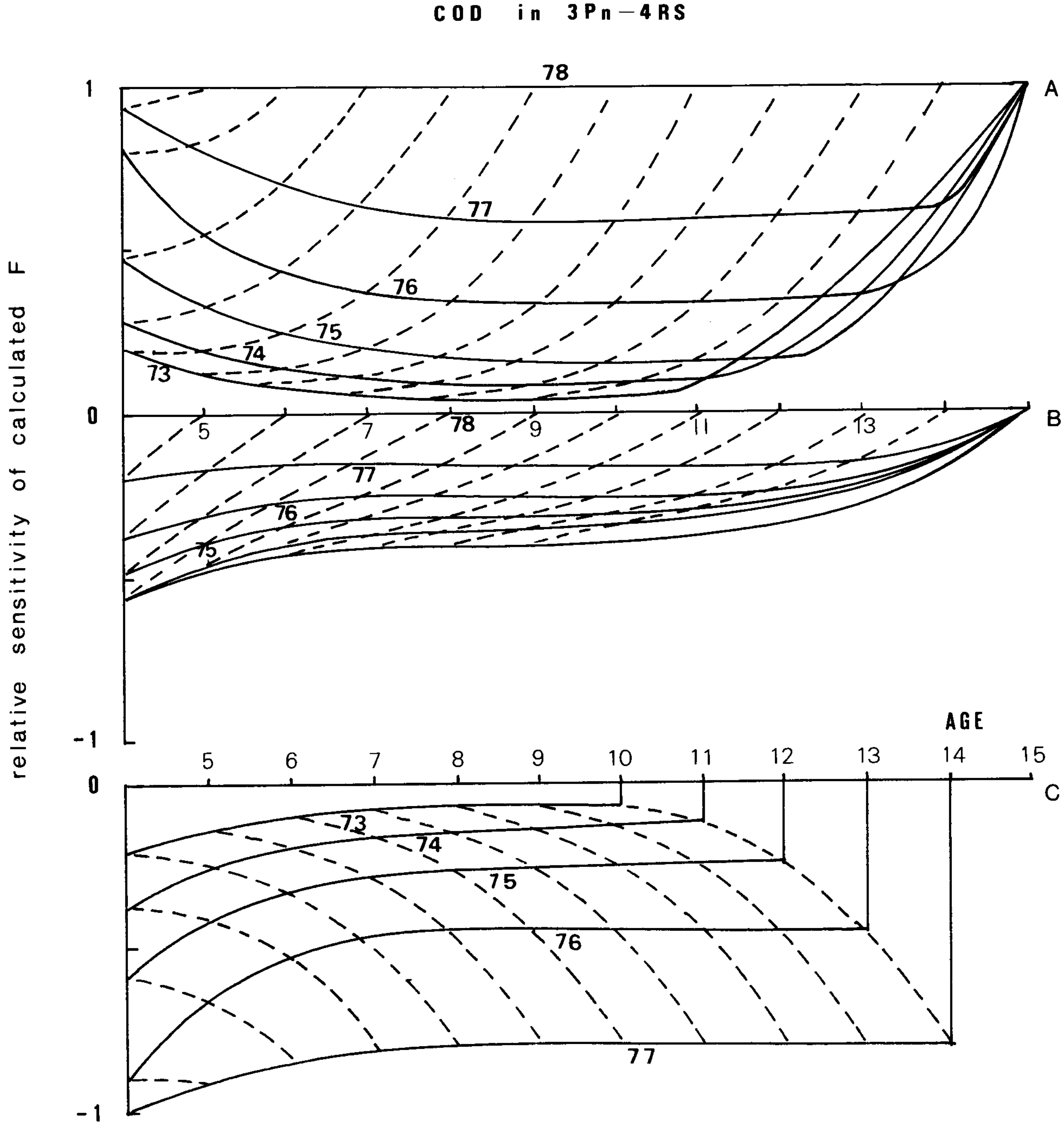


FIGURE 2. Sensitivity of calculated F-values with respect to initial parameters. A) Sensitivity to initial F's. B) Sensitivity to natural mortality. C) Sensitivity to catch-at-age values in the last year. The dashed lines indicate the sensitivities for consecutive cohorts.

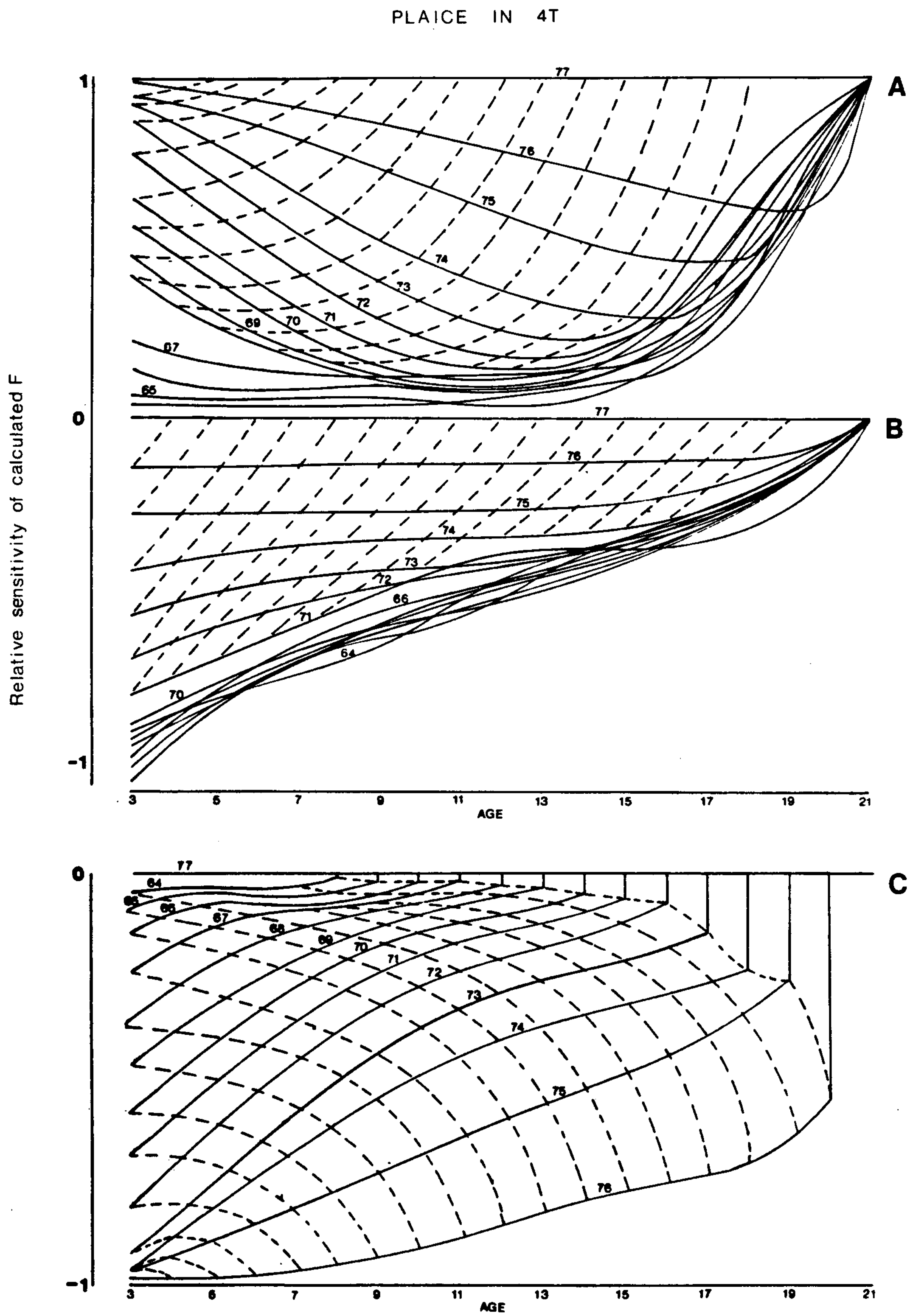


FIGURE 3 . Sensitivity of calculated F values with respect to initial parameters. A) Sensitivity to initial F values. B) Sensitivity to natural mortality. C) Sensitivity to catch-at-age values in the last year. The dashed lines indicate the sensitivities for consecutive cohorts.

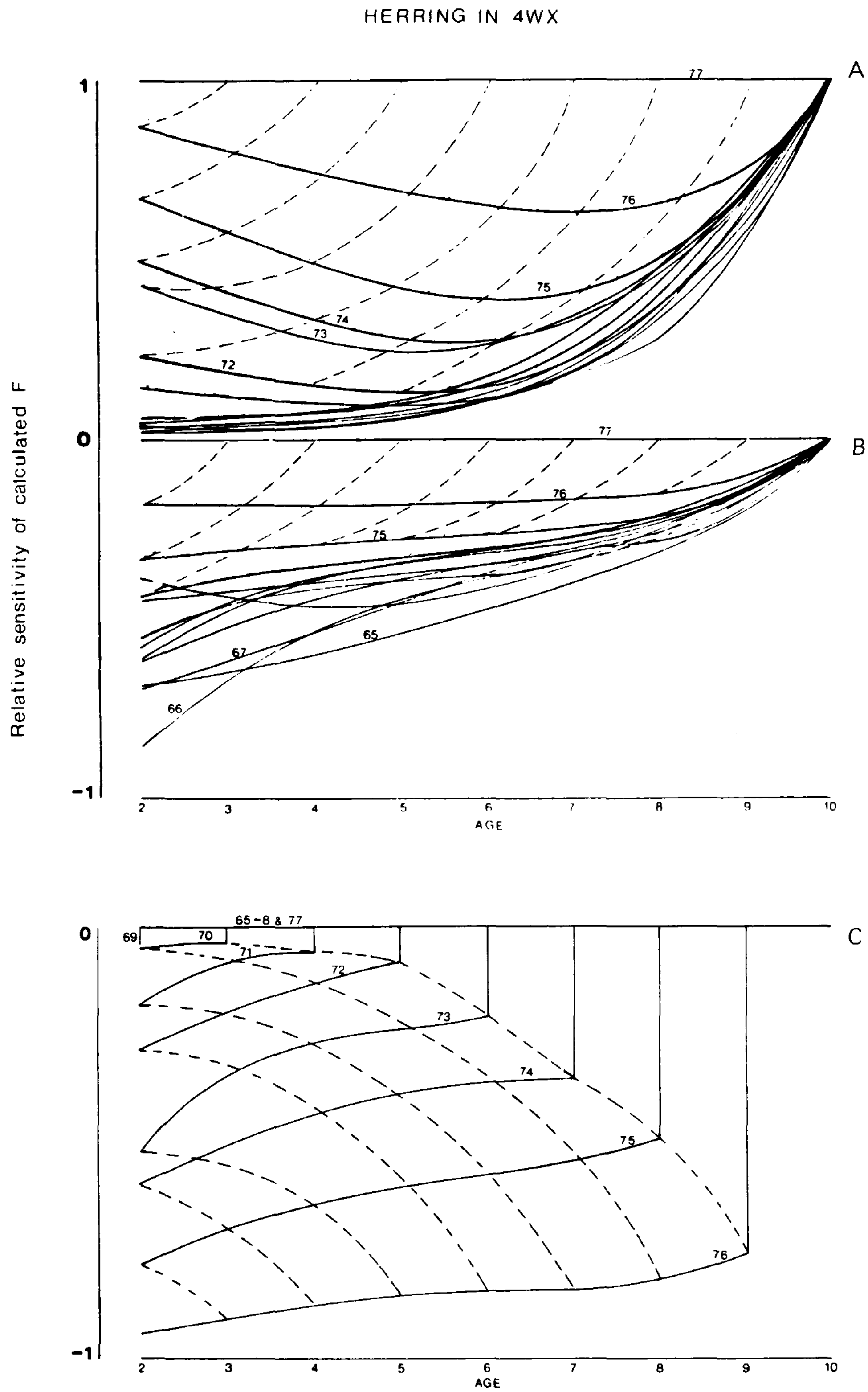


Figure 4. Sensitivity of calculated F values with respect to initial parameters. A) Sensitivity to initial F values. B) Sensitivity to natural mortality. C) Sensitivity to catch-at-age values in the last year. The dashed lines indicate the sensitivities for consecutive cohorts.

General Conclusion

Estimation of population numbers.

1. The high sensitivity of recruitment estimates and of estimated population numbers for the last years is a striking common feature of the examples. This is due to the fact that population numbers in the last year are estimated solely from the last year of catch-at-age data and from the terminal F's. Thus, good sampling is particularly important for catch in the last year if the calculated numbers-at-age are to be used as an indicator of population size. Further analysis would be necessary in order to determine the influence of the population numbers thereby calculated on the projection of future catches.

Estimation of recruitment.

2. Recruitment estimates are not sensitive to the initial F's for the last age-groups. But recruitment estimates are very sensitive to the initial F- values in the last year. This sensitivity decreasesas we go back in time and as recruitment estimates becomes more sensitive to our initial estimate of natural mortality.

3. The possibility of generating spurious trends in calculated recruitment through the sensitivity to M and terminal F's should be noted. Our three examples show that recruitment in recent years is sensitive to the F- values in the last year while recruitment in early years is more sensitive to the natural mortality. Since the input value of M is often chosen arbitrarily, we may expect spurious trends to appear in calculated recruitment when the sensitivity of recruitment to parameters changes from M to the terminal F's. Thus, routine examination of sensitivities is desirable and should be considered as an important source of information for the determination of relevant trends in calculated recruitment.

4. An accurate estimate of catch composition prior the current year is less important than other parameters in the evaluation of recruitment figures. The sensitivities of calculated recruitment to individual catches is low, except for the current year of catch data. Thus, casual misreporting of catches prior the current year is not an important source of error in the estimation of recruitment by cohort analysis. If it is persistent from year to year, misreporting will influence considerably our recruitment estimates. However, recruitment figures calculated by cohort analysis will still provide, in that case, a good relative index of recruitment. Finally, a change in the reporting practice for the current year may also generate spurious trends in recruitment. Thus, the accuracy of sampling estimates of catch in the current year, particularly for younger fishes, as well as an analysis of possible changes regarding the reporting (and/or discarding) practice for the current year, should be given prime consideration in the interpretation of trends in the calculated recruitment.

Estimation of instantaneous fishing mortalities.

5. A change in the reporting practice for the current year will influence our estimation of age-specific fishing mortalities mainly for recent years. The influence is more important for younger age-groups.

6. The sensitivities of age-specific fishing mortalities to terminal F's decrease rapidly as we reconstruct the cohorts (i.e. as we move back in time and as we move towards younger age-groups). On the other hand, the estimated fishing mortalities become more sensitive to the initial value of natural mortality as we reconstruct the cohorts. Routine application of sensitivity analysis to cohort analysis is desirable in order to decide which years are appropriate to include in possible regressions of mature F on fishing effort.

SUMMARY Cohort analysis is generally used to obtain estimates of past recruitment and of instantaneous fishing mortalities and to describe past trends in stock level. Our results indicate that cohort analysis provides reliable indices of recruitment and fishing mortality for the "far past". However, these indices may show spurious trends in the recent years. Such trends are solely the result of the procedure by which the cohorts are reconstructed. Spurious trends in recent years become a major concern when the initial value of natural mortality is chosen arbitrarily. Sensitivity analysis may be helpfull in that case by determining which period of a chosen time series is particularly sensitive to a given parameter.

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APPENDIX 1

Summary of the trends which can be observed in the calculated quantities when input parameters are perturbed. A positive sign (+) indicates that an *increase* (*decrease*) of the input parameter generates an *increase* (*decrease*) of the calculated quantity. A negative sign (-) indicates that an *increase* (*decrease*) of the input parameter generates a *decrease* (*increase*) of the calculated quantity. A nil value indicates that the input parameter does not influence the evaluation of the calculated quantity.

	CALCULATED QUANTITIES	
INPUT PARAMETER	recruitment	age-specific F's
catch-at-age		
-systematic perturb.	+	*0*
-specific perturb.	+	-
natural mortality (M)	+	-
F in last year	-	+
F in last age-groups	-	+