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Weight-at-age growth models and forecasts for Northern cod (*Gadus morhua*)

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ABSTRACT

A Von Bertalanffy growth model is presented to estimate beginning-of-year and mid-year weights-at-age for Northern cod (*Gadus morhua*) during 1983-2014 and ages 2-20. This information is required in stock assessment to estimate beginning-of-year spawning stock biomass and for evaluating fishery biomass landings using mid-year weights. Model parameters are estimated using the Department of Fisheries and Oceans (DFO) autumn Research Vessel (RV) bottom trawl survey mean weights-at-age. The model is applied by cohort but the model is formulated in a mixed-effects framework with fixed parameters for all cohorts and random auto-correlated cohort interactions. This model and time-series structure is also used to provide short-term forecasts, to 2017.

Modèles et prévisions de croissance du poids selon l'âge de la morue du Nord (*Gadus morhua*)

RÉSUMÉ

Un modèle de croissance Von Bertalanffy est présenté pour estimer le poids selon l'âge au début de l'année et au milieu de l'année de la morue du Nord (*Gadus morhua*) de 1983 à 2014 et de 2 à 20 ans. Ces renseignements sont nécessaires pour évaluer les stocks, c'est-à-dire estimer la biomasse du stock reproducteur en début d'année et évaluer la biomasse de débarquements de pêches à l'aide des poids de mi-année. Des paramètres de modèle sont estimés à l'aide des poids moyens selon l'âge obtenus par un relevé au chalut de fond effectué à l'automne sur un navire scientifique de Pêches et Océans Canada. Ce modèle est appliqué par cohorte, mais le modèle est formulé selon un cadre d'effets mixtes avec des paramètres fixes pour toutes les cohortes et des interactions de cohorte auto-corrélées à l'aléatoire. Cette structure de modèle et de série chronologique est également utilisée pour fournir des prévisions à court terme jusqu'en 2017.

INTRODUCTION

Recent stock assessments and updates of cod (*Gadus morhua*) in Northwest Atlantic Fisheries Organization (NAFO) Divisions 2J3KL (i.e. Ncod) have used spawning stock biomass (SSB) indices derived from the Fisheries and Oceans (DFO) autumn bottom-trawl research survey (e.g. DFO 2015) or SSB indices derived from an age-based survey assessment model (Cadigan 2013a) as a main indicator of stock status. SSB is computed as the age-specific survey abundance indices (direct or modelled) multiplied by individual weights and proportion mature, summed over all ages. Cadigan (2013a) calculated SSB for the beginning of the year because it is common to report stock status at that time rather than mid-year, etc. SSB at the beginning of the year may be different than SSB during the autumn survey, especially when mortality rates are high. The assessment model Cadigan (2013a) used produced estimates of beginning-of-year relative stock size (i.e. index) but additional analyses were required to estimate beginning of year maturities and weights.

The DFO autumn survey is also the main source of biological information about maturities and weights. A cohort-specific Binomial logistic regression model has been used to estimate the proportion mature as a function of age (e.g. Brattey et al. 2010) and this model can be used to estimate maturities at the beginning of the year. Another advantage of this modelling approach is to reduce the measurement error in the maturity estimates, and this was considered further in Cadigan, Morgan, and Brattey (2013). They focused on improving short-term forecasts of maturities, although their recommended model has not been used in recent assessments of Ncod. The beginning of year weights-at-age (W_{ay}) used in recent assessments have either been taken directly as the average weight-at-age from samples obtained in the autumn survey, or these autumn weights have been used for $W_{a+1,y+1}$. However, modelling of the weights could provide the same benefits as modelling the maturities. Cadigan (2013b) presented preliminary analyses of a cohort model for weights-at-age (W_{ay}) with the goal of estimating W_{ay} at the beginning of the year.

In this paper, I update and extend the modelling investigations of Cadigan (2013b). Extensions involve:

1. a greater range of ages because of the recent expansion of the age distribution of Ncod;
2. improvements in the model;
3. a focus on short-term forecasts of W_{ay}.

METHODS

VON BERTALANFFY MODEL AND ESTIMATION

The Von Bertalanffy (VonB) model for length at age *a*, denoted as *l(a)*, is

(1) $$l(a) = L_\infty - L_\infty(1-\rho_o)exp(-ka)$$

where L_∞ is the asymptotic length as $a \to \infty$, *k* is a growth rate parameter, and $\rho_o = l(0)/L_\infty$ is the size at birth relative to the maximum size. The VonB model is derived from the differential equation

(2) $$\partial l(a)/\partial a = k\{L_\infty - l(a)\}$$

The growth rate at birth (i.e. slope at the origin) is $kL_\infty(1-\rho_o) \approx kL_\infty$ and *∂l(a)/∂a* declines to zero as age increases.

Growth in weight is approximated as a power function, $W = \alpha L^b$, and the Von Bertalanffy model for weight at age a is

(3) $$w(a) = W_\infty\{1-(1-\rho_o)exp(-ka)\}^b$$

It is often the case that $b \approx 3$. I use the value b = 3.0879 from Brattey et al. (2010; pg. 6) in all analysis in this paper.

The observations are a time-series of average weights-at-age from the DFO autumn trawl surveys, which I denote as W_{oay}. I fit Eqn. (3) to the W_{oay} for each cohort with b fixed and ρ_o assumed to be a constant parameter for all cohorts. The W_∞ and k parameters are estimated for each cohort, but not freely. They are assumed to be random effects that are more similar for adjacent cohorts. I model this dependency using random walks for both $W_{c\infty}$ and k_c, where c indicates cohort. This is different from Cadigan (2013b) who assumed $W_{c\infty}$ and k_c were independent across cohorts. My approach is similar to the maturity model in Cadigan, Morgan, and Brattey (2013), where the logistic regression parameters for cohort maturity were auto-correlated across cohorts.

Conditional on the $W_{c\infty}$ and k_c random effects for all cohorts, the W_{oay} are assumed to the lognormally distributed with mean given by Eqn. (3). Let W_{ay} denote the model values for weight-at-age with cohort values of $W_{c\infty}$ and k_c, where $c = y\text{-}a$. I assume $E\{log(W_{oay})\} = log(W_{ay})$, and do not make a correction for the log transformation bias which should be small when the estimation error is small. The conditional variance is assumed to be the same for all ages and years; that is, $Var\left[\log\left(W_{oay}\right)\middle|\{W_{\infty c}, k_c\}\right] = \sigma_e^2$. I use the *{}* notation to denote sets, so $\{W_{\infty c}, k_c\}$ is the set of all VonB random effects for all cohorts. The log-likelihood term for the observed weights conditional on $\{W_{\infty c}, k_c\}$ is

(4) $$l\left(\{W_{ay}\}|\rho_o, \sigma_e^2, \{W_{\infty c}, k_c\}\right) = \sum_a \sum_y log\left(\sigma_e^{-1}\varphi_N\left[\frac{\log(W_{oay}) - \log(W_{ay})}{\sigma_e}\right]\right),$$

where φ_N is the probability distribution function (pdf) of a $N(0,1)$ random variable.

The unconditional (aka marginal) log-likelihood of the observed weights which I use to estimate model parameters (i.e. fixed effects) requires that the $\{W_{\infty c}, k_c\}$ random effects be integrated out of the joint likelihood for the observed weights and random effects. This joint likelihood is based on Eqn. (4) and log-likelihoods for $\{W_{\infty c}, k_c\}$. Each random effect is modelled as a main effect (W_∞ or k) times a cohort-specific deviation ($\delta_{c\infty}$ or δ_{ck}) and these deviations are modelled as lognormally distributed random walks, with zero log-means for the first cohort. Note that W_∞ or k are fixed-effect parameters to estimate. The variance of each random walk are denoted σ_∞^2 or σ_k^2.

The fixed-effect parameters to estimate are $\theta = (\rho_o, W_\infty, k, \sigma_e^2, \sigma_\infty^2, \sigma_k^2)$. They are estimated via maximum likelihood (MLE) based on the marginal likelihood, $L(\theta)$. Let Ψ denote a vector of all random effects, $\{\delta_{c\infty},\delta_{ck}\}$, for all cohorts. The marginal likelihood is

(5) $$L(\theta) = \iint_\Psi f_\theta(\{W_{oay}\}|\Psi)g_\theta(\Psi)\partial\Psi$$

where $f_\theta(\{W_{oay}\}\Psi)$ is the conditional joint pdf of the data, whose log likelihood is given by Eqn. (4), and $g_\theta(\Psi)$ is the joint pdf for the Ψ random walk effects.

The template model builder (TMB; Kristensen et al. 2015) package within R (R Core Team 2014) was used to implement the model. The MLE's of θ maximize $L(\theta)$. The user has to provide C++ computer code to calculate $f_\theta(\{W_{oay}\}|\Psi)$ and $g_\theta(\Psi)$ but the integration in Eqn. (5) is provided by TMB. The high dimensional integral is numerically evaluated in TMB using the Laplace approximation. The random effects Ψ can be predicted by maximizing the joint likelihood, $f_\theta(\{W_{oay}\}|\Psi)g_\theta(\Psi)$; however, these effects are not freely estimated like θ. Additional information on these procedures is provided by Skaug and Fournier (2006). TMB uses automatic differentiation to evaluate the gradient function (i.e. derivative wrt to θ) of Eqn. (5) and for the Laplace approximation. The gradient function is produced automatically from $f_\theta(\{W_{ay}\}|\Psi)$ and $g_\theta(\Psi)$. This greatly improves parameter estimation using a derivative-based optimizer. I use the *nlminb* package within R (R Core Team 2014) to find the MLE for θ.

In preliminary analysis the estimates of $W_{c\infty}$ were implausibly large. The maximum size is not well identified in the data (see below) and there is a well-known confounding in the estimation of $W_{c\infty}$ and k_c (e.g. Shelton and Mangel 2012). Estimates for these two parameters are often highly and negatively correlated. Hence, the estimated fish weight at observed ages may be reasonable even if estimates of W_∞ and k are not. However, if these parameters are poorly estimated then extrapolations of weight for ages much outside the range of observed ages may not be reliable. Such extrapolations are required in Maximum Sustainable Yield (MSY) or virgin biomass calculations and possibly other management strategy evaluations. I used a prior on the fixed effect for $W_{c\infty}$ to reduce this problem. The prior was $\log(W_\infty) \sim N(\log(40.0),sd=0.1)$.

A MORE FLEXIBLE VON BERTALANFFY MODEL

Fish may not grow specifically like the VonB model for a variety of reasons, including the impact of maturation and other age-dependent changes in metabolic processes. The type of prey cod can consume may change substantially with age which could result in different functional forms of growth rates than the VonB model. Growth rates of cod also vary seasonally. The VonB model may be generalized by replacing *ka* in Eqn. (1) with a non-negative and monotonic increasing function of age, $k(a)$. Cadigan and Brattey (2001) developed such a nonparametric model to use with growth data from tagging studies and demonstrated consistent (across years) seasonal variation in growth rates, with most growth occurring post-spawning and in the autumn.

I investigate a simple parametric alternative using $k(a) = k_1a + k_2a^2$, with $k_1,k_2 > 0$. This choice for $k(a)$ is monotonic increasing in a. I estimate k_1 and k_2 for each cohort as random walks in the same manner as in the above Von Bertanlanffy model and estimation Section. I refer to this as the VonB2 model. A prior for W_∞ was not required for this model.

RESULTS

VONB MODEL

Three models were investigated:

1. a constant W_∞ and k for all cohorts for comparison purposes;
2. a random walk in $W_{c\infty}$ and k_c across cohorts; and
3. a random walk in $W_{c\infty}$ but a constant k.

Fit statistics and parameters estimates (Table 1) indicate that model 3) was the most parsimonious. Estimates of $W_{c\infty}$ (Fig. 1) generally increased since the 1980 cohort. Although a fairly stiff prior for W_∞ was used, some of the $W_{c\infty}$ values seem too high for Ncod. The world record for an Atlantic cod, caught off Norway, is reportedly 47 kg (Daily Mail 2013) and some of the values in Fig. 1 exceed 47 kg.

The model predicts observed weights fairly well (Figs 2-4) but with some evidence of lack of fit. For example, the model under-estimates weight at ages 2-5 during 1983-89 (Fig. 2), under-estimates the growth curves of the 1989 and 1990 cohorts (Fig. 3), and there are correlated residuals for blocks of ages and years (Fig. 5). The model slightly under-estimates weights for large fish (>8 kg; Fig. 6). The R package 'segmented' (Muggeo 2008) was used to performed the segmented regression assuming a single break-point. The estimated break-point is 4.6 (std.err 0.89). The slope of the red line is 0.95 and the slope of the green line is 0.23 higher (std. err 0.05) which is highly significant. This apparent lack-of-fit is the motivation to investigate a more flexible growth model.

VONB2 MODEL

Three models were investigated:

1. a constant W_∞, k_1 and k_2 for all cohorts for comparison purposes;
2. random walks in $W_{c\infty}$, k_{c1} and k_{c2} across cohorts; and
3. random walks in k_{c1} and k_{c2} but a constant W_∞.

Fit statistics and parameters estimates (Table 2) indicate that model 3) was the most parsimonious, with a lower AIC and BIC than the other model formulations and those of the regular VonB models (Table 1). The estimate of W_∞ seems very plausible. Note that this model is different than the model 3) in the above VonB model Section which had a random walk in W_∞ but not k. Estimates of k_{c1} (Fig. 7a) generally increased since the 1990 cohort. Estimates of k_{c2} (Fig. 7b) were significantly positive for the 1989 and 1990 cohorts, and negative for the1978-82 cohorts. Note that the estimates of k_{c2} have relatively wide confidence intervals for the first and last few cohorts because there are few observed ages for these cohorts.

The model predicts observed weights fairly well (Figs 8-10) but there is still evidence of lack of fit. Similar to the VonB model, the VonB2 model under-estimates weight at ages 2-5 during 1983-89 (Fig. 8) and there are correlated residuals (Fig. 10), but the VonB2 model fits the growth curves for the1989-90 cohorts (Fig. 9) better than the VonB model (Fig. 3). The Vonb2 model also slightly under-estimated weight for large fish (>8 kg; Fig. 11), but not as much as the VonB model (Fig. 6). The slope of the red line in Fig. 11 is 0.96 and the slope of the green line is 0.15 higher (std. err 0.05) which is less of a difference than for the VonB model. Amongst the various models I investigated, I prefer version 3) of the VonB2 model.

Beginning of year predicted weights-at-age are shown in Fig. 12 and compared with observed survey weights, incremented by one age and year. The results indicate that the latter procedure under-estimates the size of small fish. A similar result was found in Cadigan (2013b). This is consistent with Cadigan and Brattey (2001) who found most individual growth for Ncod occurred in the autumn and cod probably continue to grow after the autumn survey until the beginning of the following year. ***These beginning of year predicted weights (Table 3) are the ones I propose to use for the 2015 framework assessment of Ncod***.

The predicted weights in Fig. 12 are broadly consistent with those in Cadigan (2013b) but with less between year variability (Fig. 13) as one would expect because the current model has a time-series structure whereas the model in Cadigan (2013b) did not.

DISCUSSION

The VonB2 model is proposed to provide estimates of beginning-of-year and mid-year weights-at-age for the 2015 framework assessment of Ncod. This model produced realistic values for W_∞ which suggests that extrapolations of weights to older ages much outside the range of ages in the survey data will be more reliable than those obtained using the VonB model. In the latter model I had to constrain estimates of W_∞ using a subjective prior. Such extrapolations may be required for MSY and other calculations. The VonB2 model also provided somewhat better fits to the observed weights-at-age.

Residual analyses for the VonB2 model indicated some systematic patterns that suggest further improvements may be possible. In unreported analyses I explored model variants that included years effects in the VonB coefficient of anabolism ($L\infty/k$; see Shelton and Mangel 2012) common to all cohorts, or a biphasic variant using maturity ogives (Quince et al. 2008). However, my implementation of these approaches did not result in improved growth models but this may still be a useful area for future research. Further investigations should use individual weight-at-age and length-at-age measurements rather than only the average weights as used in this paper. The sample sizes for different years and ages are probably highly variable and this is important information to include in a more rigorous statistical analysis of this data. This growth data is collected across a broad spatial scale for Ncod in the DFO autumn research survey. Spatial variation in growth rates and low sample sizes at older ages may be contributing to some of the residual patterns. A more rigorous statistical analysis of temporal and spatial variability in growth rates should also account for between-individual variation in growth rates, within-individual variation from the VonB model, measurement error in size and age, and the length-stratified (i.e. size biased) sampling design used to collect biological samples in the research survey. Additional growth information is also available from sampling of stewardship and Sentinel fisheries and from tagging studies, although an additional complication of these additional sources of data is the selectivity of the fishing gears which can introduce biased data when between-individual variation in growth rates is large. Such an investigation is well beyond the scope of this paper but may be necessary to provide more realistic estimation of size for a broad range of ages for Ncod.

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APPENDIX I - TABLES

*Table 1. Fit statistics and parameter estimates for three formulations of the Von Bertalanffy (VonB) growth model. Minimum values of AIC and BIC goodness of fit statistics are shaded in grey. The model indicated as k + W∞ did not include random cohort effects for these parameters. *cohort indicates a model with random cohort effects in the corresponding component.*

Statistic	k + W∞	k*cohort + W∞*cohort	k+ W∞*cohort
Deviance	-142.224	-237.714	-237.714
No. parms.	4	6	5
AIC	-134.224	-225.714	-227.714
BIC	-118.370	-201.932	-207.896
MSE	0.041	0.027	0.027
W_∞	44.789	46.557	46.557
k	0.064	0.066	0.066
ρ_o(%)	1.494	1.085	1.085
σ_∞	-	0.053	0.053
σ_k	-	0.000	-
σ_e	0.202	0.170	0.170

*Table 2. Fit statistics and parameter estimates for three formulations of the generalized Von Bertalanffy (VonB2) growth model. Minimum values of AIC and BIC goodness of fit statistics are shaded in grey. The model indicated as k + W∞ did not include random cohort effects for these parameters. *cohort indicates a model with random cohort effects in the corresponding component.*

Statistic	k + W∞	k*cohort + W∞*cohort	k*cohort + W∞
Deviance	-142.693	-250.372	-250.372
No. parms.	5	8	7
AIC	-132.693	-234.372	-236.372
BIC	-112.875	-202.664	-208.627
MSE	0.041	0.025	0.025
W_∞	53.726	33.879	33.879
k_1	0.059	0.065	0.065
k_2	0.000	0.002	0.002
ρ_o(%)	1.722	1.931	1.931
σ_∞	-	0.000	-
σ_{k1}	-	0.010	0.010
σ_{k2}	-	0.314	0.314
σ_e	0.201	0.164	0.164

Table 3. Northern cod weight-at-age estimates (kg) from the generalized Von Bertalanffy (VonB2) growth model.

Year	Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10	Age 11	Age 12	Age 13	Age 14	Age 15	Age 16	Age 17	Age 18	Age 19	Age 20
1983	0.01	0.08	0.21	0.44	0.78	1.26	1.89	2.56	3.43	4.35	5.83	7.47	9.37	11.27	13.43	15.29	16.51	18.06	19.55	20.98
1984	0.01	0.08	0.21	0.45	0.77	1.22	1.83	2.60	3.36	4.35	5.36	7.03	8.82	10.86	12.84	15.06	16.91	18.06	19.55	20.98
1985	0.01	0.08	0.21	0.44	0.78	1.21	1.77	2.52	3.42	4.26	5.36	6.45	8.30	10.23	12.37	14.41	16.66	18.48	19.55	20.98
1986	0.01	0.08	0.22	0.45	0.77	1.22	1.76	2.42	3.30	4.34	5.25	6.45	7.60	9.61	11.66	13.90	15.96	18.22	19.99	20.98
1987	0.01	0.08	0.22	0.46	0.78	1.21	1.77	2.40	3.17	4.18	5.34	6.31	7.60	8.79	10.96	13.10	15.41	17.48	19.72	21.42
1988	0.01	0.08	0.22	0.46	0.80	1.22	1.75	2.41	3.13	4.00	5.13	6.42	7.42	8.79	10.01	12.32	14.55	16.90	18.95	21.14
1989	0.01	0.08	0.22	0.46	0.80	1.26	1.77	2.39	3.15	3.95	4.91	6.16	7.55	8.59	10.02	11.26	13.69	15.97	18.34	20.36
1990	0.02	0.08	0.22	0.46	0.81	1.26	1.82	2.42	3.12	3.97	4.84	5.87	7.24	8.74	9.78	11.26	12.51	15.04	17.36	19.73
1991	0.02	0.09	0.23	0.47	0.81	1.27	1.83	2.49	3.16	3.93	4.87	5.79	6.89	8.36	9.95	10.99	12.52	13.76	16.38	18.71
1992	0.02	0.09	0.25	0.50	0.83	1.28	1.85	2.51	3.26	3.99	4.81	5.82	6.79	7.95	9.51	11.18	12.21	13.77	15.00	17.68
1993	0.02	0.08	0.25	0.54	0.89	1.32	1.86	2.54	3.29	4.12	4.89	5.76	6.82	7.83	9.04	10.68	12.42	13.43	15.00	16.21
1994	0.02	0.08	0.25	0.55	0.98	1.42	1.93	2.55	3.33	4.16	5.06	5.85	6.75	7.86	8.89	10.14	11.86	13.66	14.64	16.22
1995	0.02	0.08	0.24	0.53	1.00	1.59	2.09	2.66	3.35	4.21	5.11	6.06	6.86	7.78	8.93	9.97	11.26	13.04	14.88	15.83
1996	0.02	0.08	0.24	0.52	0.95	1.62	2.37	2.90	3.50	4.24	5.17	6.12	7.11	7.90	8.84	10.02	11.07	12.37	14.21	16.09
1997	0.02	0.09	0.24	0.51	0.93	1.52	2.41	3.32	3.84	4.44	5.21	6.20	7.19	8.20	8.98	9.91	11.12	12.16	13.48	15.36
1998	0.02	0.09	0.25	0.51	0.91	1.48	2.25	3.38	4.42	4.90	5.47	6.25	7.28	8.30	9.32	10.08	11.00	12.21	13.25	14.58
1999	0.02	0.09	0.26	0.53	0.90	1.44	2.18	3.13	4.51	5.67	6.06	6.58	7.35	8.40	9.43	10.46	11.18	12.08	13.30	14.32
2000	0.02	0.09	0.26	0.54	0.94	1.41	2.12	3.02	4.16	5.78	7.04	7.30	7.75	8.48	9.56	10.59	11.61	12.28	13.16	14.37
2001	0.02	0.09	0.26	0.55	0.97	1.48	2.06	2.92	3.99	5.31	7.17	8.50	8.61	8.96	9.65	10.73	11.76	12.76	13.38	14.22
2002	0.02	0.09	0.26	0.56	0.99	1.54	2.17	2.83	3.85	5.09	6.57	8.67	10.04	9.97	10.20	10.84	11.91	12.92	13.89	14.46
2003	0.02	0.09	0.26	0.55	1.01	1.57	2.26	2.99	3.72	4.89	6.29	7.93	10.23	11.63	11.36	11.47	12.04	13.09	14.07	15.02
2004	0.02	0.09	0.26	0.54	0.97	1.61	2.30	3.12	3.94	4.71	6.02	7.57	9.35	11.84	13.24	12.77	12.74	13.23	14.26	15.21
2005	0.02	0.09	0.27	0.55	0.95	1.53	2.36	3.18	4.11	4.99	5.79	7.24	8.92	10.83	13.48	14.85	14.17	14.00	14.41	15.41
2006	0.02	0.09	0.26	0.56	0.98	1.49	2.23	3.27	4.20	5.22	6.14	6.94	8.51	10.31	12.32	15.10	16.43	15.56	15.25	15.58
2007	0.02	0.09	0.26	0.55	0.99	1.54	2.17	3.07	4.32	5.33	6.43	7.37	8.15	9.84	11.74	13.83	16.70	17.97	16.92	16.47
2008	0.02	0.09	0.26	0.55	0.98	1.56	2.24	2.97	4.03	5.48	6.56	7.72	8.66	9.41	11.19	13.17	15.33	18.26	19.46	18.24
2009	0.02	0.10	0.27	0.55	0.98	1.54	2.27	3.07	3.88	5.09	6.76	7.88	9.07	9.99	10.69	12.55	14.60	16.79	19.76	20.88
2010	0.02	0.10	0.28	0.57	0.98	1.54	2.25	3.12	4.02	4.90	6.26	8.11	9.26	10.47	11.35	11.98	13.91	16.01	18.22	21.18
2011	0.02	0.10	0.29	0.60	1.02	1.55	2.25	3.08	4.09	5.08	6.00	7.49	9.53	10.68	11.89	12.72	13.28	15.26	17.39	19.59
2012	0.02	0.10	0.30	0.62	1.07	1.61	2.26	3.08	4.04	5.17	6.23	7.18	8.79	10.99	12.12	13.31	14.09	14.56	16.58	18.72
2013	0.02	0.10	0.29	0.65	1.11	1.71	2.35	3.10	4.04	5.10	6.34	7.45	8.41	10.13	12.47	13.57	14.73	15.43	15.83	17.86
2014	0.02	0.10	0.29	0.63	1.17	1.77	2.52	3.24	4.06	5.10	6.24	7.58	8.72	9.67	11.49	13.96	15.01	16.12	16.75	17.06
2015	0.02	0.10	0.29	0.63	1.13	1.88	2.61	3.48	4.26	5.13	6.25	7.47	8.88	10.04	10.97	12.86	15.43	16.42	17.48	18.04
2016	0.02	0.10	0.29	0.62	1.13	1.81	2.78	3.62	4.58	5.39	6.29	7.48	8.74	10.21	11.38	12.27	14.22	16.87	17.79	18.80
2017	0.02	0.10	0.29	0.61	1.12	1.81	2.67	3.86	4.78	5.81	6.61	7.53	8.76	10.06	11.57	12.72	13.56	15.56	18.26	19.12

APPENDIX II - FIGURES

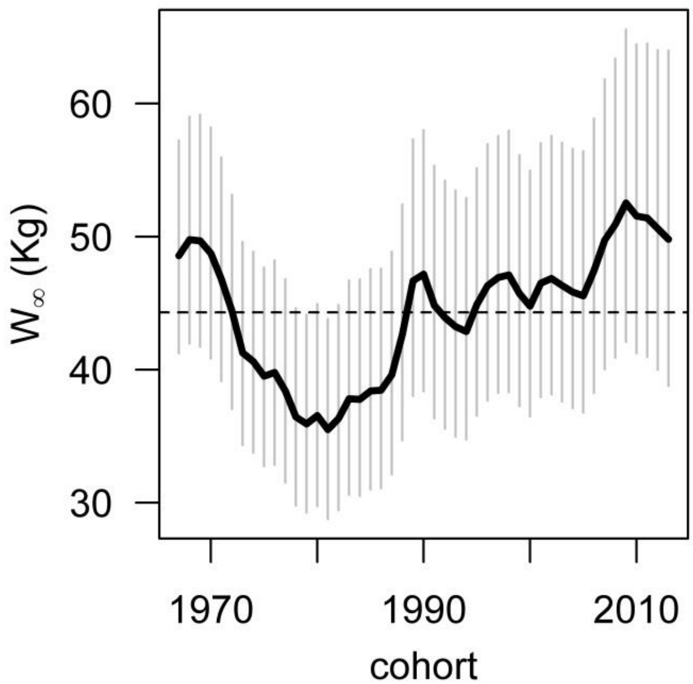

Figure 1. Solid lines connect estimates of the Von Bertalanffy Ncod W_∞ parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

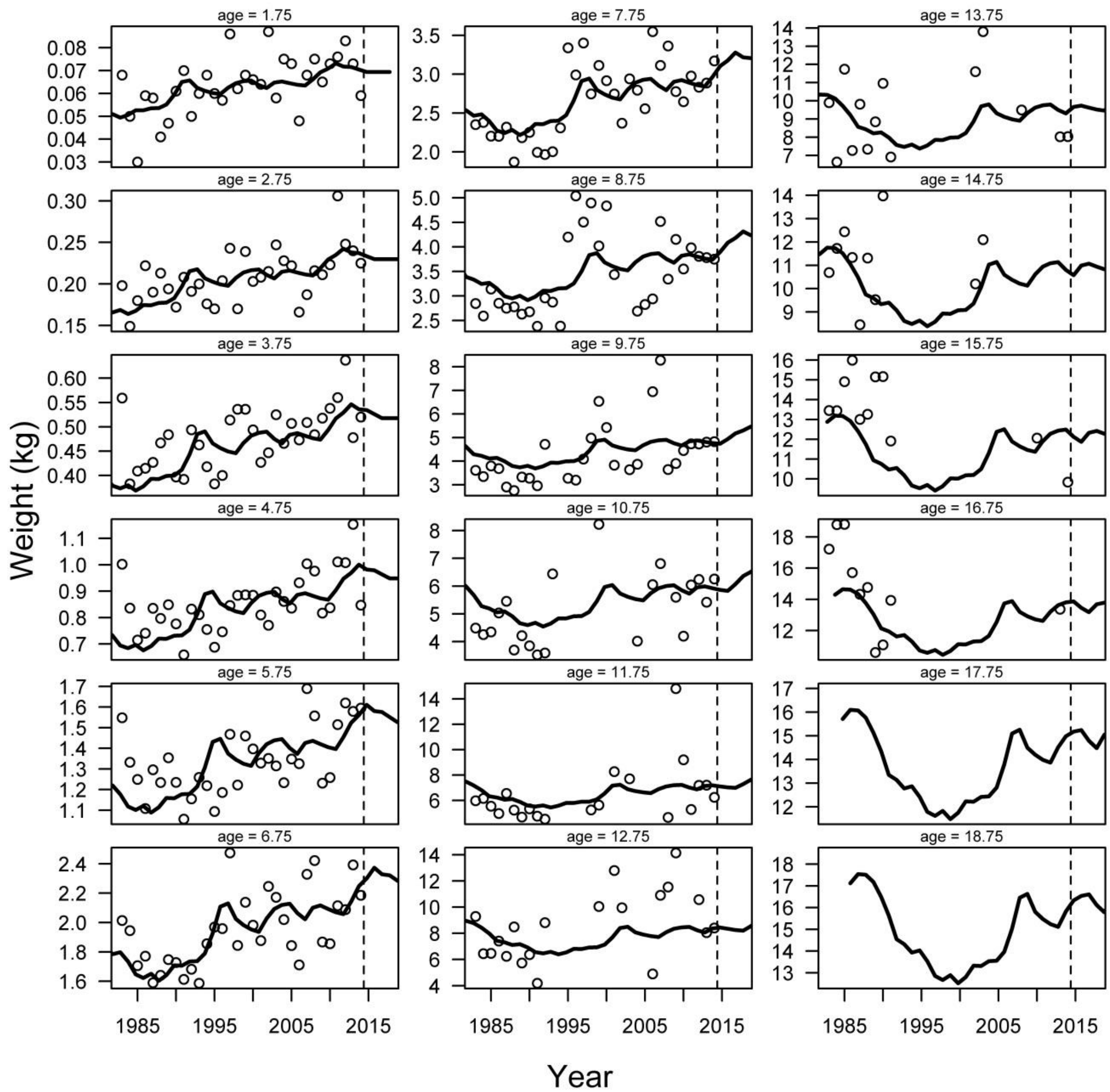

Figure 2. Von Bertalanffy model predicted versus observed weight-at-age for Ncod. Each panel shows the results for an age class. The survey occurs during the fall and the age at the top of each panel includes the fraction of year. Three year forecasts are also shown to the right of the vertical dashed lines.

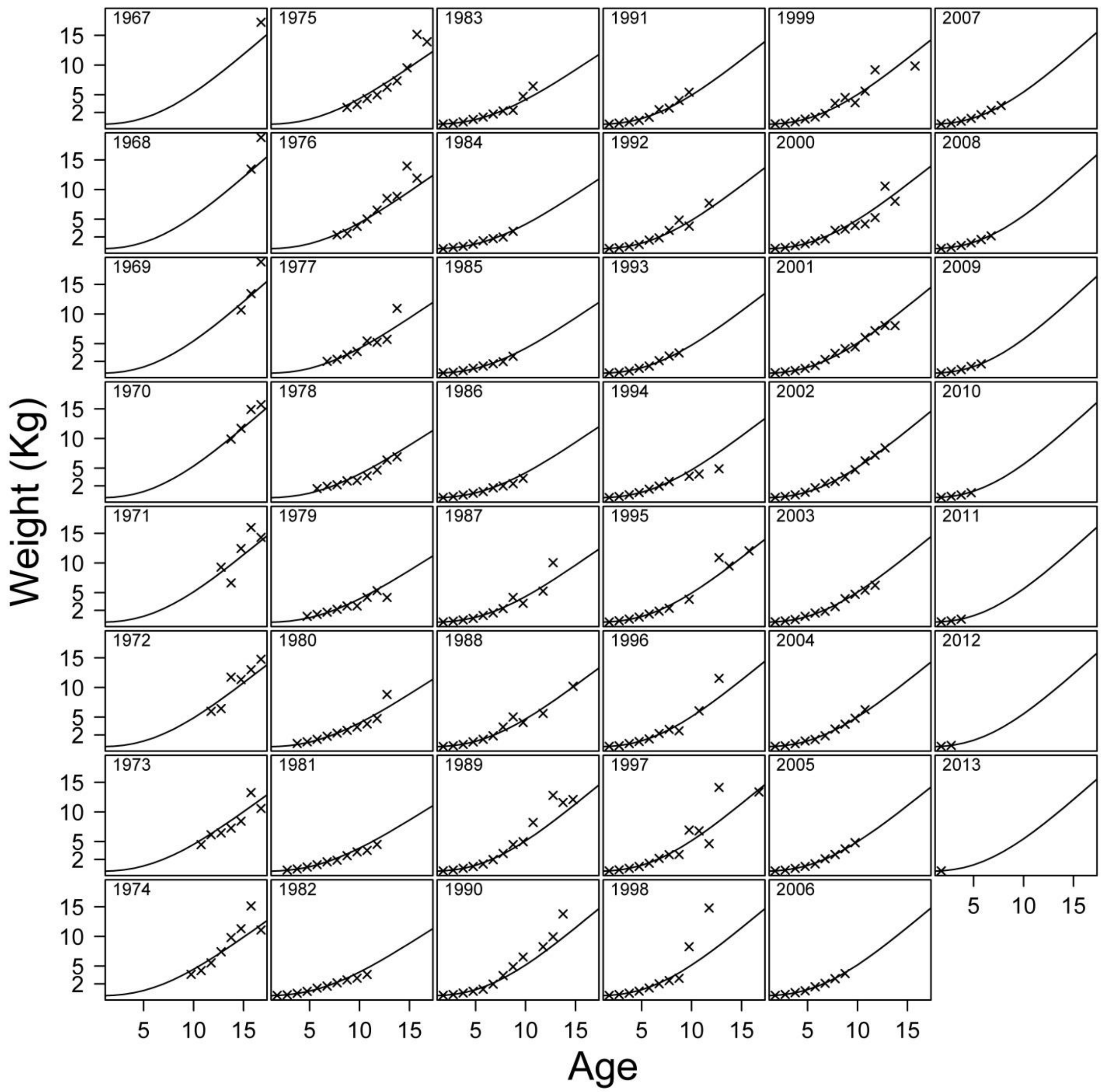

Figure 3. Von Bertalanffy model predicted versus observed weight-at-age for Ncod, by cohort.

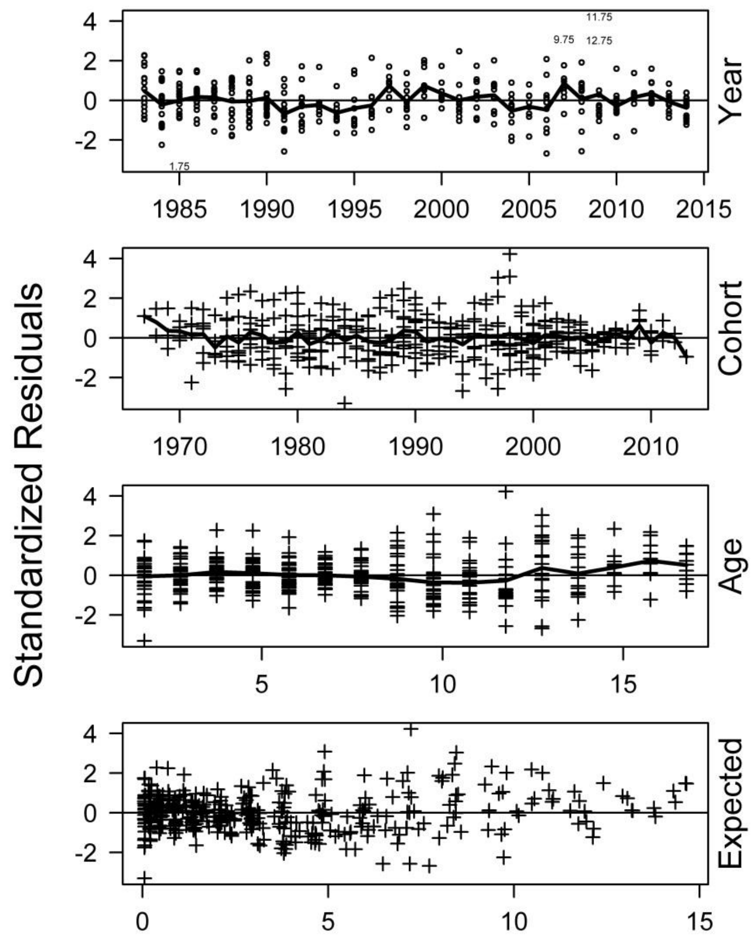

Figure 4. Standardized residuals from fitting the Von Bertalanffy model, versus year, age, cohort, and predicted value. Solid lines in the top three panels indicates the average residual each year, cohort, and age, respectively. Text in the top panel indicates ages that have residual absolute value greater than three.

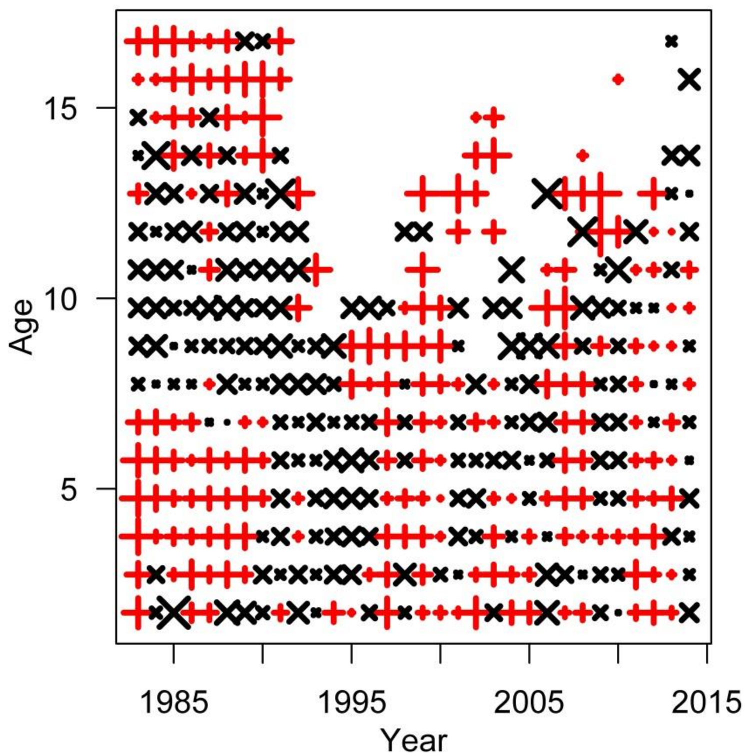

Figure 5. Matrix plot of residuals from the Von Bertalanffy model. Red +'s are positive and black ×'s are negative. The sizes of plotting symbols are proportional to the absolute value of the residuals. Blanks indicate ages and years with no sampled weights or too few (in a cohort) to fit the model.

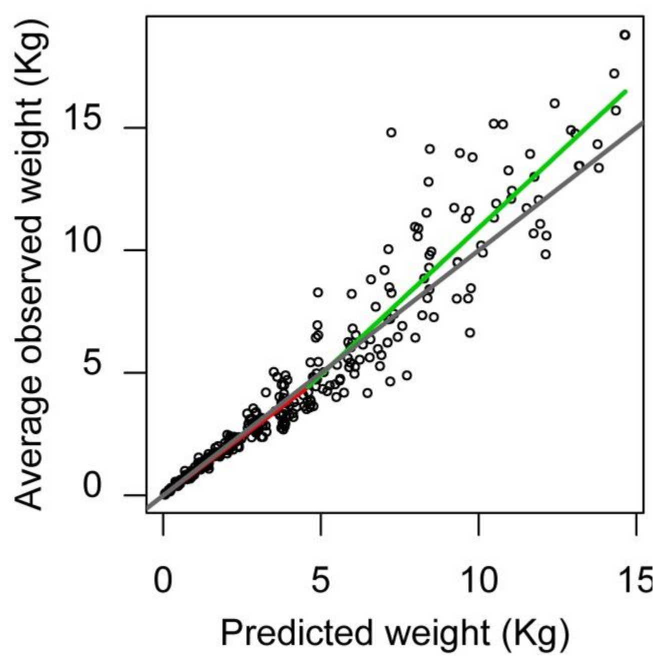

Figure 6. Von Bertalanffy model predicted versus observed weights for all ages and years. The red and green lines indicate a segmented regression of observed versus predicted weights, and the 1:1 line is in grey.

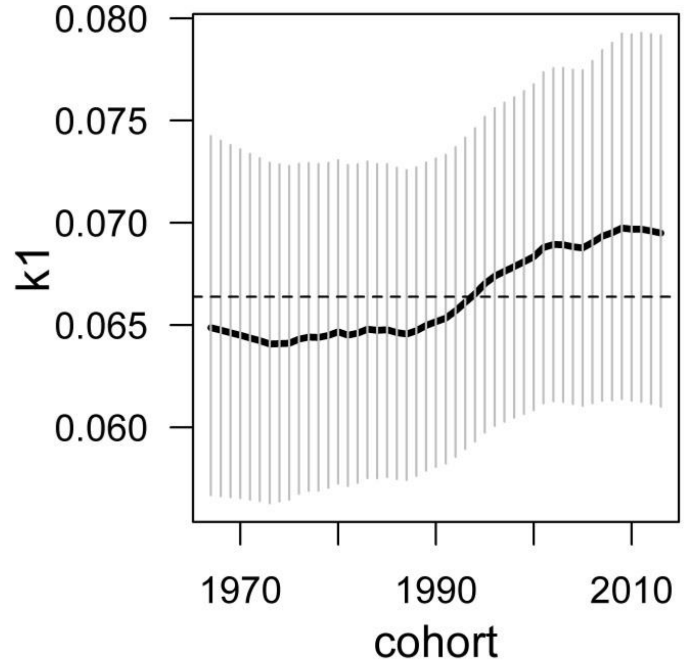

Figure 7a. Solid lines connect estimates of the VonB2 Ncod k_1 parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

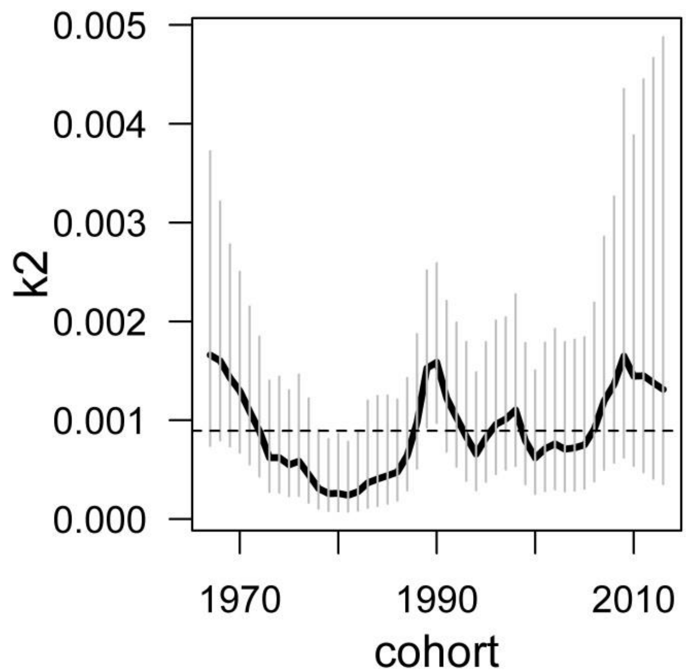

Figure 7b. Solid lines connect estimates of the VonB2 Ncod k_2 parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

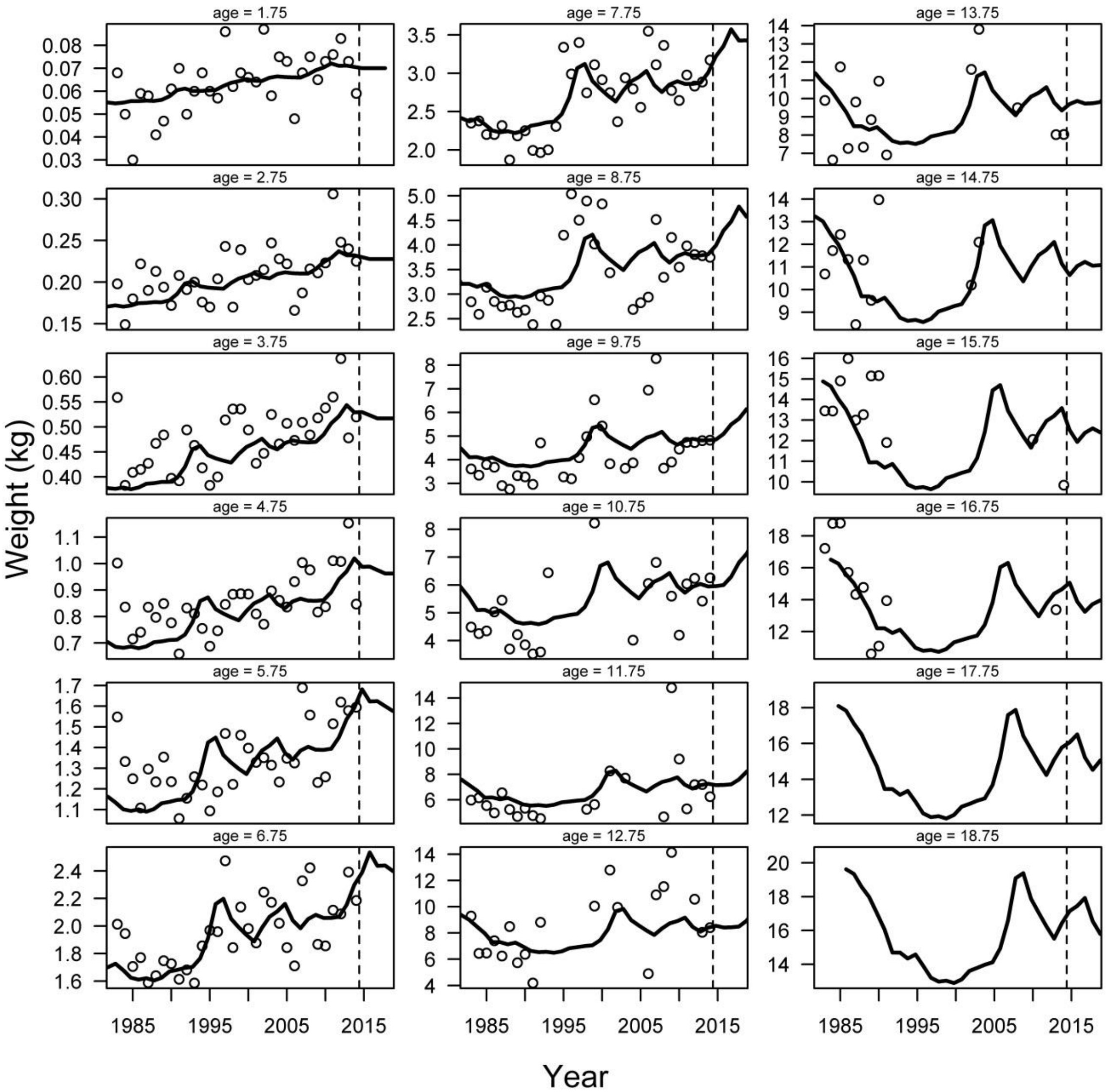

Figure 8. VonB2 model predicted versus observed weight-at-age for Ncod. Each panel shows the results for an age class. The survey occurs during the fall and the age at the top of each panel includes the fraction of year. Three year forecasts are also shown to the right of the vertical dashed lines.

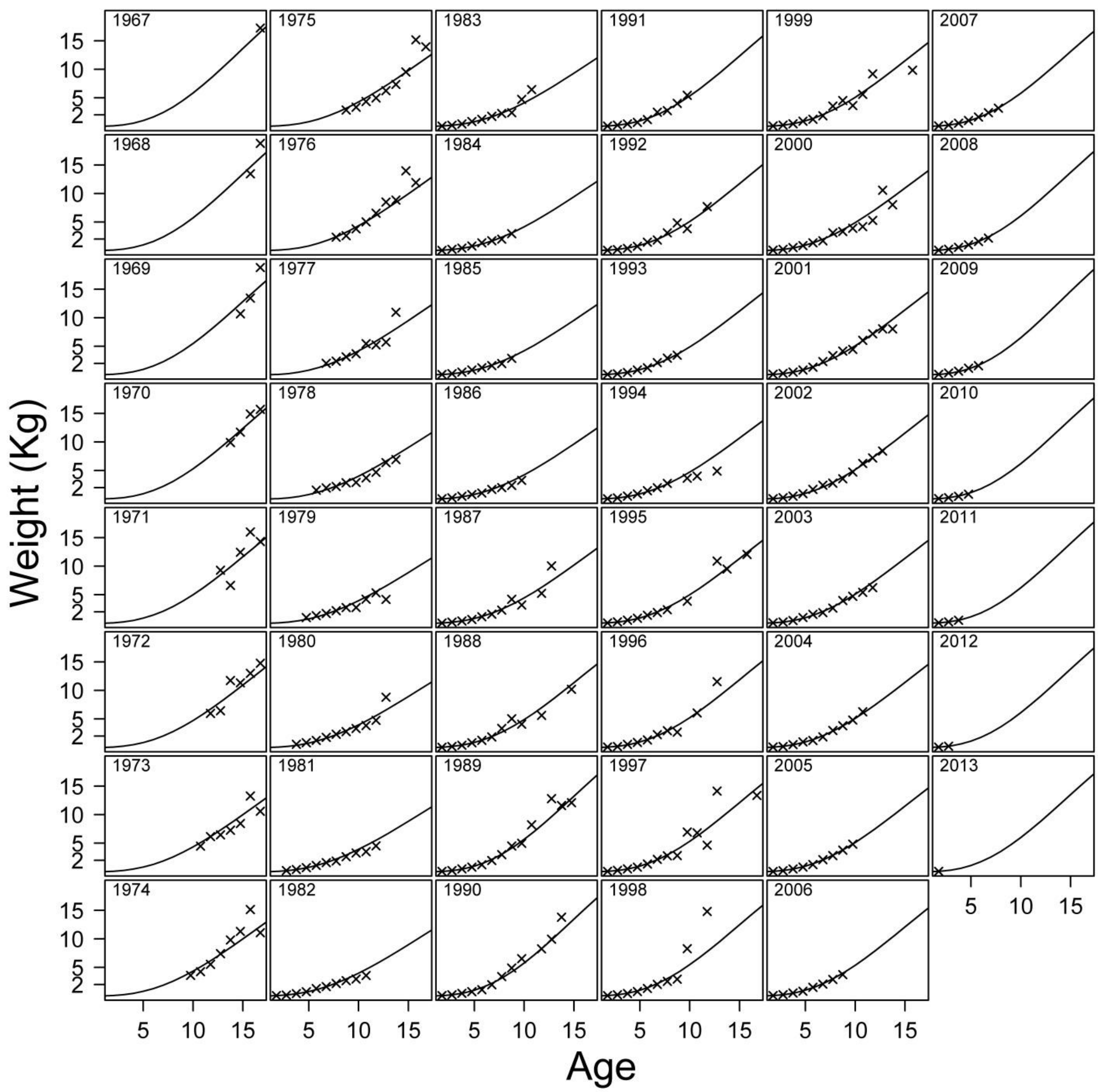

Figure 9. VonB2 model predicted versus observed weight-at-age for Ncod, by cohort.

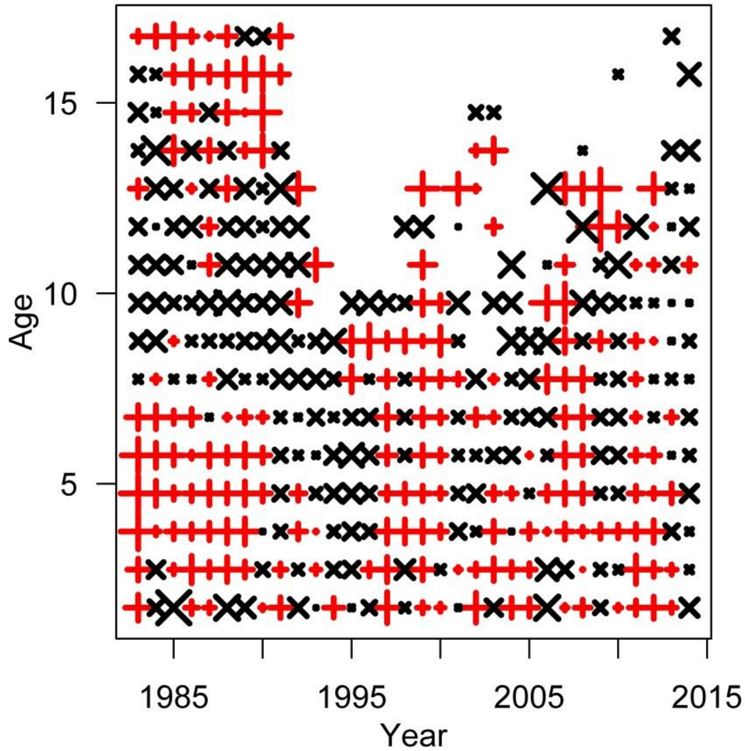

Figure 10. Matrix plot of residuals from the VonB2 model. Red +'s are positive and black ×'s are negative. The sizes of plotting symbols are proportional to the absolute value of the residuals. Blanks indicate ages and years with no sampled weights or too few (in a cohort) to fit the model.

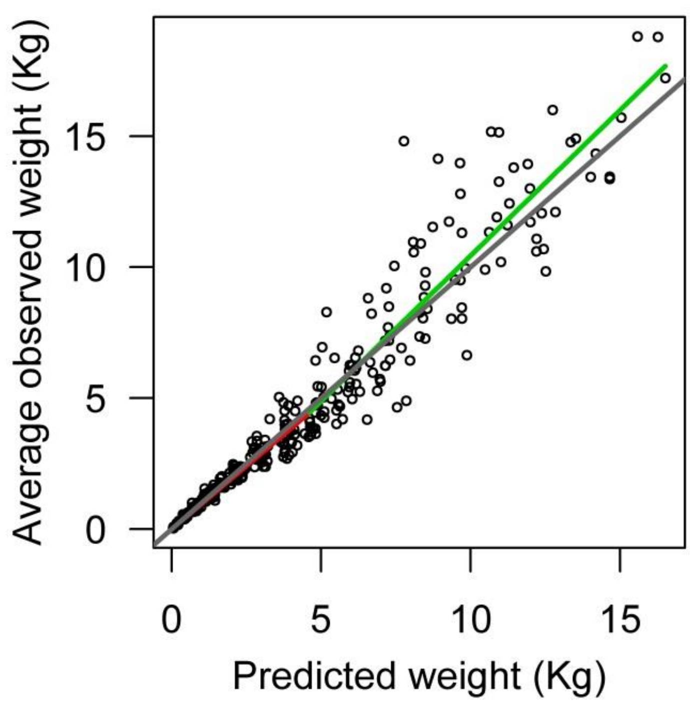

Figure 11. VonB2 model predicted versus observed weights for all ages and years. The red and green lines indicate a segmented regression of observed versus predicted weights, and the 1:1 line is in grey.

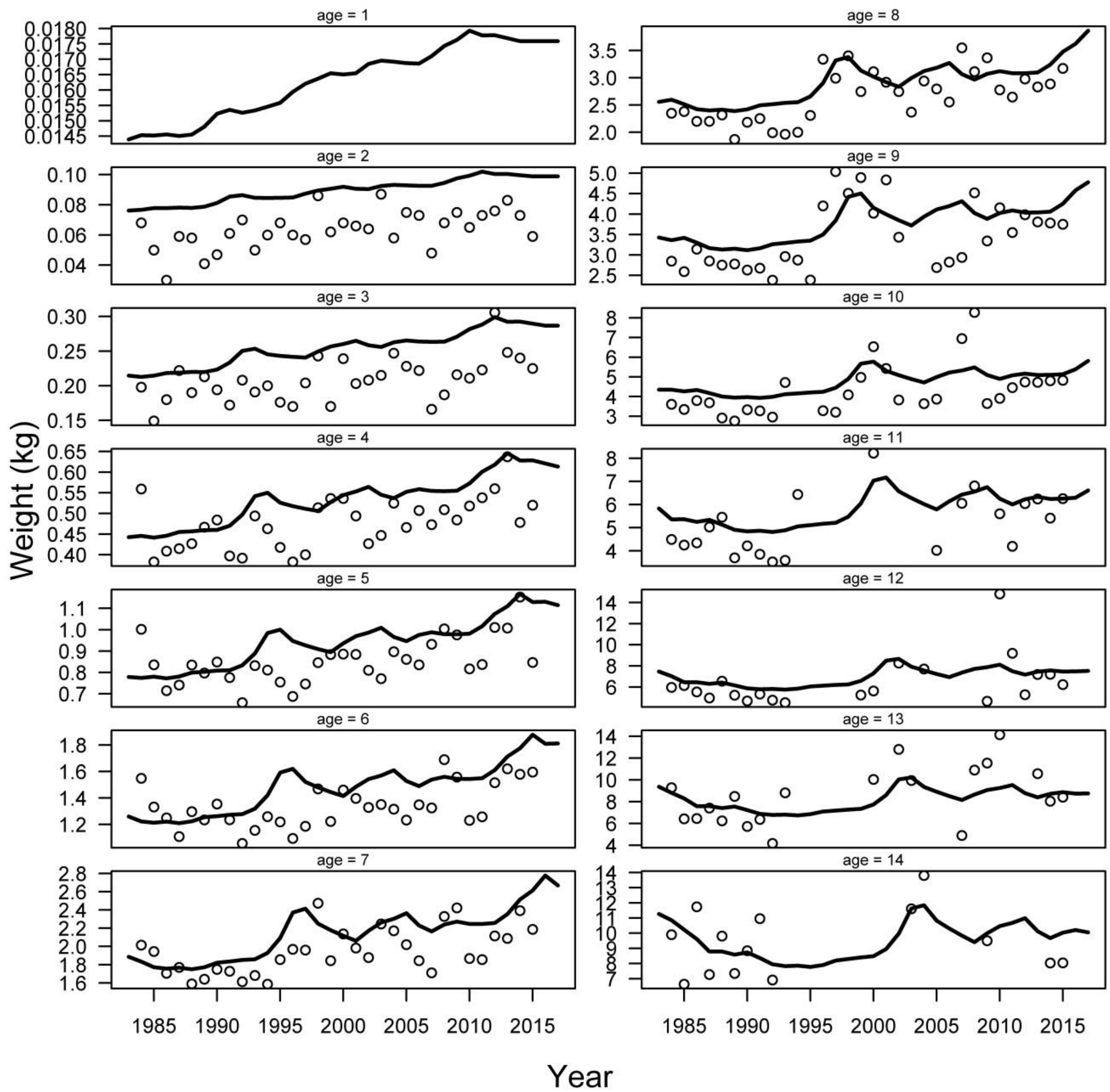

Figure 12. VonB2 model predicted beginning-of-year weights versus the sampled weights in the fall surveys, incremented by one age and year. Each panel shows the results for an age class.

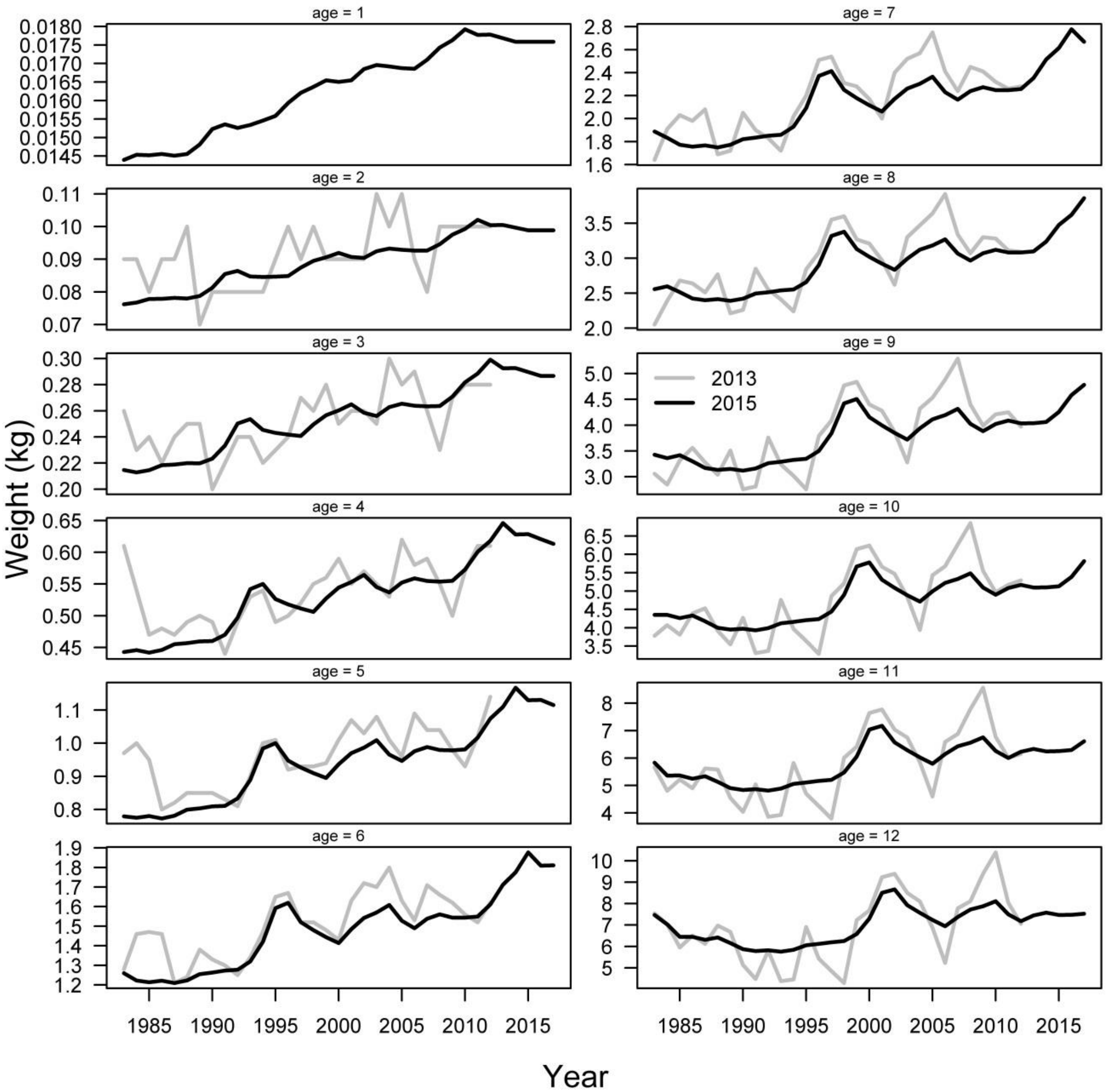

Figure 13. A comparison of VonB2 model predicted beginning-of-year weights from the 2013 analysis (Cadigan, 2013b) and the current analysis (i.e. 2015). Each panel shows the results for an age class.