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**Allocation model for landed catches  
from Baffin Bay narwhal stocks**

**Modèle d'attribution des prises  
débarquées des stocks de narvals de la  
baie de Baffin**

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**ABSTRACT**

This report presents a decision tool that can help co-managers decide on the allocation of total allowable landed catch (TALC) of Baffin Bay narwhal stocks given that multiple communities may harvest from the same summering stocks. Communities hunt narwhal during the summer when stocks are isolated in their summering aggregations and, in some cases, at other times of year from non-summer stock mixtures. The allocation tool is based on a spatial model of the source and degree of stock mixtures that are hunted and produces possible solutions that maximize the catch, particularly for communities with large historic narwhal catches, while minimizing the risk of over-exploitation of any one stock. The document also presents sensitivity analyses of the major assumption that narwhals from different stocks are available to hunters in proportion to stock size during the spring and fall migrations. Results vary depending on input variables but, in general, reducing optimized allocations by 10% or 20% further reduces the risk of exceeding a stock's total allowable landed catch.

**RÉSUMÉ**

Le rapport présente un outil de décision pouvant aider les cogestionnaires à prendre une décision concernant l'attribution du nombre total de prises débarquées autorisées des stocks de narvals de la baie de Baffin, étant donné que de multiples collectivités peuvent exploiter les mêmes stocks d'été. Les collectivités chassent le narval l'été lorsque les stocks sont isolés dans leurs regroupements d'été et, dans certains cas, à d'autres moments de l'année à partir de mélanges de stocks non estivaux. L'outil d'attribution est fondé sur un modèle spatial de la source et du degré des mélanges qui sont chassés et produit des solutions possibles permettant de maximiser les prises, particulièrement pour les collectivités ayant des prises historiques importantes de narvals, tout en réduisant au minimum le risque de surexploitation de n'importe quel stock. Le document présente également des analyses de sensibilité sur l'hypothèse principale selon laquelle les narvals de différents stocks sont disponibles pour les chasseurs en proportion de la taille des stocks pendant les migrations du printemps et de l'automne. Les résultats varient selon les variables d'intrant, mais en général, le fait de réduire les attributions optimales de 10 à 20 % permet de réduire davantage le risque de dépasser le nombre total de prises débarquées autorisées d'un stock.

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## INTRODUCTION

The Nunavut Wildlife Management Board (NWMB) plans to establish Total Allowable Harvest (TAH) levels for narwhal in Nunavut<sup>1</sup>. In preparation for this, Fisheries and Oceans Canada (DFO) presented the NWMB with science advice that narwhal summering aggregations represent distinct stocks (i.e., management units), along with sustainable catch recommendations for each. Narwhals from one summering stock may be harvested by communities located near their summering aggregations areas as well as by more distant communities during spring and fall migrations. For that reason, science advice on sustainable catch limits applies to the catch in the local summering and wintering areas and to the catch taken during migrations between them. Narwhals in the migratory herds cannot be differentiated, and the proportion of different management units represented in spring and fall community catches is unknown. Therefore, total hunting pressure on individual stocks cannot be assessed directly.

DFO Fisheries Management has requested advice on how best to determine community allocations so that harvests from each of the summering stocks are consistent with their sustainable catch recommendations. A community landed catch allocation tool has been developed to facilitate the process of apportioning narwhal catch limits to each of the Nunavut communities that hunt from Baffin Bay narwhal stocks. The tool is structured around a spatial and temporal model of narwhal mixtures based on available information on narwhal seasonal distribution. The goal is to allow allocation of the total allowable landed catch (TALC) of the four Baffin Bay stocks (Somerset Island, Admiralty Inlet, Eclipse Sound and East Baffin Island) among communities without depleting any of the stocks. This tool does not address hunting by Grise Fiord as it is not known whether the whales they harvest comprise a separate stock or their relationship with the four known Baffin Bay stocks that summer in Canadian waters. This model does not include Northern Hudson Bay narwhals because these whales are managed at the population level. The model is meant to be an easy-to-use tool for decision-making. It is intended to allow co-managers to explore the impact of allocation decisions on each of the communities harvesting these stocks.

## METHODS

### NARWHAL CATCH ALLOCATION MODEL

The Canadian portion of the Baffin Bay narwhal population is comprised of at least four summering stocks. The allocation of landed catch limits to communities harvesting these stocks would be a simple task if narwhals were only hunted during the summer open-water season (from late July to early September), a period when narwhals form relatively stable aggregations in specific summering areas of Nunavut. The only harvest which fits this ideal occurs at the western end of the range of narwhals, in hunting areas of Resolute, Gjoa Haven, Taloyoak, Kugaaruk, Igloolik and Hall Beach. Hunters from those communities most likely take only narwhals from the Somerset Island stock, although, in the past, Hall Beach hunters have also taken narwhals from Lyon Inlet in summer (Gonzalez 2001), where the Northern Hudson Bay narwhal population is distributed. In contrast, the Baffin Island communities take a substantial

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<sup>1</sup> *Total Allowable Harvest is the term used under the Nunavut Land Claims Agreement for the amount of a wildlife stock or population that can be lawfully harvested (i.e., harvest limits). The term used in this and previous science advice on narwhal hunt limits is Total Allowable Landed Catch (TALC), emphasizing that the limit is set on the landed catch from a stock, after discounting for hunting loss rates.*

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number of narwhals in the autumn, winter or spring seasons (Romberg and Richard 2005) as well as in summer. Outside of the summer period, narwhals migrate therefore the catch is probably composed of narwhals from a mixture of stocks. This applies to the communities on the northern and eastern Baffin Island coast (Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq). Two southeast Baffin communities, Pangnirtung in particular and more rarely Iqaluit, take narwhals from mixed migratory herds in spring or fall. Occasionally, Pangnirtung hunters also encounter pods of narwhals that have wandered into Cumberland Sound in the summer.

This report presents a decision tool that can help co-managers decide on the allocation of total allowable landed catches (TALCs) for the four Baffin Bay summering stocks, given that part of several communities' narwhal catches come from non-summer mixed stocks of narwhals. The allocation tool is based on a spatial model of the source and degree of stock mixtures that are hunted and produce possible solutions that maximize the catch particularly for communities with large historic narwhal catches while minimizing the risk of over-exploitation of any one stock.

### **Spatial model of narwhal seasonal distribution**

Baffin Bay narwhals aggregate in four or more areas in summer in the Qikiqtani (Baffin) and Kitikmeot (central Arctic) regions of Nunavut (Richard 2010). This allocation model pertains to the four largest Baffin Bay narwhal aggregations in Nunavut (Innes et al. 2002, Richard et al. 2010, Richard 2010): the Somerset Island (SI), Admiralty Inlet (AI), Eclipse Sound (ES) and East Baffin Island (EB) narwhal stocks (Fig.1). The allocation model does not consider the narwhals harvested by Grise Fiord which are likely from a different, but as yet unspecified, stock of narwhals. Also, it does not address the allocation of total allowable catch of Northern Hudson Bay narwhal population. That population is thought to be geographically separate year-round from Baffin Bay narwhal stocks and exhibits different genetic and contaminant profiles (de March et al. 2003, de March and Stern 2003).

The four summering stocks considered in this model (SI, AI, ES and EB) are relatively sedentary in summer and are hunted in their summer range (Fig.1) by local communities. Arctic Bay harvests from the Admiralty Inlet summering stock, Pond Inlet from the Eclipse Sound summering stock, Clyde River and Qikiqtarjuaq hunt the summering stock that occupies fiords of East Baffin Island. The fourth stock, Somerset Island, is larger in size and has a wider summer distribution than all the other summering stocks. Its summer range is densest around Somerset Island throughout summer where it is hunted by Resolute hunters (particularly in the Creswell Bay area), but the stock is also distributed further south where it is hunted by the Kitikmeot communities of Gjoa Haven, Taloyoak and Kugaaruk, and the Northern Foxe Basin communities of Igloolik and Hall Beach (Fig.1). All six communities are referred to as the Western Communities in this paper and their harvest is called the "Western annual catch".

Baffin Island communities hunt mixed stocks outside of the summer open-water season. Based on surveys and movements of tagged narwhals (Dietz and Heide-Jørgensen 1995; Dietz et al. 2001; Richard et al. 2001; Heide-Jørgensen et al. 2002, 2003a and b, 2008; Dietz et al. 2008), Arctic Bay and Pond Inlet seem likely to hunt mixed-stock herds composed of the Somerset Island, Admiralty Inlet and Eclipse Sound stocks. This mixture of stocks is referred to as the Western stock mixture (Fig.1). Tracking indicates that animals from the SI stock move offshore to the center and east side of Baffin Bay therefore it is probably not part of the Eastern stock mixture hunted along the east and southeast coast of Baffin Island by the communities of Clyde River, Qikiqtarjuaq, Pangnirtung and rarely Iqaluit. The AI and ES stocks may be subject to hunting in both non-summer mixtures.

The proportion of animals belonging to any particular stock in the non-summer community harvest is unknown, but it is assumed to be proportional to the size of each stock relative to the total number of animals in the mixture of stocks. Sensitivity of the modelling analyses to this assumption is evaluated using risk modelling.

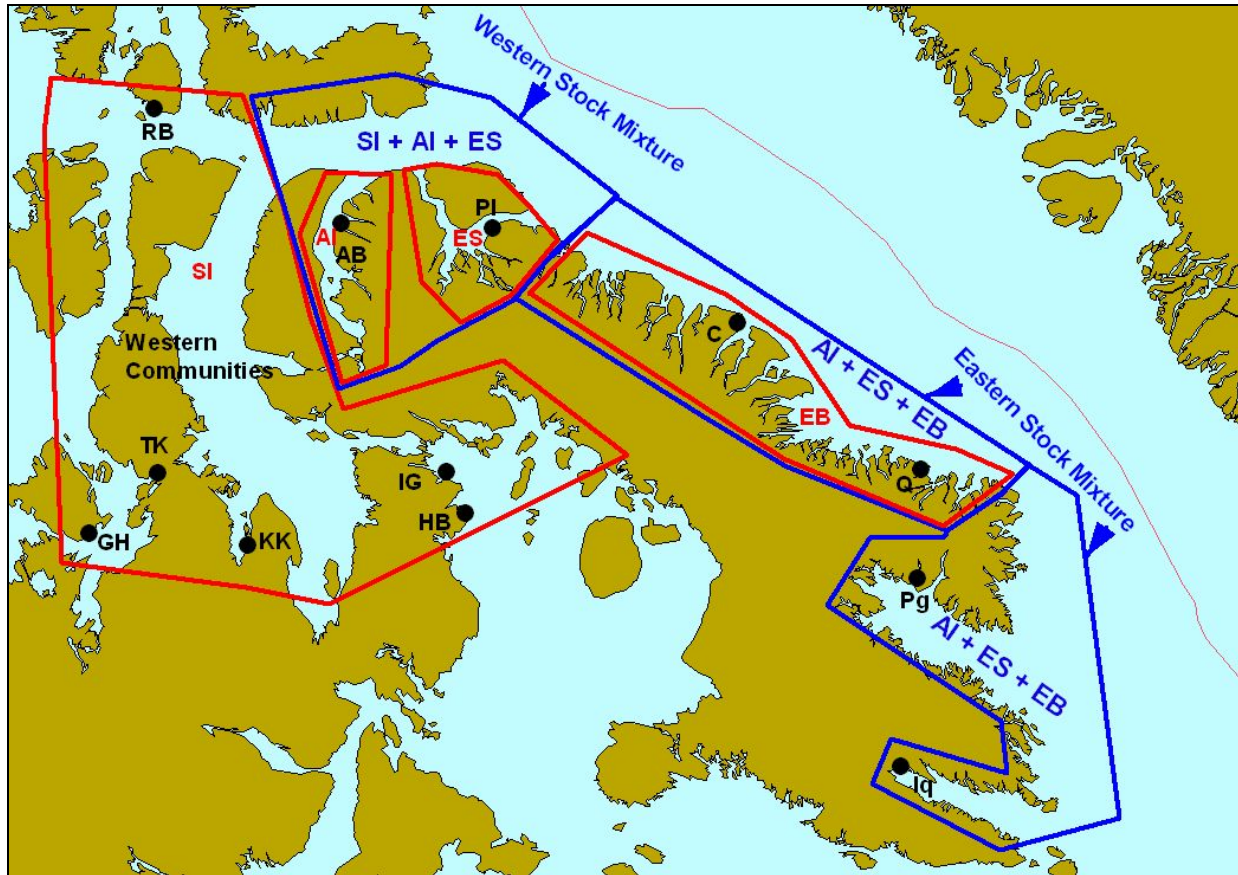


Fig. 1: Schematic representation of the summering stocks discussed in the text (in red letters: SI: Somerset Island stock; AI: Admiralty Inlet stock; ES: Eclipse Sound stock; EB: East Baffin Island stock) and of non-summer stock mixtures (in blue letters). Communities that hunt the stocks are indicated in black letters (RB: Resolute; TK: Taloyoak; GH: Gjoa Haven; KK: Kugaaruk; IG: Igloodik; HB: Hall Beach; AB: Arctic Bay; PI: Pond Inlet; C: Clyde River; Q: Qikiqtarjuaq; Pg: Pangnirtung; Iq: Iqaluit).

### **Model inputs**

Summer Proportion (SP) of the hunt by a community or set of communities (AB, PI, C and Q) is entered in the model as a decision, i.e., a chosen value between 0% and 100%. The non-summer proportion is simply 1-SP.

Because hunts by Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq during non-summer seasons are affected by logistic constraints (e.g., how long the floe edge hunt can reasonably last or how many animals can be taken in fall weather), the proportion of the total annual catch that is allocated to summer or non-summer periods is not a good variable to optimize mathematically. It is preferable that they be fixed prior to the hunting season by co-managers based on consultation with local hunters. The summer catch proportions are therefore decision

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variables in the model. It will be possible for co-managers to determine the dates surrounding the open-water period by incorporating local conditions on an annual basis.

Because the Resolute, Kitikmeot (Gjoa Haven, Taloyoak and Kugaaruk) and Northern Foxe Basin (Igloodik and Hall Beach) catches are relatively small and primarily from the largest stock, the Somerset Island stock, that also supports non-summer hunts elsewhere, the catch limit for that area, called here the “Western annual catch”, is best to remain a decision variable rather than being mathematically optimized. Finally, because the Pangnirtung and Iqaluit hunts also result in small catches, their landed catch is combined and is also best used as a decision variable.

Having made the above decisions, the allocation tool can be run to optimize the landed catch by Nunavut communities that have historically relied most on narwhal hunting for their subsistence and economic well-being: Arctic Bay, Pond Inlet, and the two East Baffin communities (Clyde River and Qikiqtarjuaq). The model uses a linear optimization tool to solve the model by finding the optimal division of annual landed catches. The optimal solution is a vector of landed catches (one catch for each of AB, PI, C and Q) that maximizes the sum of landed catches while minimizing the difference between each stock’s TALC and the total catch (TC) on it (optimized or decision). The optimization is constrained by limiting solutions to positive or zero values of TALC-TC. In other words, the optimization allows as much landed catches by those communities as possible within the limits of the TALCs of the four stocks affected, and without any one stock’s TALC being exceeded (i.e., TALC-TC  $\geq 0$ ).

### **Description of the optimization model**

For different trial vectors of community catches, the model calculates the total catch from each stock and calculates the TALC-TC as follows:

$$TC_s = SC_s + NSC_s$$

so

$$TALC_s - TC_s = TALC_s - (SC_s + NSC_s)$$

where:

TALC<sub>s</sub>: Total Allowable Landed Catch on stock s

s: Stocks “SI” (Somerset Island), “AI” (Admiralty Inlet), “ES” (Eclipse Sound), or “EB” (East Baffin Island)

TC<sub>s</sub>: Total catch on stock s

SC<sub>s</sub>: Summer catch on stock s

NSC<sub>s</sub> = Non-summer catch on stock s

and:

$$SC_s = \sum SC_c = \sum (SCP_c * AC_c)^\dagger$$

$$NSC_s = \sum ((1 - SCP_c) * NSSP_m * AC_{cm})^\ddagger$$

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*Erratum* December 2011:

<sup>†</sup>  $SC_s = SC_c = SCP_c * AC_c$  was replaced with  $SC_s = \sum SC_c = \sum (SCP_c * AC_c)$

<sup>‡</sup>  $NSC_s = (1 - SCP_c) * \sum (NSSP_m * AC_{cm})$  was replaced with  $NSC_s = \sum ((1 - SCP_c) * NSSP_m * AC_{cm})$



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where:

$SC_c$ : Summer catch by community c

c = Communities AB (Arctic Bay), PI (Pond Inlet), C (Clyde River) and Q (Qikiqtarjuaq)

$SCP_c$ : Summer catch proportion by community c

$AC_c$ : Annual catch by community c

$NSSP_m$  = Non-summer stock proportion in mixture m

m: "W" (Western Stock mixture) or "E" (Eastern Stock mixture)

$NSSP_m = SS_s / \sum (SS_{sm})$  where  $SS_s = 0$  if stock not in mixture

where:

$SS_{sm}$  = estimated stock size of stock s in mixture m

To be more specific:

$$SC_{SI} = AC_W$$

$$SC_{AI} = SCP_{AB} * AC_{AB}$$

$$SC_{ES} = SCP_{PI} * AC_{PI}$$

$$SC_{EB} = (SCP_C * AC_C) + (SCP_Q * AC_Q)$$

$$NSC_{SI} = NSSP_W * (NSC_{AB} + NSC_{PI})$$

$$NSC_{AI} = NSSP_W * (NSC_{AB} + NSC_{PI}) + NSSP_E * (NSC_C + NSC_Q + AC_{Pg\&Iq})$$

$$NSC_{ES} = NSSP_W * (NSC_{AB} + NSC_{PI}) + NSSP_E * (NSC_C + NSC_Q + AC_{Pg\&Iq})$$

$$NSC_{EB} = NSSP_E * (NSC_C + NSC_Q + AC_{Pg\&Iq})$$

where:

$SC_s$ : Summer catch from stock s

$NSC_s$  = Non summer catch from stock s

$AC_W$  = Annual catch for western communities

$AC_{AB}$  = Annual catch for Arctic Bay (AB)

$AC_{PI}$  = Annual catch for Pond Inlet (PI)

$AC_C$  = Annual catch for Clyde River (C)

$AC_Q$  = Annual catch for Qikiqtarjuaq (Q)

$AC_{Pg\&Iq}$  = Annual catch for Pangnirtung-Iqaluit (Pg&Iq)

To summarize, the allocation model allows co-managers to set decisions about the values of landed catches by communities at both ends of the range of the four stocks and set the proportions of the catch that is to be taken in the summer season. Once those decision parameters are set, the optimization model can be run to maximize the sum of  $AC_{AB}$ ,  $AC_{PI}$ ,  $AC_C$  and  $AC_Q$  while minimizing  $TALC_s - TC_s$  that are positive or zero.

The modeling was developed using the software Analytica 4.3 Professional with Optimizer ([www.lumina.com](http://www.lumina.com)). Analytica's Optimizing tools are Frontline's Premium Solver package of solver engines and handles all types of optimization problems: Linear, Quadratic, and Non-Linear. When an optimization model is run, Analytica scans the model to determine what type of optimization is required and chooses the appropriate optimization engines. Given that the equations in the present model are linear, the solutions were obtained using Frontline's LP (Linear Programming) solver engine. To obtain the optimized results presented below and illustrate the results that can be obtained for the allocation model, a slight modification was made to the above formulas. In the structured optimization version, the annual catch for both Clyde River and Qikiqtarjuaq was optimized as a set because the software was unable to optimize them separately without favouring one or the other communities. Hence a new parameter was introduced to allocate total catch between those two communities. For illustration purposes, the total catch was apportioned by 0.4 and 0.6 for Clyde River and Qikiqtarjuaq, respectively. This was an arbitrary choice to illustrate the kind of results one can

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get from the allocation tool. Other ratios could be chosen by co-managers if they choose to use this structured optimization version of the allocation model. The simpler, albeit more tedious, iterative version of the allocation model and the sensitivity analysis versions all treat the two communities separately, as in the above formulas.

## RESULTS

To illustrate the types of results that can be produced by the narwhal allocation model, four sets of scenarios were run using linear optimization in Analytica. Each set used different proportions of the summer catches for AB, PI, C and Q and a range of decisions on Western annual catches (100-500) and Pagnirtung-Iqaluit catches (20-80). The scenarios span the range of low summer catch proportions (Table 1) to high summer catch proportions (Table 4), and include summer catch proportions (Table 2) modeled after tag returns from past decades (Romberg and Richard 2005). The high summer catch proportions may be more realistic options for the future, as climate change acts to shorten the season during which floe edge hunts for narwhals can be safely conducted, with a progressively earlier fast-ice melt. Results for the four summer, catch-proportion scenarios are also shown, from low summer proportions to high proportions, in Fig 2-5 with the Western annual catch fixed at 200 narwhals.

In general, results indicate that optimal landed catch levels for Arctic Bay, Pond Inlet and for Clyde River and Qikiqtarjuaq combined can be more than 138 and, under certain scenarios for some communities, could be in excess of 300. Varying the Pagnirtung-Iqaluit catch has a noticeable effect on the optimized catches, causing them to go down as it increases (Figs 2-5). The results change little, if at all, when the Western catch (Resolute, Kitikmeot and Northern Foxe Basin) is increased above 200 until it reaches 500 or close to the TALC for the SI stock. At that point, the optimization algorithm fails to find a solution for the model without having to choose between allocating 0 to Arctic Bay or to Pond Inlet<sup>2</sup>. The Somerset Island TALC is never fully used except when the Western catch is at 500. That is because, in the other cases, the linear optimization stops when the remainder of the smaller stocks' TALCs reach 0 or 1. The full TALC of the Somerset stock cannot be allocated without risking an over-harvest of the smaller stocks that are in non-summer mixtures. Because the relative size of the SI stock is large and the summer harvests small, most scenarios with low summer catch proportions (high non-summer proportions) and Western annual catches up to 400, will only cause a small reduction in the optimized catches for AB, PI, and C and Q (Tables 3-4). In all scenarios, when the Western annual catch is set to 500, the optimized results become unstable, having to choose between two communities (AB or PI) to allocate the catch (Tables 1-4).

## SENSITIVITY ANALYSIS

The above optimization scenarios are based on the assumption that non-summer catches are taken in proportional to the size of each stock relative to the total number of animals in the mixture of stocks. For example, the Somerset Island, Admiralty Inlet, and Eclipse Sound stock mean abundances have been estimated at 45,358 narwhals, 18,049 narwhals and 20,225 narwhals, respectively. The Admiralty Inlet mean stock size represents 22% of the Western Stock mixture, therefore we assume in this model that 22% of the non-summer catch for AB and PI is from the Admiralty Inlet stock. However, this may not be the case as the proportion may vary depending on the timing of migration by different stocks and on the timing of the hunt in the

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<sup>2</sup>A Western annual catch of 500 is not a viable option, since it leaves little room for non-summer catches in the optimized total catches for the other communities. It is shown here to illustrate the point.

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spring and fall. Given the uncertainty in stock proportions, what is the risk associated with this major assumption?

To illustrate that risk, two separate sensitivity analysis models were run. In the first version, the optimized catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq in each of the above scenarios were entered as fixed values and the mixture stock proportions were modeled as lognormal distributions. The means of the lognormals were set equal to the stock proportion in each mixture and the standard errors were set iteratively until their probability densities ranged from 0 to 1, with small probability densities near 0 and 1. These uncertainty distributions were further normalized by dividing each resample of these lognormal distributions by the sum of the resampled stock proportions. This ensured that all proportion resamples summed to 1 and changed the probability density only slightly (Fig. 6). These normalized lognormals were used to model the uncertainty in stock proportions in the non-summer catches.

The probability of a total catch on a stock exceeding the TALC for each stock was calculated from the resulting distribution of the TALC-TC on each stock, that result from the resamples of the stock proportions. The risk model calculated the probability of exceeding the TALC for a given stock as the fraction of the TALC-TC probability density for values smaller than zero. In simpler terms, the model drew 10,000 possible states of stock proportions and calculated the percentage of them that exceeded the TALCs of one or more of the four stocks.

The second version of the sensitivity analysis model modified the stock proportion of the SI stock to make it more conservative. Since this is the largest stock contributing to the mixture in the first version, the distribution of its proportion was modeled so it had higher probability density in the low range, i.e., SI stock proportion  $\leq 50\%$ . This was achieved by using a normalized Gamma (1,0.2) distribution. The result is a cumulative density distribution of the SI stock proportion in the Western Stock mixture that has a median around 25.5%, while version one's lognormal has a median of 57.3% (Fig. 7). In other words, the SI stock in this more conservative version contributes less to most Western Stock mixtures drawn by resampling.

In general, the results from both versions of the sensitivity models (1-lognormal and 2-gamma) suggest that assuming the stock proportion assumption is correct results in medium to high risk of exceeding TALCs for AB, ES and EB if the whole optimized catches (100% in x-axis) are taken (Fig 8-15). The larger the summer proportions of the annual catch, the smaller the risk. Conversely, taking a larger proportion in the non-summer seasons is riskier. The risk is high for the smaller stocks (ES, AI) in the Western non-summer mixture in the gamma model runs, where the SI stock contributes fewer animals.

Co-managers can choose to discount the allocation models optimized catches to reduce that risk. Scenarios with 90% to 50% of the optimized community catches are also shown (Fig. 8-15) but with the fixed catches for western communities and Pangnirtung-Iqaluit remaining unchanged. For all lognormal model's summer proportion scenarios, a choice of 90% of the optimized community catches reduces the risk of exceeding TALCs to less than 20% for all stocks. Choosing 20% as a risk cutoff is in keeping with the spirit of the Potential Biological Removal (PBR) method where  $N_{\min}$  is set at the 0.20 percentile of population estimates. In the gamma model, allocating 80% of the optimal catch brings that probability below 20% for all scenarios but the two lowest summer proportions. Consequently, co-managers might want to consider not allowing low summer proportions for the catch and setting catches at 80% or 90% of optimized community catches. In any case, co-managers can use these two risk models to look at the consequence of the stock proportion assumption under any scenario they might choose to study.

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## DISCUSSION

This allocation modeling and the risk analysis tools were kept as simple as possible, given the complexity of the problem, so decision-making could be explored by co-managers themselves, rather than relying on an analyst and having to resort to lengthy and complex programming for each trial. Until better input data are available, more complex programming methods are less likely to be conducive to effective co-management decision analysis.

There are nevertheless a number of uncertainties with this approach. This document reflects our current view of the Baffin Bay narwhal population substructure based on current available information. The spatial model makes a number of assumptions based on published and anecdotal observations about narwhal distribution and movements. As new data are obtained, the spatial model will likely need to be modified. For example, two narwhals recently tagged near Uummannaq, Greenland, in autumn were tracked to Lancaster Sound the following spring and one transmitted long enough to make it to Somerset Island (Mads Peter Heide-Jørgensen, Greenland Nature Institute, pers.comm.). These new data already suggest that it would be unadvisable to allocate the entire TALC of the Somerset Island stock until this link is better understood. The Somerset Island stock covers a large area so caution should be used in managing this stock as there also may be some sub-stock structuring that we are not yet aware of. The East Baffin Island stock may also represent more than one summering stock. Data used to determine narwhal fidelity to summering stocks is limited to a few tagged narwhal that were tracked long enough to show that they returned to their capture area. However, year-to-year variability in use of summering areas by narwhal during their lifetime (circa 100 years) is possible and not resolved by current data. Furthermore, it is possible that climate induced changes will take place in the future that will change the seasonal distribution of narwhals. There are reasons to believe that narwhals have already responded to the thinning of summer pack ice in some parts of the region, particularly in the range of the Somerset Island stock. To date, narwhals taken by Pangnirtung and Iqaluit have genetically matched more closely with Baffin Bay narwhals (Stephen Petersen, DFO, pers. comm.), but the fact that tracked Northern Hudson Bay (NHB) narwhals have wintered in southern Davis Strait, not too far offshore of Frobisher Bay (Westdal 2008), suggests that catches in Iqaluit and perhaps also Pangnirtung should be monitored to determine if any NHB narwhals could be taken by them. Research needs to focus on methods to estimate the stock proportions in non-summer mixtures from the catch and reduce this major source of uncertainty. Finally, population estimates used in the model are dated, particularly for the Somerset Island stock. New surveys are needed to update population estimates and the corresponding TALCs, as well as the stock proportions used in the model, at least until those proportions are estimated from catch data.

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Table 1. Illustration of possible optimal annual catches for Arctic Bay, Pond Inlet, and Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.50, 0.50, 0.50, 0.50, given specific annual catches for Resolute, Kitikmeot and N Foxe Basin communities, and for Pangnirtung-Iqaluit. The side panel gives the remainder of the TALC for each stock after the allocation tool maximizes the catch under the decision parameters. In this scenario, Clyde and Qikiqtarjuaq share their total annual catch in a 0.4 / 0.6 proportional allocation for illustration purposes.

Optimized total catch by communities or group of communities							Remainder of stock TALCs			
Resolute, Kitikmeot & N Foxe Basin	Arctic Bay	Pond Inlet	Clyde River	Qikiqtarjuaq	Pangnirtung Iqaluit	Total	SI	AI	ES	EB
100	267	247	78	117	20	829	293	0	0	0
200	267	247	78	117	20	929	193	0	0	0
300	267	247	78	117	20	1029	93	0	0	0
400	242	244	78	117	20	1101	0	16	5	0
500	0	118	78	117	20	833	0	176	113	0
Resolute, Kitikmeot & N Foxe Basin	Arctic Bay	Pond Inlet	Clyde River	Qikiqtarjuaq	Pangnirtung Iqaluit	Total	SI	AI	ES	EB
100	259	238	75	112	40	824	297	0	0	1
200	259	238	75	112	40	924	197	0	0	1
300	259	238	75	112	40	1024	97	0	0	1
400	255	231	75	113	40	1114	0	3	5	0
500	0	118	75	113	40	846	0	170	106	0
Resolute, Kitikmeot & N Foxe Basin	Arctic Bay	Pond Inlet	Clyde River	Qikiqtarjuaq	Pangnirtung Iqaluit	Total	SI	AI	ES	EB
100	250	228	72	109	60	819	302	0	0	0
200	250	228	72	109	60	919	202	0	0	0
300	250	228	72	109	60	1019	102	0	0	0
400	250	228	72	109	60	1119	2	0	0	0
500	0	118	72	109	60	859	0	164	99	0
Resolute, Kitikmeot & N Foxe Basin	Arctic Bay	Pond Inlet	Clyde River	Qikiqtarjuaq	Pangnirtung Iqaluit	Total	SI	AI	ES	EB
100	242	219	69	104	80	814	307	0	0	1
200	242	219	69	104	80	914	207	0	0	1
300	242	219	69	104	80	1014	107	0	0	1
400	242	219	69	104	80	1114	7	0	0	1
500	0	118	70	104	80	872	0	158	92	0

Table 2. Illustration of possible optimal annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.43, 0.66, 0.68, 0.47, given specific annual catches for Resolute, Kitikmeot and N Foxe Basin communities, and for Pangnirtung-Iqaluit. The side panel gives the remainder of the TALC for each stock after the allocation tool maximizes the catch under the decision parameters. In this scenario, Clyde and Qikiqtarjuaq share their total annual catch in a 0.4 / 0.6 proportional allocation for illustration purposes.

Optimized total catch by communities or group of communities							Remainder of stock TALCs			
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	326	199	73	109	20	827	295	0	0	0
200	326	199	73	109	20	927	195	0	0	0
300	326	199	73	109	20	1027	95	0	0	0
400	306	202	73	109	20	1110	0	11	1	0
500	0	173	73	109	20	875	0	183	64	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	316	191	70	105	40	822	299	0	0	0
200	316	191	70	105	40	922	199	0	0	0
300	316	191	70	105	40	1022	99	0	0	0
400	312	192	70	105	40	1119	0	2	0	0
500	0	173	70	105	40	888	0	176	57	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	305	183	68	101	60	817	304	0	0	0
200	305	183	68	101	60	917	204	0	0	0
300	305	183	68	101	60	1017	104	0	0	0
400	305	183	68	101	60	1117	4	0	0	0
500	0	173	68	101	60	902	0	170	50	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	295	176	65	97	80	813	308	0	0	1
200	295	176	65	97	80	913	208	0	0	1
300	295	176	65	97	80	1013	108	0	0	1
400	295	176	65	97	80	1113	8	0	0	1
500	0	173	65	97	80	915	0	163	43	1

Table 3. Illustration of possible optimal annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.70, 0.70, 0.60, 0.60, given specific annual catches for Resolute, Kitikmeot and N Foxe Basin communities, and for Pangnirtung-Iqaluit. The side panel gives the remainder of the TALC for each stock after the allocation tool maximizes the catch under the decision parameters. In this scenario, Clyde and Qikiqtarjuaq share their total annual catch in a 0.4-0.6 proportional allocation for illustration purposes.

Optimized total catch by communities or group of communities							Remainder of stock TALCs			
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	241	233	69	103	20	766	355	0	0	0
200	241	233	69	103	20	866	255	0	0	0
300	241	233	69	103	20	966	155	0	0	0
400	241	233	69	103	20	1066	55	0	0	0
500	0	196	69	103	20	888	0	187	46	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	234	225	66	98	40	763	357	0	0	2
200	234	225	66	98	40	863	257	0	0	2
300	234	225	66	98	40	963	157	0	0	2
400	234	225	66	98	40	1063	57	0	0	2
500	196	0	66	100	40	902	0	43	176	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	226	216	64	95	60	761	360	0	0	1
200	226	216	64	95	60	861	260	0	0	1
300	226	216	64	95	60	961	160	0	0	1
400	226	216	64	95	60	1061	60	0	0	1
500	196	0	64	96	60	916	0	37	169	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	218	207	62	92	80	759	363	0	0	0
200	218	207	62	92	80	859	263	0	0	0
300	218	207	62	92	80	959	163	0	0	0
400	218	207	62	92	80	1059	63	0	0	0
500	196	0	62	92	80	930	0	30	162	0



Table 4. Illustration of possible optimal annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.90, 0.90, 0.70, 0.70, given specific annual catches for Resolute, Kitikmeot and N Foxe Basin communities, and for Pangnirtung-Iqaluit. The side panel gives the remainder of the TALC for each stock after the allocation tool maximizes the catch under the decision parameters. In this scenario, Clyde and Qikiqtarjuaq share their total annual catch in a 0.4 / 0.6 proportional allocation for illustration purposes.

Optimized total catch by communities or group of communities							Remainder of stock TALCs			
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	220	218	62	92	20	712	408	1	1	0
200	220	218	62	92	20	812	308	1	1	0
300	220	218	62	92	20	912	208	1	1	0
400	220	218	62	92	20	1012	108	1	1	0
500	220	218	62	92	20	1112	8	1	1	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	213	210	60	89	40	712	409	1	0	0
200	213	210	60	89	40	812	309	1	0	0
300	213	210	60	89	40	912	209	1	0	0
400	213	210	60	89	40	1012	109	1	0	0
500	213	210	60	89	40	1112	9	1	0	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	206	202	57	86	60	711	410	0	0	0
200	206	202	57	86	60	811	310	0	0	0
300	206	202	57	86	60	911	210	0	0	0
400	206	202	57	86	60	1011	110	0	0	0
500	206	202	57	86	60	1111	10	0	0	0
<b>Resolute, Kitikmeot &amp; N Foxe Basin</b>	<b>Arctic Bay</b>	<b>Pond Inlet</b>	<b>Clyde River</b>	<b>Qikiqtarjuaq</b>	<b>Pangnirtung Iqaluit</b>	<b>Total</b>	<b>SI</b>	<b>AI</b>	<b>ES</b>	<b>EB</b>
100	199	194	55	83	80	711	411	0	0	0
200	199	194	55	83	80	811	311	0	0	0
300	199	194	55	83	80	911	211	0	0	0
400	199	194	55	83	80	1011	111	0	0	0
500	199	194	55	83	80	1111	11	0	0	0

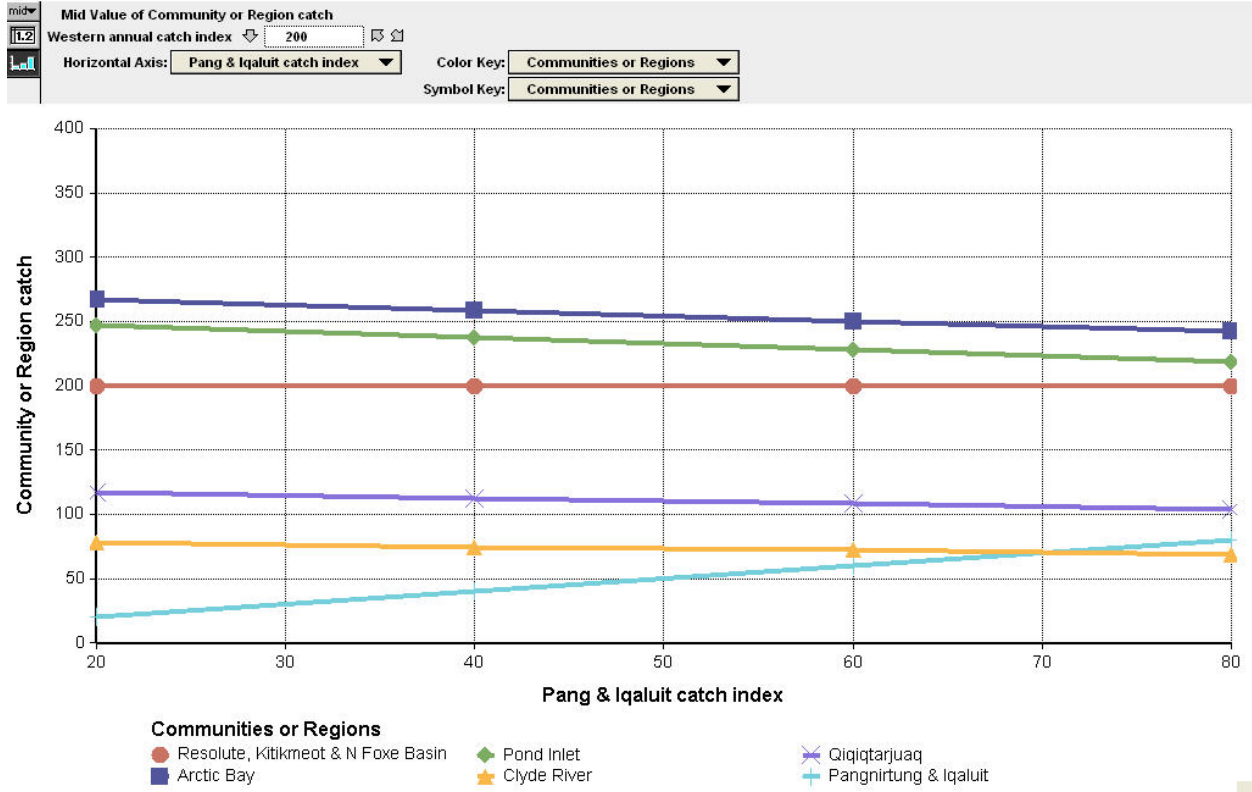


Fig. 2. Illustration of possible optimized annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.50, 0.50, 0.50, and 0.50, given a catch of 200 for Resolute, Kitikmeot and N Foxe Basin, and varying over 20-80 for Pangnirtung and Iqaluit.

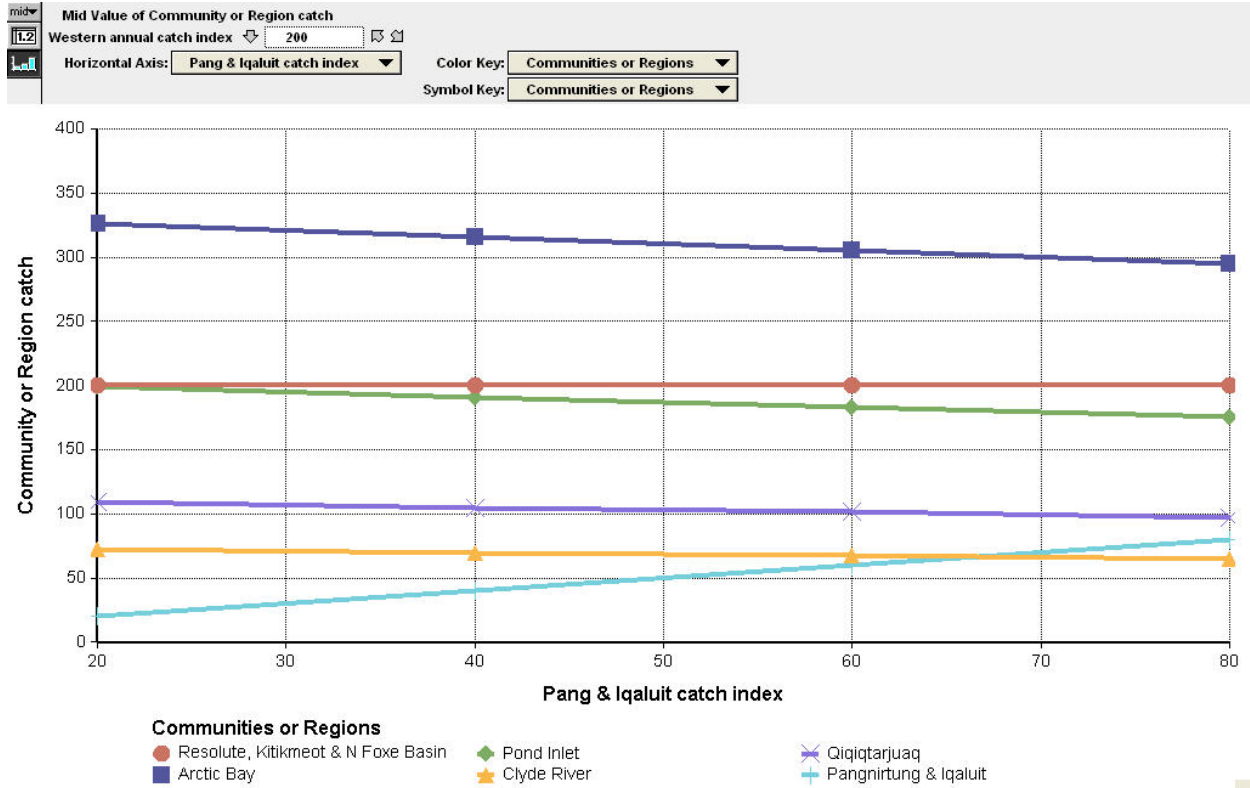


Fig. 3. Illustration of optimized annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.43, 0.66, 0.68, and 0.47, given a catch of 200 for Resolute, Kitikmeot and N Foxe Basin, and varying over 20-80 for Pangnirtung and Iqaluit.

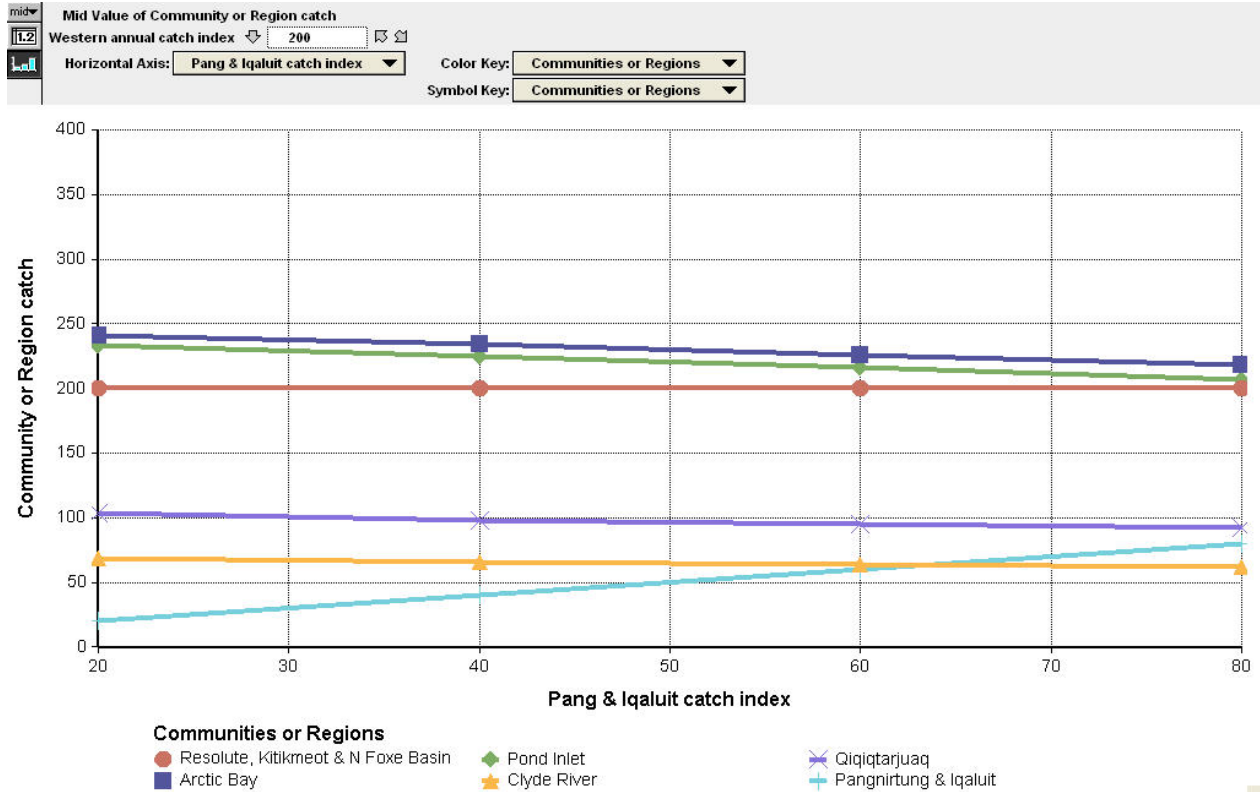


Fig.4. Illustration of optimized annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.70, 0.70, 0.60, and 0.60, given a catch of 200 for Resolute, Kitikmeot and N Foxe Basin, and varying over 20-80 for Pangnirtung and Iqaluit.

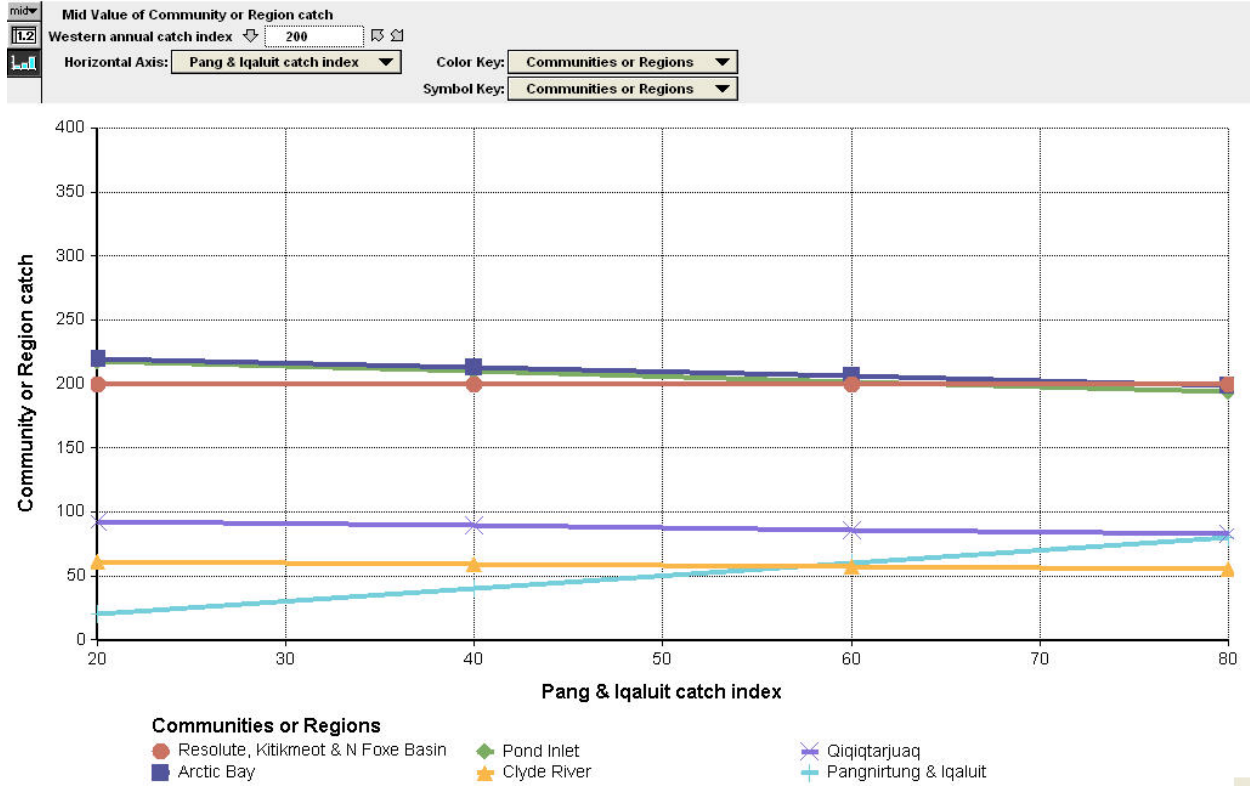


Fig.5. Illustration of optimized annual catches for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq, assuming summer proportion of catch is respectively 0.90, 0.90, 0.70, and 0.70, given a catch of 200 for Resolute, Kitikmeot and N Foxe Basin, and varying over 20-80 for Pangnirtung and Iqaluit.

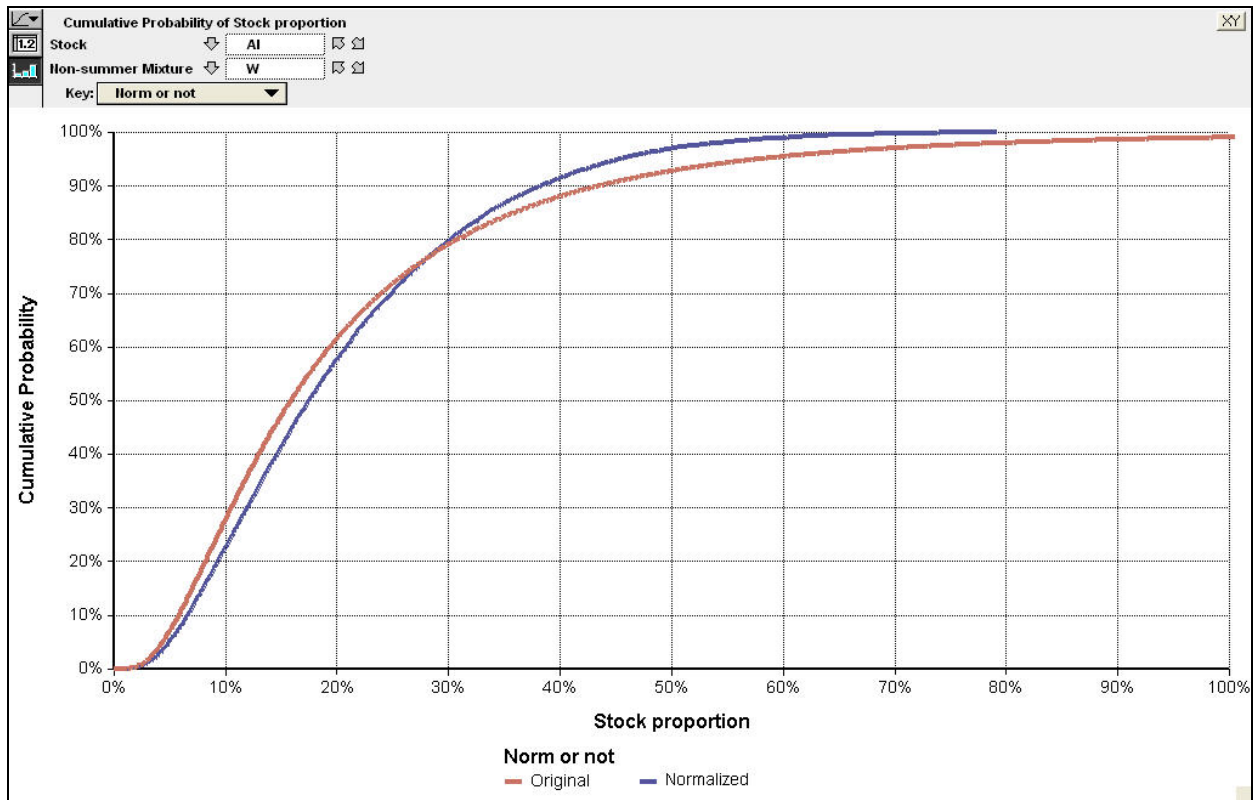


Fig 6. Cumulative lognormal distribution of the Admiralty Inlet stock size for the Western Stock mixture: red line – not normalized; blue line – normalized.

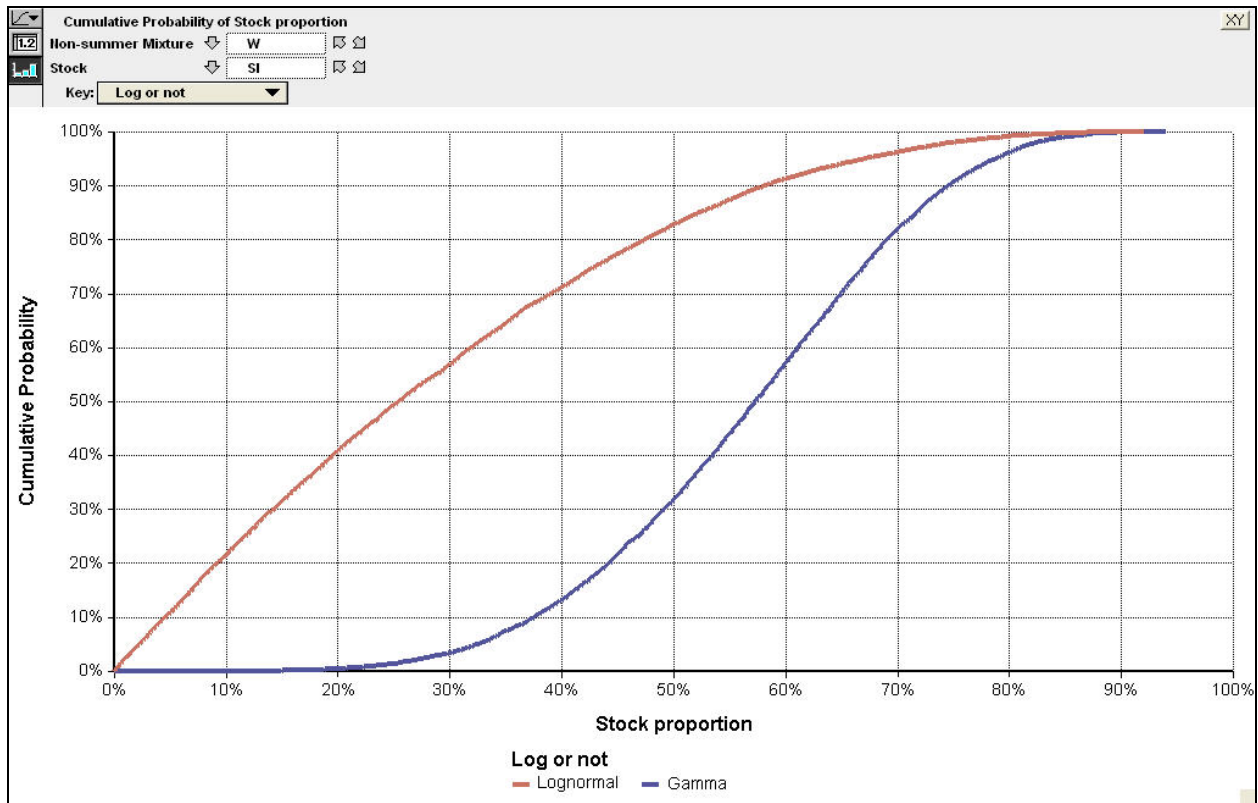


Fig. 7. Cumulative normalized Gamma distribution (red line) of the Somerset Island stock proportion for the Western Stock mixture in the sensitivity model version 1, compared to the cumulative lognormal distribution (blue line) in the sensitivity model 2.

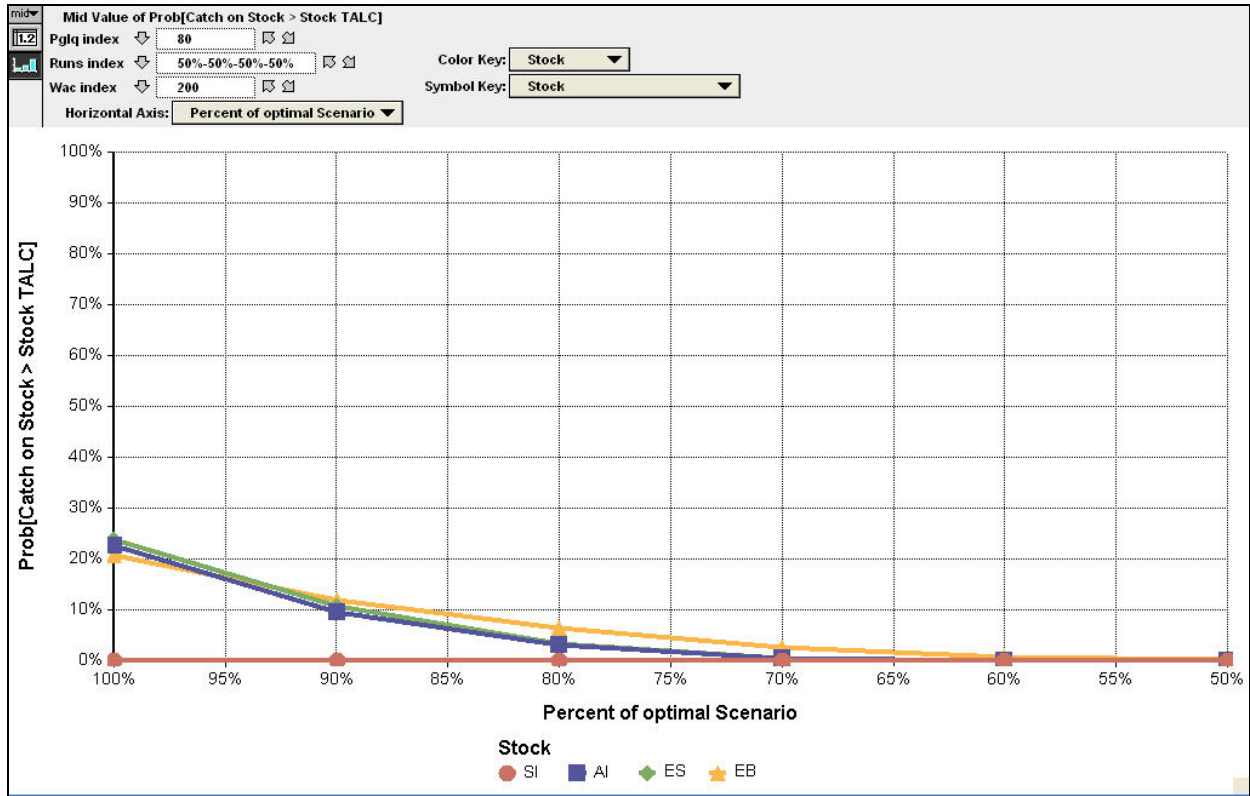


Fig. 8. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.5-0.5-0.5-0.5. Stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture.



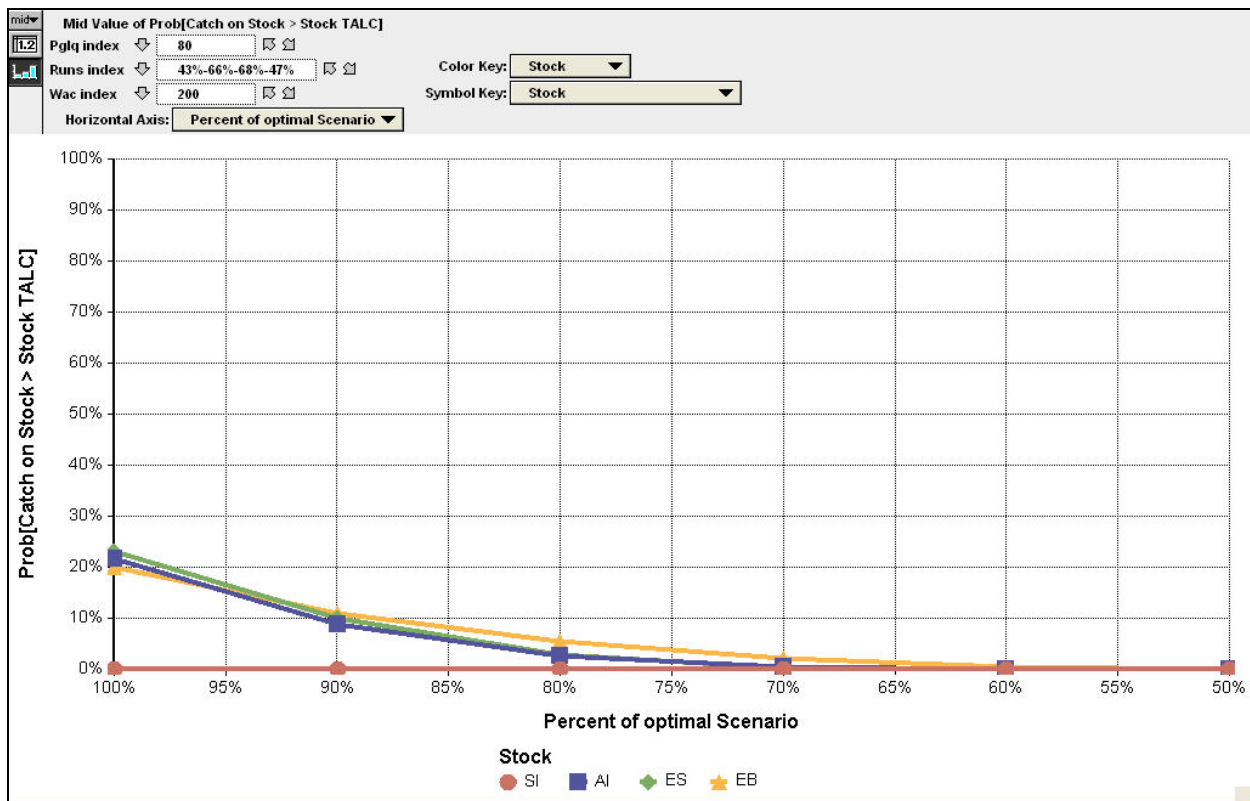


Fig. 9. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.43-0.66-0.68-0.47. Stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture.

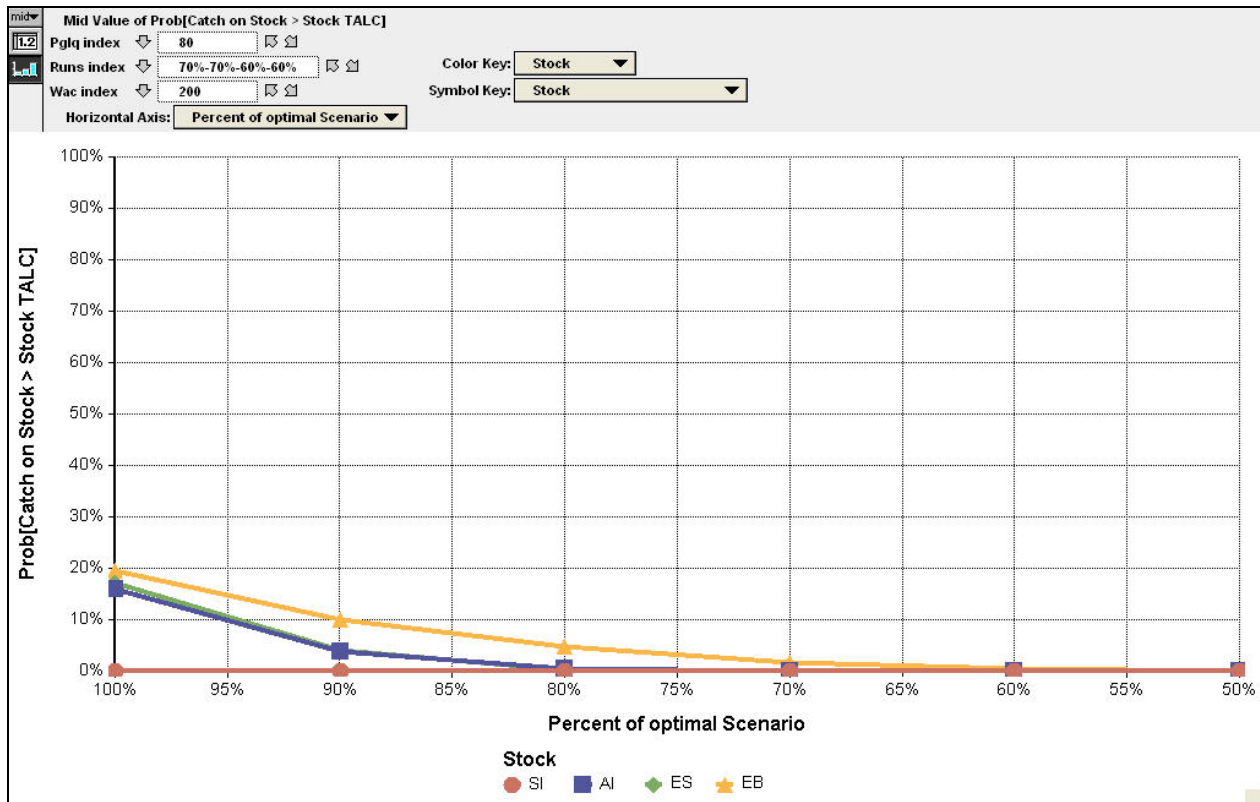


Fig. 10. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.7-0.7-0.6-0.6. Stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture.

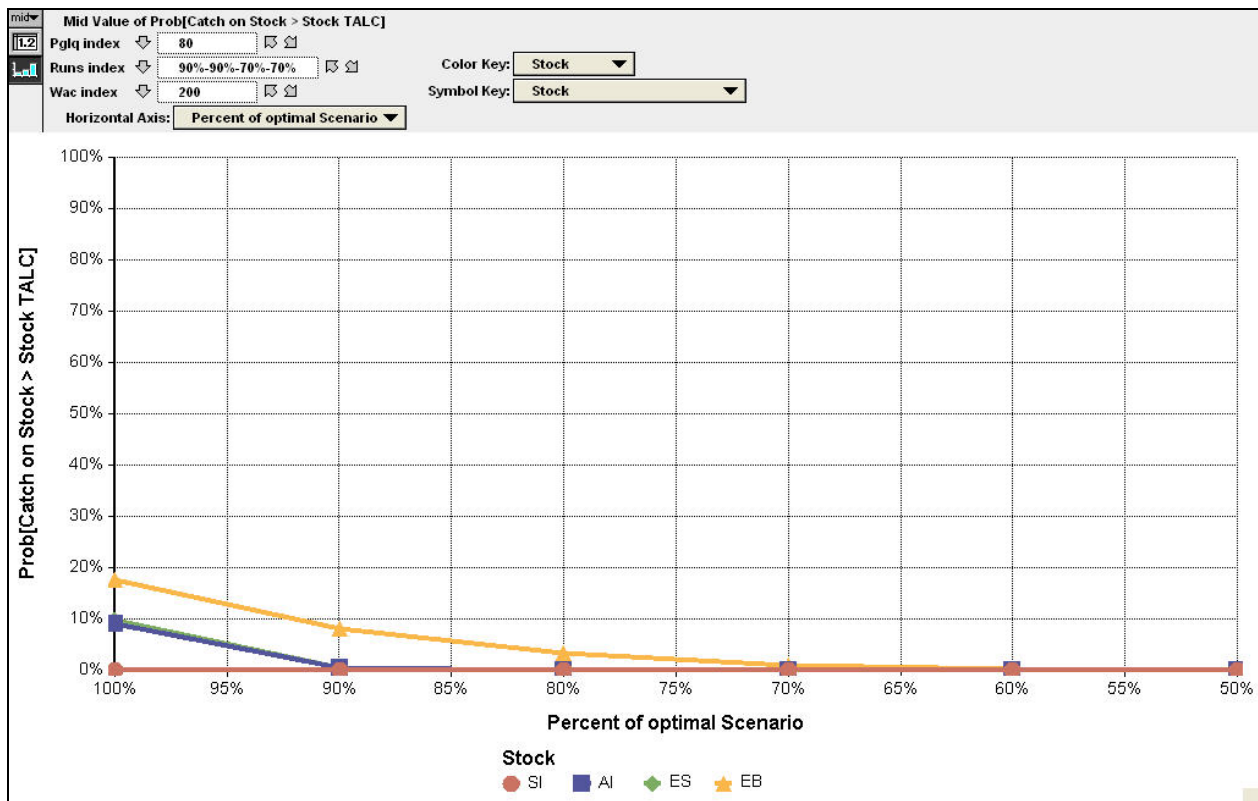


Fig. 11. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.9-0.9-0.7-0.7. Stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture.

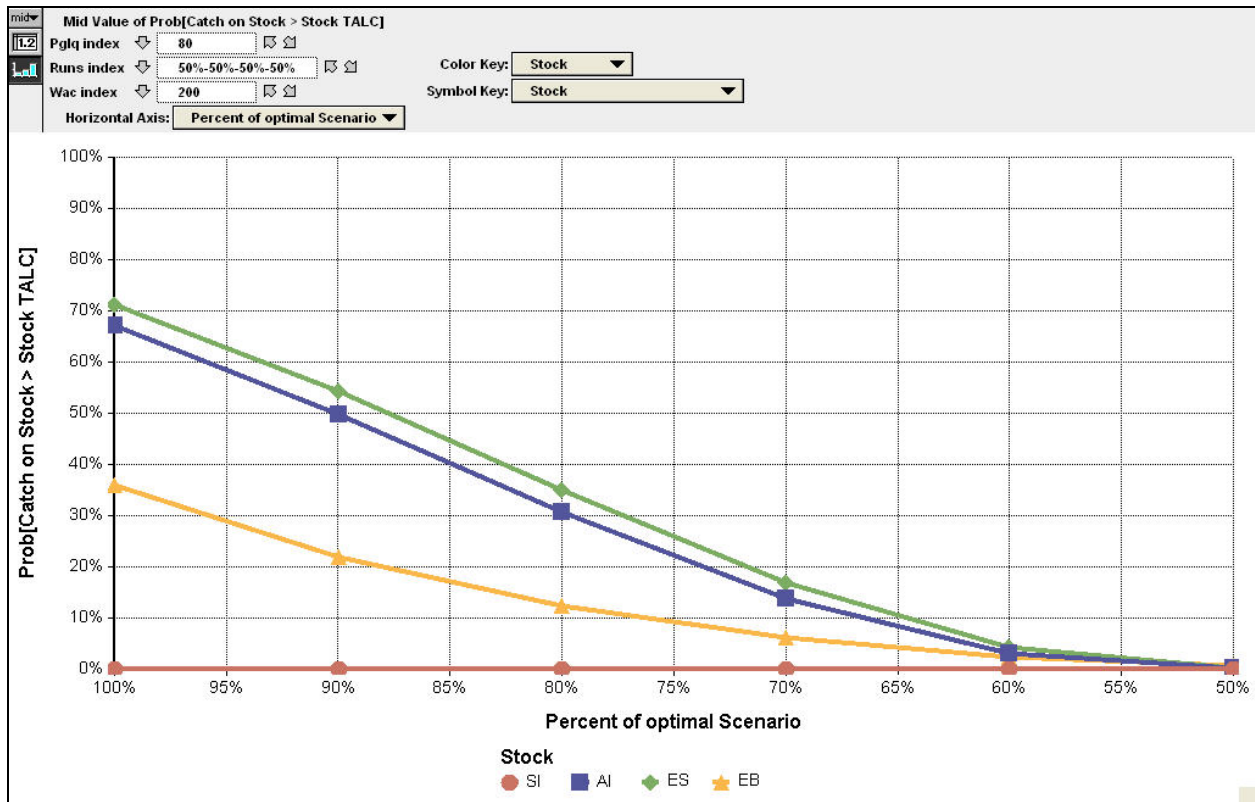


Fig. 12. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.5-0.5-0.5-0.5. Except for the SI stock, stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture. The SI stock proportion in the Western non summer mixture is a normalized gamma(1,0.2) distribution.

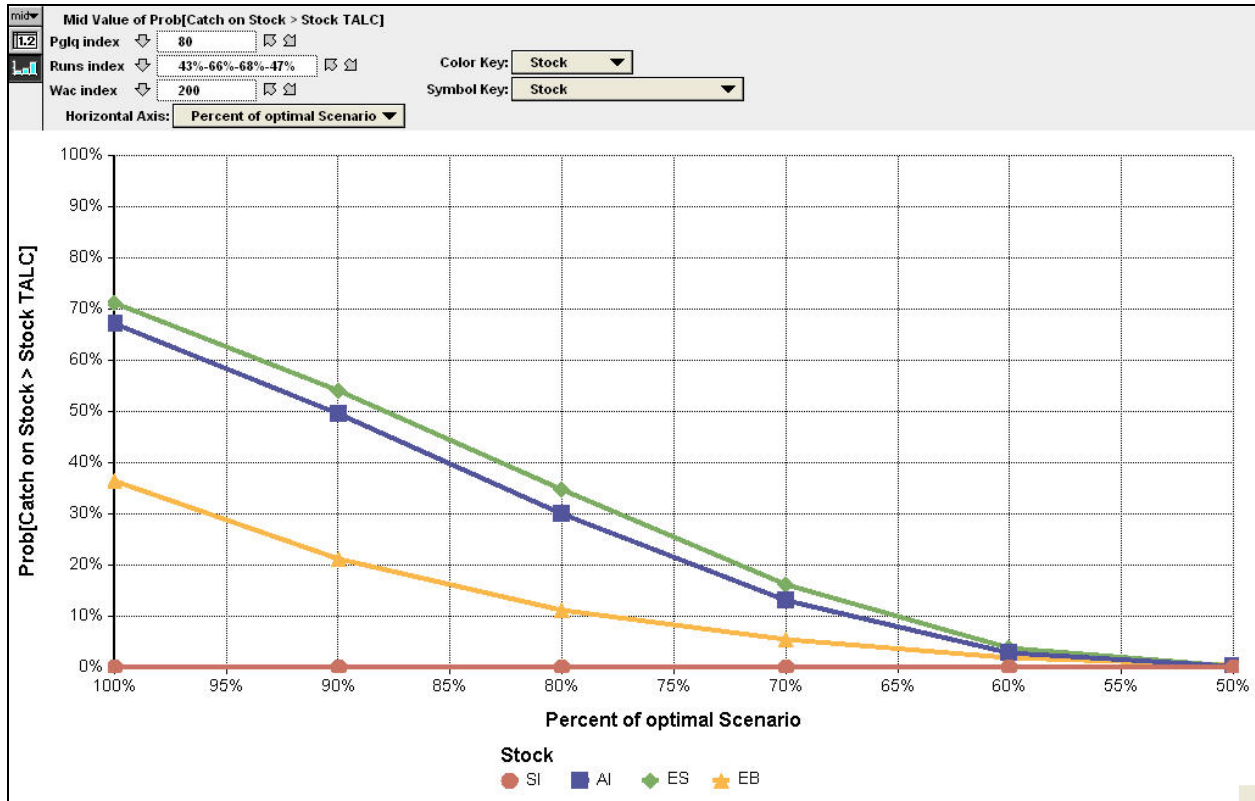


Fig. 13. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.43-0.66-0.68-0.47. Except for the SI stock, stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture. The SI stock proportion in the Western non summer mixture is a normalized gamma(1,0.2) distribution.

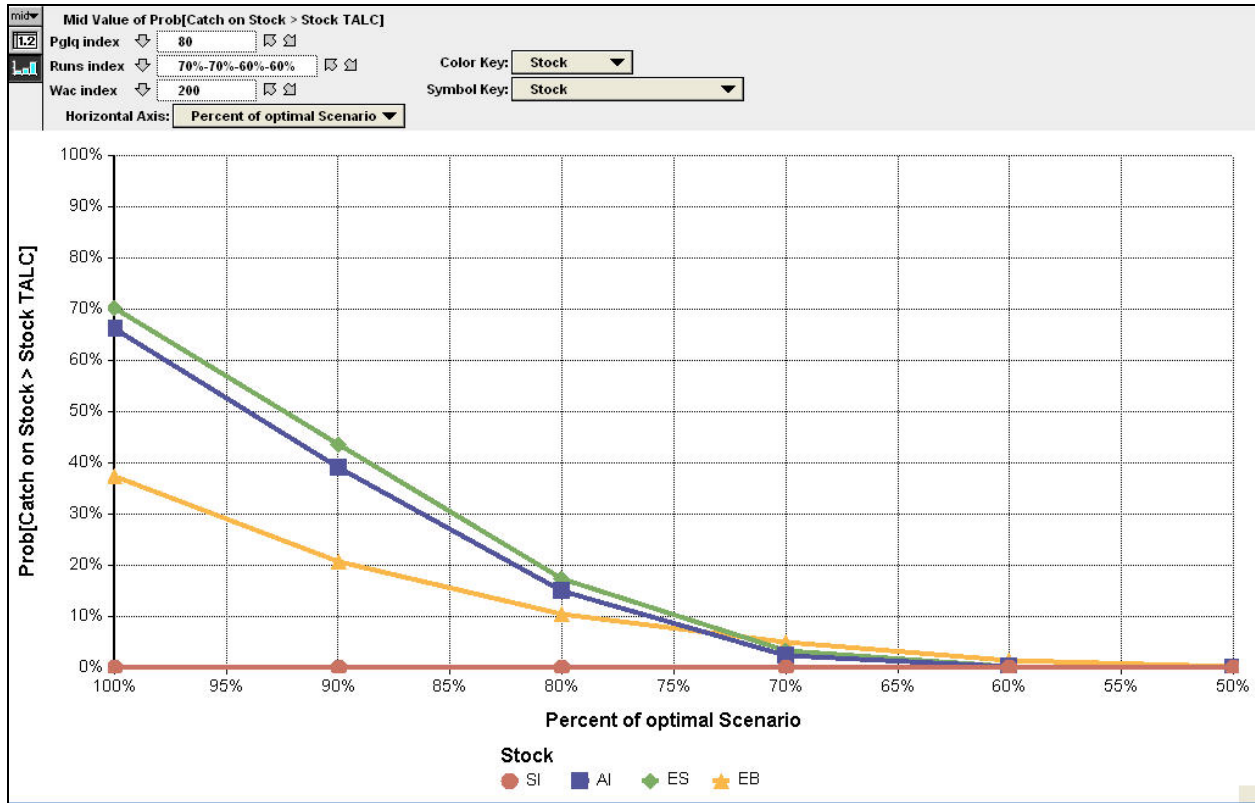


Fig. 14. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.7-0.7-0.6-0.6. Except for the SI stock, stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture. The SI stock proportion in the Western non summer mixture is a normalized gamma(1,0.2) distribution.

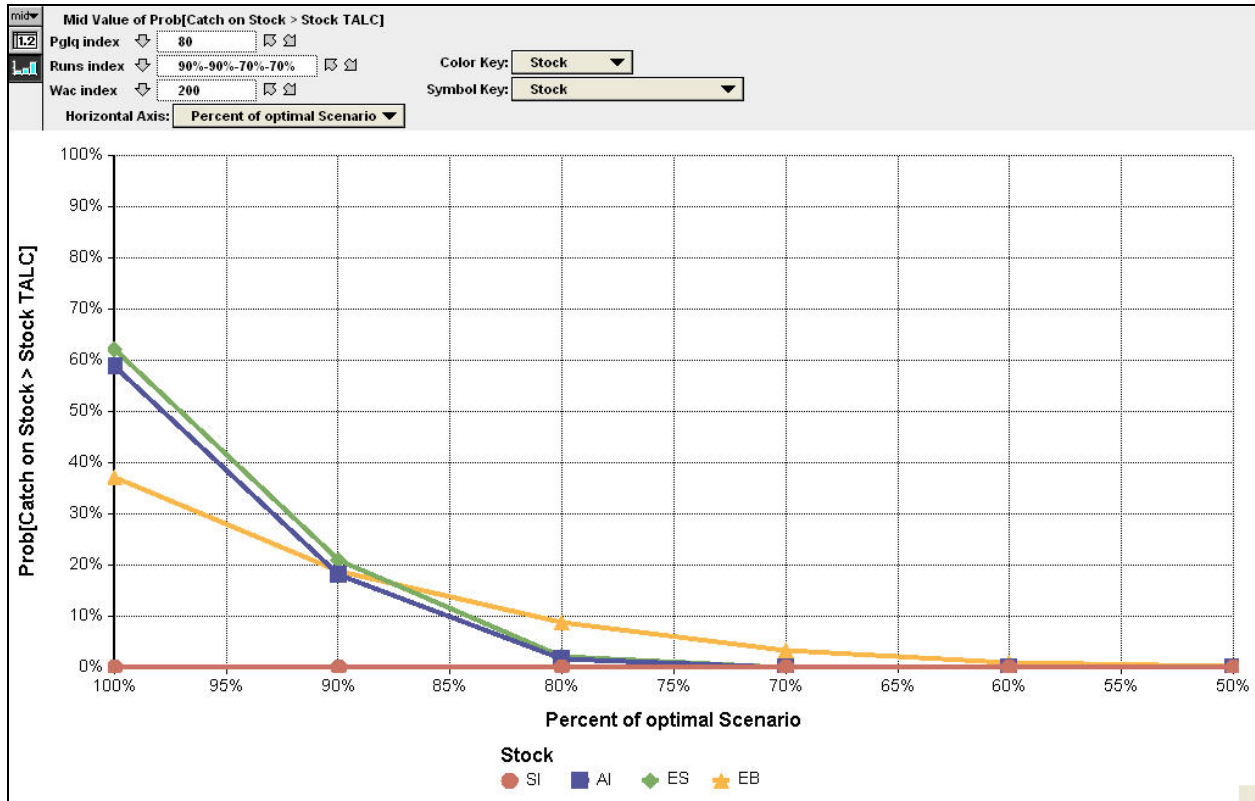


Fig. 15. Probability of exceeding stock TALCs if the optimal catches are reduced from 90% to 50% of their original (100%) values, assuming the scenario of summer catch proportions for Arctic Bay, Pond Inlet, Clyde River and Qikiqtarjuaq of 0.9-0.9-0.7-0.7. Except for the SI stock, stock proportions are set as normalized lognormals with stock proportions calculated from mean stock sizes in each non-summer stock mixture. The SI stock proportion in the Western non summer mixture is a normalized gamma(1,0.2) distribution.