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# Vessel calibration results for redfish (*Sebastes* sp.) from comparative fishing between the CCGS Teleost research vessel and the MV Cape Beaver fishing vessel.

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# Résultats de calibration des navires pour le sébaste (*Sebastes* sp.) d'après une pêche comparative entre le navire de recherche NGCC Teleost et le navire de pêche MS Cape Beaver.



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## ABSTRACT

The DFO research vessel CCGS Teleost and the fishing vessel MV Cape Beaver (or sister vessel MV Cape Ballard) have been used in stratified-random bottom trawl surveys for redfish (*Sebastes* sp.) in the management area UNIT2 (NAFO Div. 3P4V). The two vessels may have different stock-catchabilities, mainly because of differences in the vessel classes, fishing gears and fishing protocols. Adjustment or calibration of their catch rates was explored to combine survey data for each vessel into a single time-series of stock size indices. A paired-tow fishing experiment with 24 sets was carried out in August 2000 to calibrate the vessels. These data were analyzed using a generalized linear mixed model to provide a length-based conversion formula to calibrate Cape Ballard survey catches to Teleost catches. The model also accounted for within-pair random differences in the length distributions of fish encountered by each vessel. The results indicated that there was no overall significant difference in catch rates, but that there was a significant length effect such that the Teleost will catch more small fish (< 30 cm) and the Cape Ballard will catch more large fish (> 30 cm).

## RÉSUMÉ

Le navire de recherche du MPO NGCC Teleost et le navire de pêche MS Cape Beaver (ou son navire jumeau, le MS Cape Ballard) ont été utilisés dans le cadre de relevés au chalut de fond par stratification aléatoire sur les sébastes (*Sebastes* sp.) dans la zone de gestion de l'unité 2 (divisions 3P4V de l'OPANO). Les deux navires peuvent afficher des capturabilités différentes, principalement en raison des différences entre les catégories de navire, les engins de pêche et les protocoles de pêche. On a examiné l'ajustement ou le calibrage de leurs taux de prises afin de combiner les données dérivées de relevés pour chaque navire en une seule série chronologique d'indices sur la taille du stock. On a mené une expérience de pêche par trait double dont 24 traits ont été effectués en août 2000 pour calibrer les navires. Ces données ont été analysées au moyen d'un modèle linéaire mixte généralisé afin d'obtenir une formule de conversion axée sur la longueur permettant de calibrer les prises du MS Cape Ballard avec les prises du NGCC Teleost effectuées dans le cadre du relevé. Le modèle a également tenu compte des différences aléatoires dans les données appariées des distributions de longueurs des poissons observées par chaque navire. Les résultats indiquaient qu'il n'y avait en général aucune différence significative dans les taux de prises, mais qu'il y avait un effet important sur la longueur : le NGCC Teleost capture plus de petits poissons (< 30 cm) et le MV Cape Ballard capture plus de gros poissons (> 30 cm).

# INTRODUCTION

The DFO research vessel CCGS Teleost and the fishing vessel MV Cape Beaver (or sister vessel MV Cape Ballard) have been used to conduct two stratified-random bottom trawl surveys for redfish (*Sebastes* sp.) in the management area UNIT2 (NAFO Div. 3P4V). The Teleost survey was conducted from 1994-97, 2000 and 2002 primarily in late July to August. An industry survey conducted by the Groundfish Enterprise Allocation Council (GEAC) began in December 1997 on the Cape Beaver then switched to August/September from 1998 to 2001 and has continued bi-annually to 2009. The sister ship Cape Ballard conducted four of the nine surveys including the most recent two surveys in 2007 and 2009.

The two vessel-surveys likely have different catchabilities, mainly because of the different vessel classes, fishing gears and fishing protocols. Therefore, adjustment or calibration of catch rates was explored to derive an unbiased index of stock size, thus enabling the utilization of the full time span of each series. The objective is to adjust catches from one vessel so that they are comparable with catches from the other vessel. Commonly (e.g. Pelletier, 1998) the adjustment factor is based on paired-tow comparative fishing experiments where both vessels are fished close together so that differences in catches are primarily related to differences in vessel/gear catchability and not differences in stock densities fished by both vessels. A paired-tow fishing experiment was carried out between the Teleost and the Cape Beaver in August 2000 to provide adjustment factors. Note that it is thought that the Cape Beaver and Cape Ballard have the same catchability. The experiment yielded 24 sets that could be used to estimate and adjust for differences in catches.

The survey catch at a particular site is usually considered to provide an unbiased measure of a fraction ($q$) of the stock at that site. If $R_{vi}$ is the catch by vessel $v$ at survey site $i$, and if $N_i$ is the stock abundance at site $i$, then it is assumed that $E(R_{vi}) = q_v N_i$; that is, on average and in the long run $R_{vi}$ will equal $q_v N_i$, which is referred to as trawlable abundance. The fraction $q_v$ is often referred to as the survey catchability. It should be constant from site to site but for many reasons $q_v$ may vary for different vessels and gears. In a single vessel statistical survey, the sampling design-weighted (e.g. Särndal et al., 1992) average catch provides an unbiased estimate of the average trawlable abundance over the entire survey area; that is, average catch is an index of stock size. If two or more vessels are used in a survey and these vessels have different catchabilities then the design-weighted average survey catch is not an unbiased index of stock size. Some adjustment is required.

If the ratio of catchabilities were known then the catches from one vessel could be adjusted to approximately correspond to the catches that would have occurred if the other vessel was used. For the specific example of this paper, let $B$ index the Cape Beaver vessel and let $T$ index the Teleost. Define the Teleost relative efficiency as

$$\rho = \frac{q_T}{q_B}.$$

If $\rho$ were known then an unbiased index of average stock size based on survey catches from both vessels is

$$I = M^{-1}\left(\sum_{i\in s_T}\pi_i^{-1}R_{Ti} + \rho\sum_{i\in s_B}\pi_i^{-1}R_{Bi}\right), \quad \text{(A)}$$

where $s_T$ and $s_B$ are the sets of observations for each vessel, $\pi_i$ is the survey design-weight for the i'th observation, and $M$ is the total number of sites in the survey region. If stratified random sampling is used, with $m_h$ samples selected from $M_h$ total sites in strata $h$, then $\pi_{i\epsilon h} = m_h/M_h$. It can be shown that $E(I) = q_T\overline{N}$ where $\overline{N}$ is average stock size over the entire survey region. Hence, the $\rho$–adjusted $I$ in Equation (A) is an unbiased index of stock size.

In practice the survey analysis is length and age based. This involves straight-forward modifications of the above procedures.

Paired-tow catches from comparative fishing can be used to estimate $\rho$. The basic data obtained from such studies are the catches $R_{ij}$ obtained at the $i$th paired-tow station ($i$=1,…,$n$) by vessels $j = T$ or $j = B$. Pelletier (1998) reviewed several estimation procedures for this kind of data. Cadigan and Dowden (2010) advocated a conditional approach. If the stock size is the same at each tow station within a pair and if the catches are Poisson distributed then Cadigan and Dowden (2010) showed that the distribution of $R_{iT}$ given the total $R_i = R_{iT} + R_{iB}$ is Binomial with expectation $R_ip$. The probability $p$ that a fish captured at tow station $i$ was taken by the Teleost is a function of only $\rho$,

$$p = \frac{\rho}{1+\rho}.$$

Note that $log\{p/(1-p)\} = log(\rho) = \beta$ which can easily be estimated using a Binomial logistic regression model with only an intercept parameter (i.e. $\beta$). Confidence intervals for $\rho$ can be obtained by exponentiating intervals for $\beta$. Note that in practice there may be additional complications because of sub-sampling of catches, etc., and these will be accounted for in the **Methods** section.

Usually there will be some differences in stock densities encountered by both vessels in paired-tow comparative fishing. It is impossible to ensure that exactly the same number and size distribution of fish are available to both vessels. In addition, catchability ($q_v$) may vary randomly from site to site (see Cadigan and Dowden, 2010). This introduces additional variability in the survey catches that should be accounted for when estimating differences in catch rates. Cadigan and Dowden (2010) showed that bias in simple logistic regression estimates of $\rho$ can occur when these types of additional variability exist but are not accounted for. Cadigan and Dowden (2010) showed that mixed-effects logistic regression can provide unbiased estimates of $\rho$. In the next section we develop this model for the redfish comparative fishing data.

## METHODS

Comparative Fishing trials were conducted between the Cape Beaver and the DFO research vessel Teleost for about two days in August 2000. The Cape Beaver deployed an Engel 170 trawl with a 30 mm liner in the lower 7 m of the codend, and a standard tow of 30 minute duration on bottom at 3.5 knots (vessel speed). The Teleost deployed a Campelen 1800 survey trawl with a 12.7 mm liner in the lower 7 m of the codend, and a standard tow of 15 minutes on bottom at 3.0 knots (vessel speed). Both vessels utilized SCANMAR net monitoring systems to determine bottom contact and gear configuration (doors and/or wingspread). There were no special tow positions for these trials. The Teleost joined the Cape Beaver during its progress of conducting the industry survey in 3P4V and set up for comparative fishing at its stations. On level bottom vessels towed side by side at a distance no greater than 0.5 nautical miles but as close as was safely possible, with vessels alternating port and starboard positions relative to each other on consecutive tows. The two vessels towed on the same course, and net deployment was planned such that the midpoint of each vessels tow should correspond as closely as possible. On slopes of high relief, where side by side tows were not feasible due to depth differences, one vessel towed ahead of the other, alternating the lead vessel on a tow-by-tow basis. This was done in such a way that the end of the tow on the trailing vessel occurred at a position just before the start of the tow on the leading vessel. The same depth range for each paired tow was maintained as close as possible between vessels. In terms of biological sampling, redfish were measured for fork length to the nearest cm.

Methods are first derived for total (pooled) catch per set, and extended to catch-at-length data later in this section.

The basic assumption for paired-tow catches is $E(R_T) = \rho E(R_B)$. This is based on the assumption that exactly the same stock sizes are fished by both vessels. That is, if $E(R_T) = q_T N$, $E(R_B) = q_B N$, and $\rho = q_T/q_B$ then $E(R_T) = \rho q_B N = \rho E(R_B)$. If there is sub-sampling of catches or variations in the target tow distance then the model is

$$\frac{E(R_T)\times 0.8}{f_T d_T} = \frac{\rho E(R_B)\times 1.8}{f_B d_B} \tag{1}$$

where $f_j$ and $d_j$ are the sub-sampling fraction and tow distance of vessel $j = T,B$, 0.8 n. mi. is the target tow distance for the Teleost, and 1.8 n. mi. is the target tow distance of the Cape Beaver. Equation (1) can be re-written,

$$E(R_T) = \rho E(R_B)\times \exp(z),\;\; z = \log\left(\frac{1.8\, f_T d_T}{0.8\, f_B d_B}\right). \tag{2}$$

We refer to $z$ as the offset. If $f_T = f_B$, $d_T = 0.8$ and $d_B = 1.8$ then $z = 0$ and $E(R_T) = \rho E(R_B)$ in Equation (2). Note that we do not raise catches for sub-sampling or adjust catches for tow distance because this artificially changes samples sizes and causes problems for statistical inferences. Adjustments for sub-sampling and tow distances are incorporated in our analyses using offsets.

If the stock densities fished by both vessels at a paired-tow station are different then the model is

$$E(R_T \mid \delta) = \rho E(R_B \mid \delta) \times \exp(z + \delta), \tag{3}$$

where $\delta$ is the log ratio of stock densities. Cadigan and Dowden (2010) showed that it is reasonable to assume that $\delta$ has a normal distribution, $\delta \sim N(0, \sigma^2)$. If the conditional distributions $R_T|\delta$ and $R_B|\delta$ are both Poisson then the distribution of $R_T$ conditional on the total $R = R_T + R_B$ and $\delta$ has a Binomial distribution,

$$R_T \mid \delta, R \sim Bin(R, p),\ p = \frac{\rho \exp(z + \delta)}{1 + \rho \exp(z + \delta)} = \frac{\exp(\beta + z + \delta)}{1 + \exp(\beta + z + \delta)}, \tag{4}$$

where *log(ρ)* = *β*. Cadigan and Dowden (2010) presented arguments why this conditional distribution is appropriated for paired-tow data. Note that

$$\log\left(\frac{p}{1 - p}\right) = \beta + z + \delta. \tag{5}$$

Equations (4) and (5) define a Binomial Generalized Linear Mixed Model (GLMM) for which there are many software packages available for estimation. We used Version 9.1.3 of SAS/STAT (SAS, Cary, NC.) PROC NLMIXED software which fits nonlinear mixed models, including Binomial logistic regression, using marginal maximum likelihood estimation (MLE). Cadigan and Dowden (2010) showed that this approach provided unbiased estimation of $\rho$ and accurate confidence intervals (CIs); that is, in simulations the CIs provided by PROC NLIMXED covered the true value of $\rho$ with the intended probability.

PROC NLMIXED provides empirical Bayes estimates of the random effects (i.e. $\hat{\delta}$), which can be added to the fixed effect parameter estimate ($\hat{\beta}$) to estimate the i'th paired-tow specific relative efficiency, $\hat{\rho}_i = \exp(\hat{\beta} + \hat{\delta}_i)$. This is useful for diagnostic purposes; however, for calibrating other independent survey catches the estimate of relative efficiency that is appropriate to use is $\hat{\rho} = \exp(\hat{\beta})$.

It is straight-forward to modify Equation (4) to examine for length (*l*) effects with survey catches-at-length. The Binomial conditional distribution for the Teleost catch-at-length from the *l*'th paried tow ($R_{Til}$) is

$$R_{Til} \mid (\delta, R_{il} = R_{Til} + R_{Bil}) \sim Bin(R_{il}, p_{il}),\ p_{il} = \frac{\exp(z_i + \beta_{il})}{1 + \exp(z_i + \beta_{il})}. \tag{6}$$

Two simple models were used for $\beta_l$ = $log(\rho_l)$; either a linear or quadratic function of standardized length,

$$l_{std} = \frac{l - \bar{l}}{s_l}, \tag{7}$$

where $\bar{l}$ was the mean length of total catch from both vessels and for all sets, and $s_l$ was the standard deviation of total catch. Random set effects were included for both slopes and intercepts. The GLMM linear model was

$$\beta_{il} = \beta_0 + \delta_{0i} + (\beta_1 + \delta_{1i}) l_{std}, \tag{8}$$

where $\delta_{01},\ldots,\delta_{0n}$ are independent $N(0,\sigma_0^2)$ random effects and $\delta_{11},\ldots,\delta_{1n}$ are independent $N(0,\sigma_1^2)$ random effects. This model involves four parameters to estimate ($\beta_0, \beta_1, \sigma_0^2, \sigma_1^2$). Cadigan and Dowden (2010) showed how this model could account for random within-pair variations in the length distributions of fish encountered by each vessel. The GLMM quadratic model was

$$\beta_{il} = \beta_0 + \delta_{0i} + (\beta_1 + \delta_{1i}) l_{std} + (\beta_2 + \delta_2) l_{std}^2, \tag{9}$$

where $\delta_{ij}$ are independent $N(0,\sigma_j^2)$, $i = 1,\ldots,n$, and $j = 1,2,3$.

Models are compared via the likelihood. In this paper the term fit refers to -2×loglikelihood.

## RESULTS

Total catch per set and other sampling information is presented in Table 1 and Figure 1 in the **Appendix**. The Cape Beaver caught more redfish than the Teleost in 15 of 24 tows. The Cape Beaver towed slightly more than twice the distance of the Teleost, so all other things being equal one would expect the Cape Beaver to catch more redfish. However, the Teleost caught more small fish than the Cape Beaver (Figure 1).

It is useful to first examine for vessel effects based on total (i.e. pooled) catch per set. This can help when interpreting length-based results.

## POOLED MODELS

The GLMM estimate of $\beta$ based on Equation (4) was -0.1430, with a 95% CI (-0.3677, 0.08164). This indicates that the relative efficiency of the Teleost compared to the Cape Beaver was $\rho$ = exp(-0.143) ≅ 87%, but $\rho$ was not significantly different from one. The random effects estimate of tow specific $\rho$'s (Figure 2) varied about one. Note that the model assigned large effects to sets 1 and 3. The estimate of $\beta$ with sets 1 and 3 removed was -0.1599, with a 95% CI (-0.2959, -0.02397) which still suggests that $\rho$ was not significantly different from one. The largest change was in the estimate of $\sigma^2$; it decreased from 0.265 with sets 1 and 3 to 0.076 when these sets were removed. However, estimates of $\beta$ seemed reasonably robust to these anomalous sets.

The estimate of $\beta$ based on the simple Binomial Generalized Linear Model (GLIM) based on Equation (2), with no random set effects, was -0.1667, with a 95% CI (-0.2183, -0.1150). This is similar to the GLMM estimate, but the GLIM results indicate that $\rho$ was significantly different from one. However, the GLMM fit was substantially better (> 400 chi-square units) than the GLIM fit suggesting that there was significant Binomial over-dispersion in the data. Hence, the GLIM CIs are probably not accurate.

## LENGTH-BASED MODELS

The linear model GLMM indicated the slope was significant but the intercept was not (Table 2). This is consistent with the results from the pooled model and also Figure 1. The model fit (1693.6; Table 2) was significantly better than the model fit with no length effects (1804.1). For most lengths $\rho_l$ was significantly different from one (Figure 3). These estimates indicate that the Teleost was more efficient at catching small fish (<30 cm) than the Cape Beaver, but it was the reverse case for large fish. The results in the shaded regions of Figure 3 are extrapolations outside of the range of observed lengths, and are therefore speculative. The raised length distributions of catches from all 24 sets (Figure4) agree fairly well, although there are some discrepancies at smaller sizes. This is also evident in the model residuals (Figure 5), but this figure suggests that the discrepancies may be minor given the variability in the data.

Nonetheless a quadratic GLMM was fit in an attempt to improve the conversion. The fit was significantly better compared to the linear GLMM (Table 2). The difference in fit was 8.2 which is significant; that is, $\Pr(\chi_2^2 > 8.2) = 0.01657$. Within the range of most sampled lengths the estimates $\rho_l$ and CIs (Figure 6) for were qualitatively similar to the results in Figure 3. The lowest $\rho_l$ in Figure 6 was 62% at 42.3 cm. The linear GLMM $\rho_l$ was 48% at this same length. The quadratic GLMM extrapolations are unreliable because they have very wide confidence intervals. This model produced a closer conversion between the Teleost and Cape Beaver raised catches (Figure 7) and the slight positive trend in residuals at smaller lengths in Figure 5 was removed in Figure 8.

# DISCUSSION AND CONCLUSIONS

The recommended conversion formula based on the quadratic calibration GLMM is to raise Cape Ballard catches-at-length by multiplying with the factor:

$$\rho_l = \exp\left(-0.2173 - 1.0018 \times \mathrm{l}_{\mathrm{std}} + 0.9699 \times \mathrm{l}_{\mathrm{std}}^2\right),\ l_{std} = \frac{l - 32.5474}{19.5801}. \quad (9)$$

Note that this is in addition to any data adjustments used to deal with subsampling of catches or variable tow distances. We do not recommend Equation (9) for lengths less than 13 cm or lengths greater than 44 cm which were the 95% quantiles of the observed lengths in the 24 paired-tow comparative fishing data. Calibration of such survey catches-at-length is speculative but if it must be done then the following equation should be used:

$$\rho_l = \exp(-0.1954 - 1.0785 \times l_{std}),\ l_{std} = \frac{l - 32.5474}{19.5801}. \qquad (10)$$

If the stock assessment is sensitive to survey catches less than 13 cm or greater than 44cm then it is recommended to do more comparative paired-tow fishing in areas with such small or large redfish.

It is also recommended to derive and compare indices with and without calibration of vessels to assess how sensitive stock size indices are to calibration coefficients. If the sensitivity is large then more careful regression modeling of the data will likely be required. Fryer et al. (2003) used spline methods to model relative efficiency nonparametrically, and this could be a useful approach for the redfish data. Also, if the sensitivity is large then additional comparative fishing data will be required.

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# APPENDIX

## FIGURES

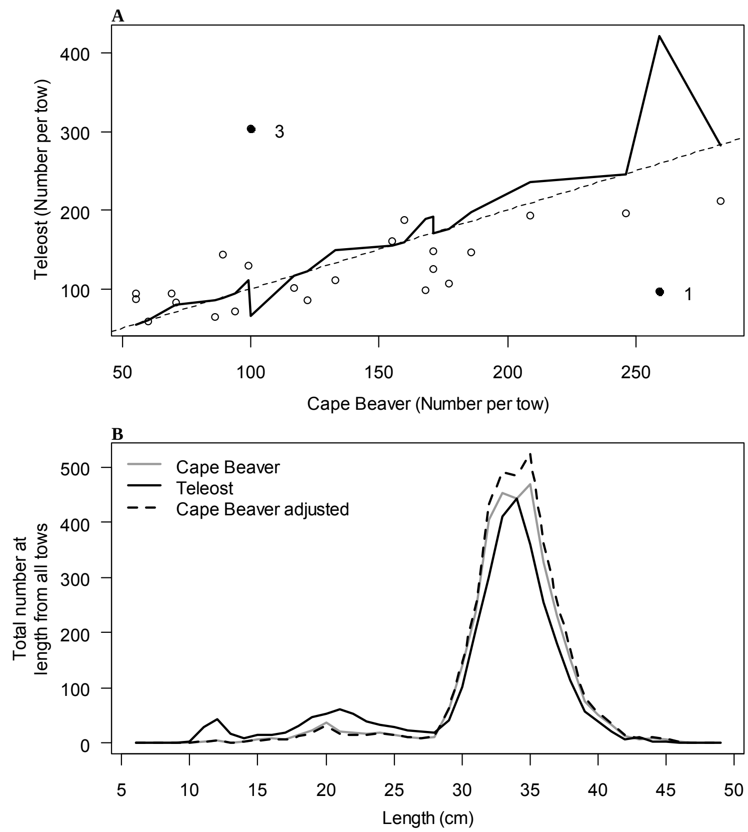


*Figure 1: Top panel A: Teleost catch versus Cape Beaver catch. Both catches are actual and not adjusted for sub-sampling or tow distances. Black circles are potential outliers, with the set number indicated to the right. The solid line shows the offsets, exp(z) (see Equation 2). The dashed line is a 1:1 reference line. Bottom panel B: Total catch at length (over all sets). The Cape Beaver adjusted catch was scaled by the offset before summing over sets. Differences in Teleost catches and Cape Beaver adjusted catches reflect differences in relative efficiency ($\rho$).*

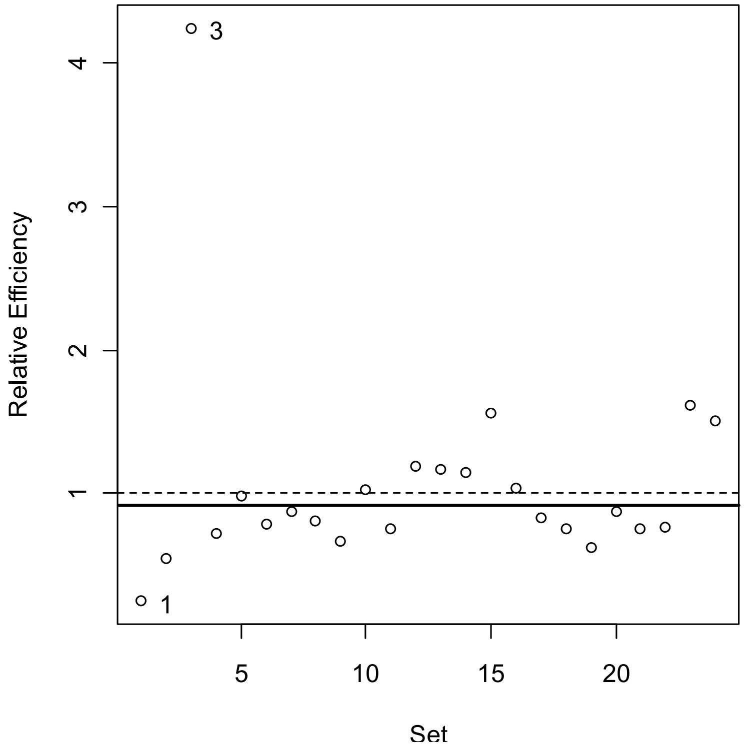


*Figure 2: Random effect estimates of relative efficiency (ρ) for each set. Sets 1 and 3 are labeled. The solid vertical line indicates the fixed (i.e. mean) estimate of ρ for all sets. A dashed reference line at one is shown.*

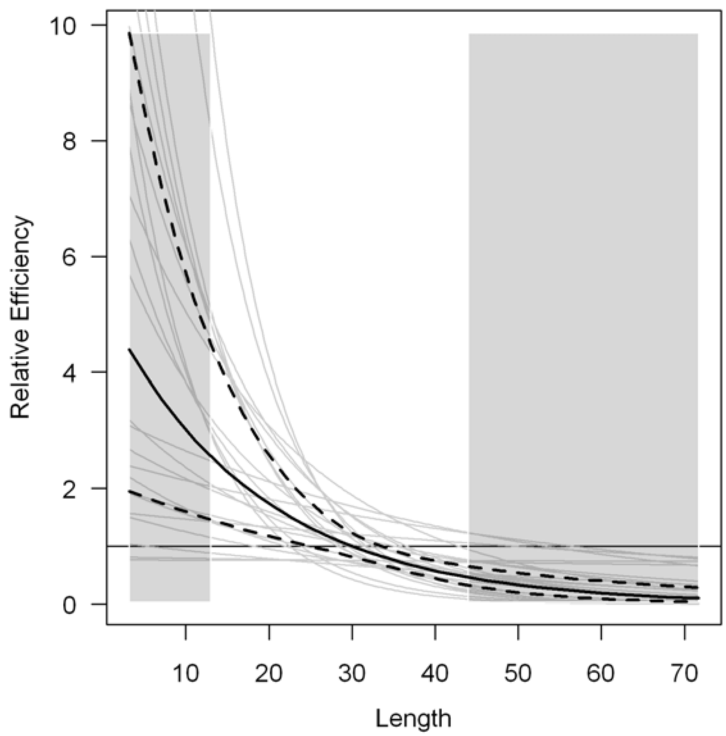


*Figure 3: Length-based estimates of relative efficiency ($\rho_l$; solid curve) with 95% confidence intervals (dashed lines), based on the linear GLMM. A horizontal reference line at one is shown. The grey lines show the random effect estimates of tow-specific $\rho_l$ 's. The white area indicates the 95% range of lengths; that is, only 2.5% of observed lengths were in the left-hand shaded region, and 2.5% of lengths were in the right-hand shaded region.*

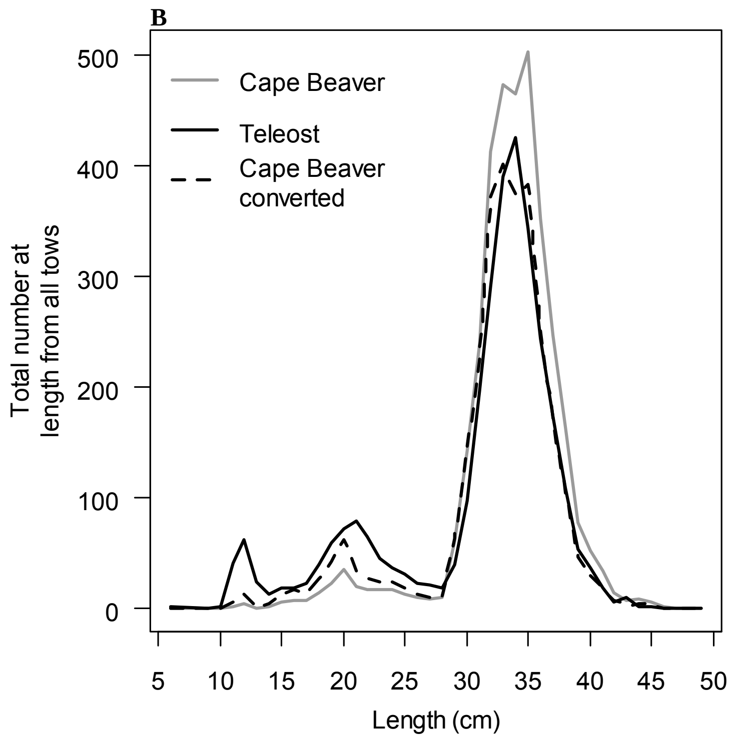


*Figure 4: Catches-at-length from all paired-tows that were raised for sub-sampling and adjusted for tow distance. The Cape Beaver converted catches are also adjusted by the linear GLMM estimates of relative efficiency.*

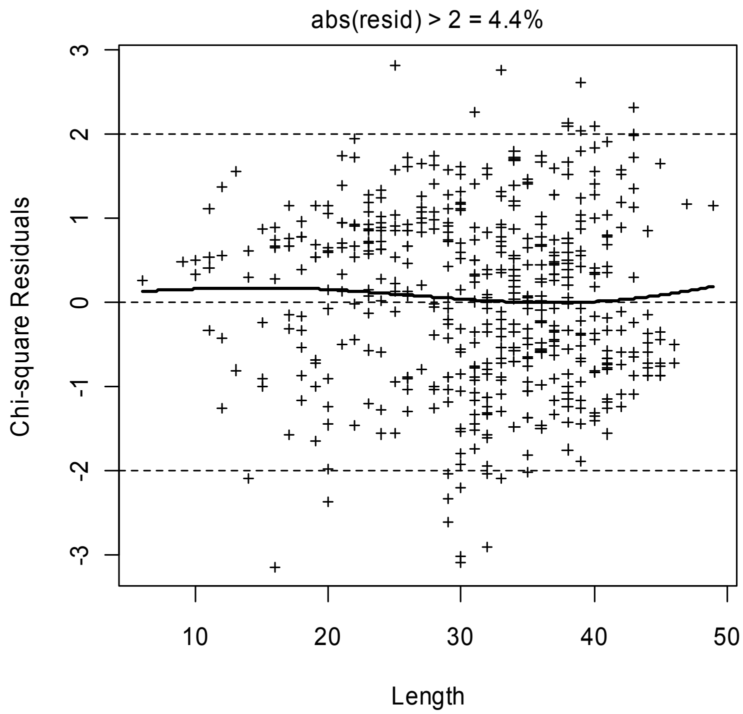


*Figure 5: Chi-square standardized residuals versus length, from the linear GLMM. The solid line shows the trend in residuals, estimated from the R function loess(). Dashed reference lines at 2, 0, and -2 are shown. The percent of residuals that exceed ±2 is shown at the top.*

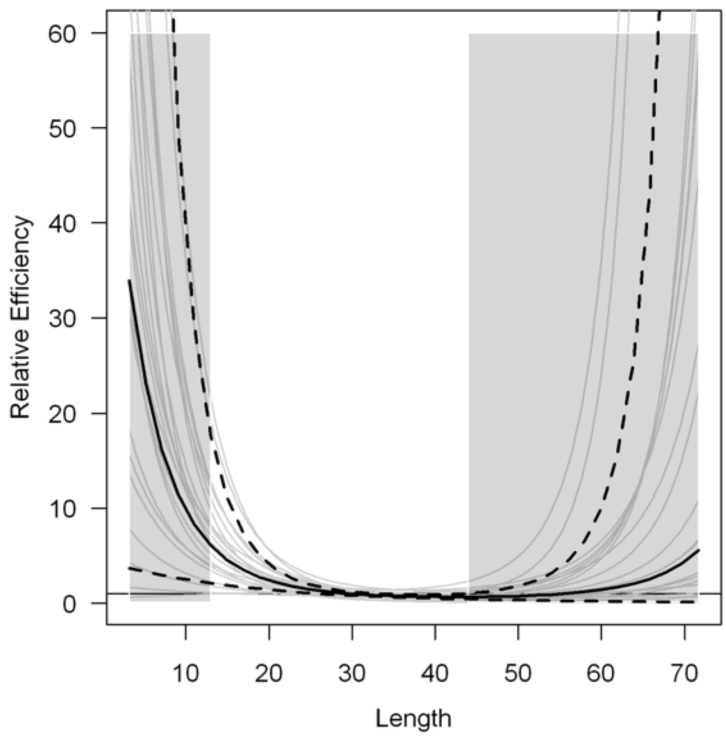


*Figure 6: Length-based estimates of relative efficiency ($\rho_l$; solid curve) with 95% confidence intervals (dashed lines), based on the quadratic GLMM. A horizontal reference line at one is shown. The grey lines show the random effect estimates of tow-specific $\rho_l$ 's. The white area indicates the 95% range of lengths; that is, only 2.5% of observed lengths were in the left-hand shaded region, and 2.5% of lengths were in the right-hand shaded region.*

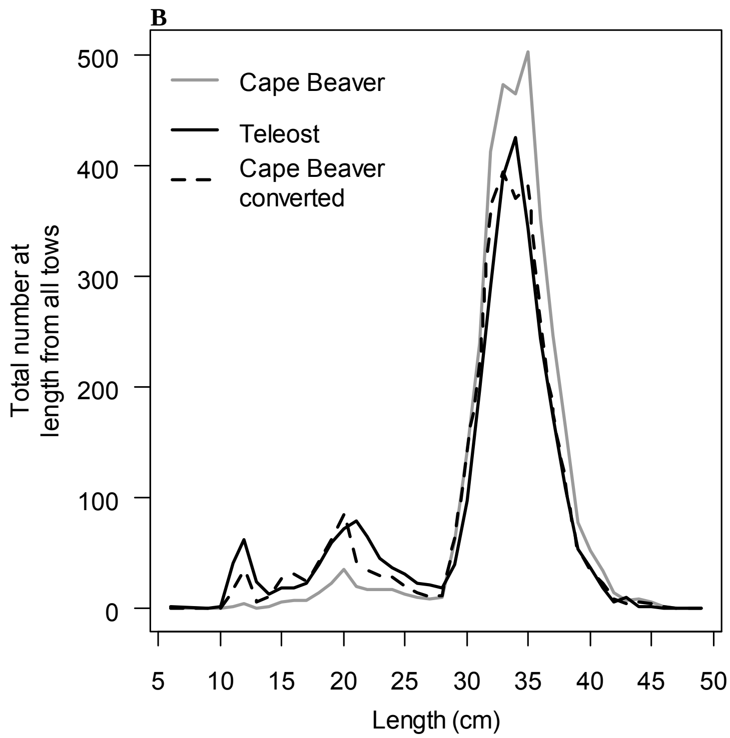


*Figure 7: Catches-at-length from all paired-tows that were raised for sub-sampling and adjusted for tow distance. The Cape Beaver converted catches are also adjusted by the quadratic GLMM estimates of relative efficiency.*

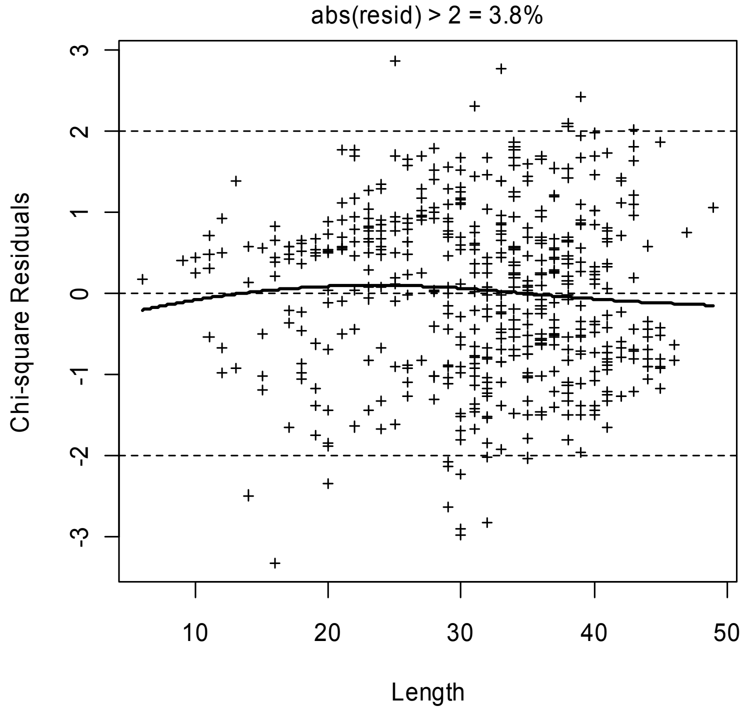


*Figure 8: Chi-square standardized residuals versus length, from the quadratic GLMM. The solid line shows the trend in residuals, estimated from the R function loess(). Dashed reference lines at 2, 0, and -2 are shown. The percent of residuals that exceed ±2 is shown at the top.*

## TABLES

*Table 1. Cape Beaver vs Teleost Comparative Fishing Total Catch*

| SET | BEV tow dist | TEL tow dist | BEV subsamp | TEL subsamp | BEV n | TEL n | total n |
|---|---|---|---|---|---|---|---|
| 1 | 17 | 8 | 0.61520 | 1.00000 | 259 | 97 | 356 |
| 2 | 17 | 9 | 1.00000 | 1.00000 | 168 | 99 | 267 |
| 3 | 17 | 8 | 1.00000 | 0.66157 | 100 | 303 | 403 |
| 4 | 17 | 8 | 1.00000 | 1.00000 | 122 | 86 | 208 |
| 5 | 17 | 8 | 1.00000 | 1.00000 | 60 | 59 | 119 |
| 6 | 17 | 8 | 1.00000 | 1.00000 | 94 | 72 | 166 |
| 7 | 17 | 8 | 1.00000 | 1.00000 | 117 | 102 | 219 |
| 8 | 17 | 8 | 1.00000 | 1.00000 | 246 | 196 | 442 |
| 9 | 17 | 9 | 1.00000 | 1.00000 | 171 | 125 | 296 |
| 10 | 17 | 9 | 1.00000 | 1.00000 | 71 | 83 | 154 |
| 11 | 17 | 9 | 1.00000 | 1.00000 | 133 | 111 | 244 |
| 12 | 17 | 9 | 1.00000 | 1.00000 | 69 | 94 | 163 |
| 13 | 17 | 8 | 1.00000 | 1.00000 | 160 | 188 | 348 |
| 14 | 17 | 9 | 1.00000 | 1.00000 | 99 | 129 | 228 |
| 15 | 17 | 8 | 1.00000 | 1.00000 | 89 | 144 | 233 |
| 16 | 17 | 8 | 1.00000 | 1.00000 | 155 | 161 | 316 |
| 17 | 17 | 9 | 1.00000 | 1.00000 | 209 | 194 | 403 |
| 18 | 18 | 9 | 1.00000 | 1.00000 | 186 | 146 | 332 |
| 19 | 17 | 8 | 1.00000 | 1.00000 | 177 | 107 | 284 |
| 20 | 17 | 8 | 1.00000 | 1.00000 | 171 | 148 | 319 |
| 21 | 17 | 8 | 1.00000 | 1.00000 | 283 | 212 | 495 |
| 22 | 17 | 8 | 1.00000 | 1.00000 | 86 | 64 | 150 |
| 23 | 17 | 8 | 1.00000 | 1.00000 | 55 | 94 | 149 |
| 24 | 17 | 8 | 1.00000 | 1.00000 | 55 | 87 | 142 |

*Table 2. Linear GLMM results.*

Fit Statistics

| | |
|---|---|
| -2 Log Likelihood | 1693.6 |
| AIC (smaller is better) | 1701.6 |
| AICC (smaller is better) | 1701.7 |
| BIC (smaller is better) | 1706.3 |

Parameter Estimates

| Parameter | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper | Gradient |
|---|---|---|---|---|---|---|---|---|---|
| Intercept | -0.1954 | 0.09013 | 22 | -2.17 | 0.0413 | 0.05 | -0.3823 | -0.00846 | -0.00001 |
| slope | -1.0785 | 0.2495 | 22 | -4.32 | 0.0003 | 0.05 | -1.5959 | -0.5611 | 0.000223 |
| vint | 0.1692 | 0.05683 | 22 | 2.98 | 0.0070 | 0.05 | 0.05134 | 0.2871 | -0.00108 |
| vslope | 0.8947 | 0.4227 | 22 | 2.12 | 0.0458 | 0.05 | 0.01815 | 1.7713 | -0.00022 |

*Table 3. Quadratic GLMM results*.

Fit Statistics

| | |
|---|---|
| -2 Log Likelihood | 1685.4 |
| AIC (smaller is better) | 1697.4 |
| AICC (smaller is better) | 1697.6 |
| BIC (smaller is better) | 1704.5 |

Parameter Estimates

| Parameter | Estimate | Standard Error | DF | t Value | Pr > \|t\| | Alpha | Lower | Upper | Gradient |
|---|---|---|---|---|---|---|---|---|---|
| Intercept | -0.2173 | 0.09038 | 21 | -2.40 | 0.0255 | 0.05 | -0.4052 | -0.02930 | -0.00209 |
| slope1 | -1.0018 | 0.2459 | 21 | -4.07 | 0.0005 | 0.05 | -1.5132 | -0.4903 | 7.341E-6 |
| slope2 | 0.9699 | 0.4481 | 21 | 2.16 | 0.0421 | 0.05 | 0.03804 | 1.9017 | 0.000542 |
| vint | 0.1681 | 0.05687 | 21 | 2.96 | 0.0076 | 0.05 | 0.04981 | 0.2863 | 0.000615 |
| vslope1 | 0.7383 | 0.4338 | 21 | 1.70 | 0.1035 | 0.05 | -0.1637 | 1.6404 | -0.00027 |
| vslope2 | 1.1180 | 1.3302 | 21 | 0.84 | 0.4101 | 0.05 | -1.6484 | 3.8844 | -0.00009 |