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Assessment and Management advice for Pacific hake in U.S. and Canadian waters in 2009

Évaluation du merlu du Pacifique dans les eaux américaines et canadiennes en 2009 et avis sur sa gestion

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Abstract

This is an alternative stock assessment model for Pacific hake (*Merluccius productus*) in Canadian and U.S. Waters for 2009. The assessment model is parameterized in terms of management variables maximum sustainable yield (C^*) and the fishing mortality rate (F^*) that achieves C^*, and provides catch advice in the form of a decision table based on the risk of exceeding fishing mortality rate and spawning biomass targets. The stock assessment model was fit to historical time series information on relative abundance (from a fisheries independent acoustic survey) and age-composition information from fisheries independent and dependent sources. The likelihood components in the statistical fitting criterion were each weighted using the maximum likelihood estimates of the variance conditional on a prior distribution for the proportion of the total variance associated with observation errors. Parameters were estimated using both maximum likelihood and Bayesian approaches. The decision table is based on a Bayesian interpretation of the data. Sensitivity analysis was conducted using alternative prior distributions for key model parameters; catch advice decreased by as much as 28% if a higher proportion of observation error was assumed. Catch advice was very sensitive to alternative prior distributions for the instantaneous natural mortality rate. There was also some retrospective bias in estimates of current spawning stock biomass, fishing mortality rate and the relative strength of the 1999 year class, but virtually no bias in the estimates of unfished spawning stock biomass, survey catchability coefficients and productivity parameters. As each subsequent year of data is removed in the retrospective analysis, the strength of the 1999 year class and spawning stock biomass was reduced, while estimates of fishing mortality increased (i.e., err on the conservative side).

Removals for the 2008 coast wide fishery were estimated at 322,017 metric tons, which is roughly 88.2% of the permissible catch in 2008. The fishing season was protracted in 2008; fishing operations in Canada terminated at the end of December 2008. Median estimates of spawning stock depletion at the start of the 2009 is estimated at 34.7% of unfished states with a 90% credible interval of 12.3–69.4%. Median estimates of instantaneous fishing mortality in 2008 was 0.735 (nearly double the estimate of F^*) with a 90% credible interval of 0.303–3.265. Catch advice for the 2009 fishery based on median parameter values and the Pacific Fisheries Management Councils 40/10 harvest rule is 174,000 metric tons, and the 25th and 75th percentiles are 72,000 and 308,000 metric tons respectively. Catch advice based on the 50th percentiles were 91,000 and 256,000 metric tons.

Résumé

Le prsent modèle est un modle de rechange d'évaluation des stocks de merlus du Pacifique (*Merluccius productus*) dans les eaux canadiennes et américaines pour 2009. Lemodèle d'évaluation est paramétré en fonction du rendement maximale soutenu selon les variables de gestion (C^*) et du taux de mortalité par pêche (F^*) qui atteint C^*et qui contient un avis concernant les prises sous la forme d'une table de décision fondée sur le risque de dépassement du taux de mortalité par pêche et des objectifs en matière de biomasse reproductrice. Le modèle d'évaluation des stocks a été adapté à l'information historique de la série chronologique sur l'abondance relative (à partir d'un relevé acoustique indépendant des pches) et l'information sur la composition en âge des pêches de sources indépendantes et dépendantes. Chacune des composantes de probabilité des critères de rajustement statistique a été mesurée à l'aide des estimations du maximum de probabilité de la variance conditionnelle à une distribution a priori pour la proportion de la variance totale associée aux erreurs d'observation. Les paramètres ont été évalués en utilisant à la fois le maximum de probabilité et les méthodes bayésiennes. La table de décision est fondée sur une interprétation bayésienne des données. Une analyse de sensibilité a été menée en utilisant d'autres distributions a priori pour les paramètres clés du modèle ; les avis concernant les prises ont diminué à un niveau aussi bas que 28%, si on suppose une proportion plus élevée d'erreurs d'observation. Les avis concernant les prises étaient très sensibles aux autres distributions a priori pour le taux de mortalité naturelle instantané. Les estimations de la biomasse du stock reproducteur actuel, du taux de mortalité par pêche et de la force relative de la classe d'âge 1999 montraient un certain biais rétrospectif, mais pratiquement aucun biais dans les estimations de la biomasse du stock reproducteur non exploité, les coefficients du potentiel de capture des relevés et les paramètres de la productivité. Au fur et à mesure que chaque année de données ultérieure est éliminée de l'analyse rétrospective, la force de la classe d'âge 1999 et la biomasse du stock reproducteur ont diminué, tandis que les estimations de mortalité par pêche ont augmenté (c.-à-d. quelles penchent du côté conservateur).

Les prélèvements pour la pêche de 2008 d'un bout à l'autre de la côte étaient estimés à 322,017 tonnes métriques, ce qui représente environ 88.2% des prises permises en 2008. La saison de pêche a été prolongée en 2008 ; les opérations de pêches au Canada se sont terminées à la fin de décembre2008. L'estimation médiane de l'épuisement du stock reproducteur au début de 2009 était évalue 34.7% des états non exploités avec un intervalle de confiance de 90% : 12.3 - 69.4%. L'estimation médiane de la mortalité instantanée par pêche en 2008 était de 0.735 (près du double de l'estimation de F^*) avec un intervalle de confiance de 90% : 0.303 - 3.265. L'avis concernant les prises pour la pêche de 2009, fondé sur des valeurs paramétriques médianes, et la règle d'exploitation 40/10 du Pacific Fisheries Management Council sont de 174,000 tonnes métriques et les 25^e et 75^{e}percentiles sont de 72,000 et 308,000 tonnes métriques respectivement. L'avis concernant les prises sur le 50^e percentile d'avoir excédé F^*était également de 174,000 tonnes métriques et les 25^e et 75^e percentiles étaient de 91,000 et de 256,000 tonnes métriques.

Executive summary

This is an alternative assessment model (TINSS) that directly estimates the management variables C^*(the maximum sustainable yield) and F^*(the fishing mortality rate that produces C^*). The model was implemented in the AD Model Builder software and is based on the methods in Martell et al. (2008). The structural assumptions are similar to that of SS: a Beverton-Holt stock recruitment relationship is assumed, it is assumed that the population was at an unfished state in 1966, and the model is conditioned on historical catch information. The data for TINSS was extracted from the input files use for Stock Synthesis and the catch and catch-age information from U.S. and Canadian fisheries are aggregated into a single fishery and the selectivity curve for this aggregate fishery is assumed to be asymptotic. I also assume an asymptotic selectivity curve for the fisheries independent acoustic trawl survey. In contrast to previous assessments, the assessment attempts to reduce the amount of prior information on key population parameters and subjective weighting of data that ultimately defines the catch advice. Model parameters were estimated using both maximum likelihood methods and Bayesian methods. Catch advice is based on the Bayesian view of the model parameters.

There was a substantial change in the likelihood kernel used for the age-composition data in comparison to the assumed model structure in last years assessment prepared by Martell (2008). In last years assessment, a robust normal approximation to the multinomial distribution was used as the likelihood for the age composition data. This is the same likelihood function that is used in Multifan CL (see Fournier et al., 1990; Martell et al., 2008). In this years assessment I adopted a more objective approach and used the multivariate logistic kernel (see Richards and Schnute, 1998) where the conditional maximum likelihood estimate of the variance was used to weight the age-composition data in both the commercial fishery samples and the acoustic survey samples.

In summary the maximum likelihood estimate of spawning stock depletion (male and female) in 2009 is 25.9% and recent fishing mortality rates exceed median estimates of F^*(Table 1, median F^*=0.36). Median estimates of spawning stock depletion at the start of 2009 is estimated at 34.7% and the 5% and 95% quantiles for the spawning stock depletion is 12.3% and 69.4%, respectively. Estimates of the female spawning stock biomass at the start of 2009 range from 0.213 to 1.687 million mt with a median estimate of 0.721 million mt. Recent trends in fishing mortality rates have been increasing owing to the disappearance of the 1999 year class and above average landings in the commercial fisheries. Estimates of fishing mortality in 2008 range from 0.303 to 3.265 with a median value of 0.735.

Catch advice is presented in a decision table format where measures of risk include the probability of exceeding the target fishing morality rate (F^*), probability of a decline in the 2010 spawning stock biomass and the probability of the spawning stock biomass falling below SB_{MSY}, 40% and 25% of the unfished levels (Table 2). Arbitrary levels of probability we defined for risk averse (P=0.25), risk neutral (P=0.5) and risk prone (P=0.75). Based on the risk neutral policy of not exceeding the fishing mortality, a recommended ABC for the 2009 Pacific hake fishery is 174,000 mt; the risk averse policy calls for an ABC of 91,000 mt and the risk prone policy is 256,000 mt.

Risk neutral (defined here as the median estimate) catch advice for 2009 fishery was based on the Pacific Fisheries Management Councils 40/10 adjustment rule is 174,000 mt, and the 25$^{\text{th}}$ and 75$^{\text{th}}$ percentiles are 72,000 mt and 308,000 mt, respectively.

In summary, a 2009 catch greater than 100,000 mt resulted in a fairly significant probability of overfishing (P $\geq$ 0.25), further declines in spawning stock biomass in 2009 (P $\geq$ 0.5), a significant

Table 1: Median estimate and 5% and 95% credible intervals for the female spawning stock biomass (million mt), spawning stock depletion, and fishing mortality rates in 1966 and recent years. These estimates are based on sampling the joint posterior distribution using MCMC, chain length 1,000,000 with systematic samples drawn every 200 iterations.

	Spawning stock biomass			**Depletion**			**Fishing Mortality**		
Year	median	5%	95%	median	5%	95%	median	5%	95%
1966	2.019	1.527	2.814	1.000	1.000	1.000	0.062	0.044	0.083
2004	1.792	1.305	2.604	0.884	0.662	1.176	0.168	0.104	0.277
2005	1.277	0.913	1.891	0.630	0.475	0.845	0.232	0.147	0.341
2006	0.969	0.664	1.564	0.481	0.353	0.674	0.350	0.206	0.547
2007	0.794	0.470	1.444	0.394	0.265	0.587	0.459	0.238	0.874
2008	0.764	0.348	1.572	0.379	0.211	0.644	0.735	0.303	3.265
2009	0.721	0.213	1.687	0.347	0.123	0.694			

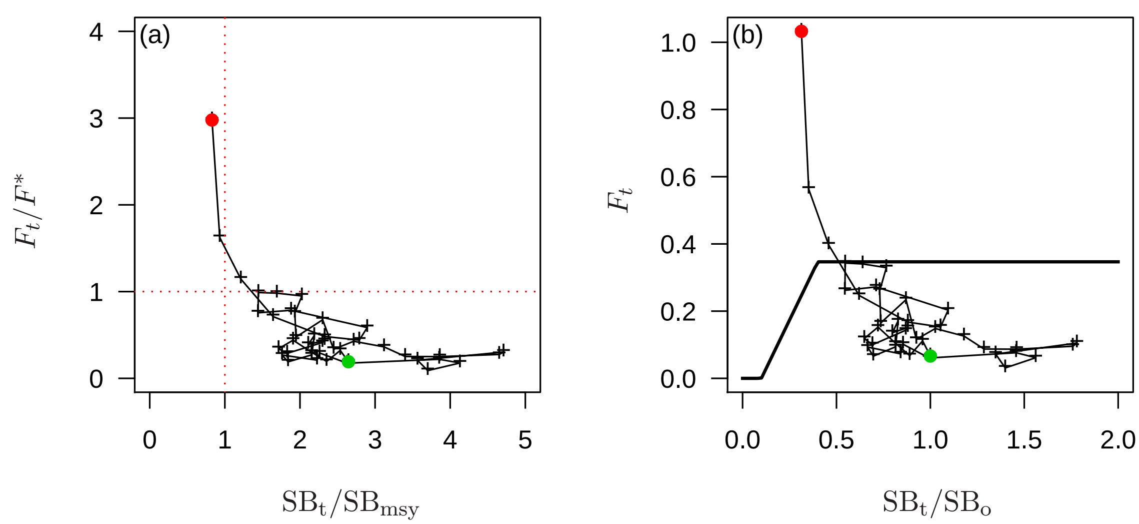

Figure 1: Maximum likelihood estimates of the spawning stock biomass relative to the unfished spawning stock biomass versus the fishing mortality rate relative to F^*(a). In panel (b) the inferred harvest control rule (thick line) and the spawning stock biomass depletion levels versus maximum likelihood estimates of historical fishing mortality rates. Green circles indicate the start of the series (1966) and red indicates the end of the series (2008).

Table 2: Decision table for catch advice. The risk level represents the probability of exceeding a specified management target for a given ABC option. The interpretation of this table is as follows; if the management goal is not to exceed the target fishing mortality rate of F^*in 2009 with a 0.25 probability, then the ABC option should be set at 0.091 million mt or less. If the management target is prevent further decline in spawning stock biomass with a 0.5 probability then the ABC should be set at 0.113 million mt or less.

Risk level	$F_{09} \geq F^*$	$SB_{10} \leq SB_{09}$	$SB_{10} \leq SB_{MSY}$	$SB_{10} \leq SB_{40}$	$SB_{10} \leq SB_{25}$
0.25	0.091	0.000	0.000	0.000	0.110
0.50	0.174	0.113	0.035	0.000	0.443
0.75	0.256	0.259	0.457	0.409	0.776

probability of reducing the spawning stock biomass below $\mathrm{SB}_{\mathrm{MSY}}$ (P $\geq$ 0.55), and there is roughly a P $\geq$ 0.25 that the spawning stock biomass will fall below SB_{25}. With no fishery in 2009, there is a P $\leq$ 0.18 chance that the spawning stock biomass will fall below SB_{25}.

There is some retrospective bias in the estimates of spawning stock biomass, fishing mortality and age-1 recruits; this bias is in a safe direction where removal of the most recent data tends to reduce estimates of spawning stock biomass and increase estimates of fishing mortality. It is likely that the time invariant selectivity assumption is the source of this retrospective bias.

Management advice is fairly insensitive to assumed prior distributions for the variance partitioning parameter ρ that partitions the total error into observation and process error components. The advice was also fairly insensitive to the ad hoc prior distributions for $F^*$and C^*; a 20% change in the prior distribution resulted in catch advice that was plus or minus 15,000-20,000 tons. Management advice was hypersensitive to the assumed prior distribution for the instantaneous natural mortality rate. The global scaling of the assessment is positively correlated with the assumed value of M. The age-composition data along with structural assumptions (i.e., asymptotic selectivity) imply that M is greater than the previously assumed value of 0.23.

1 Introduction

Previous assessments of Pacific hake (*Merluccius productus*) have been troubled by the lack of contrast in the acoustic survey data that allow for the estimation of the unfished biomass (B_o) and the steepness of the stock recruitment relationship. To cope with the lack of information in the acoustic survey data, the assessments have proceeded by fixing the value (h) of steepness for stock recruitment relationship and presented two alternative scenarios for the acoustic survey scaling parameter q. Fixing these parameters is necessary due to the lack of contrast in the acoustic survey data; however, it also results in a gross under-estimation of the uncertainty in model results and estimates of the reference points used in the determination of Acceptable Biological Catch (ABC).

At present, uncertainty in parameters that define the harvest control rule is only represented by the uncertainty associated with size selectivity parameters in the various commercial fisheries as well as the acoustic survey itself. The parameters that define the underlying production function include the instantaneous natural mortality rate (M), the steepness of the stock recruitment relationship (h) and a measure of population scale (usually the unfished spawning stock size or B_o). In previous assessments, h and M are fixed, and the population scale is determined by the combined effects of selectivity in the acoustic survey and the survey scaler q (which is fixed at two different values). For example for a given value of q, estimates of the unfished biomass increase as the acoustic survey selectivity becomes more dome-shaped, and vice-versa.

Historically, management advice has been based on the application of the 40-10 harvest control rule. Three critical pieces of information were required to apply the harvest control rule: 1) an estimate of F_{MSY} and B_{MSY} which is approximated by F_{40} and B_{40}, respectively, 2) an estimate of the current level of depletion in the spawning stock biomass, and 3) a biomass forecast based on historical recruitment or the underlying stock recruitment relationship. Accurate estimates of F_{MSY} require accurate estimates of M and h, which are difficult to obtain in many (if not all) fisheries assessments; therefore a proxy F_{40} (which is the fishing mortality rate that reduces the spawning potential ratio to 40% of its unfished state) was used to approximate F_{MSY}. This approximation has been shown to achieve nearly 80% of the maximum yield over a wide range of stock recruitment parameters with a variety of stock recruitment models (Clark, 1991, 2002). Similarly, B_o was also difficult to estimate; therefore, the spawning potential ratio (SPR) is used as a measure of mortality rates. The current level of depletion is determined by comparing the ratio of present day spawning biomass to the estimated unfished spawning biomass. Finally, the forecast was based on current levels of depletion and estimates of h.

There are a few unresolved problems and inconsistencies in the input data for Stock Synthesis II (SS2) or any other age-structured model. First there is a large inconsistency between information in the age-compositions and the acoustic survey biomass index. The age compositions suggest a buildup of biomass through the late 1980s owing to the strong 1980 and 1984 cohorts, yet the biomass index is relatively flat during this time period. Furthermore, (Helser et al., 2008) documented a clear contradiction in the age-composition information between the US, Canadian and Fisheries independent surveys. Each of these independent data sets contradict each other in terms of information content with respect to estimated model parameters in SS2.

In contrast to previous assessments for Pacific hake, this assessment attempts to reduce the amount of prior information that is used on key population parameters that ultimately defines the harvest control rule and catch advice. To do this, I have implemented a age-structured model that is parameterized from a management oriented perspective, where the leading parameters are $C^*$and F^*. The population model is structurally similar to that of SS2, where I assume that the stock is at its unfished state in 1966, recruitment follows a Beverton-Holt stock-recruitment relationship, and the model is conditioned on the historical catch information. The fundamental differences between the two approaches is that I make no prior assumptions about the survey q, and no direct prior assumptions about the steepness of the stock recruitment relationship. The model parameterization is such that there is an implied prior for the steepness of the stock recruitment function; however, this prior is very diffuse in comparison to 2008 SS2 implementation. Another fundamental difference is the treatment of the data. In this application, catch data from U.S. and Canadian operations are aggregated into a single fishery, and it is assumed that selectivity curve for the aggregate fishery and the acoustic trawl survey is asymptotic.

2 Methods

A summary of the input data and complete technical description of the model is provided in Appendix A and B, respectively. For technical details on the acoustic trawl surveys, please refer to Fleischer et al. (2005). For a more detailed description of the fishery and historical management of the fishery see Helser and Martell (2007) for more details. The purpose of this section is three fold: 1) summarize the modeling approach, 2) provide documentation for informative prior distributions, and 3) provide a technical description on how the reference points and catch advice is formulated.

2.1 Modeling approach

The principle difference between the assessment here, and that of last years assessment using Stock Synthesis (SS), is that the leading parameters in this model pertain to the management parameters F^*(the fishing mortality rate that produced the maximum sustainable yield) and C^*(the maximum sustainable yield). Whereas, SS estimates the unfished biomass B_o and the steepness of the stock recruitment relationship h.

The approach was to fit and age-structured population dynamics model to time series information on relative abundance, proportions-at-age in the commercial fishery, and proportions-at-age from the acoustic trawl survey index using a Bayesian estimation framework. The commercial catch and age-composition information from Canada and the U.S. has been combined to represent a single fishery. The aggregation of the commercial catch data has the potential to create a bias in the predicted-age composition because it assumes that the age-specific fishing mortality rates between the two countries has been relatively consistent over time.

The objective function contains 5 major components: 1) the negative loglikelihood of the relative abundance data, 2) the negative loglikelihood of the catch-at-age proportions in the commercial fishery, 3) the negative loglikelihood of the catch-at-age proportions in the acoustic survey, 4) the prior distributions for model parameters, and 5) two penalty functions that constrain the estimates of steepness to lie between 0 and 1, and prevent exploitation rates exceeding 1. Note that the value of the penalty functions was 0 for all samples from the posterior distribution. The joint posterior distribution is defined by equation (T20.6). This distribution was numerically approximated using the Markov Chain Monte Carlo routines built into AD Model Builder (Otter Research, 1994). Posterior samples were drawn systematically every 200 iterations from a chain of length

1,000,0000 (the first 1000 samples were dropped to allow for sufficient burnin). Convergence was diagnosed using various test provided in the R-package CODA (R Development Core Team, 2006), as well as, running medians and visual inspection of the trace plots. Where possible, we provide comparisons between the maximum likelihood estimates and median estimates from the marginal posterior distributions. Catch advice is based on the samples from the joint posterior distribution (T20.6).

2.1.1 Assumptions

There is no *a priori* assumption about the scaling parameter for the acoustic biomass survey (q), and the biomass index was treated as a relative abundance index that is directly proportional to the survey vulnerable biomass as the beginning of the year. It is assumed that the observation errors in the relative abundance index are lognormally distributed. Fishing mortality in the assessment model is conditioned on the observed total catch weight (combined US and Canada catch), and it is assumed that total catch is known and reported without error. I further assume that fishing mortality and natural mortality occur simultaneously. Age-composition information is assumed to come from a multivariate logistic distribution where the predicted proportion-at-age is a function of the predicted population age-structure and the age specific vulnerability to the fishing gear. The likelihood for the age-composition data was evaluated at the conditional maximum likelihood estimate of the variance (i.e., no subjective weighting scheme was used to scale likelihood for the age-composition information). No aging errors were assumed in this assessment.

Historical observations on mean weight-at-age shows systematic changes, where the average weights-at-age have declined from the mid 1970s and increased again slightly late 1990s (Figure 2). A number of the historical cohorts have a growth trajectories that initially increase from age-2 to age-8 then decline or stay relatively flat (e.g., 1977 cohort in Figure 2). Given these data, there are at least three alternative explanations for the observed decreases in mean weight-at-age: 1) changes in condition factor associated with food availability, 2) intensive size selective fishing mortality with differential fishing mortality rates on faster growing indivuals, and 3) apparent changes in selectiviity over time. All three of these variables are confounded, and it is not possible to capture decreasing weight-at-age using the von Bertalanffy growth model and a fixed allometric relationship between length and weight. As such, the assessment model herein uses the observed mean weight-at-age data from the commercial fishery to scale population numbers to biomass.

The structural assumptions of the model assume that recruitment follows a Beverton-Holt type model and the process error terms are represented by a vector of deviation parameters (ω_j) that are assumed to be lognormally distributed. Both fishing mortality and natural mortality are assumed to occur simultaneously; instantaneous fishing mortality is based on the Baranov catch e equation where the analytical solution for F_t is found using an iterative Newton-Rhaphson method with a fixed number of iterations to ensure the proper derivative information is carried forward in the autodiff libraries. Selectivity, or vulnerability-at-age, to the fishing gear is assumed to be age-specific, time-invariant, and is represented by an asymptotic function (T17.5). Age-specific fecundity is assumed to be proportional to the product of body-weight and the proportion-at-age that are sexually mature.

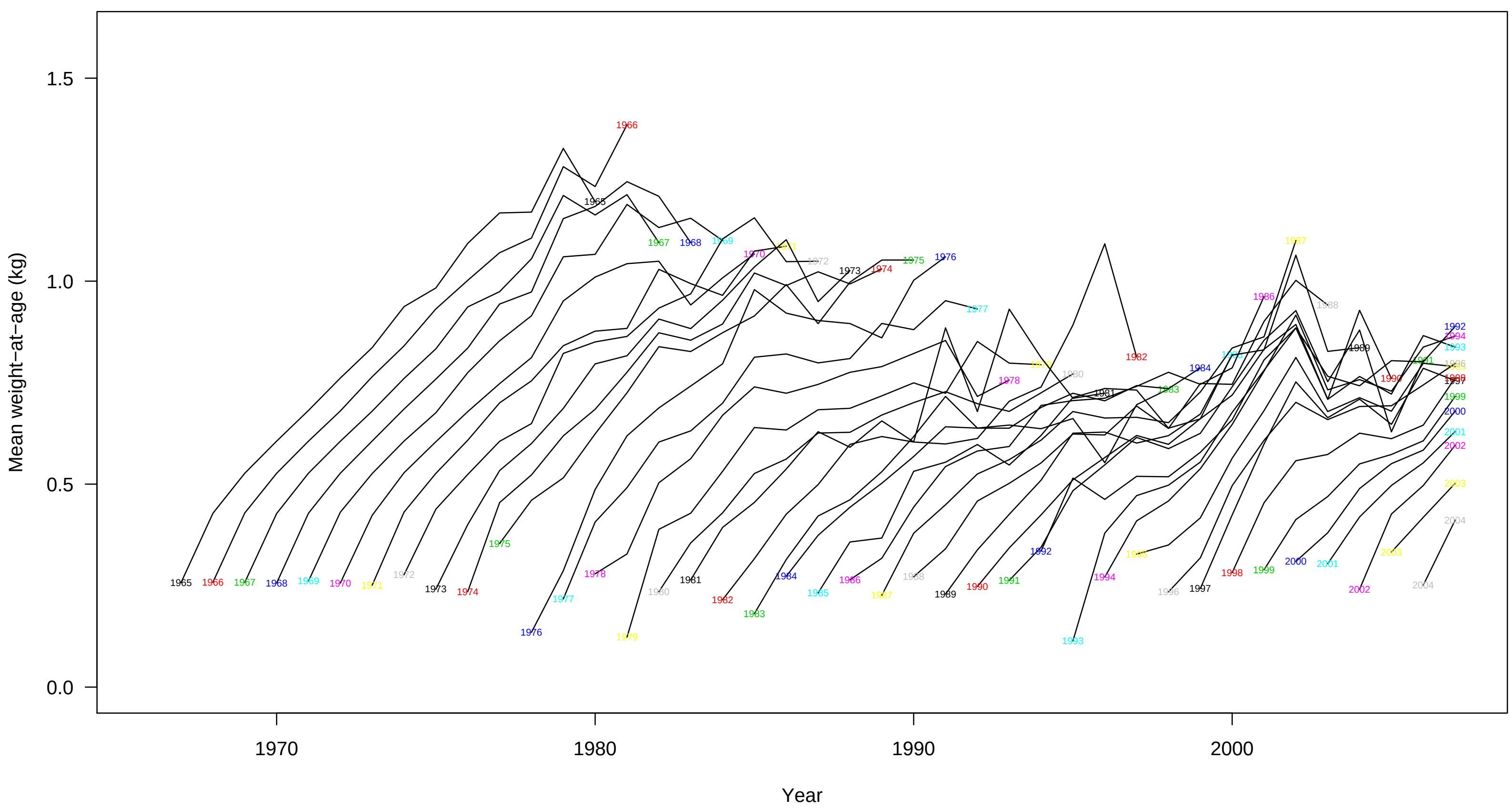

Figure 2: Observed mean weights-at-age by cohort in the commercial catch. Text labels for each line represent the cohort year.

2.2 Prior distributions

The underlying production function is defined by three key population parameters (C^*, F^*, and M) and the parameters that define age-specific selectivity ($v_a = f(\hat{a}_h, \hat{\gamma})$). Informative lognormal prior distributions were used for C^*, F^*, and M where the log means and log standard deviations are given in Table 3. These prior distributions were developed on an *ad hoc* basis and not necessarily derived from meta-analytic work that is the typical source of prior information.

The global scaling parameter in this model is C^*; the maximum long-term sustainable yield. Since 1966, the average annual landings removed from this population is 218,963.5 mt, and in the last decade 282,408.7 mt. We assume a rather diffuse lognormal prior for $C^*$with mode corresponding to 200,000 mt and a standard deviation of 500,000 mt. This prior is even more diffuse in comparison to last years assessment by (Martell, 2008). This represents a 95% confidence interval of roughly 96,300 mt to 684,000 mt. Assigning a prior density for $C^*$is nearly equivalent to assigning a prior density for the global scaling parameter q.

Table 3: Prior distributions for model parameters.

Parameter	prior density	range	μ	σ	a	b
C^*	lognormal	(0.01-3.0)	0.2568	0.5		
F^*	lognormal	(0.01-0.9)	0.35	0.262		
M	lognormal	(0.05-0.9)	0.23	0.1		
$\hat{a}, \bar{a}$	uniform	(0.0-14.0)				
$\hat{\gamma}, \bar{\gamma}$	uniform	(0.05-5.0)				
ρ	beta	(0.01-0.99)			3.5	31.5
φ	inverse gamma	(0.02-100)			7.5	5.78

A lognormal prior was assumed for M with a mean corresponding to 0.23 (which is the assumed fixed value in Helser and Martell (2007)) and a standard deviation of 0.1. This roughly corresponds to a 95% confidence interval of 0.19 and 0.28 for M, which is lower than the range reported in (Bailey et al., 1982, Table 10).

Uniform prior distributions were assumed for the selectivity parameters for the commercial fishery and the acoustic trawl survey. These parameters are bounded between 0 and 14 years for the age at 50% vulnerability and 0.05 and 5.0 for the standard deviation in age at 50% vulnerability.

In comparison with Helser and Martell (2007), a prior probability for $F^*$is nearly equivalent to a prior probability for steepness h. A lognormal prior was assumed for F^*, with a mean corresponding to 0.35 and a standard deviation of 0.262 (corresponds to a 95% confidence interval of 0.21 and 0.59). To derive the prior for F^*, a steady state age-structured model was developed to calculate spawning potential ratio based on growth parameters from Francis et al. (1982), a natural mortality rate of 0.23, and a logistic selectivity curve ($\hat{a} = 3.13, \hat{\gamma} = 0.8$). Arbitrarily, it was assumed that production is maximized somewhere between SPR=0.3 and SPR=0.45, and the corresponding values for F_{30} and F_{45} were then calculated. Based on the growth-maturity, natural mortality, and assumed selectivity the values correspond to $F_{30} = 0.48$ and $F_{45} = 0.25$, which were then assumed to be the 10th and 90th percentiles for a lognormal distribution. Note that the Spawning potential ratio curve is insensitive to the assumed value of steepness (Figure 3) and that F_{40} is the assumed proxy for F^*that is used by the Pacific Fisheries Management Council.

The transition from (C^*,F^*)$\Rightarrow$(B_o, h), that is carried out using the algorithm described in Table

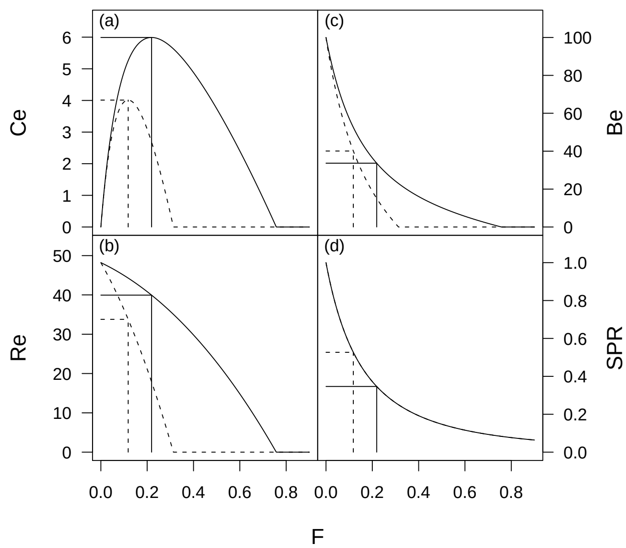

Figure 3: Relationship between equilibrium fishing mortality rate and yield (a), recruitment (b), biomass(c) and spawner per recruit(d) with an assumed value of h = 0.75 and $h = 0.5$. The vertical lines in each panel represent estimates of F^*(solid lines), F_{45}, and F_{30} (dotted lines). Note that the y axis scaling is arbitrary (i.e. B_o was assumed at 4 units of biomass).

17, implies a prior density for the steepness parameter in the stock recruitment relationship. The implied prior density for h used in this assessment is shown in Figure 4. Note that in the Beverton-Holt stock recruitment model, values of h range between 0.2 and 1.0, where 0.2 implies that recruitment is nearly proportional to spawner/egg production, and 1.0 implies that recruitment is unrelated to spawner/egg production. The implied prior for h is sensitive to two key model components: the assumed prior distribution for F^*, and the age at which fish recruit to the fishery relative to the age at which fish mature. Larger values of F^*imply a more productive stock and higher values of h for given selectivity and maturity schedules. Similarly, if fish recruit to the fishery prior to maturing then the levels of recruitment compensation (or h) must increase for a given value of F^*. Therefore, a critical piece of information is the maturity-at-age and weight-at-age schedules used to develop the age-specific fecundity relationship.

2.3 Reference points and catch advice

Catch advice in this model is based on a modified 40:10 harvest control rule, where the modification is to fish at F^*, rather than F_{40}. Unless otherwise stated, the reference point calculations and catch advice is based on the most recent information about growth (Table 15) and maturity-at-age information from Dorn and Saunders (1997).

The reference points for the harvest control rule are $F^*$and SB_{40}. Recall that $F^*$is the fishing mortality rate that produces the maximum sustainable yield, and this differs from that assumed in the previous assessments where F_{40} was used. $F^*$is estimated as a leading parameter, and

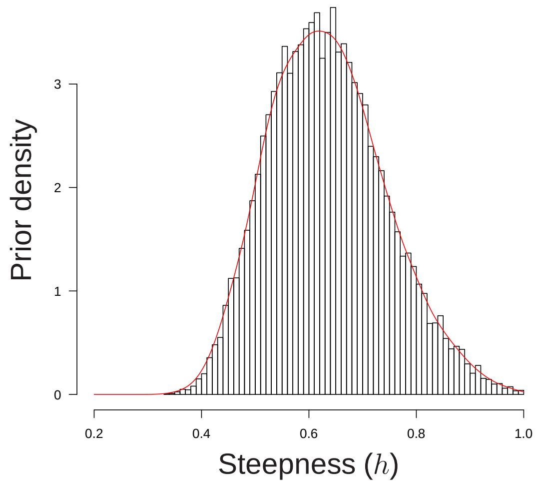

Figure 4: Implied prior for the steepness parameter in the stock recruitment relationship. Note that steepness is derived from the leading parameters Θ; therefore, any assumed prior information for Θ results in an implied prior for derived quantities such as h.

$\mathrm{SB}_{40}$is 40% of the unfished spawning biomass (SB_{o}). An alternative (but as it turns out, less conservative) harvest rule would be to use $\mathrm{SB}_{\mathrm{MSY}}$as the reference point in the harvest control rule, where $\mathrm{SB}_{\mathrm{MSY}}$=$R_e\phi_e$ evaluated at $F^*$and C^*.

Catch advice was generated by projecting the stock abundance forward to 2010 by applying catch options between 0 and 750,000 mt tons over 25 equally spaced intervals and then calculating various management objectives for each of the 5,000 samples from the joint posterior distribution. It was assumed in each simulated projection that the total catch option was fully utilized and implemented without error. In the stock projections, age-1 recruits for 2007-2010 were generated using the underlying Beverton-Holt stock recruitment model with annual lognormal recruitment deviates with standard deviation equal to the current estimate of standard deviation in the process errors (τ).

A decision table for catch advice (ABC options) was developed using measures of overfishing (probability that the ABC option will result in a fishing mortality rate that exceeds F^*), and four measures of spawning stock depletion. The first measure is the probability that the spawning stock biomass in 2010 will be less than the spawning stock biomass in 2009, and the second measure is the probability that the spawning stock biomass in 2010 will be less than $\mathrm{SB}_{\mathrm{MSY}}$. The third measure is the probability that the spawning stock biomass will be less than SB_{40}, and the fourth measure is the probability that the spawning stock biomass will fall below SB_{25}. For each sample from the joint posterior distribution the projection model loops over 25 increments of this ABC ranging from 0 to 750,000 mt and then calculates the corresponding fishing mortality rates and levels of spawning stock depletion. We then score the fishing rate and spawning stock depletion on a 0 or 1 scale (0 not overfishing or spawning stock biomass greater than or equal to management target) and fit a binomial (link logit) model versus ABC option to these data. The result is a sigmoid like curve or the cumulative probability of an ABC option versus management objective can be assessed.

For specified levels of risk, ABC options for each management objective are then provided in a decision table. This cumulative probability distribution is also compared to the cumulative density function of catch advice produced by the 40/10 harvest control rule.

3 Results

Maximum likelihood estimates of the vulnerable biomass, fishing mortality rates, age-1 recruits and historical landings are summarized in Fig. 5. During the late 1960 and 1970s, annual landings averaged 169,000 tons and the corresponding fishing mortalities were less than 0.18 per year. During the 1980s catches increased from 90,000 tons to just over 300,000 tons and the fishing mortality rates during this period averaged less than 0.11 per year. Two exceptionally strong cohorts (1980, 1984) were responsible for a large increase in the vulnerable biomass during this time period. The vulnerable biomass peaked in the mid 1980s declined steadily to a low of 1.35 million tons in 2000 (Table 4). During this time period, there were no significant recruitment events (Fig. 5c), and also during this time period annual landings increased from 110,000 tons in 1985 to nearly 312,000 tons in 1999. The 1999 cohort was an exceptional year class, and the vulnerable biomass more than doubled from 1.35 millon tons in 2000 to 2.75 million tons in 2004 as a result. Catches declined as this year class recruited to the fishery, resulting in a reduction in fishing mortality to 0.15 in 2003. Catches increased again, reaching 360,000 tones in 2005 and 2006 resulting in an sharp increase in fishing mortality. As the 1999 year class passed through the fishery and was not replaced with another exceptional year class, catches remained high. Vulnerable and spawning biomass reached their historical minima following the 2008 fishery, and estimated fishing mortality in 2008 reached an extremely high value nearly 3 times the estimate of F^*.

Table 4: Maximum likelihood estimates of vulnerable biomass (B_t), spawning biomass (SB_t) and depletion, , landings (C_t millions mt), instantaneous fishing mortality rates (F_t), 2+ and 3+ biomass ($B_{t,2+}, B_{t,3+}$), and total catch over 2+ and 3+ biomass ($C_t/B_{t,2+}$, $C_t/B_{t,3+}$), from 1966 to the begining of 2009.

Year	B_t	SB_t	SB_t/SB_0	C_t	F_t	$B_{t,2+}$	$B_{t,3+}$	$C_t/B_{t,2+}$	$C_t/B_{t,3+}$
1966	2.77	3.54	1.00	0.14	0.07	4.32	3.73	0.03	0.04
1967	2.63	3.40	0.96	0.21	0.11	4.17	3.58	0.05	0.06
1968	2.44	3.15	0.89	0.12	0.07	3.69	3.39	0.03	0.04
1969	2.34	2.89	0.81	0.18	0.11	3.36	2.99	0.05	0.06
1970	2.09	2.55	0.72	0.23	0.15	3.02	2.66	0.08	0.09
1971	1.77	2.29	0.65	0.15	0.12	2.91	2.35	0.05	0.07
1972	1.64	2.45	0.69	0.12	0.10	4.04	2.41	0.03	0.05
1973	1.76	3.08	0.87	0.16	0.14	3.95	3.78	0.04	0.04
1974	2.21	3.12	0.88	0.21	0.15	3.60	3.22	0.06	0.07
1975	2.22	2.82	0.80	0.22	0.14	3.93	2.79	0.06	0.08
1976	1.99	2.93	0.83	0.24	0.17	3.68	3.39	0.06	0.07
1977	2.09	2.88	0.81	0.13	0.09	3.36	3.06	0.04	0.04
1978	2.07	2.46	0.69	0.10	0.07	2.62	2.55	0.04	0.04
1979	1.91	2.35	0.66	0.14	0.09	3.73	2.11	0.04	0.07

Table 4: *(continued)*

Year	B_t	SB_t	SB_t/SB_0	C_t	F_t	$B_{t,2+}$	$B_{t,3+}$	$C_t/B_{t,2+}$	$C_t/B_{t,3+}$
1980	1.75	2.98	0.84	0.09	0.07	3.81	3.76	0.02	0.02
1981	2.15	3.02	0.85	0.14	0.10	3.25	3.05	0.04	0.05
1982	2.36	3.52	0.99	0.11	0.06	7.40	2.85	0.01	0.04
1983	2.38	5.16	1.46	0.11	0.08	7.04	6.95	0.02	0.02
1984	3.77	5.53	1.56	0.14	0.06	5.89	5.85	0.02	0.02
1985	4.53	4.96	1.40	0.11	0.03	5.07	4.90	0.02	0.02
1986	3.66	4.78	1.35	0.21	0.07	9.41	3.83	0.02	0.06
1987	3.32	6.32	1.78	0.23	0.11	8.34	8.30	0.03	0.03
1988	4.32	6.25	1.76	0.25	0.10	6.88	6.50	0.04	0.04
1989	4.61	5.19	1.46	0.31	0.09	5.82	5.21	0.05	0.06
1990	3.83	4.57	1.29	0.26	0.09	5.78	4.64	0.04	0.06
1991	3.31	4.19	1.18	0.31	0.13	4.83	4.53	0.06	0.07
1992	2.83	3.65	1.03	0.30	0.15	4.86	3.59	0.06	0.08
1993	2.38	3.29	0.93	0.20	0.12	4.14	3.68	0.05	0.05
1994	2.34	3.09	0.87	0.36	0.23	3.56	3.26	0.10	0.11
1995	2.12	2.62	0.74	0.25	0.17	3.12	2.68	0.08	0.09
1996	1.74	2.60	0.73	0.31	0.26	3.77	2.82	0.08	0.11
1997	1.68	2.72	0.77	0.33	0.33	3.68	3.03	0.09	0.11
1998	1.56	2.27	0.64	0.32	0.34	2.90	2.42	0.11	0.13
1999	1.43	1.94	0.55	0.31	0.34	2.39	2.08	0.13	0.15
2000	1.35	1.93	0.55	0.23	0.26	2.52	1.96	0.09	0.12
2001	1.38	2.52	0.71	0.24	0.27	5.71	2.12	0.04	0.11
2002	1.66	3.89	1.10	0.18	0.20	5.40	5.21	0.03	0.04
2003	2.43	3.75	1.06	0.21	0.15	4.21	3.96	0.05	0.05
2004	2.75	3.13	0.88	0.33	0.17	3.27	3.23	0.10	0.10
2005	1.99	2.20	0.62	0.36	0.25	2.61	2.15	0.14	0.17
2006	1.34	1.63	0.46	0.36	0.40	1.91	1.74	0.19	0.21
2007	0.90	1.25	0.35	0.30	0.57	1.78	1.24	0.17	0.24
2008	0.67	1.10	0.31	0.32	1.05	1.51	1.29	0.21	0.25
2009	0.54	0.92	0.26			1.16	1.02		

The maximum likelihood estimate of the 2009 spawning stock biomass is 0.92 millon tons, which corresponds to a depletion level of 0.26 (Fig. 6ab, Table 4). This is well below the management target of 0.4. By comparison, the estimated level of depletion in the assessment by Helser and Martell (2007) was 0.309.

In this assessment we assume a constant age-selectivity curve for both the commercial and acoustic surveys (Fig. 7c). This is markedly different from previous assessments and other assessments run in parallel (i.e., Stock Synthesis) where selectivity is allowed to vary over specified time blocks. The conditional maximum likelihood estimates of the standard deviations for the age-composition data is 1.93 and 3.10 for the commercial and acoustic surveys, respectively. These are very large errors in the age-composition information and more emphasis (in terms of contribution to the likelihood component) is placed on the commercial age-composition information. For

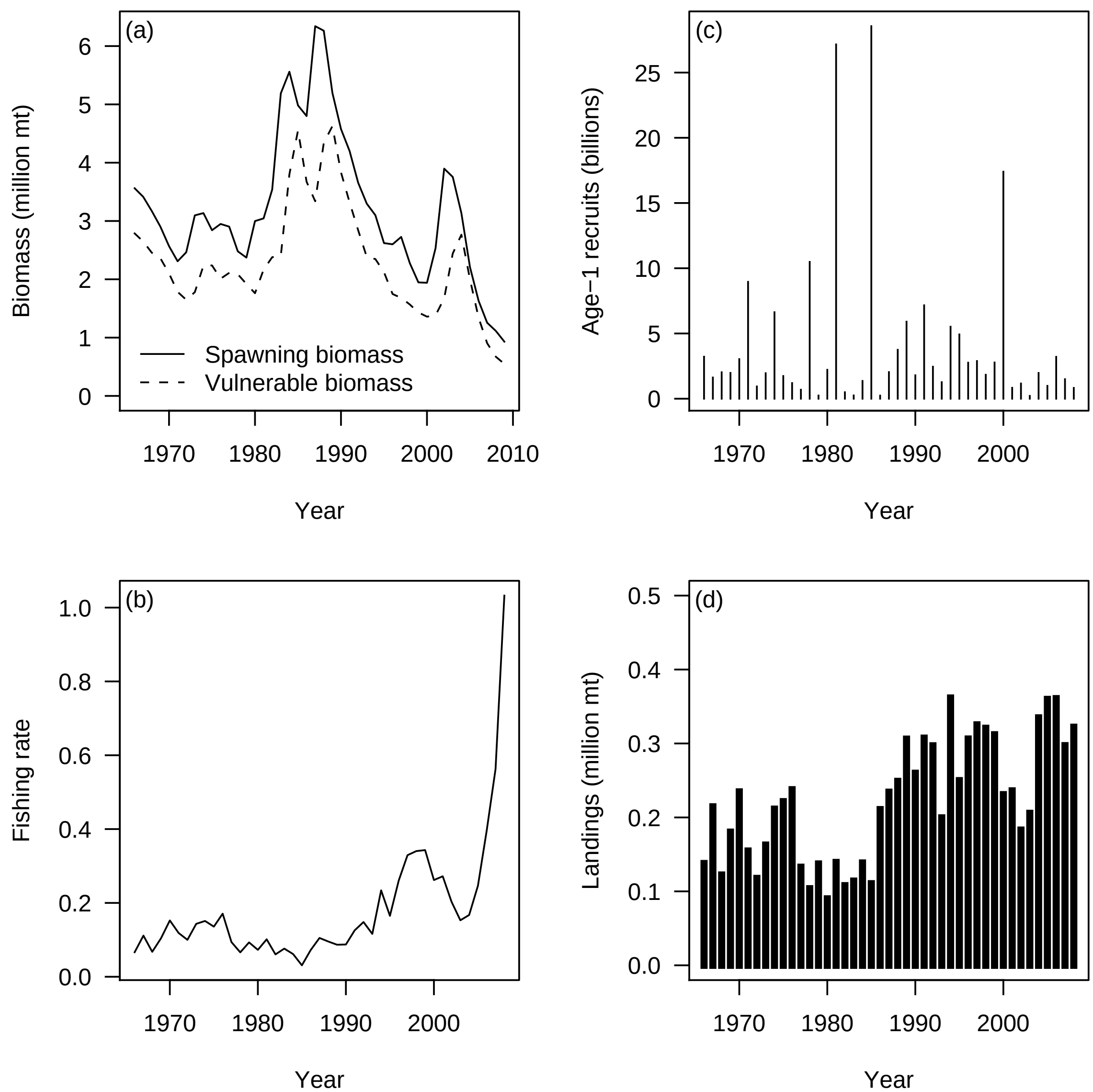

Figure 5: Maximum likelihood estimates of vulnerable and spawning biomass (panel a), fishing mortality (b), age-1 recruits (c) and the observed historical landings (d) for U.S. and Canadian fisheries combined.

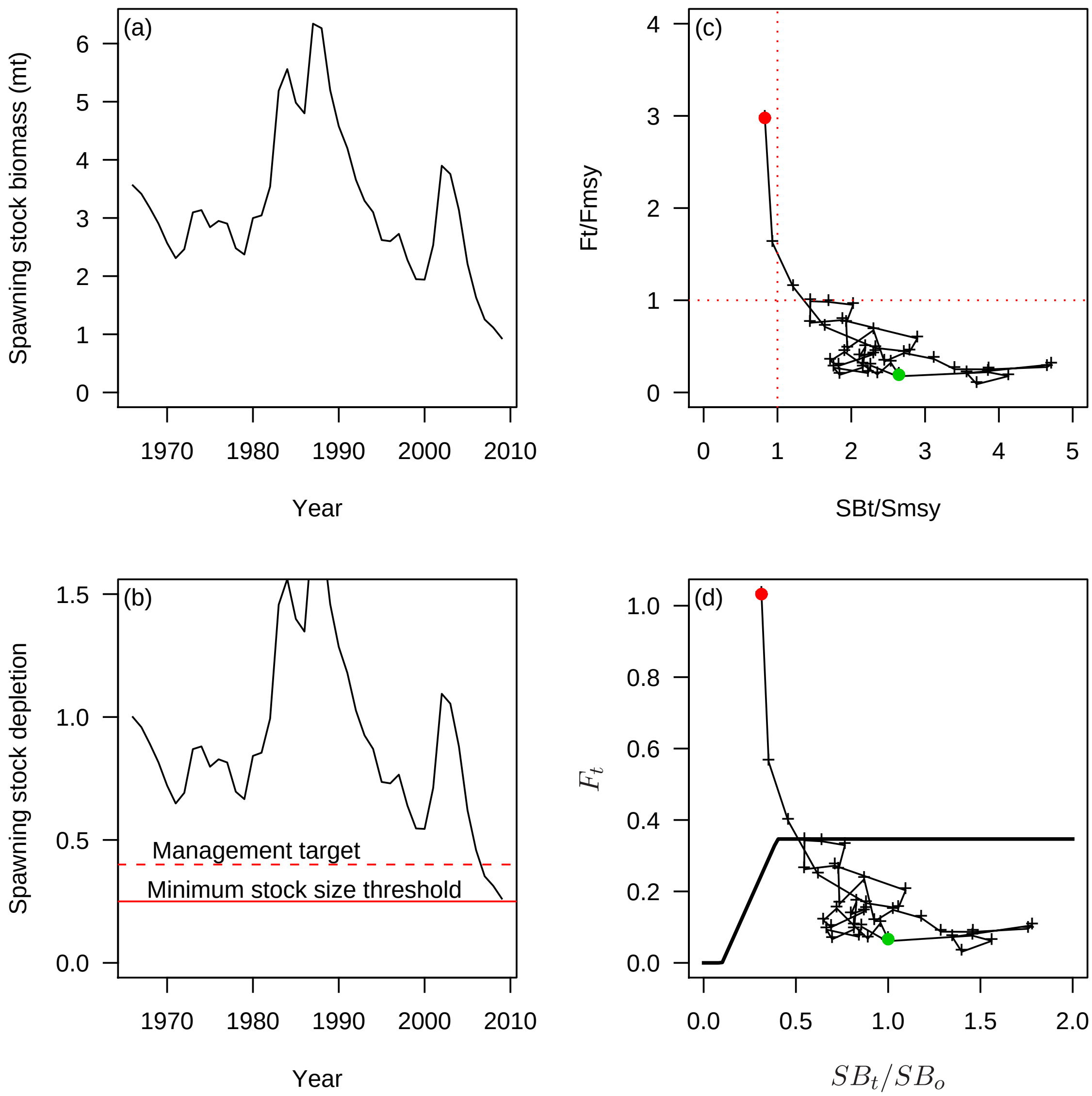

Figure 6: Maximum likelihood estimates of spawning stock biomass (a), spawning biomass depletion (b), the ratio of fishing mortality rates to C^*versus the spawning stock biomass to S_{msy} (c) and the harvest control rule (d). Note that the spawning stock biomass calculations include both male and females.

the acoustic trawl survey, reasonable fits were obtained in most years, with strong exceptions in 1983, 1989, 1992 and 1995 with nearly all positive residuals for ages 2–12 (Figs. 8-9). Since 1998, residual values in the acoustic survey age-composition are much smaller, and primarily negative for younger ages and positive for older ages.

In the commercial fishery, a time-invariant asymptotic selectivity curve was assumed and surprisingly good fits were obtained to the older age-classes in the commercial catch-age proportions (Figs. 10-11), with the exception of the persistent under-estimate of the proportions-at-age in the plus group in the late 1970s (this owes to an initialization of the numbers-at-age using a stable age distribution with a $Z = M$). The largest residual variation in the commercial age-composition data occurred in ages 2 and 14 (Fig. 12). The model tends to under estimate the 1980 and 1984 cohorts at age-2 and over estimate the 1979 and 1983 cohorts at age-2. For the plus group, after 1984 there is no strong positive or negative residuals and no persistent pattern that would better suggest a dome-shaped selectivity curve; however, this residual pattern is in part determined by the instantaneous natural mortality rate M and lower values of M would lend more support for a hypothesis of a dome-shaped selectivity curve in the commercial fishery. Observed proportions-at-age are nearly all positive for the 2001 fishery with the exception of age-14. In 2000-2001, fish did not show up in the Canadian zone and the Canadian fleet operated in non-traditional fishing grounds in the north and landed older fish in comparison to the US fishery.

Overall, the constant selectivity assumption fits the commercial catch-age data reasonably well (Fig 10). There is a marked pattern in the residuals that appear to correspond to an aging error pattern around above average cohorts prior to the 1980 cohort (Fig 12). In the year proceeding the above average cohorts, much larger proportions-at-age are observed in comparison to what is predicted for fish younger than age-11. This could be partly explained by ageing errors (where the strong cohorts are persistently over-aged by 1-year). Also there are some negative residuals for age-15+ from 1977 to 1083; these residuals arise due to the initialization of the numbers-at-age in 1966 where I assumed a stable age-distribution. Finally, in 2001 hake failed to show up in the traditional fishing grounds in Canada. The commercial fleet operated in non-traditional waters further to the North (Queen Charlotte sound) and landed much larger/older hake in comparison to the US fleet. This change in distribution of fishing operations shows up as a series of positive residuals for nearly all age s in 2001 (Fig. 12) and probably is the source of the negative residuals for ages 4-7 in the late 1990s due to the time-invariant selectivity curve assumption.

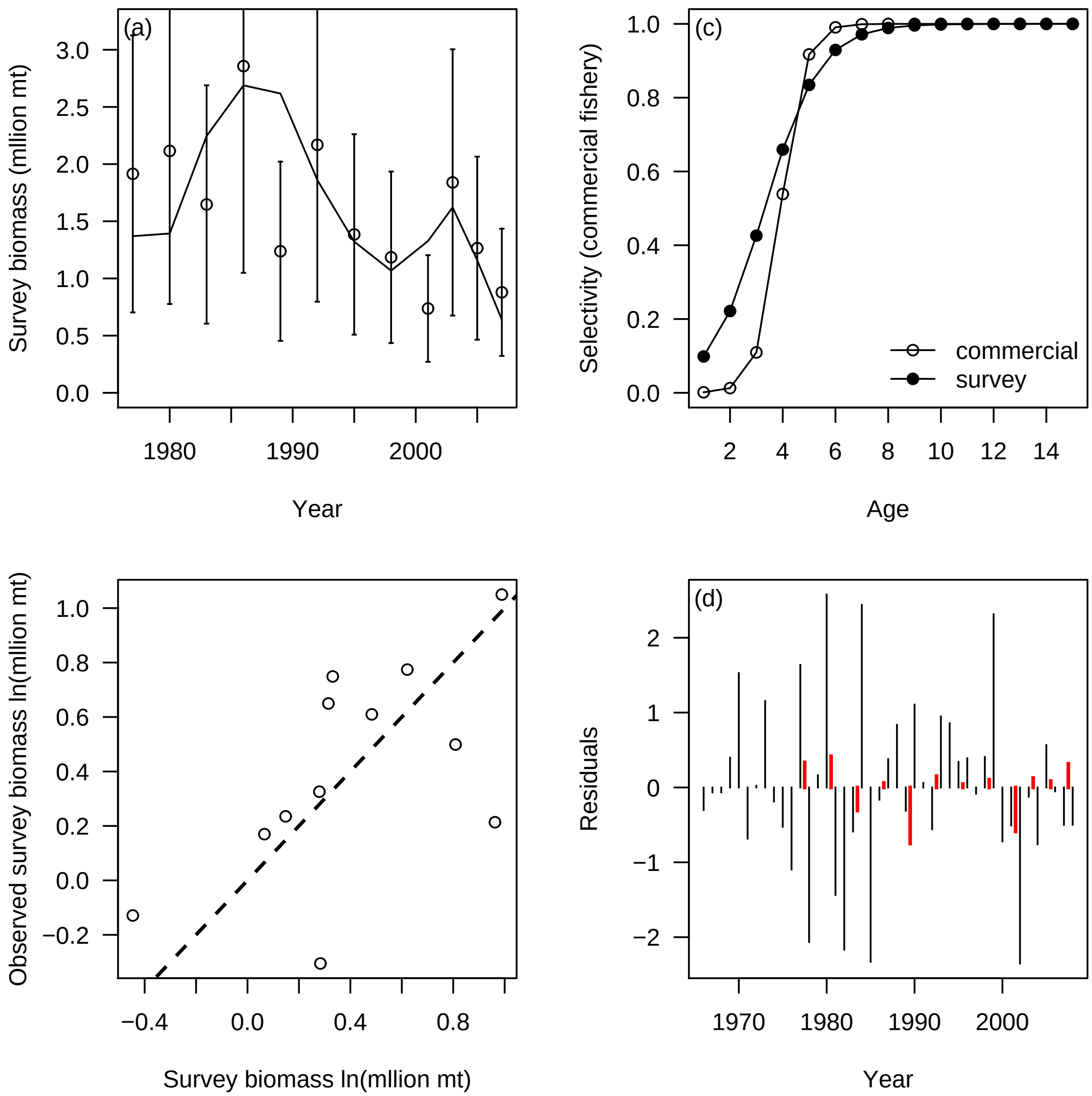

Figure 7: Predicted and observed survey biomass estimates (panel a-b, 1:1 line shown in panel b) based on the maximum likelihood fit to the data. Approximate 95% confidence intervals are shown for the survey points in panel (a) based on the estimated standard deviation in the survey. The estimated selectivity curves for commercial and survey selectivity (c), and the residuals between abundance indices (thick bars in panel d) and recruitemnt deviations (thin bars in panel d).

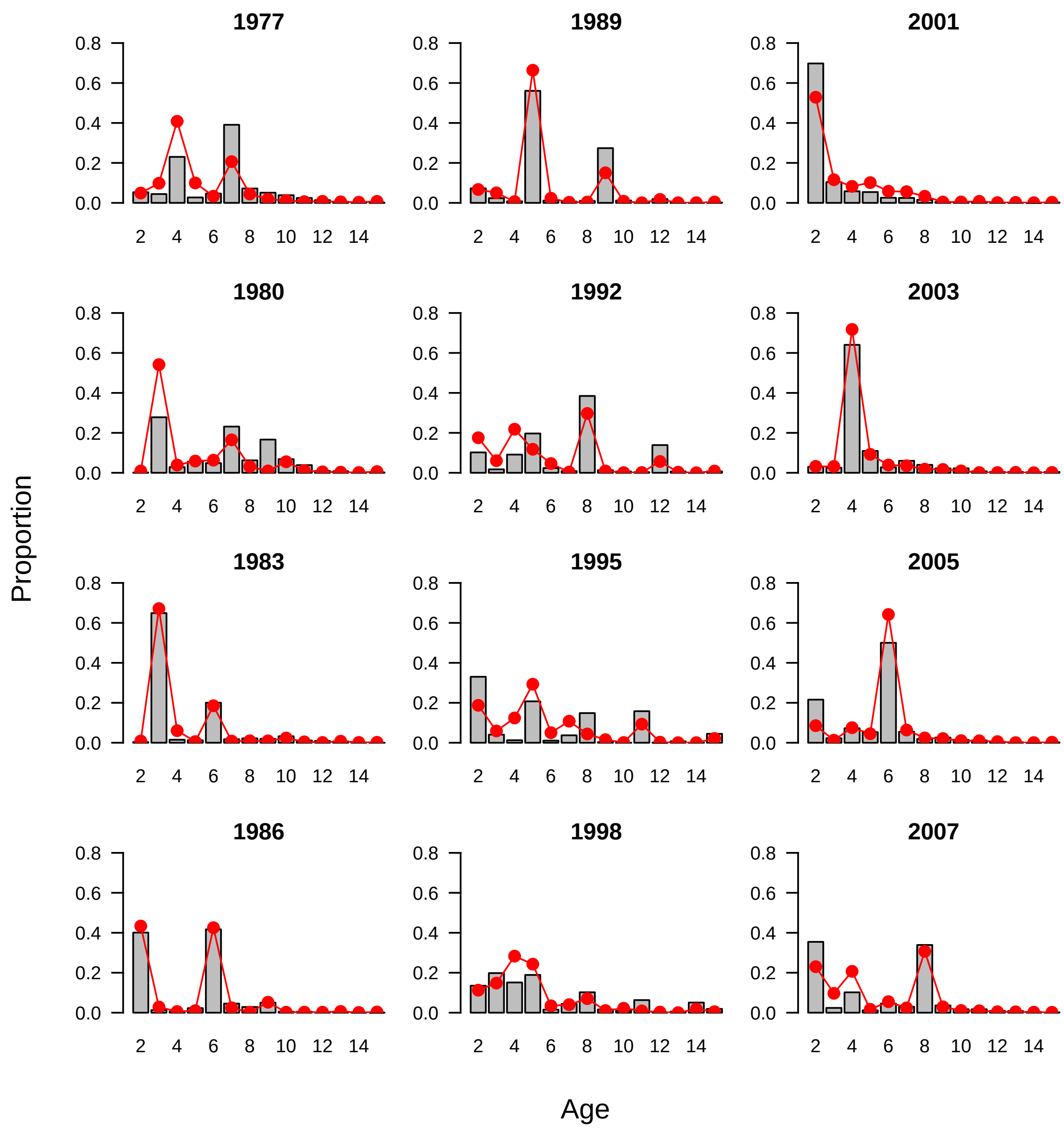

Figure 8: Observed (bars) and predicted (lines) proportions-at-age in the acoustic trawl surveys.

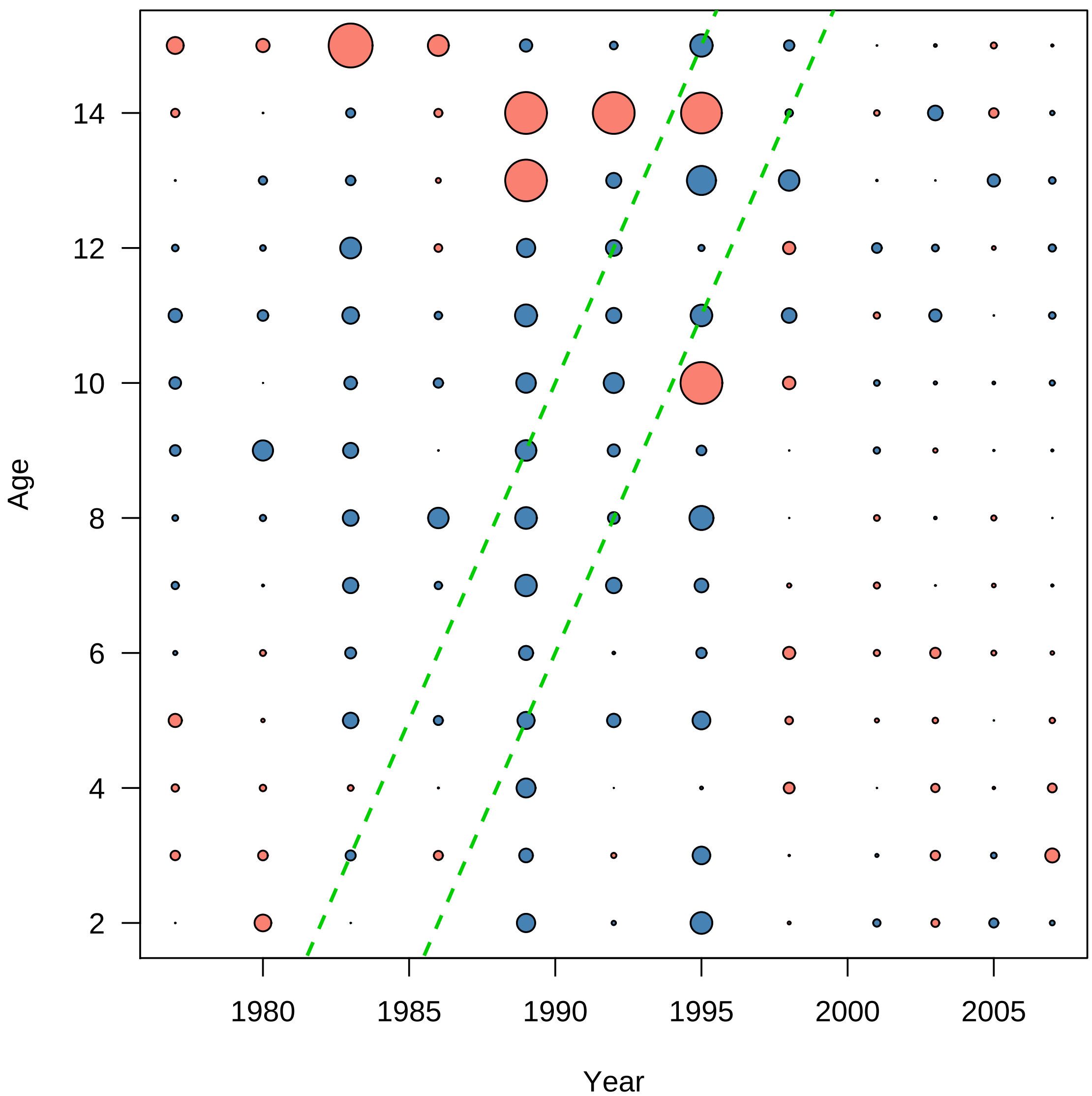

Figure 9: Bubble plots of the multivariate logistic residuals for the proportions-at-age in the acoustic trawl surveys. Diameter of the circle is proportional to the natural log of the residual, blue is positive (i.e., observed is greater than predicted). Dashed lines track the 1980 and 1984 cohorts.

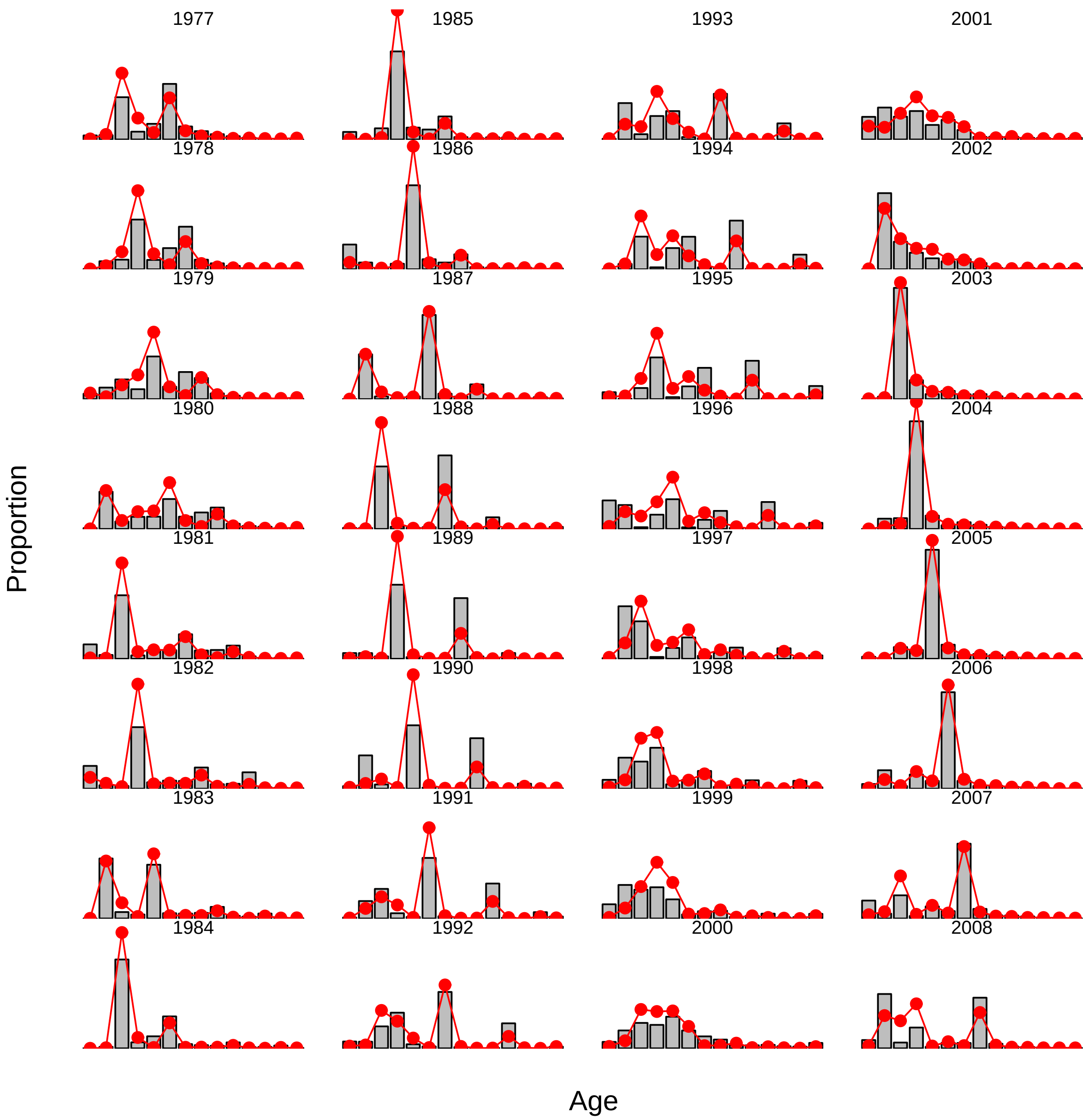

Figure 10: Observed (bars) and predicted (lines) proportions-at-age in the commercial age compositions.

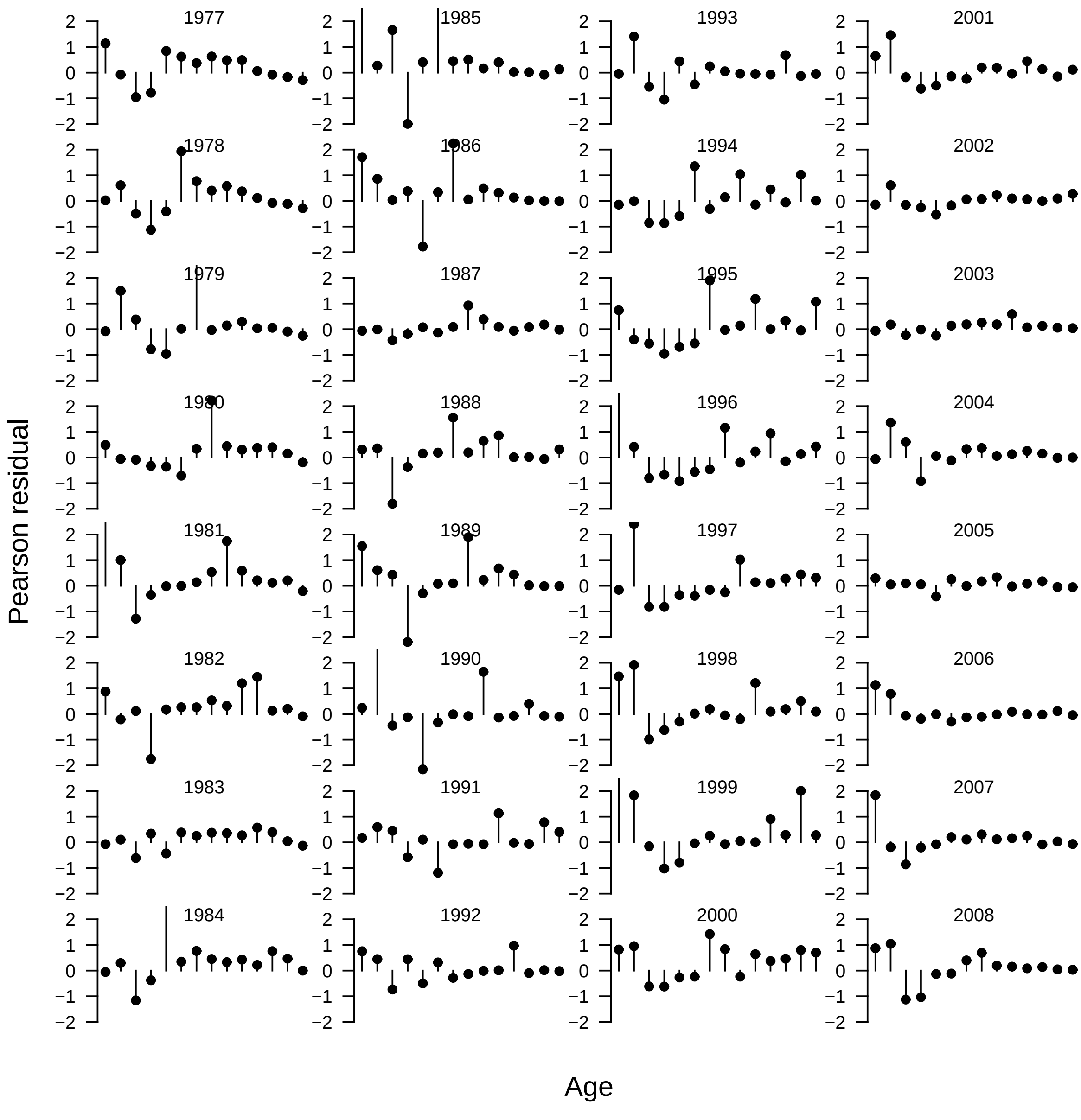

Figure 11: Pearson residuals for the proportions-at-age in the commercial age compositions.

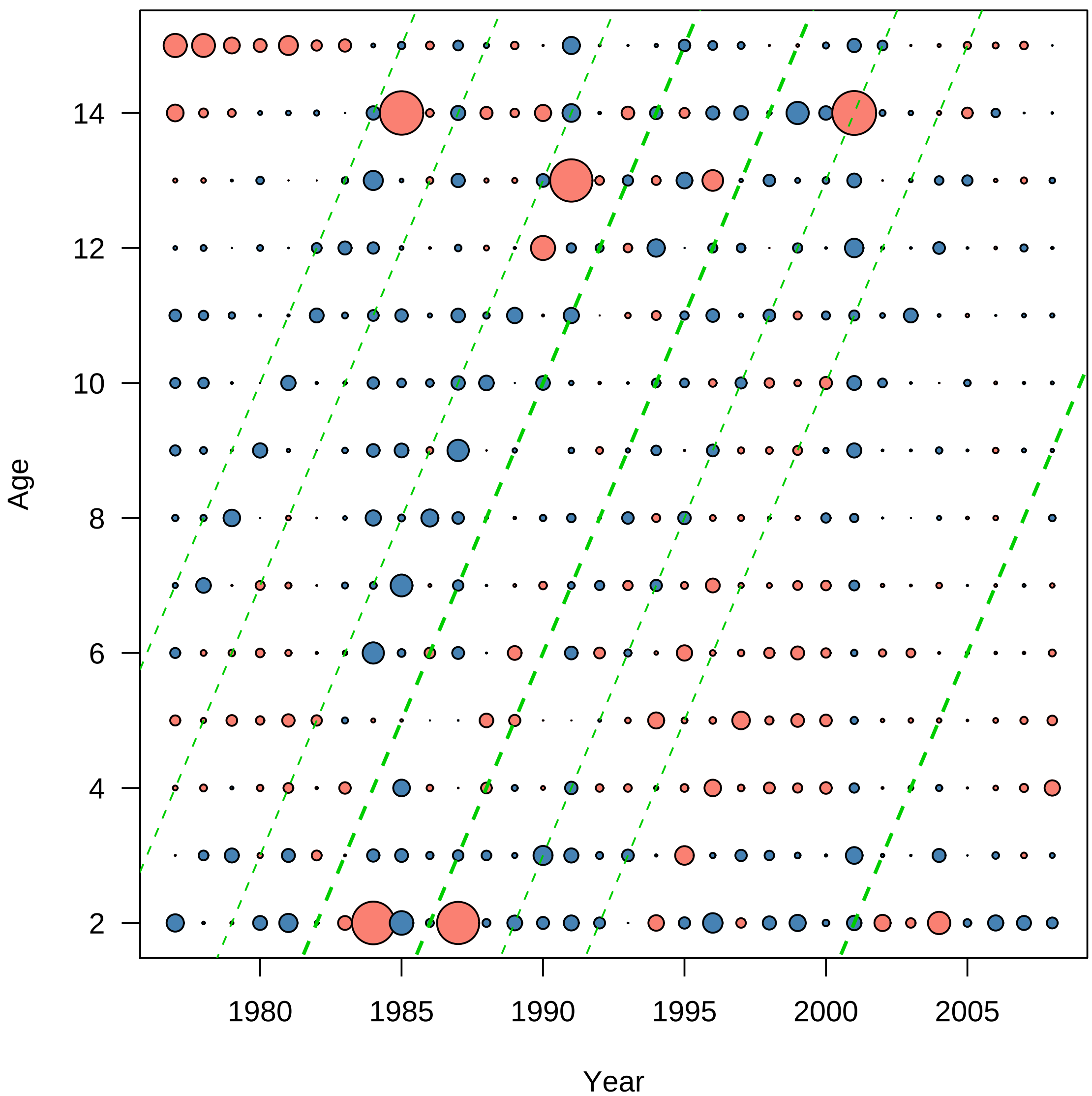

Figure 12: Bubble plots of pearson residuals for the proportions-at-age in the commercial age compositions. Dashed lines follow above average cohorts and the 1980, 1984 and 1999 cohorts are shown in bold dashed lines, positive residuals shown in blue, negative residuals shown in red.

3.1 Results from posterior integration

As reported in Martell et al. (2008), there is insufficient trend information, and an apparent contradiction between the age-composition and trend information to reliably estimate overall population scale and productivity parameters (in this case $C^*$and F^*, and in previous assessments B_o and h). The relative abundance indices are relatively flat, with a slight downward trend between 1986 and 2007. Such one-way trip information is insufficient to resolve parameter confounding between B_o and h, yet this information can be surprisingly informative about MSY (Walters and Martell, 2004).

The marginal posterior density for F^*reflects the assumed prior information for F^*(Fig. 13). The median estimate for $C^*$is 0.253 million mt (Table 5), which is similar to the assumed prior mean of 0.256 million mt; however, the 95% credible intervals for $C^*$are less than the assumed prior (Fig. 13). Hence there is some information in the data about C^*, but this information is confounded with estimates of $F^*$and the instantaneous natural mortality rate (Table 6).

Median estimates of M=0.347 are also much higher than the assumed prior mean of 0.23 (Table 5). Information to estimate M comes from the age-composition information and is slightly positively correlated with the age at 50% vulnerability parameters ($\hat{a}$ and $\bar{a}$) in the selectivity curves. Note that if a dome-shaped selectivity curve was assumed, then estimates of M would likely decrease owing to the disappearance of older animals due to reduced selectivity. The median estimate of the age at 50% recruitment to the commercial and survey gears is 4.0 and 3.1 years respectively (Table 5). Also, note the higher standard deviation (spread) of the survey selectivity parameter relative to the commercial selectivity parameters. In particular, the standard deviation in the logistic selectivity curve is sufficiently large that a high proportion of age-2 fish are recruited to the survey gear. The median estimate of the variance ratio ρ is 0.278 and the inverse of the total variance φ^{-2} is 0.777 which corresponds to standard deviations of 0.358 and 0.926 for the observation errors and process errors, respectively (Table 5 and Table 7). There is a negative correlation between the inverse total error φ^{-2} and the proportion of observation error ρ. As values of ρ increases more of the total error is allocated to observation error in the surveys and the proportion of the process error remains relatively stable (i.e., information in the age-composition data are informative about process errors, Fig. 14).

Trends in the median estimates of vulnerable biomass and spawning stock biomass are exactly the same as the maximum likelihood estimates; however, in absolute terms the median estimates are slightly higher than the maximum likelihood estimates (Fig. 15a). Thus, uncertainty in biomass estimates is not normally distributed. In comparison to Helser and Martell (2007), uncertainty is much greater in this assessment owing to the large amount of uncertainty admitted in the global scaling parameter (C^*) and productivity parameter (F^*). Although the survey catchability coefficient (q) is not directly comparable with the assumed values in Helser and Martell (2007), the range of uncertainty in this assessment is much larger than the two options explored in previous assessments (Table 7).

Trends in historical recruitment are also comparable with Helser and Martell (2007), and the median estimates are slightly higher than the maximum likelihood estimates (Fig. 16). The overall uncertainty in annual recruitment is also proportional to the overall uncertainty in the global scaling as well as uncertainty in the estimates of M. The largest cohorts in the past are the 1980, 1984, and 1999, and the 2005 cohort is estimated to be slightly above the long term median historical recruitment but below the long-term average recruitment. There is a substantial amount of uncertainty in the estimates of age-1 recruits, and this uncertainty owes to the assumed uncertainty

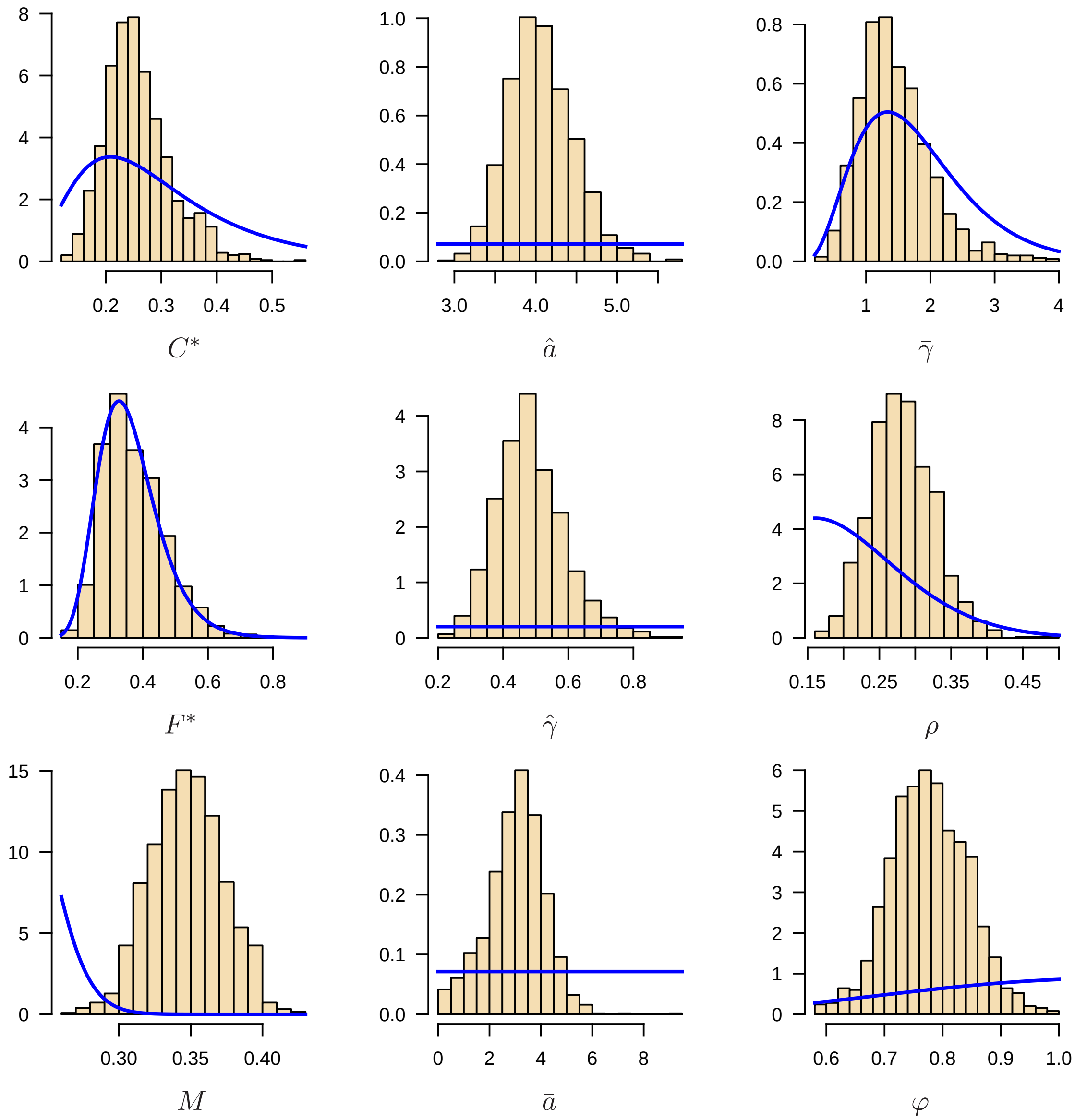

Figure 13: Marginal posterior (histograms) and prior distributions (lines) for key model parameters. Means and variances for the prior distributions are summarized in Table 3.

Table 5: Maximum likeliood estimates (MLE) of model parameters with asymptotic estimates of the standard deviation and median estimates with corresponding 2.5% and 97.5% quantiles from the marginal posterior distributions. Medians and quantiles are based on 5,000 samples from the joint posterior distribution.

	MLE		**Marginal densities**		
	Mean	Std	Median	2.5%	97.5%
C^*	0.223	0.051	0.253	0.161	0.396
F^*	0.347	0.087	0.362	0.226	0.600
M	0.333	0.027	0.347	0.297	0.398
$\hat{a}$	3.931	0.391	4.029	3.362	4.934
$\hat{\gamma}$	0.444	0.098	0.474	0.308	0.710
$\bar{a}$	3.311	0.771	3.115	0.617	5.079
$\bar{\gamma}$	1.044	0.423	1.375	0.601	2.828
ρ	0.269	0.046	0.278	0.205	0.375
φ^{-2}	0.834	0.073	0.777	0.653	0.913

Table 6: Correlation among key model parameters based on 5,000 samples from the posterior distribution.

	C^*	F^*	M	$\hat{a}$	$\hat{\gamma}$	$\bar{a}$	$\bar{\gamma}$	ρ	φ^{-2}
C^*	1.000								
F^*	0.478	1.000							
M	0.412	−0.093	1.000						
$\hat{a}$	−0.284	0.003	0.121	1.000					
$\hat{\gamma}$	−0.258	0.000	0.037	0.952	1.000				
$\bar{a}$	−0.034	0.007	0.156	0.116	0.067	1.000			
$\bar{\gamma}$	−0.020	−0.016	0.027	0.050	0.047	0.045	1.000		
ρ	−0.174	0.006	−0.072	0.072	0.084	−0.153	0.094	1.000	
φ^{-2}	0.009	0.011	−0.009	−0.123	−0.129	0.081	−0.068	−0.507	1

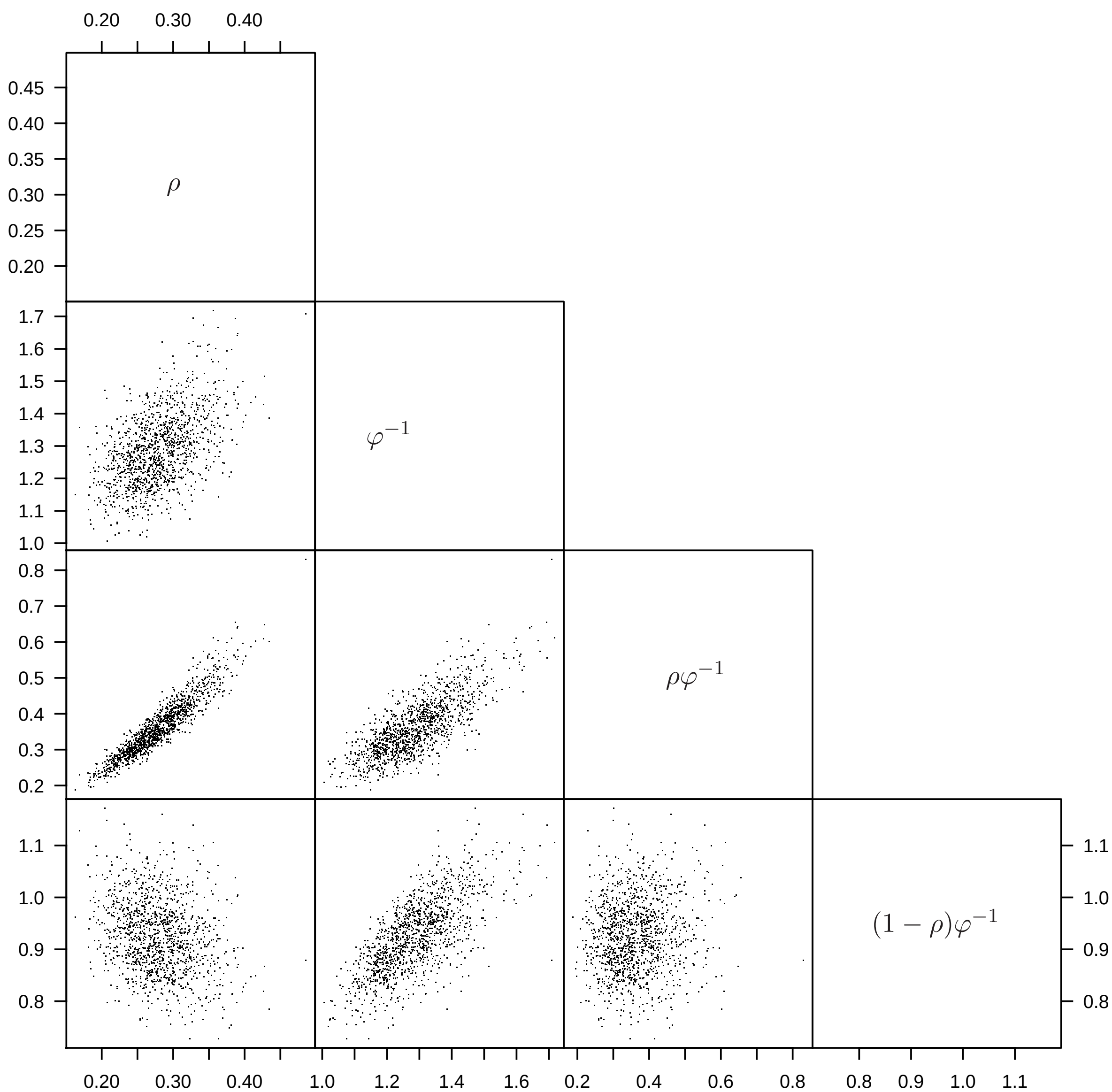

Figure 14: Pair plot of 1250 samples from the joint posterior distribution of the variance components, where the standard deviation in observation and process errors is given by $\rho\varphi^{-1}$ and $(1-\rho)\varphi^{-1}$, respectively.

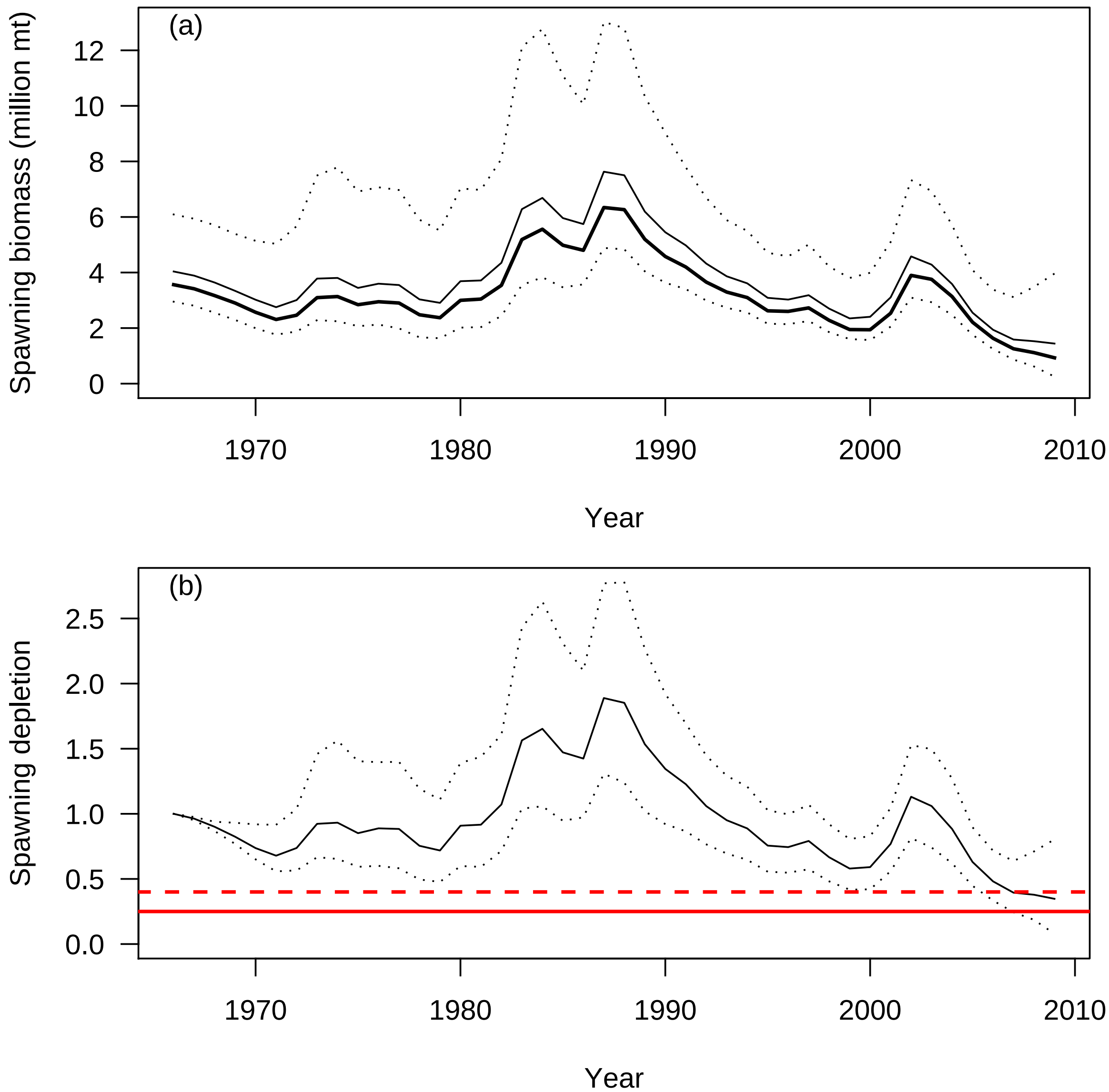

Figure 15: Maximum likelihood estimates (thick line) and median estimates (thin line) of the spawning stock biomass (a) and spawning stock depletion level with 40% and 25% horizontal reference lines (b). The dotted lines represent the 0.025 and 0.975 quantiles based on 5,000 systematic samples from the joint posterior distribution.

Table 7: Modal and median estimates of derived quantities of management interest. Medians and quantiles are based on 5,000 systematic samples from the joint positerior distribution, and the modal estimates correspond to the maximum likelihood estimates.

Derived quantity & Reference piont	Mode	Median	5%	95%
Survey catchability coefficient (q)	0.567	0.457	0.288	0.728
Steepness (h)	0.43	0.412	0.341	0.525
Spawning stock depletion (2009)	0.261	0.347	0.123	0.694
2009 ABC from 40/10 rule	0.114	0.175	0.005	0.553
Unfished total biomass (B_0)	4.727	5.394	3.954	7.713
Unfished 3+ biomass ($B_{0,3+}$)	3.745	4.252	3.209	5.954
Unfished spawning stock biomass (SB_0)	3.562	4.038	3.055	5.629
Unfished female spawning biomass	1.781	2.019	1.527	2.814
Spawning stock biomass at MSY (SB_{MSY})	1.346	1.558	1.095	2.274
Female spawning biomass at MSY	0.673	0.779	0.548	1.137
Spawning stock biomass in 2009 (million mt)	0.931	1.442	0.426	3.374
Female spawning stock biomass in 2009 (million mt)	0.465	0.721	0.213	1.687
Standard deviation in surveys (σ)	0.323	0.358	0.254	0.515
Standard deviation in process errors (τ)	0.876	0.926	0.819	1.05

in the instantaneous natural mortality rate (M). In comparison to previous assessments the average long-term recruitment is higher; however, both the MLE and median estimates of M are substantially higher than the previously assumed value of 0.23.

The residual pattern from the acoustic abundance index was consistent across all 5,000 samples from the joint posterior distribution (Fig. 17). The 1989 and 2001 acoustic survey biomass estimates are roughly 60% below the predicted biomass. The greatest residual variation is in the 2007 biomass estimate, and this uncertainty owes to the uncertainty in recent recruitment. The median estimate of the survey catchability coefficient q was 0.457 with a 5% and 95% credible intervals of 0.288 and 0.728, respectively (Table 7). These estimates of q are significantly lower than Helser and Martell (2007); however, in the previous years assessment a dome-shaped selectivity curve for the acoustic survey was assumed and as much as 20% of the older fish were assumed to be “cryptic” biomass.

The median estimate of the spawning stock biomass in 2009 is 1.442 million mt (Table 7) and the modal estimate is 0.931 million mt. Less than 10% of 2009 spawning stock biomass it consists of the 1999 cohort (Fig. 18b) and as much as 70% of it consists of the smaller cohorts produced in 2004 and later. Absent any significant recruitment, the spawning stock biomass is expected to decline rapidly as the 1999 cohort continues to disappear.

Catch advice based on the 40/10 harvest control rule (ABC in 2009) is highly uncertain, ranging from 5,000 mt to 553,000 mt (5$^{\mathrm{th}}$th and 95$^{\mathrm{th}}$ percentiles, Table 7). The median estimate for the 40/10 rule is 174,000 mt and the modal estimate is 113,000 mt. The marginal posterior samples for the 2009 ABC based on the 40/10 adjustment is highly skewed with a long tail and reflects the huge amount of uncertainty in the 2009 vulnerable biomass estimate.

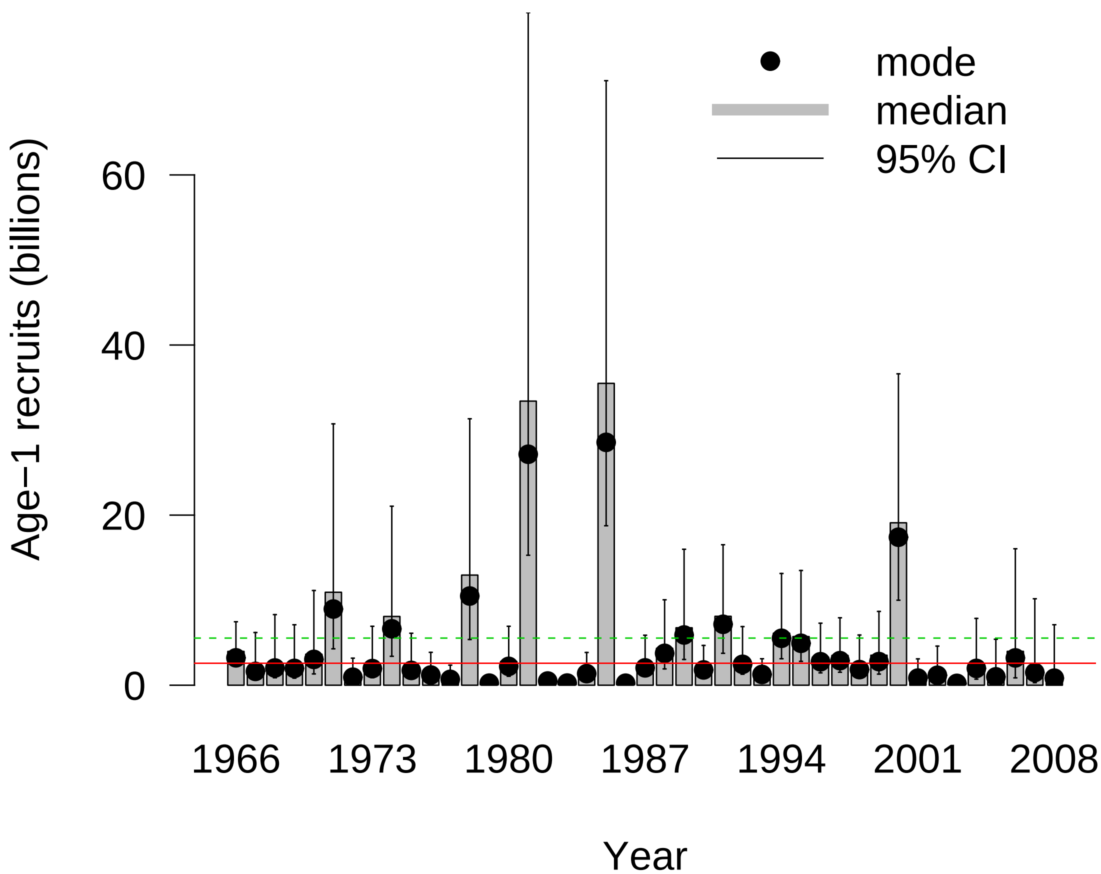

Figure 16: Median (bars) and maximum likelihood (circles) estimates of age-1 recruits, error bars represent the 0.025 and 0.975 quantiles based on 5,000 systematic samples from the joint posterior distribution. Long term average and median recruitment levels are shown as dashed and solid horizontal lines, respectively.

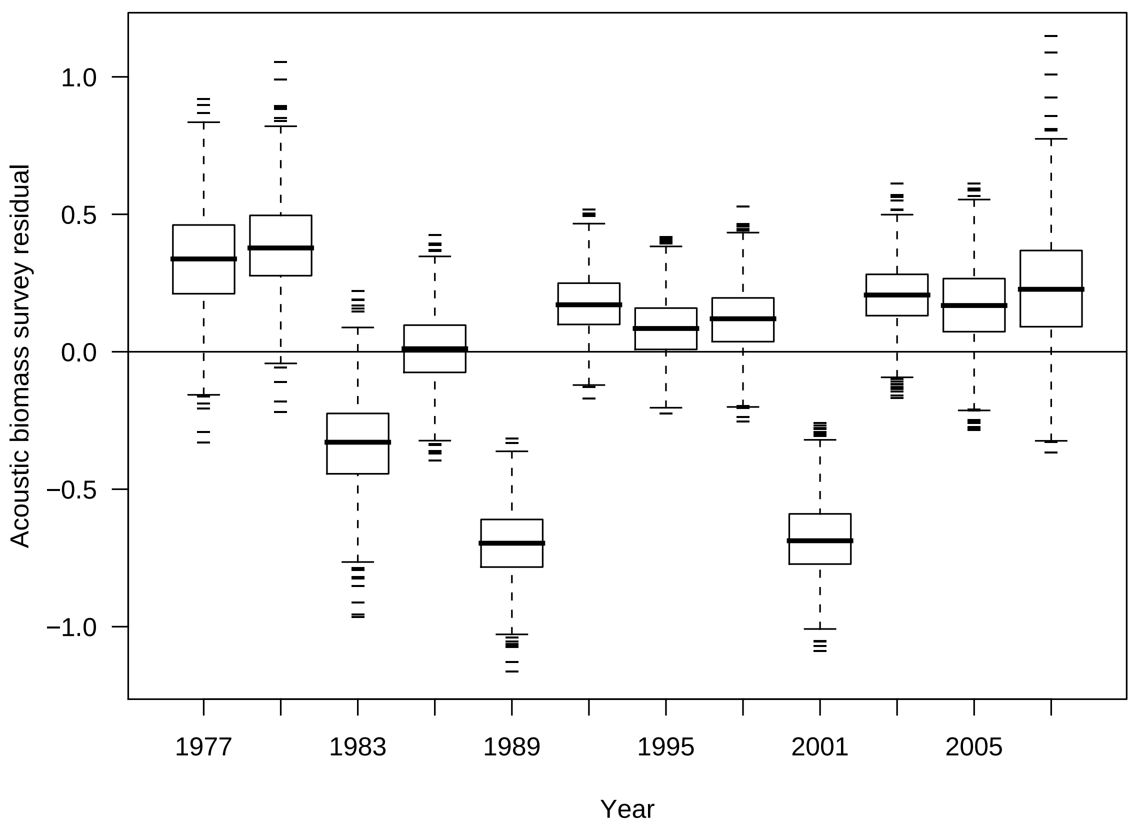

Figure 17: Boxplots of the marginal posteriors for the residuals in the acoustic survey.

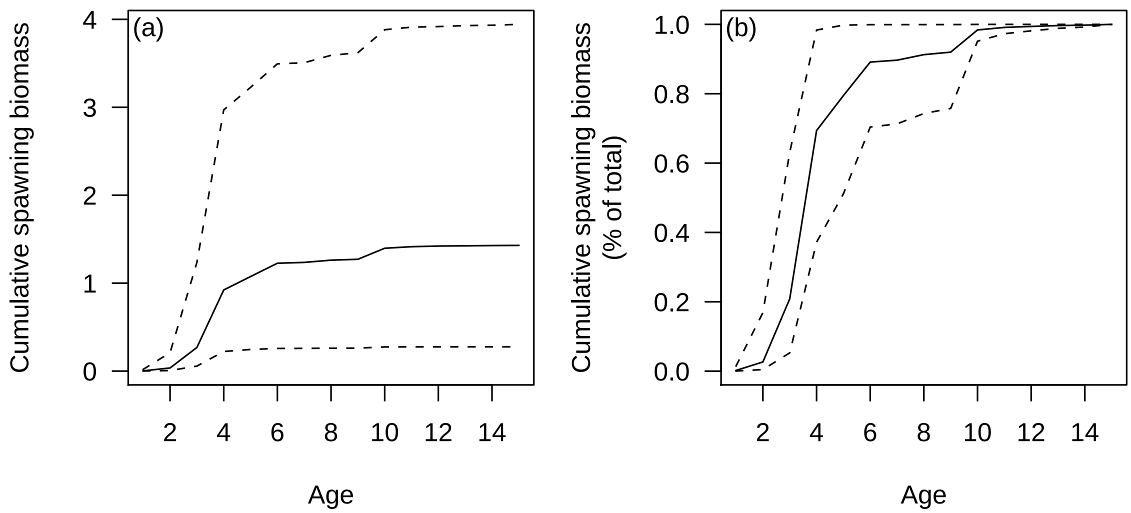

Figure 18: Cumulative spawning stock biomass at-age in 2009. Panel (a) is the cumulative total biomass where the solid line represents the median estimate, and the dashed lines represent the 0.025 and 0.975 quantiles. The cumulative spawning biomass-at-age relative to the total biomass is shown in panel (b).

3.2 Risk analysis

Five different criteria were examined in developing risk profiles for various catch options in 2009. The first criterion is the probability of the fishing mortality rate exceeding the estimated value of F^*(Fig. 19a). First, let 0.25, 0.5 and 0.75 probabilities represent definitions of risk averse, risk neutral, and risk prone, respectively. The risk averse ABC option for the 2009 fishing season based on exceeding the target fishing rate of $F^*$is 91,000 mt (Table 8). The risk neutral and risk prone ABC options are 174,000 and 256,000 mt, respectively. The second criterion is the probability of the spawning stock declining between 2009 and 2010 (Fig. 19b). Under this criterion the risk averse to risk prone ABC options are 0, 113,000 and 259,000 mt, respectively (Table 8 column 3). The third criterion examines the probability that the spawning stock biomass in 2010 will remain below the estimate of $\mathrm{SB}_{\mathrm{MSY}}$ (Fig 19c). Under this criterion the probability of the spawning stock falling below $\mathrm{SB}_{\mathrm{MSY}}$ is fairly high with no fishery (P=0.47); the risk neutral and risk prone policies call for ABCs of 35,000 and 457,000 mt (Table 8). The last two criteria criterion examines the probability that the spawning stock will fall below the management target SB_{40} and SB_{25} (Fig 19d). Under these criterion, the risk averse policy calls for 0 catch and 110,000 mt for the SB_{40} and SB_{25} policies, respectively.

In summary, catch options in excess of 200,000 mt result in a fairly significant probability of overfishing (P $\geq$ 0.5), further declines in spawning stock biomass over present levels, and a significant probability of reducing the spawning stock biomass below $\mathrm{SB}_{\mathrm{MSY}}$ (P $\geq$ 0.6). Catch options less than 200,000 mt reduce the odds of the spawning stock biomass falling below SB_{25} level (P $\leq$ 0.3).

3.3 Retrospective analysis

Retrospective analysis was conducted to examine the sensitivity of spawning biomass, fishing mortality rates and age-1 recruits to to the addition of new data (Figure 20). There is a slight retrospective bias in more recent years of spawning stock biomass; there is a downward bias in spawning biomass. For example as data are removed from estimates of spawning stock biomass in 2002 become smaller. As more data has accumulated the strength of the 1999 cohort continues to increase as indicated by the estimates of age-1 recruits in the year 2000 (Figure 20). Due to the fixed selectivity curve, it is possible that the strength of recent cohorts (e.g., 2005 cohort) could increase over time as these fish fully recruit to the fishing gear.

Retrospective estimates of unfished spawning stock biomass SB_o and the parameters that defined the underlying production function are relatively stable (Figure 21). The most recent maximum likelihood estimate of survey q is 0.57, and the largest retrospective estimate is 0.62 using data up to 2006 (Figure 21a). Survey q (which is a derived variable) is negatively correlated with unfished spawning stock biomass (Pearson $r = -0.56$) and positively correlated with survey selectivity parameters (Pearson $r = 0.61$). Thus, changes derived estimates of survey q are confounded with selectivity parameters and estimates of unfished biomass are relatively stable.

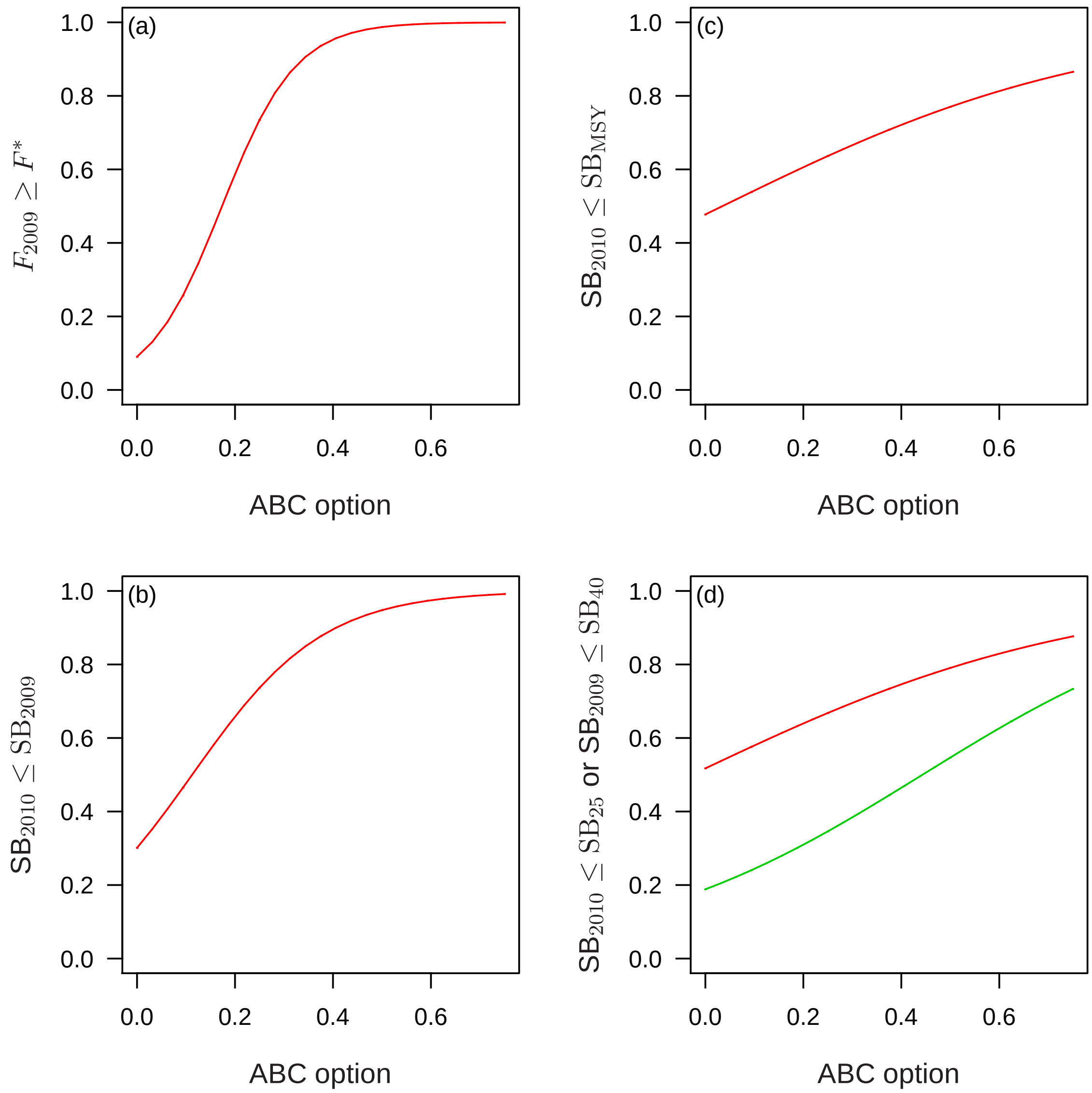

Figure 19: Probability of $F_{2009} > F^*$(panel a) versus ABC option, (b) probability of a decline in spawning biomass ($\mathrm{SB}_{2010} < \mathrm{SB}_{2009}$) versus ABC option, (c) probability of the SB_{2010} falling below $\mathrm{SB}_{\mathrm{msy}}$, and (d) probability of SB_{2010} falling below SB_{25} (bottom line) or SB_{40} (middle line) and the probability of the SB_{2010} is below SB_{2000} (top line) which corresponds to the lowest biomass estimate in previous assessments.

Table 8: Decision table for catch advice. The risk level represents the probability of exceeding a specified management target for a given ABC option. The interpretation of this table is as follows; if the management goal is not to exceed the target fishing mortality rate of F^* in 2009 with a 0.25 probability, then the ABC option should be set at 0.091 million mt or less. If the management target is prevent further decline in spawning stock biomass with a 0.5 probability then the ABC should be set at 0.113 million mt or less.

Risk level	$F_{09} \geq F^*$	$SB_{10} \leq SB_{09}$	$SB_{10} \leq SB_{MSY}$	$SB_{10} \leq SB_{40}$	$SB_{10} \leq SB_{25}$
0.05	0.000	0.000	0.000	0.000	0.000
0.10	0.009	0.000	0.000	0.000	0.000
0.15	0.043	0.000	0.000	0.000	0.000
0.20	0.069	0.000	0.000	0.000	0.023
0.25	0.091	0.000	0.000	0.000	0.110
0.30	0.110	0.000	0.000	0.000	0.186
0.35	0.127	0.030	0.000	0.000	0.255
0.40	0.143	0.058	0.000	0.000	0.320
0.45	0.159	0.086	0.000	0.000	0.382
0.50	0.174	0.113	0.035	0.000	0.443
0.55	0.189	0.139	0.112	0.052	0.504
0.60	0.204	0.167	0.191	0.133	0.566
0.65	0.220	0.195	0.273	0.218	0.631
0.70	0.237	0.226	0.361	0.309	0.700
0.75	0.256	0.259	0.457	0.409	0.776
0.80	0.278	0.298	0.568	0.523	0.864
0.85	0.304	0.344	0.702	0.661	0.969
0.90	0.339	0.406	0.880	0.845	1.110
0.95	0.395	0.506	1.167	1.141	1.336

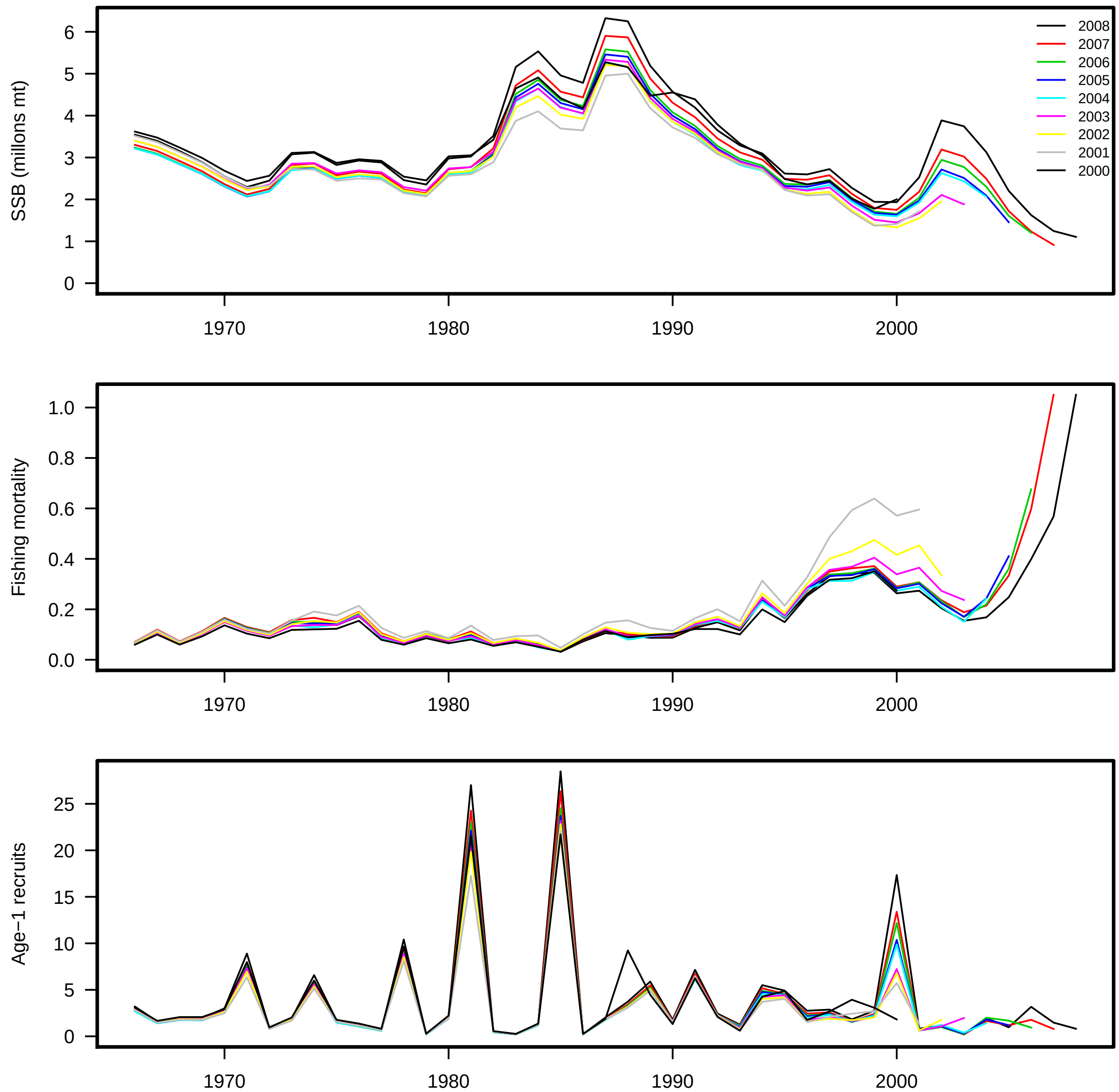

Figure 20: Retrospective maximum likelihood estimates of spawning stock biomass, instantaneous fishing mortality and age-1 recruits based on removal of data from 2008 to 2000.

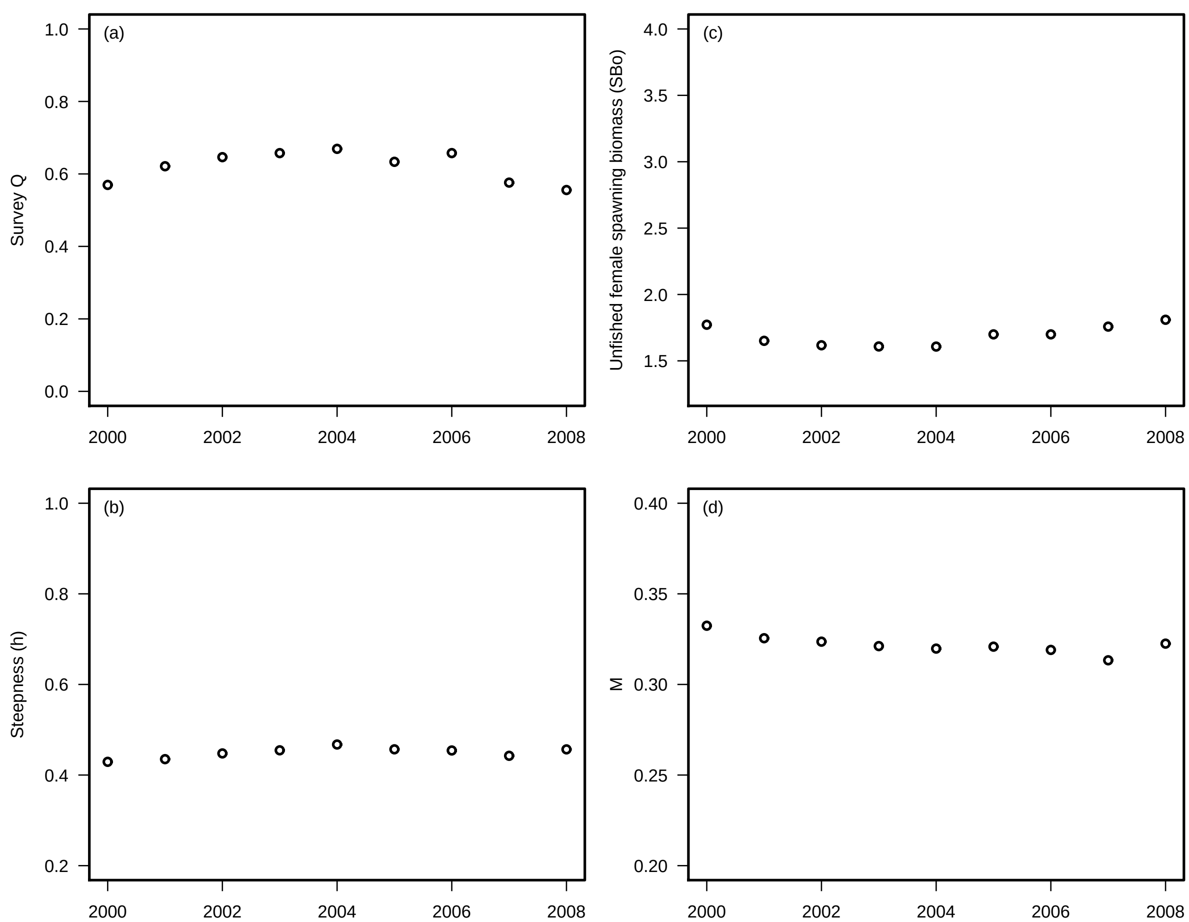

Figure 21: Retrospective maximum likelihood estimates of key parameters. Note that the y-axis for the unfished female spawning stock biomass spans the historical range of biomass estimates in 1966 from stock assessments dating back to 1991.

Table 9: Maximum likelihood estimates of unfished female spawning stock biomass (SB_o), C^*, instantaneous natural mortality rate (M) and Acceptable Biological Catch (ABC t) versus assumed expected value of ρ with a standard deviation equal to 0.1 in the prior distribution.

$E(\rho), \sigma_\rho = 0.1$	SB_o	C^*(million t)	M	40/10 ABC (mt)	ΔABC
0.1	1.79	0.224	0.33	117,089	3,291
0.2	1.78	0.223	0.33	113,798	-
0.3	1.74	0.227	0.33	98,646	(15,151)
0.4	1.69	0.210	0.33	79,179	(34,618)
0.5	1.65	0.203	0.32	62,130	(51,667)

3.4 Sensitivity to priors

3.4.1 Prior for ρ

In the previous assessment of TINSS (Martell, 2008) a major influence on the estimates of unfished biomass in 1966 was the relative weighting of the age-composition data and the assumed variances in the recruitment deviations and observation errors. The assessment herein makes fewer subjective assumptions about how much weight to place on the age-composition data, and the catch advice is partially influenced by the assumed prior distribution on the variance ratio ρ that partitions the total error in to observation and process error components. The assumed beta prior for ρ has an expected value of 0.2 (i.e., 20% of the total error is observation error), and a standard deviation of 0.1. As the assumed proportion of observation errors increases the overall catch advice decreases (Table 9). Estimated rate parameters (e.g., M and F^*) are relatively insensitive to the assumed prior distribution for ρ. The global scaling parameters (e.g., C^*) is somewhat sensitive to the assumed value of ρ; catch advice varies by less than 52,000 tons over a wide range of hypotheses about ρ.

3.4.2 Prior for F^*

I also examined the sensitivity of maximum likelihood estimates of the catch advice, based on the 40/10 adjustment, to alternative assumptions about the prior distribution for F^*(see Fig. 22). Increasing or decreasing the mean value for the F^*prior by 20% and maintaining the same standard deviation of the lognormal prior results in a ABC estimate that is roughly 16,000 mt higher or lower, respectively. Increasing the prior standard deviation from 0.262 to 0.5 results in a minor reduction of 1,500 mt. Overall, the catch advice is fairly robust to the specified prior distribution for F^*(Table 10).

3.4.3 Prior for C^*

Catch advice was slightly sensitive to the assumed mode of the prior distribution for C^*. As the mode of the prior distribution for $C^*$was decreased by 20% from 200,000 metric tons to 160,000 metric tons, the 2009 catch advice (maximum likelihood estimate of ABC based on the 40/10 rule) decreased from 113,000 tons to 93,000 tons (roughly and 18% decease in ABC). As the mode of the prior for $C^*$was increased by 20% to 240,000 metric tons, the catch advice increased by 11,000 metric tons (roughly a 10% increase in ABC, Table 11). Maximum likelihood estimates

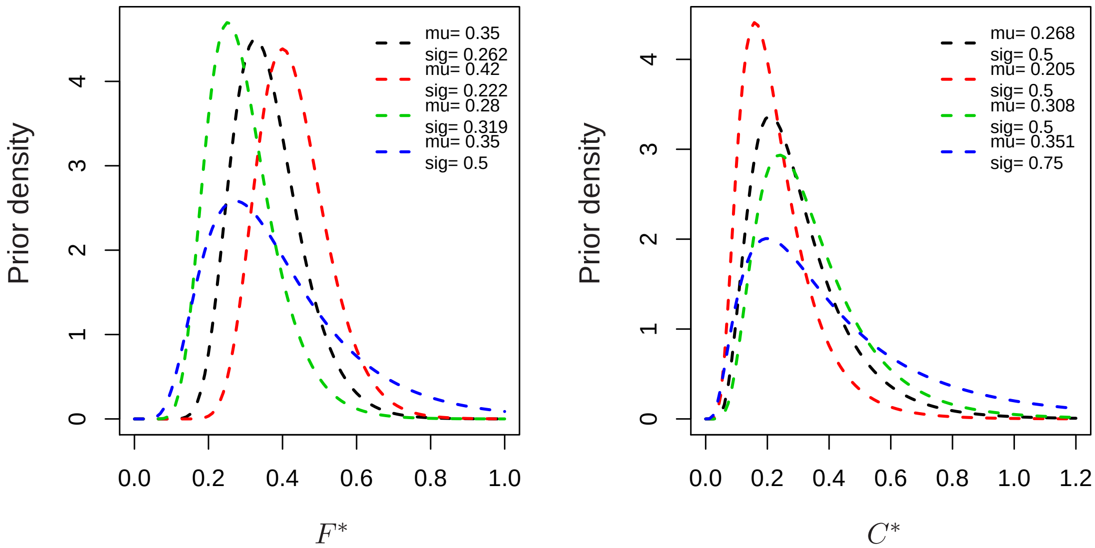

Figure 22: Alternative prior distributions for $F^*$and C^*in the sensitivity analysis presented in Tables 10 and 11. Note that the black distribution corresponds to the assumed distribution that was used to generate the catch advice.

Table 10: Sensitivity of catch advice (40/10 ABC in metric tons) to alternative prior distributions forF^*. Note the results here correspond to the MLE estimates.

Prior parameters						
μ	σ	C^*	F^*	M	40/10 ABC(mt)	ΔABC
0.35	0.262	0.223	0.347	0.333	113,798	-
0.28	0.319	0.202	0.284	0.336	97,745	(16,053)
0.42	0.222	0.242	0.413	0.331	129,824	16,026
0.35	0.5	0.221	0.340	0.333	112,209	(1,589)

Table 11: Sensitivity of catch advice (40/10 ABC in metric tons) to alternative prior distributions for C^*. Results correspond to the maximum likelihood estimates.

Prior parameters							
mode	μ	σ	C^*	F^*	M	ABC	∆ABC
0.208	0.256	0.5	0.223	0.347	0.333	113,798	
0.16	0.205	0.5	0.211	0.335	0.330	93,614	(20,184)
0.24	0.308	0.5	0.230	0.353	0.334	125,588	11,790
0.2	0.351	0.75	0.224	0.348	0.333	115,452	1,654

of C^*were also sensitive to the mode of the prior distribution, but estimates of $F^*$and M were relatively insensitive.

3.4.4 Prior for M

Management advice and the global scaling are extremely sensitive to the assumed prior value for the instantaneous natural mortality rate. There is a fairly strong positive correlation between M and $C^*$and virtually no correlation between M and F^*(Table 6). As the mean of the prior for M increases, the overall scaling of the population increases along with the catch advice. For example changing the mean of the prior for M from 0.23 to 0.28 results in an increase in $C^*$from 223,000 mt to 260,000 mt. The catch advice for 2009 increases from 113,000 mt to 217,000 mt. Reducing the standard deviation for the prior on M from 0.1 to 0.05 results in a overall reduction in $C^*$from 223,000 mt to 177,000 mt, and the catch advice based on the 40/10 adjustment is 0 mt because the estimated depletion level is 9%.

4 Discussion

Uncertainty in previous assessments of Pacific hake was under-represented due to the use of assumed fixed values for the steepness of the stock recruitment relationship and survey catchability coefficients. This assessment attempts to integrate over this uncertainty by using less informative prior information for these key parameters. The relative abundance indices alone lack sufficient information to resolve confounding between the global scaling and stock productivity. Addition of the age-composition information further confounds this problem because there appears to be some conflict between expected trends in abundance due to the exceptional 1980 and 1984 cohorts and the downward trend in abundance between the 1986 and 1989 survey points. Helser et al. (2008) also reported similar contradictions in the age composition information between the US and Canadian fishery as well as the fisheries independent survey. Previous assessments have omitted the 1986 survey due to pre- and post-survey calibration problems. However, it appears that the 1986 survey point is consistent with trends inferred from the age-composition data, but the 1989 survey point is inconsistent with these trends. Also, the 2001 survey points is considerably low relative to estimated trends in abundance.

In the previous assessment by Martell (2008), the catch advice was extremely sensitive to the relative weighting of the age-composition information. Minor changes in the assumed effective sample size (e.g., from 10 to 33) resulted in a near doubling of the catch advice (e.g., 142,000 mt to 305,000 mt). In this assessment, I've attempted to remove this subjectivity by using a less informative likelihood for the age-composition data, where the conditional maximum likelihoods of the variance terms are used to weight the age-composition information (see Schnute and Richards, 1995, for more details on this method). The standard procedure of using the mulitinomial distribution and iterative re-weighting procedures (as described in McAllister and Ianelli, 1997) for weighting age-composition fails in cases where there is complete contradictions in 2 or more independent sets of proportion-at-age data. When independent sets of age-composition information are contradictory, the iterative re-weighting procedure fails to converge to an effective sample size.

It is clear that there have been changes in selectivities over time for the commercial gears in the two different countries. Evidence for this is not hard to find; for example, interannual variation in northward migration has profound effects of selectivity, age-specific estimates of F continue to increase for strong cohorts in the VPA models (Sinclair and Grandin, 2008). Treating the selectivity curves as constant over time (whether or not a logistic or dome-shaped selectivity curve is assumed) will obviously affect estimates of relative cohort strengths. Under the mulitinomial likelihood of last years assessment, down weighting the age-composition data was necessary to reduce the amount of retrospective bias, but this down weighting was completely subjective. The multivariate logistic model is much more robust to weighting problems as this likelihood kernel can be evaluated at its conditional maximum likelihood estimate of the variance; this is also known as a concentrated likelihood (Harvey, 1990).

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A Input data

Table 12: Combined historical landings (mt) for the U.S. and Can. fisheries, mean age of the catch, and survey abundance indices (millions mt) from the acoustic-trawl survey.

Year	C_t	$\bar{a}$	I_t	Year	C_t	$\bar{a}$	I_t
1966	137700			1988	248804	6.68	
1967	214375			1989	305916	6.76	1.238
1968	122180			1990	259792	6.78	
1969	180131			1991	307258	7.20	
1970	234584			1992	296910	7.00	2.169
1971	154612			1993	199435	6.77	
1972	117546			1994	361529	7.75	
1973	162639			1995	249770	7.92	1.385
1974	211259			1996	306075	6.59	
1975	221360			1997	325215	5.86	
1976	237521			1998	320619	5.77	1.185
1977	132693	6.24	1.915	1999	311855	5.41	
1978	103639	6.59		2000	230820	6.25	
1979	137115	6.52		2001	235962	5.05	0.737
1980	89936	6.74	2.115	2002	182911	4.69	
1981	139121	6.09		2003	205582	4.86	1.840
1982	107734	6.50		2004	334672	5.37	
1983	113924	5.68	1.647	2005	359661	6.18	1.265
1984	138441	5.67		2006	360683	6.41	
1985	110401	5.78		2007	297098	6.63	0.879
1986	210617	5.92	2.857	2008	322017	5.96	
1987	234147	6.33					

Table 13: Age-composition (reported in percentages) of the combined U.S. and Can. commercial catch from 1977-2007. Age-15 represents a plus group.

Year	age.2	age.3	age.4	age.5	age.6	age.7	age.8	age.9	age.10	age.11	age.12	age.13	age.14	age.15
1977	2.50	2.69	26.05	4.81	9.66	34.25	8.02	5.13	3.26	2.04	1.11	0.41	0.05	0.03
1978	0.24	4.97	5.95	30.66	5.78	13.07	26.26	5.94	3.69	2.10	0.73	0.42	0.18	0.03
1979	3.33	7.13	12.15	6.13	26.31	7.70	16.73	13.04	3.55	2.13	0.85	0.48	0.28	0.19
1980	0.55	22.96	4.64	7.49	7.58	18.46	7.57	10.17	13.21	3.22	1.83	1.41	0.50	0.40
1981	8.89	2.38	39.16	2.11	5.45	5.36	15.13	5.11	5.44	8.16	1.55	0.60	0.57	0.09
1982	14.03	2.14	1.67	37.90	3.92	4.96	4.78	13.05	2.75	2.99	10.08	0.87	0.54	0.32
1983	0.03	37.00	3.97	2.46	33.15	3.26	3.06	3.48	7.15	1.66	1.17	3.06	0.40	0.16
1984	0.00	0.93	54.74	3.71	7.42	19.67	2.63	2.00	1.62	3.67	0.74	0.85	1.73	0.29
1985	4.66	0.54	6.89	54.27	7.32	6.13	14.19	1.46	0.85	1.33	1.26	0.23	0.00	0.87
1986	15.27	4.19	0.87	3.44	51.77	6.21	4.18	9.22	1.31	1.08	0.67	1.09	0.18	0.53
1987	0.00	27.64	1.64	0.39	1.68	51.92	3.36	1.56	9.10	0.40	0.19	0.43	1.21	0.48
1988	0.69	0.60	38.51	1.39	0.80	1.11	45.35	1.99	0.72	7.20	0.13	0.16	0.06	1.30
1989	3.55	3.53	1.52	45.65	1.05	0.44	0.60	37.41	1.49	0.59	3.60	0.09	0.07	0.42
1990	1.55	20.50	2.59	0.44	39.05	0.65	0.25	0.20	31.10	0.36	0.00	3.06	0.01	0.24
1991	0.62	10.75	18.24	3.25	0.96	37.33	1.40	0.13	0.15	21.55	0.51	0.00	3.89	1.22
1992	4.21	4.10	13.53	21.97	2.51	1.18	34.73	0.74	0.13	0.21	15.40	0.20	0.04	1.05
1993	0.43	22.43	3.25	14.46	17.50	1.49	0.79	28.17	0.72	0.05	0.05	9.93	0.06	0.67
1994	0.04	3.31	20.15	1.23	13.10	20.07	1.21	0.43	30.01	0.20	0.43	0.03	9.06	0.73
1995	4.26	0.20	6.86	25.72	1.25	7.88	19.30	1.79	0.31	23.66	0.37	0.26	0.02	8.11
1996	17.60	14.76	1.09	8.82	18.33	1.03	5.65	11.15	0.66	0.34	16.63	0.01	0.11	3.81
1997	0.44	32.38	23.09	1.13	6.71	13.17	1.86	3.73	6.93	1.14	0.14	6.49	0.66	2.12
1998	5.46	19.11	16.70	25.22	2.71	5.39	10.92	1.17	1.79	5.15	0.58	0.13	4.82	0.87
1999	8.76	20.68	17.76	19.25	11.80	2.53	4.45	4.81	0.94	1.66	2.97	0.66	0.85	2.90
2000	3.99	11.01	15.66	14.45	19.49	11.00	7.38	5.39	1.89	1.87	2.16	1.31	1.07	3.34
2001	13.94	19.67	14.09	17.52	8.97	12.05	5.85	1.59	1.70	1.69	1.03	0.90	0.00	1.01
2002	0.05	46.94	17.03	10.23	6.78	4.90	6.29	3.81	0.89	0.66	0.98	0.11	0.40	0.95
2003	0.14	1.55	68.57	11.69	3.13	5.11	3.00	3.12	1.78	0.83	0.24	0.48	0.08	0.28
2004	0.00	6.34	6.67	66.37	8.25	2.33	4.17	2.60	1.39	1.02	0.32	0.24	0.15	0.15
2005	1.27	0.46	7.22	5.45	67.17	8.69	2.45	2.95	2.20	0.93	0.78	0.22	0.04	0.18
2006	2.88	11.36	1.61	8.71	4.77	59.43	4.83	1.68	1.80	1.21	0.85	0.46	0.15	0.23
2007	11.06	3.01	14.27	1.56	7.45	4.55	46.09	5.91	1.95	1.87	1.30	0.41	0.41	0.15
2008	5.11	33.47	3.58	12.82	0.94	3.37	3.31	31.20	2.85	1.17	0.87	0.58	0.38	0.36

Table 14: Age-composition (percent) from acoustic surveys from 1977-2007. Note that these data are the conditional age-length data multiplied by the length frequencies and collapsed over the size intervals and represent a summary of the conditional age-length data (age 15 represents a plus group).

Year	age.2	age.3	age.4	age.5	age.6	age.7	age.8	age.9	age.10	age.11	age.12	age.13	age.14	age.15
1977	5.31	4.41	23.03	2.71	4.68	39.08	7.21	5.10	3.84	2.45	1.35	0.55	0.17	0.11
1980	0.16	27.80	2.84	5.60	4.84	23.14	6.23	16.63	6.84	3.84	0.92	0.78	0.18	0.20
1983	0.36	64.90	1.50	1.25	20.05	1.75	2.17	1.92	3.25	1.15	0.87	0.70	0.14	0.00
1986	40.10	1.29	0.54	2.28	41.70	4.55	2.85	5.02	0.52	0.49	0.13	0.43	0.06	0.02
1989	7.25	2.35	0.79	56.08	1.15	0.67	0.94	27.39	1.18	0.16	1.87	0.00	0.00	0.17
1992	10.21	1.73	9.12	19.69	2.37	0.86	38.46	1.29	0.67	0.34	13.89	0.67	0.00	0.71
1995	33.02	4.07	1.25	20.71	1.08	3.73	14.85	0.31	0.00	15.78	0.04	0.72	0.00	4.46
1998	13.50	19.82	15.12	18.89	1.54	4.37	10.21	1.64	0.94	6.31	0.14	0.55	5.08	1.89
2001	69.78	10.41	5.79	5.42	2.57	2.49	1.52	0.50	0.52	0.34	0.21	0.20	0.05	0.21
2003	3.01	2.53	64.05	10.95	2.75	6.01	3.96	2.20	2.23	0.73	0.43	0.44	0.31	0.42
2005	21.57	2.27	7.24	5.30	50.03	5.49	1.86	2.61	1.48	1.17	0.49	0.27	0.04	0.19
2007	35.45	2.39	10.19	1.18	4.57	3.01	33.88	3.62	1.74	1.71	0.92	0.80	0.37	0.17

Table 15: Assumed mean weights-at-age in the commercial catch.

Year	age 2	age 3	age 4	age 5	age 6	age 7	age 8	age 9	age 10	age 11	age 12	age 13	age 14	age 15
1966	0.258	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1967	0.258	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1968	0.259	0.428	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.155	1.213	1.269	1.590
1969	0.258	0.429	0.527	0.606	0.681	0.762	0.837	0.935	0.988	1.079	1.154	1.212	1.269	1.591
1970	0.256	0.428	0.527	0.606	0.680	0.763	0.837	0.935	0.989	1.079	1.155	1.213	1.269	1.589
1971	0.261	0.428	0.527	0.606	0.682	0.762	0.838	0.936	0.988	1.079	1.156	1.213	1.269	1.591
1972	0.256	0.431	0.527	0.606	0.680	0.761	0.837	0.935	0.987	1.077	1.153	1.211	1.267	1.592
1973	0.251	0.423	0.526	0.606	0.680	0.765	0.836	0.935	0.991	1.081	1.155	1.214	1.270	1.582
1974	0.277	0.431	0.528	0.606	0.685	0.760	0.841	0.937	0.987	1.079	1.159	1.215	1.271	1.600
1975	0.241	0.438	0.527	0.605	0.676	0.759	0.833	0.932	0.983	1.073	1.145	1.204	1.261	1.593
1976	0.235	0.400	0.524	0.608	0.679	0.775	0.835	0.936	1.002	1.093	1.162	1.223	1.277	1.554
1977	0.354	0.455	0.533	0.605	0.700	0.748	0.853	0.944	0.974	1.070	1.168	1.218	1.275	1.653
1978	0.135	0.460	0.523	0.600	0.649	0.754	0.812	0.915	0.973	1.055	1.106	1.170	1.231	1.573
1979	0.217	0.287	0.515	0.619	0.686	0.822	0.841	0.951	1.060	1.154	1.211	1.282	1.327	1.435
1980	0.279	0.407	0.487	0.624	0.684	0.796	0.850	0.877	1.010	1.066	1.184	1.163	1.233	1.196
1981	0.123	0.328	0.491	0.619	0.725	0.776	0.816	0.864	0.884	1.043	1.189	1.245	1.213	1.385
1982	0.235	0.389	0.503	0.604	0.688	0.839	0.873	0.907	0.934	1.029	1.049	1.132	1.209	1.095
1983	0.264	0.355	0.428	0.563	0.631	0.742	0.827	0.855	0.883	0.969	0.994	0.941	1.155	1.095
1984	0.215	0.393	0.429	0.531	0.669	0.699	0.796	0.873	0.894	0.953	1.104	0.965	1.008	1.100
1985	0.181	0.316	0.455	0.526	0.639	0.740	0.813	0.979	0.914	1.020	1.035	1.156	1.074	1.067
1986	0.273	0.314	0.426	0.537	0.562	0.633	0.724	0.821	0.921	0.992	0.989	1.102	1.048	1.086
1987	0.232	0.374	0.421	0.499	0.629	0.626	0.683	0.746	0.799	0.903	0.895	1.023	0.950	1.049
1988	0.264	0.357	0.443	0.461	0.598	0.591	0.628	0.687	0.775	0.809	0.895	0.998	0.993	1.026
1989	0.226	0.317	0.367	0.502	0.531	0.617	0.656	0.670	0.717	0.789	0.896	0.860	1.052	1.030
1990	0.272	0.379	0.443	0.531	0.568	0.617	0.604	0.604	0.701	0.749	0.822	0.880	1.002	1.052
1991	0.229	0.341	0.449	0.543	0.554	0.641	0.716	0.599	0.885	0.728	0.724	0.854	0.952	1.060
1992	0.248	0.338	0.458	0.525	0.582	0.598	0.638	0.638	0.612	0.679	0.698	0.851	0.716	0.931
1993	0.263	0.343	0.426	0.502	0.560	0.593	0.547	0.638	0.645	0.704	0.931	0.679	0.798	0.756
1994	0.335	0.344	0.424	0.510	0.552	0.608	0.694	0.620	0.689	0.636	0.739	0.812	0.725	0.794
1995	0.114	0.515	0.484	0.511	0.625	0.623	0.679	0.706	0.713	0.724	0.661	0.892	0.711	0.771
1996	0.271	0.379	0.462	0.547	0.565	0.628	0.621	0.663	0.712	0.736	0.705	0.553	1.092	0.724
1997	0.328	0.409	0.472	0.519	0.615	0.620	0.601	0.692	0.665	0.741	0.732	0.743	0.696	0.813
1998	0.234	0.350	0.458	0.497	0.518	0.587	0.598	0.619	0.637	0.651	0.775	0.638	0.735	0.734
1999	0.243	0.318	0.417	0.538	0.554	0.578	0.625	0.661	0.672	0.748	0.727	0.746	0.661	0.786
2000	0.282	0.424	0.496	0.564	0.647	0.677	0.658	0.740	0.719	0.818	0.746	0.835	0.786	0.820
2001	0.289	0.454	0.599	0.608	0.681	0.778	0.780	0.806	0.854	0.832	0.831	0.901	0.863	0.962
2002	0.310	0.413	0.558	0.752	0.702	0.812	0.916	0.885	0.885	0.927	0.893	1.064	1.002	1.100
2003	0.304	0.380	0.469	0.573	0.664	0.659	0.679	0.732	0.709	0.766	0.752	0.709	0.827	0.941
2004	0.241	0.419	0.489	0.550	0.625	0.709	0.691	0.713	0.757	0.765	0.742	0.880	0.928	0.836
2005	0.333	0.426	0.497	0.550	0.573	0.611	0.647	0.693	0.679	0.728	0.721	0.803	0.629	0.761
2006	0.251	0.418	0.497	0.552	0.584	0.607	0.646	0.786	0.745	0.798	0.838	0.868	0.802	0.805
2007	0.241	0.408	0.512	0.580	0.618	0.639	0.641	0.697	0.779	0.743	0.776	0.796	0.805	0.863
2008	0.211	0.366	0.516	0.592	0.646	0.671	0.692	0.719	0.759	0.842	0.802	0.795	0.800	0.789

B Model description and documentation

The stock assessment model used herein consists of 4 major components: 1) a component for initializing the model based on steady-state conditions, 2) a component for updating the state variables, 3) a component that relates the state variables to observations on relative abundance and composition information, and 4) a statistical criterion for evaluating how likely these data are for a given set of model parameters. We have broken the description of the assessment model into these four components and use a series of tables to document model equations. Symbols and their definitions are defined in Table 16; furthermore, we have divided the estimated parameter set into life-history parameters Φ and population parameters Θ for clarity.

I have adopted a management oriented approach tho the parameterization of the age-structured model where the leading parameters that define population scale and productivity correspond to MSY (hereafter C^*) and Fmsy (hereafter F^*). The basic idea here is to change the question to how likely are the data given $C^*$and $F^*$and derive the corresponding B_o and slope of the stock recruitment relationship rather than the traditional approach of estimating these values directly. There are a few statistical advantages of using this approach (i.e., reduced confounding between the leading parameters Schnute and Richards, 1998), but perhaps the biggest advantage is to increase the transparency by which the application of informative priors influence model results (Martell et al., 2008).

Table 16: Description of symbols and indices used in TINSS

Symbol	Description
Indices	
i, j, k, l	index for age,year, fleet, and size interval
Estimated population parameters (Θ)	
F^*	Optimal fishing mortality rate
C^*	Maximum sustainable yield
M	Instantaneous natural mortality rate
a_{h_k}	Age at 50% selectivity
γ_k	Standard deviation in selectivity
Estimated life-history parameters (Φ)	
l_∞	mean asymptotic length
k	growth coefficient
t_o	age at 0 length
a, b	parameters for length-weight relationship
λ_1, λ_2	parameters for standard deviation in length-at-age
Derived variables	
B_o	unfished steady-state biomass
κ	recruitment compensation ratio (Goodyear, 1980)
R_e	equilibrium age-1 recruitment
$\iota_i, \hat{\iota}_i$	survivorship to age i, unfished and fished
ϕ_E, ϕ_e	eggs per recruit, unfished and fished
ϕ_B, ϕ_b	vulnerable biomass per recruit, unfished and fished
ϕ_q	vulnerable biomass available to the fishery

B.1 Model initialization

To initialize the model, we must first derive B_o and κ from $C^*$and F^*as well as other life-history parameters Φ and the vulnerability schedule. In other words, first we must transform the management parameters $C^*$and F^*into population parameters B_o and κ. This transformation starts with the equilibrium yield equation (e.g. Fig 23a), differentiating this function with respect to F_e, setting this equation equal to 0 and solving for κ (for the full derivation see Martell et al., 2008). Next substitute κ back into the equilibrium recruitment equation to obtain estimates of the unfished biomass B_o.

An alternative way to envision this transformation is to think about it graphically. For any given model (e.g., a simple production model or a complex age-structure model) we can derive a system of equation that results in the equilibrium yield for any specified equilibrium fishing mortality rate. This same system of equations can also be used to derived equilibrium values of recruitment (e.g., Fig 23b), equilibrium biomass (e.g., Fig 23c) and the spawners per recruit (Fig. 23d). The traditional approach would then differentiate the catch equation with respect to F_e, solve this expression for F_e to determine the corresponding value of F^*, then substitute the corresponding F^*into the catch equation and calculate C^*conditional on estimates of B_o and κ. What differs in the management oriented approach is that we estimate $C^*$and F^*directly and then derive B_o and κ conditional on the estimates of $C^*$and F^*.

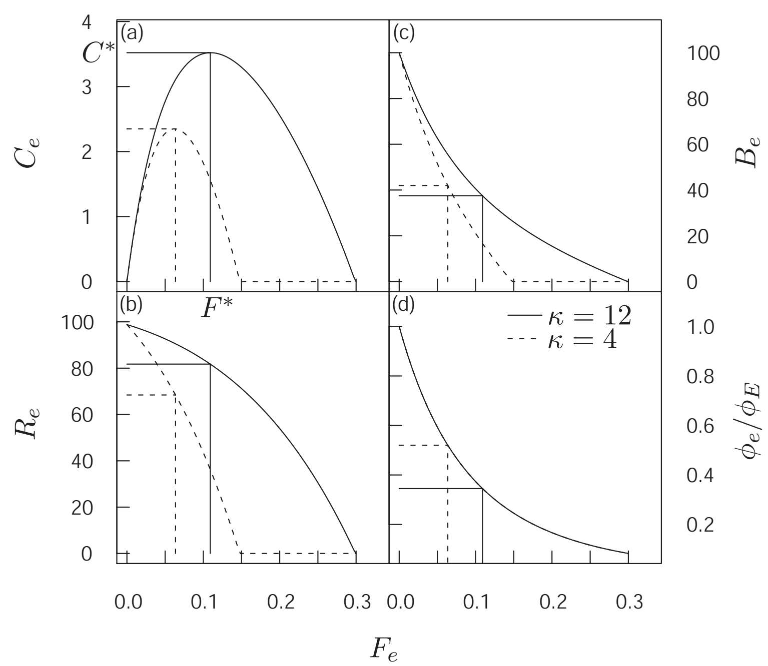

Figure 23: Relationship between equilibrium values for yield (a), recruitment (b), biomass (c) and spawners per recruit (d) versus instantaneous fishing mortality rate for a hypothetical stock with high ($\kappa = 12$) and low ($\kappa = 4$) recruitment compensation parameters.

The system of equation used to derive B_o and κ are laid out in Table 17. The purpose of laying out the equations in a tabular format is two fold, 1) documentation of the model structure and 2) to provide an algorithm or pseudo code in which to implement the model. First given initial estimates of the life-history parameters Φ (T17.2), calculate the corresponding age-schedule information (T17.3)–(T17.6). Note that this does not assume that growth or maturity is constant over time, only that some average, or steady state, growth occurred for the cohorts that are used to initialize the numbers-at-age. Next, calculate the survivorship (T17.7) of an individual recruit based on the instantaneous natural mortality rate M. These survivorship functions (T17.7) and (T17.8) are used to calculate the per recruit incidence functions for unfished and fished conditions, respectively. An incidence function is the sum of age-specific schedules that express the population units on a per recruit basis. For example the total biomass per recruit is given by (T17.10) and the total unfished biomass is the product $R_o\phi_E$. For notational purposes the prefix ϕ denotes an incidence function and the corresponding subscript denotes the type of incidence function (see Table 16 for definitions); we also use upper and lower case subscripts to denote unfished and fished conditions, respectively.

The eggs per recruit for unfished and fished conditions are defined by (T17.9), the biomass per recruit by (T17.10), and the vulnerable biomass per recruit available to the fishery is defined by (T17.11). Note that we assume both natural and fishing mortality operate simultaneously and ϕ_q represents the Barnov catch equation. To derive κ, we differentiate

$$C_e = F_e R_e \phi_q \tag{1}$$

with respect to F_e and solve this equation for κ. Using the chain rule, the derivative of (1) is

$$\frac{\partial C_e}{\partial F_e} = R_e\phi_q + F_e\phi_q\frac{\partial R_e}{\partial F_e} + F_e R_e\frac{\partial \phi_q}{\partial F_e} \tag{2}$$

To derive the recruitment compensation parameter (T17.12) it is necessary to substitute (T17.11) and (T17.13) into (2), set the corresponding expression equal to zero and then solve for κ. The partial derivatives for (T17.12) are defined in Table 18. Equation (T17.13) is the equilibrium recruits that corresponds to the equilibrium fishing mortality rate F_e and (T17.14) corresponds to the unfished biomass.

B.1.1 Initialization with multiple fleets

Although the catch data are aggregated into a single fleet for this assessment, the following describes an algorithm for implementing the management oriented approach for multiple fleets that have different age-specific fishing mortality rates. In essence, the algorithm derives F-multipliers for each fleet.

The catch equation (1) considers a single fishery with a unique vulnerability-at-age curve. In the case of multiple fisheries with different vulnerability-at-age curves, it is necessary to allocate the proportion of the total fishing mortality (F^*) to each fleet such that the sum of catches from each fleet is equal to C^*. For example, consider two separate fishing fleets A and B and assume that fleet A harvest younger fish that fleet B and that the allocation of $C^*$is assigned equally to each fleet. In this case a higher proportion of F^*would be assigned to fleet B because this fleet harvest fewer, older fish, in comparison to fleet A which harvest more abundant younger fish. Thus, if some sort of allocation agreement exists between two or more fleets, a multiplier on the fishing mortality rate must be used to allocate the total catch among these fleets. For a given allocation arrangement (e.g., where the fraction of C^*assigned to fleet k is denoted as Λ_k), the equilibrium catch of fleet k can be represented as:

$$\Lambda_k C^* = \tau_k F^* R_e \phi_q^{(k)} \tag{3}$$

where τ_k is the fleet specific multiplier on F^*, R_e is defined in (T17.13), and $\phi_q^{(k)}$ is the fleet specific vulnerable biomass per recruit which is defined as

$$\begin{aligned}
\phi_q^{(k)} &= \sum_i \frac{\hat{\iota}_i w_i v_{i,k}}{Z_i} \left(1 - e^{-Z_i}\right), \\
\text{where} \quad Z_i &= M + F^* \sum_k \tau_k v_{i,k}, \\
\hat{\iota}_i &= \begin{cases} 1 & i = 1 \\ \hat{\iota}_{i-1} e^{-Z_{i-1}} & i > 1. \end{cases}
\end{aligned} \tag{4}$$

Note that τ_k appears multiple times in (4) in the Z_i and $\hat{\iota}_i$ terms, as well as the derivation of R_e (see eq. T17.13), and there is no analytical solution for τ_k (at least that we could find using symbolic math languages). Therefore, τ_k must be solved for iteratively. Solving (3) for τ_k results in an update of τ_k:

$$\tau_k = \frac{\Lambda_k C^*}{R_e F^* \phi_q^{(k)}} \tag{5}$$

A simple algorithm to numerically calculate τ_k proceeds as follows

1. set initial values of the fishing multiplier equal to the allocation proportion: $\tau_k = \Lambda_k$ (Note that if the vulnerability-at-age curves are the same for each fleet, then τ_k is exactly equal to Λ_k, i.e., the vulnerable biomass per recruit is the same for all fleets).

2. calculate the age-specific total mortality rates for all fleets combined

$$Z_i = M + F^* \sum_k \tau_k v_{i,k}.$$

3. calculate survivorship ($\hat{\iota}_i$), and per-recruit incidence functions that lead to R_e (eqs. T17.8–T17.13) based on the age-specific total mortality rate in step 2.

4. for each fleet k, calculate the vulnerable biomass per-recruit ($\phi_q^{(k)}$) using (4).

5. update τ_k using (5), and repeat steps 2-5 until estimates of τ_k converge (Note this take 6-20 iterations depending on how different the vulnerability-at-age curves are for each fleet.

6. Check that the sum catches for each fleet equal C^*.

The algorithm outline above is based on the allocation arrangement among the various fleets (Λ_k) and is not intended to optimize the allocation arrangement based on differences in vulnerability among the various fishing fleets. This is an entirely different policy issue that is not addressed here. If there is no formal allocation arrangement, then historical catch proportions to each fleet could be used as a starting point for values of Λ_k. Recall, that the approach adopted here is to simple express the population parameters B_o and κ as analytical functions of management parameters $C^*$and F^*.

B.2 Updating state variables

Equations used to update the state variables are defined in Table 19. We aggregate the catch data from the CAN and US fisheries into a single catch time series (T19.1) and treat both fisheries as a single fishery with the same selectivity pattern over time. This data simplification reduces the number of estimated parameters but further assumes that the relative mortalities imposed by the two different fisheries has been constant over time. We also aggregate the catch-age samples from the commercial fisheries ($A_{i,j}$) into a single catch age matrix. Catch-age data for the US portion of the fishery are available back to 1976, and age-composition information for the CAN portion of the fishery are available back to 1988. The age-compositions were combined from 1988 to 2006 using a weighted average, where the weights are the proportions landed by each nation.

Process errors are represented as a vector of annual recruitment deviations ω_j which are assumed to be lognormal with an estimated variance τ^2. These annual deviations are estimated parameters and included in the objective function calculation with a bias correction term for the log-normal distribution (T20.1).

The relative abundance data (I_j) corresponds to the abundance index derived from the acoustic surveys, and here we assume these indices are proportional to abundance and use the conditional maximum likelihood estimate of the scaling parameter in the calculation of the residuals (T19.13). I assume that observation errors in the acoustic survey data are lognormal and the likelihood function for acoustic survey data are given by (T20.2).

Residuals between the observed proportions and predicted proportions-at-age for each fleet (the joint US and CAN fleet and the fisheries independent surveys) were assumed to come from a multivariate logistic distribution. Age composition information are generally thought to arise from a multinomial distribution where the probability of sampling a fish of a given age is conditioned on the product of proportions-at-age in the population and the probability of sampling a fish age-i given the sampling gear. However, the multinomial likelihood kernel generally results in errors that are unrealistically small due to the large samples taken for ageing (Schnute and Richards, 1995). The advantage of the multivariate logistic distribution is that the likelihood kernel can be weighted by the conditional maximum likelihood estimate of the variance; this is given by the mean squared error of the residual terms $\eta_{i,j,k}$ for each fleet k. The likelihood of the age composition information for both fleets k (commercial and acoustic survey) is given by (T20.3).

Table 17: Steady-state age-structured model assuming unequal vulnerability-at-age, age-specific natural mortality, age-specific fecundity and Beverton-Holt type recruitment.

Parameters

$$\Theta = (C^*, F^*, M, \hat{a}, \hat{\gamma}); \quad C^* > 0; F^* > 0; M > 0 \tag{T17.1}$$

$$\Phi = (l_\infty, k, t_o, a, b, \dot{a}, \dot{\gamma}) \tag{T17.2}$$

Age-schedule information

$$l_i = l_\infty(1 - \exp(-k(a - t_o))) \tag{T17.3}$$

$$w_i = a(l_i)^b \tag{T17.4}$$

$$v_i = (1 + \exp((\hat{a} - a)/\hat{\gamma}))^{-1} \tag{T17.5}$$

$$f_i = w_i(1 + \exp((\dot{a} - a)/\dot{\gamma}))^{-1} \tag{T17.6}$$

Survivorship

$$\iota_i = \begin{cases} 1, & i = 1 \\ \iota_{i-1}e^{-M}, & i > 1 \\ \dfrac{\iota_{i-1}}{1 - e^{-M}}, & i = A \end{cases} \tag{T17.7}$$

$$\hat{\iota}_i = \begin{cases} 1, & i = 1 \\ \hat{\iota}_{i-1}e^{-M - F^* v_{i-1}}, & i > 1 \\ \dfrac{\hat{\iota}_{i-1}}{1 - e^{-M - F^* v_i}}, & i = A \end{cases} \tag{T17.8}$$

Incidence functions

$$\phi_E = \sum_{i=1}^{\infty} \iota_i f_i, \quad \phi_e = \sum_{i=1}^{\infty} \hat{\iota}_i f_i \tag{T17.9}$$

$$\phi_B = \sum_{i=1}^{\infty} \iota_i w_i, \quad \phi_b = \sum_{i=1}^{\infty} \hat{\iota}_i w_i v_i \tag{T17.10}$$

$$\phi_q = \sum_{i=1}^{\infty} \frac{\hat{\iota}_i w_i v_i}{M + F^* v_i} \left(1 - e^{(-M - F^* v_i)}\right) \tag{T17.11}$$

Derived variables

$$\kappa = \frac{\phi_E}{\phi_e} - \frac{F^* \phi_q \dfrac{\phi_E}{\phi_e^2} \dfrac{\partial \phi_e}{\partial F^*}}{\phi_q + F^* \dfrac{\partial \phi_q}{\partial F^*}} \tag{T17.12}$$

$$R_e = \frac{C^*}{F^* \phi_q} \tag{T17.13}$$

$$B_o = \phi_B \frac{R_e(\kappa - 1)}{\kappa - \phi_E/\phi_e} \tag{T17.14}$$

Table 18: Partial derivatives, based on components in Table 17, required for the derivation of κ and B_o using the Beverton-Holt recruitment model.

Mortality & Survival

$$Z_i = M + F^* v_i \tag{T18.1}$$

$$S_i = 1 - e^{-Z_i} \tag{T18.2}$$

Partial for survivorship

$$\frac{\partial \hat{\iota}_i}{\partial F^*} = \begin{cases} 0, & i = 1 \\ e^{-Z_{i-1}} \left(\frac{\partial \hat{\iota}_{i-1}}{\partial F^*} - \hat{\iota}_{i-1} v_{i-1} \right), & i > 1 \\ \frac{e^{-Z_{i-1}}}{1 - e^{-Z_i}} \left(\frac{\partial \hat{\iota}_{i-1}}{\partial F^*} - \hat{\iota}_{i-1} v_{i-1} \right) - \hat{\iota}_{i-1} e^{-Z_{i-1}} v_i e^{-Z_i}, & i = A \end{cases} \tag{T18.3}$$

Partials for incidence functions

$$\frac{\partial \phi_e}{\partial F^*} = \sum_{i=1}^{\infty} f_i \frac{\partial \hat{\iota}_i}{\partial F^*} \tag{T18.4}$$

$$\frac{\partial \phi_q}{\partial F^*} = \sum_{i=1}^{\infty} \frac{w_i v_i S_i}{Z_i} \frac{\partial \hat{\iota}_i}{\partial F^*} + \frac{\hat{\iota}_i w_i v_i^2}{Z_i} \left(e^{-Z_i} - \frac{S_i}{Z_i} \right) \tag{T18.5}$$

Partial for recruitment

$$\frac{\partial R_e}{\partial F^*} = \frac{R_o}{\kappa - 1} \frac{\phi_E}{\phi_e^2} \frac{\partial \phi_e}{\partial F^*} \tag{T18.6}$$

Table 19: Statistical catch-age model using the Baranov catch equation and C^* and F^* as leading parameters.

Data

$$C_j = C_j^{\mathrm{US}} + C_j^{\mathrm{CA}} \tag{T19.1}$$

$$I_j, A_{i,j,k} \tag{T19.2}$$

Parameters

$$\Theta = (C^*, F^*, M, \hat{a}, \hat{\gamma}, \bar{a}, \bar{\gamma}, \{\omega_j\}_{j=1}^{J-1}, \rho, \vartheta^2) \tag{T19.3}$$

$$\sigma^2 = \rho\vartheta^2, \quad \tau^2 = (1-\rho)\vartheta^2, \quad \sum_t \omega_t = 0 \tag{T19.4}$$

Unobserved states

$$N_{i,j}, B_j, E_j, F_j \tag{T19.5}$$

Initial states (t=1)

$$N_{i,j} = B_o/\phi_B \iota_i \tag{T19.6}$$

State dynamics (t>1)

$$E_j = \sum_i N_{i,j} f_i \tag{T19.7}$$

$$Z_{i,j} = M + F_j v_i \tag{T19.8}$$

$$\hat{C}_j = \sum_i \frac{N_{i,j} w_i F_j v_i \left(1 - e^{-Z_{i,j}}\right)}{Z_{i,j}} \tag{T19.9}$$

$$F_{j_{i+1}} = F_{j_i} - \frac{\hat{C}_j - C_j}{\hat{C}'_j} \tag{T19.10}$$

$$N_{i,j} = \begin{cases} \dfrac{s_o E_{j-1}}{1+\beta E_{j-1}} \exp(\omega_j - 0.5\tau^2) & i = 1 \\ \\ N_{i-1,j-1} \exp(-Z_{i-1,j-1}) & i > 1 \end{cases} \tag{T19.11}$$

$$B_j = \sum_i N_{i,j} w_i v_i \tag{T19.12}$$

Residuals

$$\epsilon_j = \ln\left(\frac{I_j}{B_j}\right) - \frac{1}{n}\sum_{j\in I_j} \ln\left(\frac{I_j}{B_j}\right) \tag{T19.13}$$

$$\eta_{i,j,k} = \ln(p_{i,j,k}) - \ln(\bar{p}_{i,j,k}) - \frac{1}{I-1}\sum_{i=2}^{I} [\ln(p_{i,j,k}) - \ln(\bar{p}_{i,j,k})] \tag{T19.14}$$

Table 20: Likelihoods and priors used in the statistical estimation of Θ from Table 19.

Negative log-likelihoods

$$\ell(\Theta)_1 = \sum_{j=1}^{J-1} \left[\ln(\tau) + \frac{\omega_j^2 - 0.5\tau^2}{2\tau^2} \right] \tag{T20.1}$$

$$\ell(\Theta)_2 = \sum_{j \in I_j} \left[\ln(\sigma) + \frac{\epsilon_j^2}{2\sigma^2} \right] \tag{T20.2}$$

$$\ell(\Theta)_3 = \sum_k \left\{ (I-2) J_{j \in k} \ln \left(\frac{1}{(J_{j \in k} - 2) I} \sum_{j=1}^{J_{j \in k}} \sum_{i=2}^{I} \eta_{i,j,k}^2 \right) \right\} \tag{T20.3}$$

$$\ell(\Theta) = \sum_{i=1}^{3} \ell(\Theta)_i \tag{T20.4}$$

Constraints

$$\kappa > 1.0 \tag{T20.5}$$

Posterior distribution

$$P(\Theta) \propto \exp[-\ell(\Theta)] p(C^*) p(F^*) p(M) p(\rho) p(\vartheta^2) \tag{T20.6}$$