Predicting snow crab (*Chionoecetes opilio*) abundance using kriging with external drift with depth as a covariate

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PREDICTING SNOW CRAB (Chionoecetes opilio) ABUNDANCE USING
KRIGING WITH EXTERNAL DRIFT WITH DEPTH AS A COVARIATE

by

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ABSTRACT

Ordinary kriging (OK) has long been used to provide resource distribution maps and abundance indices for snow crab in the southern Gulf of St. Lawrence. To counterbalance biases along the margins of the study areas, the data set was supplemented with null data values around the study area, called pseudo-zeroes (OK_{pz}), but issues of subjectivity and estimation bias were again raised. An alternative method, kriging with external drift (KED) using depth as a predictor of local density, is proposed. All three methods are compared. OK_{pz} is found to be overly biased with respect to OK and KED. KED is shown to provide results intermediate to OK and OK_{pz} and to have the desired spatial properties. A complete description of all steps of the geostatistical analyses are provided, including variogram estimation, kriging and all relevant methodological aspects.

RÉSUMÉ

Le krigeage ordinaire (KO) est utilisé depuis longtemps pour founir des cartes de distribution et des indices d'abondance du crabe des neiges dans le sud du Golfe du Saint-Laurent. Afin de corriger des biais positifs en bordure de l'aire d'étude, l'ensemble de données usuel a été complémenté par un ensemble de données nulles entourant l'aire d'étude, appelées pseudo-zéros (KO_{pz}). Cependant, certains ont questionné la subjectivité de ces valeurs et souligné la possibilité de biais. Une méthode alternative, nommé krigeage avec dérive externe (KDE), incorporant la profondeur de l'eau pour prédire la densité locale, est proposée. Les trois méthodes sont comparées. On démontre que KO_{pz} est biaisé par rapport au KO et KDE. Avec le KDE, on obtient des résultats qui sont intermédiaires à ceux KO et KO_{pz}, et présentant les propriétés spatiales désirées. Toutes les étapes de l'analyse géostatique sont explicitement décrites, incluant l'estimation du variogramme, le krigeage et tous les aspects méthodologiques pertinents.

1.0 INTRODUCTION

Geostatistical analyses of snow crab trawl survey data have been used for estimating commercial snow crab (*Chionoecetes opilio*) abundance indices and mapping its distribution in the southern Gulf of St. Lawrence (sGSL) (Conan et al. 1988; Moriyasu et al. 1998) annually since 1988. This method models and applies the spatial correlation between data points in an effort to provide more precise estimates of abundance and distribution.

Ideally, the region under study is a well-defined area overlaid by an appropriate sampling design. However, in practice such factors as bottom topography, water depth and suitable habitat have all been used to define the boundaries of the study area and, to a lesser extent, the particular locations of sampling stations within the study area. Given the considerable cost of sampling, marginal sampling sites where no crab were expected to be present were generally excluded from the sampling design in favour of having more sites within the study region. This sampling bias results in a positive predictive bias along the margins of the study area and also resulted in an increased in predictive variance of local abundance estimates. This and other problems relating to estimation along marginal areas are collectively referred to as *edge-effects* in the geostatistical literature.

A number of methods have been used to counteract these edge-effects, one of the simplest being to place virtual sampling stations with zero-point values, colloquially called 'pseudo-zeroes', along the margins where the assumption of nil densities of crab is thought to be reasonable. However, depending on the properties of geostatistical model being used, pseudo-zeroes may have too large an influence in regions of the study area which are dominated by their margins, such as narrow straits or bays (DFO, 2006). This issue was raised by industry representatives during the Regional Assessment Process (RAP) in 2005 and formally discussed during a DFO-hosted workshop (DFO, 2006). As an alternative solution, we propose a modification of the currently used geostatistical model by incorporating a secondary variable, namely water depth, into the analysis. This method is called kriging with external drift (KED).

The goal of this paper is to compare the performance of KED with that of two previously used methods: ordinary kriging (OK) and ordinary kriging with inclusion of pseudo-zeroes (OK_{pz}) in the analysis. Each method is applied to two areas, the first spanning a substantial portion of the southern Gulf of St. Lawrence, and the second a smaller area dominated by its coastal margins. Point density data from the 1988-2005 trawl surveys will be used as part of an overall comparative analysis while data from the 2003 survey year will be the focus of a more detailed analysis. This paper will also provide for the first time, a detailed description of all analytical steps used in the estimation of southern Gulf snow crab abundance indices and resource distribution.

2.0. MATERIALS AND METHODS

2.1. DATA

The sGSL snow crab bottom trawl survey has been performed annually from mid-July to September since 1988, after most of the fishery has taken place. Due to financial limitations, only a limited survey was performed in 1996. The number of sample stations has increased from 155 in 1988 to over 300 stations in 2005, along with the extent of the study area. The study area was stratified as 10 by 10 minute regular grids and a number of stations (usually 1 or 2) were chosen within each cell. At each station a Nephrops-type bottom trawl was towed for ~5 minutes at a speed of ~2 knots. Trawl net behavior was monitored using either a Scanmar® (1988-1998) or Netmind® (1999-present) hydro-acoustic sensors relaying information in real-time onto an on-board computer. Local point density estimates (number of crab per square kilometer) are estimated by dividing the number of crab of a target category by the swept area as estimated from the trawl acoustic sensor data. The commercial category, defined as hard-shelled males having a carapace width larger than 95 millimeters, was used in the following analyses. Nearly all tows are found at depths ranging from 30 to 170 meters with the larger proportion being concentrated between 50 and 85 meters.

In addition to the snow crab survey data, an accessory data set, gathered on the sGSL September multi-species bottom trawl survey 2000-2005 (Hurlbut & Clay 1990;

Benoit et al. 2003), was used to illustrate the empirical relationship between snow crab abundance and depth. As the size and shell hardness of snow crab were not recorded throughout the time series, the total number of snow crab was used. Most tows of the sGSL September multi-species bottom trawl survey are found within the 15 to 150 meter range with the majority falling between 20 and 80 meters, thus allowing for better characterization of the depth-abundance relationship at shallower depths than the sGSL snow crab survey. This data set was not used in any geostatistical analysis.

2.2. STUDY AREA

The performance of three kriging methods (OK, OK_{pz} and KED) was evaluated in two areas: the snow crab area 12 fishing area which spans a substantial portion of the southern Gulf, and sector 1, a portion of area 12 which was highlighted as an area being particularly susceptible to edge-effects (Fig. 1).

2.3. MODEL

A basic geostatistical model assumes the existence of a covariance function which describes the relationship between random variables in a defined space. The term *kriging* usually refers to the estimation of the value of a target random variable at an unsampled location. This kriged estimate is a minimum error-variance unbiased estimator (MVUE), obtained as a linear combination of surrounding samples, weighted by the covariance function. The covariance function is intimately related to the theoretical *variogram*, which rather models the semivariance of the difference between pairs of random variables as a function of the distance which separates them. We will now provide a more formal definition of the geostatistical model to be applied.

2.3.1. Variogram model

Let x and x' be two arbitrary sample locations within a defined study region, separated by a distance $h = \|x - x'\|$, usually taken to be the Euclidean metric. We suppose that $Z(x)$ and $Z(x')$ are random functions of the locations x and x' which follow the following assumptions:

(1.1)
$$E[Z(x)] = E[Z(x')] = \mu$$

(1.2)
$$Var[Z(x') - Z(x)] = 2\gamma(h)$$

Assumption (1.1) states that the expected value of the random functions $Z(x)$ and $Z(x')$ within the study region are everywhere equal and (1.2) states that the variance of the difference between $Z(x)$ and $Z(x')$ is strictly a function of their separation distance h, also referred to as the *lag* distance. When $\gamma(h)$ is bounded, the only case considered in this study, $Z(x)$ is said to follow a second-order stationary process, referring to the fact that both the mean and covariance between variables do not depend on their specific location, but only their lag distance. When $\gamma(h)$ is unbounded, $Z(x)$ is said to be *intrinsic*. The function $\gamma(h)$ is called a variogram function and is usually formulated as an increasing function of h, expressing our intuition that locations which are in proximity to each other in space should more closely resemble each other (their difference has a smaller variance) than areas which are farther apart. Note that $\gamma(h)$ is in fact half the variance in (1.2) and thus actually models the *semivariance* rather than the variance.

The form of the variogram model used in the following analyses is given by

(2)
$$\gamma(h) = \begin{cases} 0, & h = 0 \\ a + \dfrac{3}{2}\dfrac{ch}{b} - \dfrac{c}{2}\left(\dfrac{h}{b}\right)^3, & h \le b, \\ a + c, & h > b \end{cases}$$

where $a \ge 0$, $b > 0$ and $c \ge 0$. This model is called a *spherical* variogram model in the literature (Fig. 2), though other models are commonly used such as the *exponential* or *Gaussian* models (Goovaerts, 1997). Its individual parameters relate to particular spatial features of the model: a is called the nugget value, representing the residual variance at each location since $\lim_{h \to 0^+} \gamma(h) = a$, often heuristically interpreted as a combination of micro-scale spatial variation and measurement error; b is called the *range* of the variogram model and relates to correlation length of the spatial model; and the sum $a + c$ is called the *sill* of the variogram which is the maximum semivariance attained for all

$h > b$; it is also equal to $Var[Z(x)]$. If $c = 0$, then the variogram is a constant function of distance and there is no spatial correlation between the data. Under a Gaussian assumption, this situation corresponds to the independence hypothesis required by most standard statistical models.

The theoretical covariance between data at different locations, expressed by the covariance function $C(h)$, is related to the variogram model by the expression $\gamma(h) = C(0) - C(h)$.

2.3.2. Empirical semivariogram and model fitting

Once a variogram model has been selected, its parameters must be estimated from the data. Let the data be represented by the pair $(x_i, Z(x_i))$ for $i = 1, \ldots, n$ where the x_i's are the sample locations and $Z(x_i)$ are the observed values of the spatial process at each location. As an estimator of the variogram, Matheron (1962) defined the empirical semivariogram $\gamma_e(h)$ as

$$(3) \qquad \gamma_e(h) = \frac{1}{2N(h)} \sum_{(l,m)|h_{lm} \approx h}^{n} [Z(x_i) - Z(x_j)]^2$$

where $N(h)$ is the number of data pairs within a specified range, usually taken to be regularly spaced lag distance intervals, labelled as h_i for $i = 1, \ldots, k$ where k is the number of lags. In this paper, lag intervals were set at regular 3-kilometre intervals for k=25 lags for all analyses, for a maximum distance of 75 kilometres. The empirical semivariogram is a useful tool for exploring the spatial autocorrelation between the data, though some key points, most notably the correlation between the binned semivariances, must be kept in mind when interpreting or fitting the data (Diggle et al. 2003).

2.3.3. Model fitting and parameter estimation

Parameters for the variogram model may be estimated using a number of statistical methods, such as maximum likelihood, though in the geostatistical literature, the variogram is generally fitted according to the empirical variogram via the method of

least-squares. Following this approach, the objective function to be minimized, written as a function of the model parameters is

$$(4) \qquad S(a,b,c) = \sum_{i=1}^{k} N(h_i) g(h_i) [\gamma(h_i \mid a,b,c) - \gamma_e(h_i)]^2$$

where h_i represents the average distance in lag distance bin i, k is the number of bins, $g(h_i)$ is weight function decreasing with distance, here chosen as $g(h_i) = \frac{1}{h_i^2}$ and $N(h_i)$ is the number of data pairs in the ith lag distance interval. The goal is to lend more weight to those variogram values which are estimated with many data pairs separated by smaller distances, given that these are the most influential values used in the kriging step.

It is often observed that variograms fitted to homologous data, say a time series over the same study area, show considerable variation in their parameter estimates. This variation is due to two factors. The first stems from the fact that the spatial relationship between variables may change from data set to data set and thus differences in the fitted variograms would appropriately reflect actual changes in the spatial relationship. The second factor, more problematic from an inference point of view, is the fact that simulation, maximum likelihood and Bayesian-based studies have shown that the parameters within the variogram model are often highly correlated, implying that many different combinations of parameters may yield similar fits (Diggle & Ribeiro, 2007). This implies that small changes in the spatial characteristics of the data may have a large influence on the resulting variogram. Furthermore, these variogram differences, while not necessarily entailing large changes in the goodness of fit of the model, may visibly alter the character of the interpolation maps which are produced.

In order to counterbalance this effect, each variogram for a given year was averaged over a three-year period in the following manner. The empirical variograms $\gamma_e^{(j)}(h)$ were first calculated for each year j for each lag intervals. The resulting variograms $\gamma_e^{(j)}(h)$ were then standardized by dividing the semivariance by the sample variance for each year j. The standardized semivariances were then averaged for year j-2, j-1 and j. The resulting averaged standardized empirical variogram was then scaled according to the sample variance for year j. When fitting this averaged empirical

variogram, the total numbers of data pairs in each lag interval for the target year j were used as weights.

2.3.4. Ordinary kriging (OK)

Once a variogram model is defined, one may use it to find the expected value of the variable of interest at an unsampled location as a linear combination of the values surrounding it, a process referred to as *kriging*, named after D.G. Krige (1951) who laid some of the foundations of geostatistics while working in the South African mining industry.

We suppose that the observed random variable may be decomposed, as follows:

$$Z(x) = \alpha + W(x),$$

where α is a constant and $W(x)$ is a zero-mean second-order stationary process (Eqns 1.1 and 1.2) corresponding to the correlated residual structure. The observed variables $Z(x)$ may be thought of as the response variable in a regression model with mean α and correlated residuals $W(x)$.

The goal is to estimate the value of $Z(x)$ at an unsampled location x. Under the assumptions outlined in the above model, one may express $Z(x)$ as a linear combination of the observed data values. This *kriged* estimate may be written as

(5)
$$Z(x) = \sum_{i=1}^{n} \lambda_i Z(x_i), \text{ with } \sum_{i=1}^{n} \lambda_i = 1$$

where the λ_i s are called *kriging weights*, calculated by solving the following $(n+1) \times (n+1)$ linear system, given in matrix form:

(6)
$$\begin{bmatrix} \mathbf{K} & \mathbf{1} \\ \mathbf{1^T} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda} \\ \mu \end{bmatrix} = \begin{bmatrix} \mathbf{k_x} \\ 1 \end{bmatrix},$$

where $\mathbf{K}$ is the $n \times n$ matrix data covariance matrix, equal to the covariance matrix of the residual matrix $W(x)$, whose elements are defined according to the covariance function $K_{ij} = C\left(\|x_i - x_j\|\right)$ for $i, j = 1,\ldots,n$; $\mathbf{1}$ is a n-element vector of ones,

$\boldsymbol{\lambda} = \left(\lambda_1, \ldots, \lambda_n\right)^T$ is the vector of kriging weights to be estimated; μ is the Lagrange multiplier and $\mathbf{k_x}$ is an n-element covariance vector between the n data and the point x, again modeled by the covariance function. The kriging variance at location x is given by

$$(7) \qquad \sigma_{OK}^2(x) = C(0) - \sum_{i=1}^{n} \lambda_i C\left(\|x_i - x\|\right) - \mu(x).$$

To obtain abundance indices within a specified area, one may extend the kriged estimate beyond a point estimate to that over an area (usually a specified polygon). For example if the $Z(x)$ are observed densities at specific locations, then the total abundance T_A within an area A is given by the integral

$$E[T_A] = \int_A Z(x) dx$$

and its variance is given by:

$$Var[T_A] = \int_A \int_A Cov\left(\|x - x'\|\right) dx dx'.$$

Generally, spatial predictions are calculated over a regular grid and plotted, resulting in a *kriged map*. By default, a 100x100-point grid was used.

2.3.5. Ordinary kriging with pseudo-zeroes (OKpz)

The ordinary kriging algorithm in the previous section was also applied to an extended version of the data set, comprised of the original data set having n observed locations and data values $\left(x_i, Z(x_i)\right)$ for $i = 1, \ldots, n$ plus a set of k additional locations with null density values, called pseudo-zeroes, written symbolically as $\left(x'_{n+j}, Z\left(x'_{n+j}\right) = 0\right)$ for $j = 1, \ldots, k$. These additional locations were placed at approximately regular intervals just beyond the boundary of the study area (Fig. 3). The variogram was fit using only observed data, while the ordinary kriging itself was performed using the extended data set.

2.3.6. Kriging with an external drift (KED)

Rather than supplement the data with zero values, kriging with external drift (KED) seeks to compensate for the scarcity of samples along the edges of the study area by incorporating secondary information to improve predictions in these areas. KED assumes that, rather than being constant, the mean μ is a function of location, specifically a linear combination of one or more secondary variables called *drift functions* which vary according to location. In the usual form we may write

$$Z(x) = \alpha(x) + W(x)$$

where $\alpha(x) = a_0 + a_1 Y_1(x) + \ldots + a_p Y_p(x)$, $a_0, \ldots, a_p$ are linear coefficients and $Y_1(x), \ldots, Y_p(x)$ are drift functions and $W(x)$ is defined as before. To be useful the drift functions either need to be everywhere known or can be easily and precisely interpolated throughout the study area. They are also generally smoother than the observed variable $Z(x)$. The KED linear system is given in matrix form as

$$(8) \qquad \begin{bmatrix} \mathbf{K} & \mathbf{F} \\ \mathbf{F^T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{k_x} \\ \mathbf{f} \end{bmatrix}$$

where $\mathbf{F}$ is a $n \times (p+1)$ matrix of drift function values at each location x_i, which includes a column of ones if the function $\mu(x)$ includes an intercept a_0, $\mathbf{0}$ is a $(p+1) \times (p+1)$ matrix of zero values, $\boldsymbol{\mu} = (a_0, \ldots, a_p)^T$ is the vector of Lagrange coefficients and $\mathbf{f} = (1, Y_1(x), \ldots, Y_p(x))^T$, the value of each drift function at location x to be estimated.

In this paper we will use depth as a secondary variable, so setting $d(x)$ as a drift function which represents the depth for any location x, the mean function is equal to $\mu(x) = a_0 + a_1 d(x)$. The coefficients a_0 and a_1 are implicitly estimated along with the kriging weights. The last two equations of the kriging system, i.e. $\mathbf{F^T \lambda = f}$, ensure that the estimate will be unbiased whatever the values of a_0 and a_1.

Given that the mean varies as a function of location x, one may ask how well the covariance matrix $\mathbf{K}$, also the covariance matrix of the residuals $W(x)$, is approximated by the variogram model. If one of the slope coefficients a_0 or a_1 is large over the whole study area, it will be necessary to find other ways to estimate the variogram model. Among the many methods available, cross-validation (see below) may be useful to select among candidate variogram models. In our particular case, the correlation between depth and the observed density $Z(x)$ is generally weak, leading to estimates of a_0 or a_1 which are correspondingly small, so the covariance matrix $\mathbf{K}$, as calculated from the fitted variogram, can be used as a safe approximation for that of the residual matrix $W(x)$. Thus an identical variogram was used for all three kriging methods in each given year.

2.3.7. Depth covariate in KED

The choice of water depth as a predictor of local snow crab density is less based in its direct relevance to local density, than its availability and its correlation with a number of important environmental variables, such as bottom temperature, salinity and bottom type. These variables may directly or indirectly impact local abundance by affecting larval settlement, recruitment, mortality or migration. Unfortunately, recall that external drift covariates in KED need to be available at all unsampled locations within the study area. Thus while some variables, such as bottom temperature or bottom type are locally known or observed locally, they may not be used using the KED method.

As its name implies, snow crab is an epibenthic species exhibiting a preference for cold water temperatures ranging from -1°C to +3°C (Comeau et al., 1991) though temperatures up to +7°C are physiologically tolerated and the lethal limit is about +15°C (Foyle et al., 1989; Hardy et al. 1994). This narrow range of cold temperatures naturally occurs in the sGSL within what is called the cold intermediary layer (CIL), a stratification of the water column which occurs during the spring, summer and fall seasons. This layer is effectively sandwiched between the warm surface waters and the relatively warm deeper waters. Much of the bottom in the sGSL bathes in this CIL, making an ideal habitat for snow crab, which is thus bounded away from the shoreline by the warm, shallow waters on the south and west (west and south shores of the Gaspé

Peninsula, New Brunswick, north shores of Prince Edward Island and eastern shores of Cape Breton Island) and to the northeast by the warmer, deep waters of the Laurentian Channel.

The relationship between temperature and water depth, along with the snow crab's preference for cold waters implies a relationship between water depth and local density. This was verified empirically by plotting the mean snow crab count per tow for the snow crab survey data (data pooled from 1988-2005) as well as the groundfish September multi-species survey data set (data pooled from 2000-2005). This latter data was used in order to corroborate the trend observed at shallow depths as it sampled shallower areas than the snow crab trawl survey. For this and the above mentioned reasons, water depth was chosen as a predictor of local density.

2.3.8. Local data neighbourhoods

Rather than kriging values at unknown locations by considering the global sample, we may both further relax the stationarity assumption (1.1) and reduce computation time by considering local neighborhoods of sample points. Thus it is only assumed that the mean is constant, or follows a linear function in the KED case, within a local neighborhood rather than globally and better predictions are obtained in a manner analogous to a series of linear models approximating a complex function. In cases where there is no relationship between $E[Z(x)]$ and depth, the KED system will converge towards the OK solution.

The approach that we have adopted here is to use a maximum of 32 nearest neighbors for each interpolated point, with a maximum of 8 per quadrant. The latter constraint limits the impact that points in a given direction may exert, especially along the edges of the study area. Previous testing using cross-validation show that the optimum number of local neighbors in our case usually lies between 20 and 40, with values within this range showing little quantitative difference. This approach was used for all kriging methods compared.

Because a drift model is implicitly fitted to the data when using KED, the local neighborhood must contain a sufficient number of data points to ensure a stable estimate

of the drift function parameters and avoid degenerate configurations which would lead to a singular kriging matrix. For instance, if all points in a given local neighborhood had the same depth, then the two columns of **F** would be proportional to each other, leading to a singular kriging matrix (i.e. with zero determinant).

2.4. MODEL EVALUATION

The performance of each kriging method, either ordinary kriging with pseudo-zeroes (OK_{pz}) or without (OK), or kriging with external drift (KED), were compared each other in a number of ways. Distribution maps, corresponding to predicted kriged density estimates over the sGSL study area, were plotted for each method using the 2003 commercial male trawl survey data set. The relative performance of each kriging method was evaluated using every pairwise combination (i.e. OK_{pz}-OK, KED-OK and KED-OK_{pz}), by spatial differencing and identifying areas where high discrepancies occurred. Spatial estimates were also compared on bivariate plots is order to identify global systematic differences. Pairwise predictive differences were plotted as a function of water depth to illustrate the role which depth played in the estimated values of each method. Kriged abundance estimates for area 12 from 1988-2005 were plotted for each kriging method to show temporal trends.

The bias and precision of spatial predictive estimates from each kriging method was evaluated using 'leave-one-out' cross-validation (Isaaks and Srivastava, 1989), also called the jackknife in the statistical literature (Efron, 1979). This method, in the context of geostatistical predictive power, is applied by removing each sample in turn, say $Z(x_i)$, and estimating its expected value $Z^*(x_i)$ at that location using the given kriging method. Intuitively, if the estimated values compare favourably, then the predictive ability of the method being tested is deemed acceptable. We will use two statistics based on the bias and its variability to compare each kriging method. The mean cross-validated bias is defined as

$$(9) \qquad \bar{B} = \frac{1}{n} \sum_{i=1}^{n} Z(x_i) - Z^*(x_i)$$

while the mean absolute error will be a robust measure of the scale of the errors

(10)
$$\overline{E} = \frac{1}{n}\sum_{i=1}^{n}\left|Z(x_i) - Z^*(x_i)\right|$$

The $\overline{B}$ and $\overline{E}$ summary statistics were generated for the two reference areas, Area 12 and its component sector 1 (Fig. 1) for the time period 1988-2005 as well as for 2003 in particular.

Cross validation is a general statistical method which may be used for evaluating other aspects of a geostatistical model, such as the adequacy of competing variogram models, underlying assumptions, structural aspects (Delhomme 1978) such as isotropy or anisotropy (Delhomme 1978), and the size of the local neighborhood to be used in kriging.

2.5. SOFTWARE

All geostatistical analyses were performed using MPOGEOS in a graphical-user interface implemented by one of the authors (D. Marcotte) in the MATLAB® interpreter language. For OK and OK$_{pz}$, the program uses the *kt3d* function from GSLIB, the geostatistical software library (Deutsch & Journel, 1992). For KED and all variogram computations, the program uses specifically developed MATLAB® functions.

3.0. RESULTS

3.1. ABUNDANCE VERSUS DEPTH

The mean counts per tow of total snow crab versus depth for the snow crab (Fig. 4) and September multi-species (Fig. 5) surveys show similar trends. Very few crab are present at shallow depths and these become progressively more abundant in the 20 m to 50 m range. The maximum observed mean densities occur in the 70 to 120-meter range while the densities slowly taper off at deeper water depths. The September multi-species survey has more samples at shallower and deeper water depths than does the snow crab survey and confirms that very few snow crab are present at depths shallower than 40 meters or beyond 200m. The consistency of these observations is supported by the small sampling errors. As for the snow crab survey, snow crab counts are shown to slowly taper

off in deeper waters, although since the number of samples at these depths is limited and the abundance is highly variable, the associated sample errors are correspondingly large.

3.2. VARIOGRAM

The averaged empirical variogram and fitted spherical variogram used in the 2003 analyses are shown in Figure 6. The nugget value was estimated to be nearly zero at $a = 4.13 \cdot 10^{-9} \approx 0$, the range parameter $b = 9.932\,\text{km}$ and the sill $a + c = 9.283 \cdot 10^{6}$.

3.3. KRIGED MAPS

The kriged density estimates for the 2003 commercial males over the southern Gulf are presented in Figure 7 for each of the three kriging methods used. The spatial distribution is broadly similar for all three kriging methods, with the main concentrations occurring around the Bradelle Bank, an area north of Prince Edward Island, and also off the western coast of Cape Breton. The crab concentrations obtained using OK and OK_{pz} appear spotty and more disjointed, with many concentrations centered about individual sample stations, than those of KED, which seem to be coalesced into larger contiguous units. Both OK_{pz} and KED show the desired tapering off to null densities along the boundaries of the study area, while in OK, relatively high densities are often extrapolated up to the edges of the study area. This effect is especially visible all along the north coast of PEI. We note that a large area within the OK_{pz} has consistently lower estimates than those of either OK or KED. This is readily apparent in the much reduced area of the 3000-4000 density contour layer.

3.4. DIFFERENCE MAPS

The arithmetic differences for each pairwise combination of the three kriging methods are shown in Figure 8. The OK_{pz}-OK difference map shows that the inclusion of pseudo-zeroes in the OK method yielded local density estimates which were consistently lower than those of OK using only the original data. As expected, the largest differences lie near the edges of the study area. However, negative differences are present throughout most of the study area, corresponding to the area within which the pseudo-zeroes were

included in the 32-point local data neighborhood. A large central portion shows identical results (i.e. zero-difference) with OK, its mottled appearance being due to slight numerical fluctuations in the algorithms used.

The KED-OK difference map shows large negative differences along the edges of the study area showing that inclusion of depth as a predictor results in consistently lower estimates. Many differences are visible within the study area, partially the result of the contrast between the spotty character of the OK kriged map and the more unified KED map, as well as differences between the predicted local densities based on the depth covariate. A similar pattern is apparent in the KED-OK$_{pz}$ difference map, except that the KED estimates are shown to be generally larger than those of OK$_{pz}$, resulting in mainly positive differences within the map.

3.5. BIVARIATE PLOTS

The kriged estimates over the regular grid which lie within the sGSL study area for each pairwise combination of the three kriging methods were plotted in Figure 9. Each spatial estimate from one kriging method was plotted versus that of another in a bivariate plot. The plot of OK$_{pz}$ versus OK shows that the former estimates are always lower than those of the latter. The plot of KED versus OK estimates shows that estimates from the two methods are roughly similar, though as observed on the density and difference maps, there are many local fluctuations resulting in a fairly diffuse cloud. The plot of KED versus OK$_{pz}$ estimates shows that the KED estimates area generally consistently larger than those of OK$_{pz}$.

Spatial differences between methods are plotted versus the depth at which they are estimated. The OK$_{pz}$-OK differences, as mentioned before, are consistently negative, with the largest differences occurring over shallower waters, corresponding to the edge effects observed in the density and difference maps. The KED-OK difference plot shows that KED estimates area generally lower in shallower waters while the pattern is more complicated in deeper waters, although there are discrepancies which are not so easily explained in the observed pattern. The KED-OK$_{pz}$ difference plot shows that the larger KED estimates primarily occur in shallower waters. We note that there are areas within

the differences map, notably the area about the Bradelle Bank, which show marked differences for the KED versus either OK or OK_{pz}. The KED locally models the depth-density relationship while the latter methods exclusively rely on autocorrelation of observed data values.

3.6. CROSS VALIDATION

Sector 1 and area 12 cross-validation summary statistics for 1988-2005 for sector are presented in Table 1. These show that while the mean absolute errors ($\overline{E}$) are fairly similar in all three cases, the cross-validated bias estimates ($\overline{B}$) for OK_{pz} tend substantially towards the negative, while those of OK and KED are very similar. The same set of summary statistics, along with kriged abundance and mean density estimates are presented in Table 2 for the 2003 data set. For this data, the inclusion of pseudo-zeroes with respect to the OK scenario resulted in a decrease in the kriged mean density of 20.7% for sector 1 and 10.0% for the Area 12 as a whole, while the KED resulted in a smaller decrease of 8.1% for sector 1 and 1.9% for Area 12.

3.7. TIME SERIES

The time series (1988-2005) of estimated mean abundance indices for each kriging method is presented in Figure 11. Differences between the different methods are generally small although they vary according to the mean abundance and the number of survey data available for each year. In general, the KED estimate is intermediate between those of OK and OK_{pz}, but generally nearer to the OK estimate. However, the overall trends predicted by each method are very similar. Furthermore, the larger differences observed in the earlier half of the time series are likely due in part to the more restricted coverage of past surveys with respect to the presently-defined study area, resulting in larger estimation errors as well as larger edge effects. We note that as the survey expanded within the study area in the later years, the differences between the three kriging methods became less pronounced. Differences between variograms (not shown) were also observed during the time series. However, the same type of variogram model

(spherical isotropic) consistently provided a good fit to experimental variograms. Its parameters (the range *b*, the ratio *a/c*) were also quite stable.

4.0 DISCUSSION

Empirical relationships between snow crab counts and depth in both the snow crab and September multi-species surveys support the assumption that local densities along the edges were biased in OK. These show that shallow areas have very few snow crab and that their frequency increases with depth. These curves also show that the frequencies also taper off in deeper waters, corresponding to the Laurentian Channel in the southern Gulf. In accordance with these observations KED, which performs local regressions of density on depth, also projected low densities along the margins of the study area. These projections are lower than those of OK, which are based solely on autocorrelations between the data and do not incorporate any secondary information.

Similarly, pseudo-zeroes were originally added to the ordinary kriging method in order to counterbalance perceived overestimations of local density along the sparsely sampled edges of the study area. With respect to OK, the degree to which a kriged estimate at a given location will be lowered by pseudo-zeroes will depend on the number of pseudo-zeroes, their proximity to the estimated location, the size (number of data and spatial extent) of the kriging neighborhood, the range *b* of the variogram and the relative importance of the nugget effect (i.e. the scale of the ratio *a/c*). Moreover, these factors may interact. For example, pseudo-zeroes located at a distance less than the range from an estimated point will have a significant influence when 1) it is included in the local kriging neighborhood and 2) the ratio *a/c* is large or the ratio *a/c* is small and the pseudo-zero is not screened by any true data (i.e. there is no data point lying close to a line connecting the pseudo-zero to the estimated point). Higher relative nugget effect yields, relatively, more weight to points located far from the estimated point as a high ratio *a/c* diminishes the 'screening effect' of kriging (Chilès and Delfiner, 1999). For instance, as the 2003 distribution maps have shown for a local neighborhood of 32 points, the influence of pseudo-zeroes is felt a fair distance within the study area. Most of these issues were not directly addressed when pseudo-zeroes were initially proposed.

While OK$_{pz}$ did induce the desired tapering all along the edges of the study area, it was shown in the present analyses that the resulting estimates were negatively biased with respect to KED, which models the relationship between density and depth explicitly. The extent of this relative bias is readily seen in the density and difference maps, the bivariate kriged density plots, the cross validation summary statistics tables and the time-series of kriged abundance estimates for area 12.

The differences between the three kriging methods are larger for earlier years than later ones. Also, the earlier part of the time-series in fact has larger errors, since the survey design then covered only a portion of the present study area and fewer data were available. We note that as the survey expanded within our study area, the abundance estimates from each kriging method become more and more similar, though KED still follows the pattern of being intermediate to OK and OK$_{pz}$.

KED has been shown to yield values which are intermediate between the assumed positive biases of the OK method, both along the margins of the study area as well as the resulting mean densities and abundances. OK$_{pz}$ has been shown to yield lower estimates than either OK or KED, though strong effects associated with the pseudo-zeroes are mainly observed for small areas which are dominated by their margins (e.g. sector 1). Comparisons revealed that the adjustments brought about by OK$_{pz}$ were overly conservative and not supported by the observed trends of density versus depth seen in KED. We believe that KED's approach of locally modeling of density and depth, yielding realistic tapering along the margins of the study area without overcompensating, qualifies KED as being the most desirable of the three methods.

5.0. ACKNOWLEDGEMENTS

6.0. REFERENCES

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Table 1: Cross-validation summary statistics for commercial snow crab densities for all years 1988-2005.

Area	Statistic	OK	OK$_{pz}$	KED
Sector 1	Mean observed	2927	2927	2927
	Bias ($\bar{B}$) (#/km^2)	28.1	-1184	33.1
	MAE ($\bar{E}$) (#/km^2)	1904	1951	1982
Area 12	Mean observed	3262	3262	3262
	Bias ($\bar{B}$) (#/km^2)	6.3	-451.5	-52.9
	MAE ($\bar{E}$) (#/km^2)	2300	2247	2299

Table 2: Kriging and cross-validation summary statistics for 2003 commercial snow crab. Kriging standard errors are shown in parentheses.

Area	Statistic	OK	OK$_{pz}$	KED
Sector 1	Kriged abundance (10^6)	8.00 (1.12)	6.33 (1.12)	7.33 (1.12)
	Kriged density (#/km^2)	2308 (323)	1831 (324)	2122 (324)
	Mean observed	2755	2755	2755
	Bias ($\bar{B}$) (#/km^2)	-567.8	-1327	-577.4
	MAE ($\bar{E}$) (#/km^2)	1574	1902	1498
Area 12	Kriged abundance (10^6)	105.9 (6.2)	95.3 (6.2)	103.9 (6.2)
	Kriged density (#/km^2)	2823 (165)	2540 (165)	2769 (165)
	Mean observed	3088	3088	3088
	Bias ($\bar{B}$) (#/km^2)	-144.9	-571.9	-151.8
	MAE ($\bar{E}$) (#/km^2)	2060	2101	1956
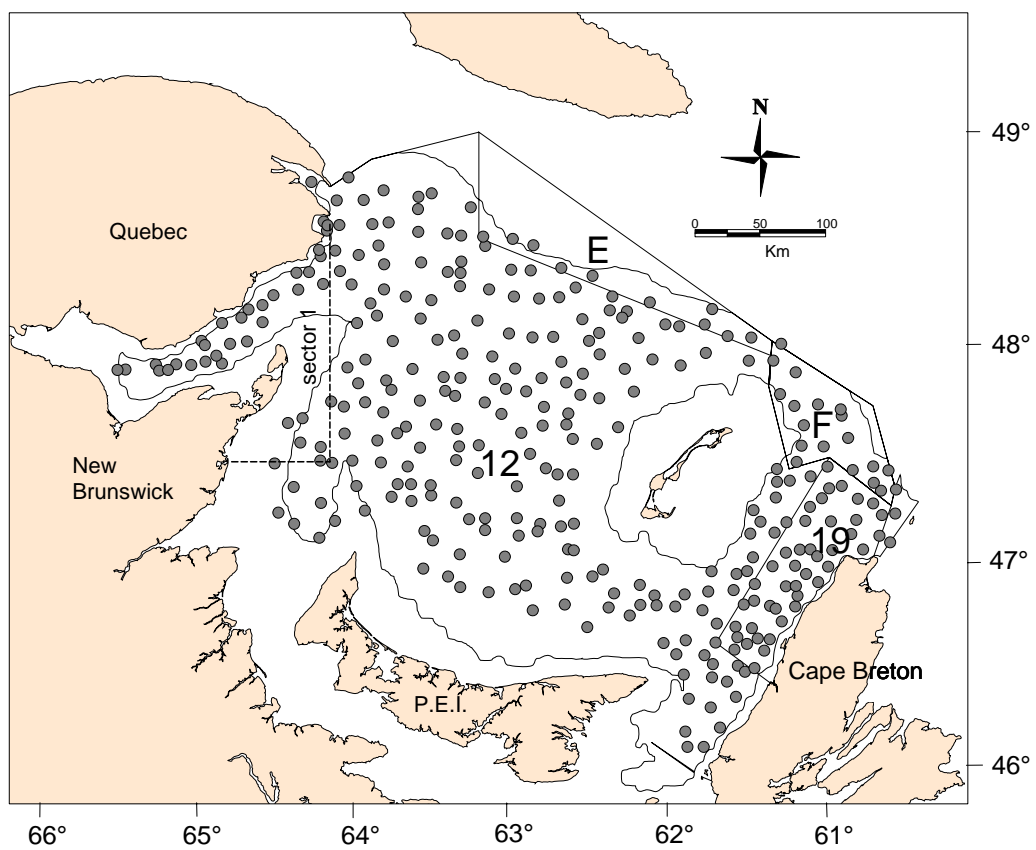
Figure 1: Snow crab trawl survey sample distribution and fishing areas in the southern Gulf of St. Lawrence for 2003.
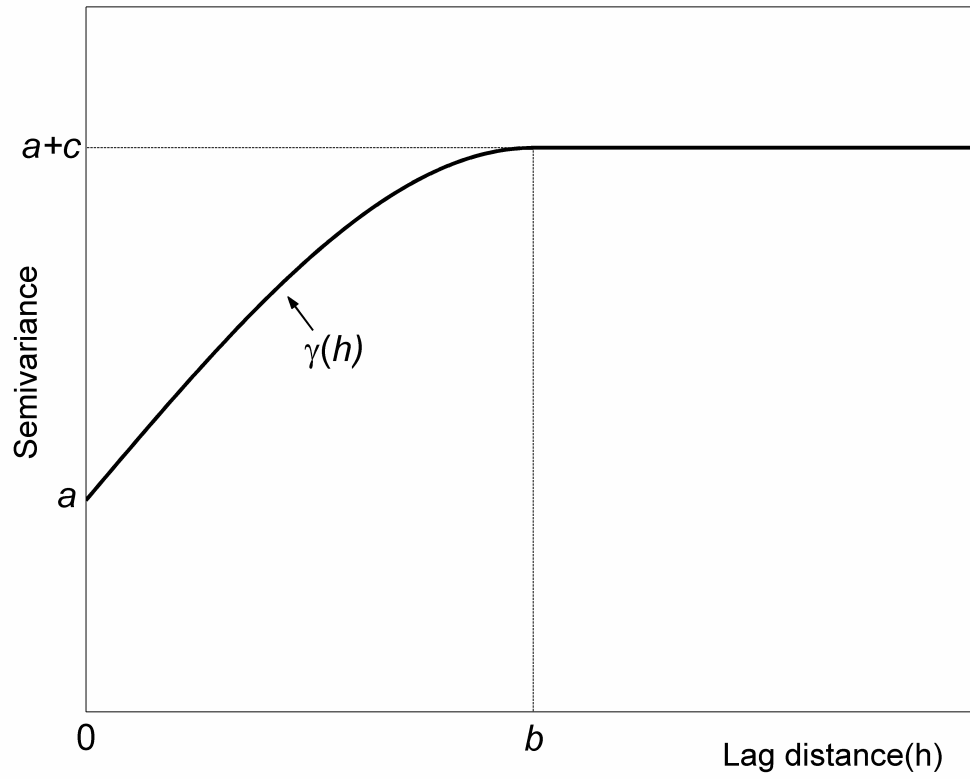
Figure 2: Sample spherical variogram (solid line) showing the relationship between the curve $\gamma(h)$ and its parameters: the nugget value a, the range b and the sill $a+c$ (dashed lines).
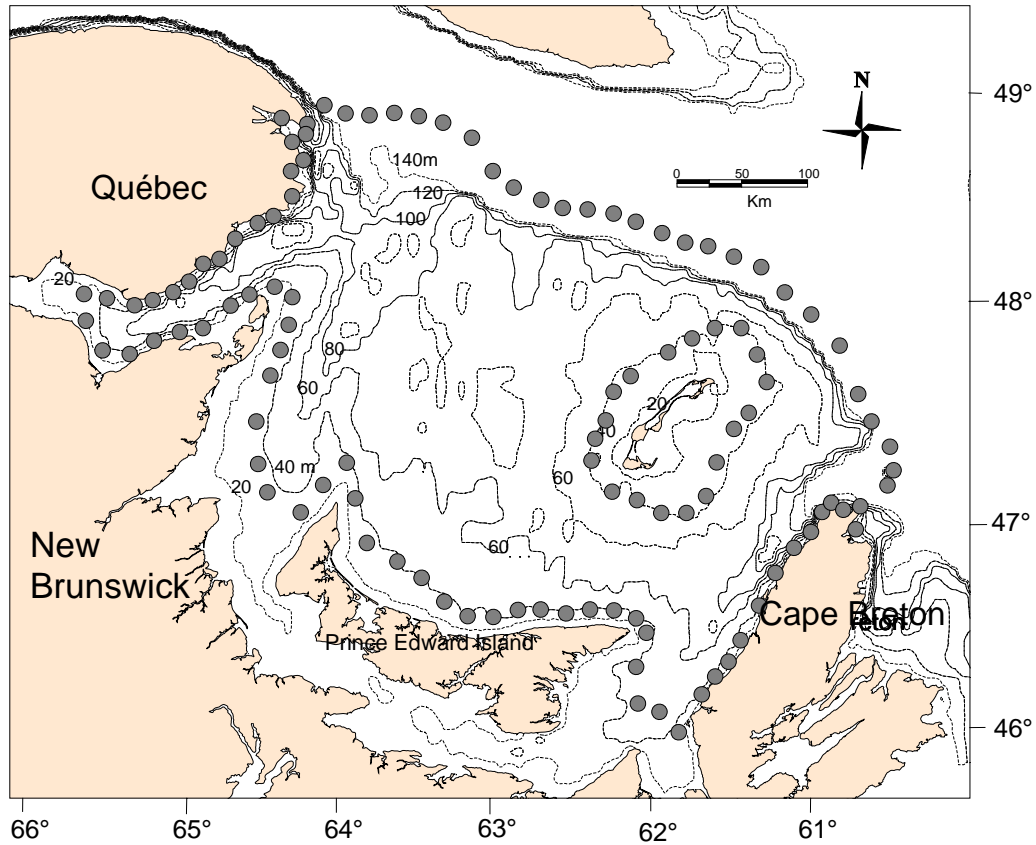
Figure 3: Bathymetry contours and location of pseudo-zeroes used in the OK_{pz} analysis.
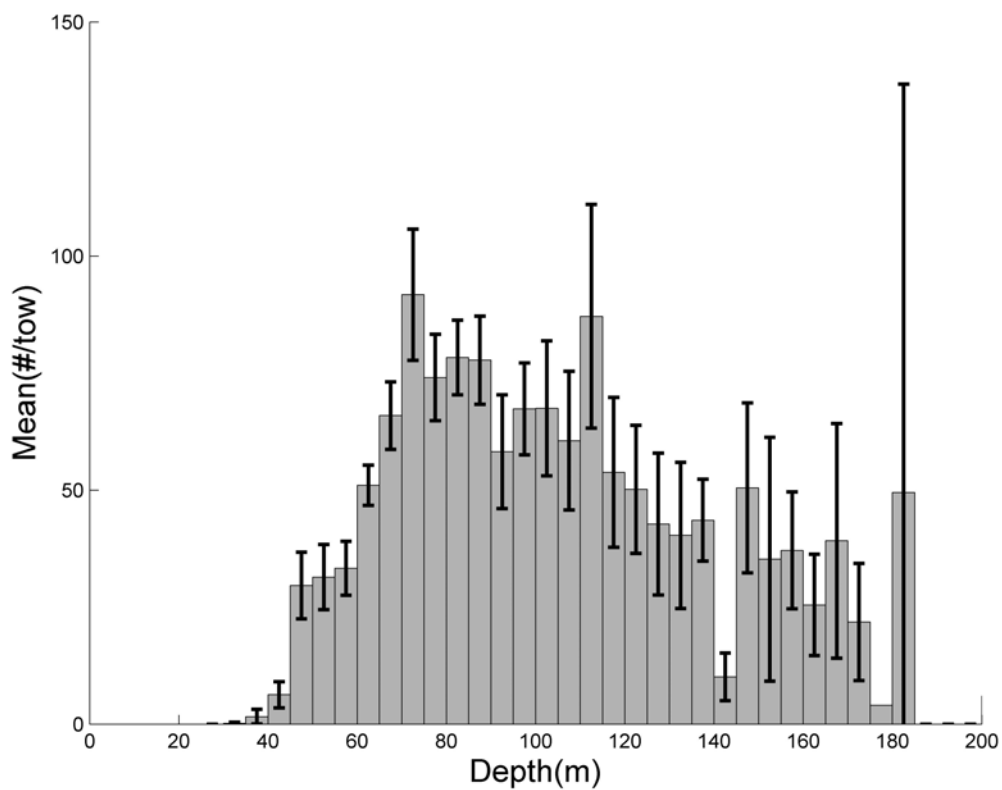
Figure 4: Depth-abundance relationship using total snow crab counts per tow with 95% confidence intervals using sGSL snow crab survey data (1988-2005). For clarity, 1.4% of tows at depths greater than 200m are omitted.
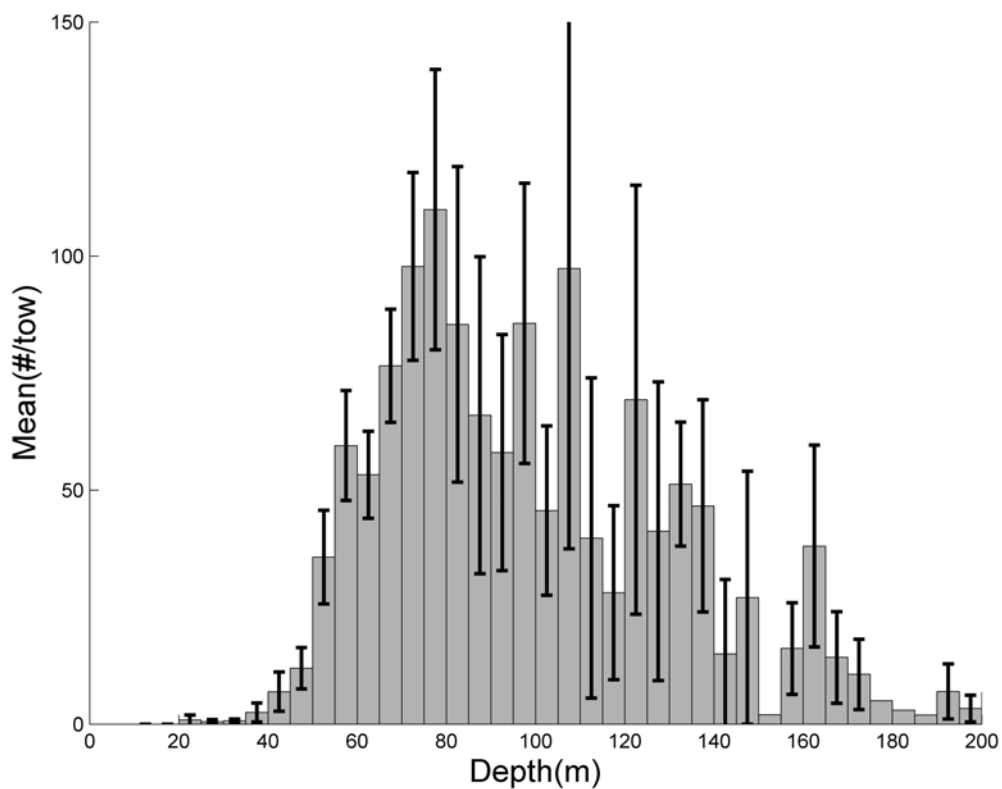
Figure 5: Depth-abundance relationship using total snow crab counts per tow with 95% confidence intervals using sGSL September multi-species survey data (2000-2005). For clarity, 7.8% of tows at depths greater than 200m are omitted.
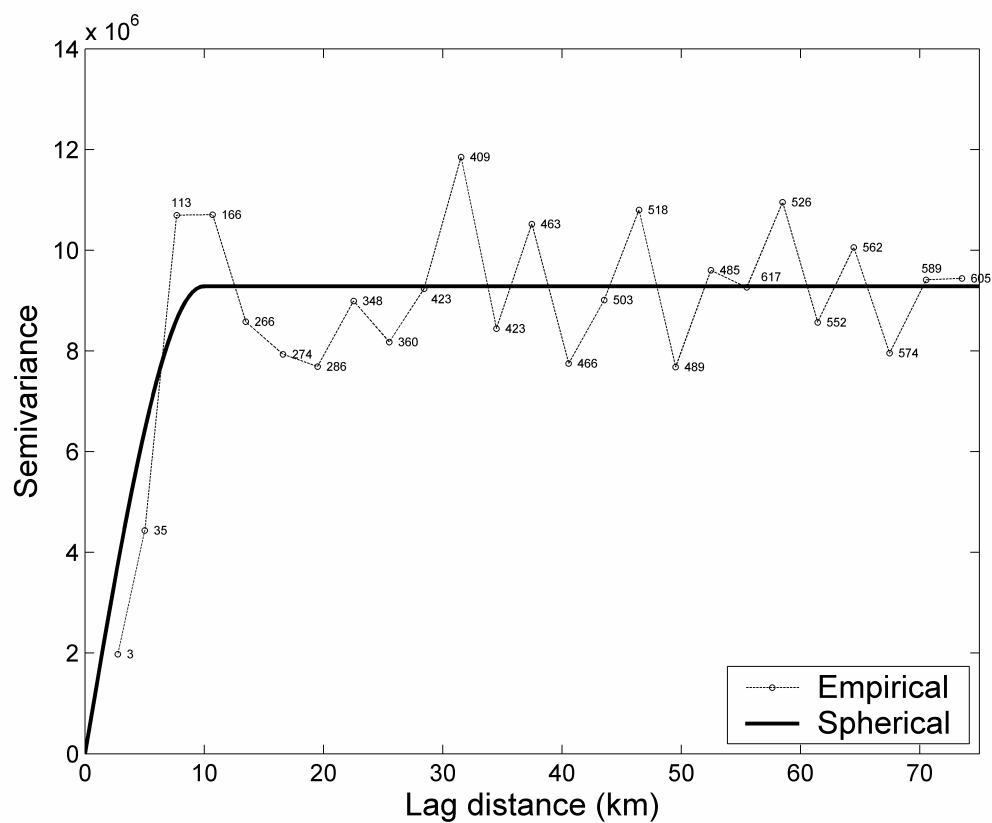
Figure 6: Averaged empirical (dashed line) and fitted spherical variogram (solid line) for 2003 commercial male data for the southern Gulf of St. Lawrence. Estimated parameter values are $a \approx 0$, $b = 9.932$ and $c = 9.283 \cdot 10^6$. The numbers next to the data points refer to the number of data pairs for each lag interval.
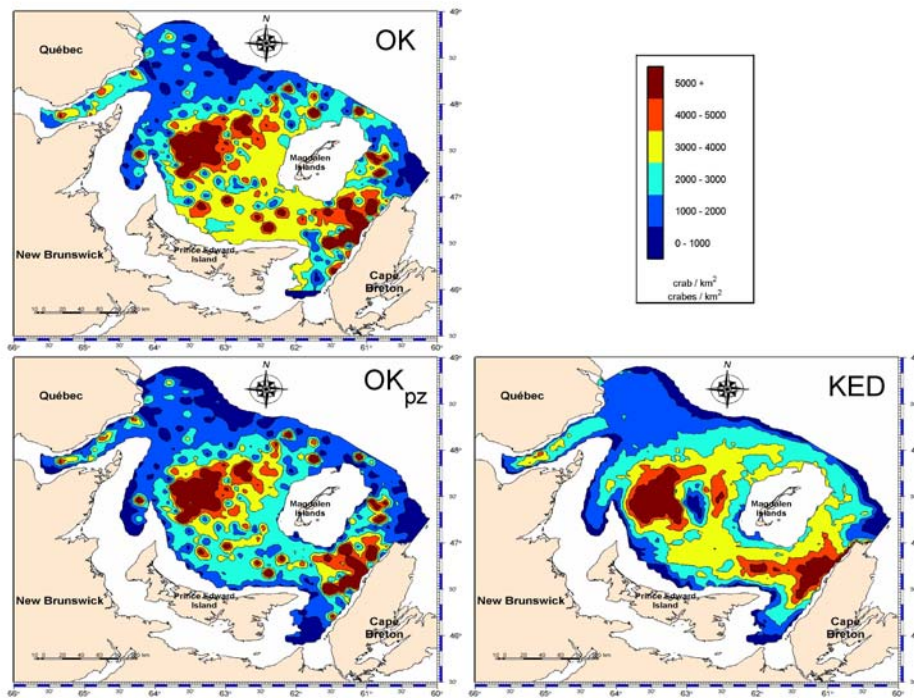
Figure 7: Distribution maps for 2003 commercial snow crab using ordinary kriging (OK), OK with pseudo-zeroes (OK$_{pz}$) and kriging with external drift (KED).
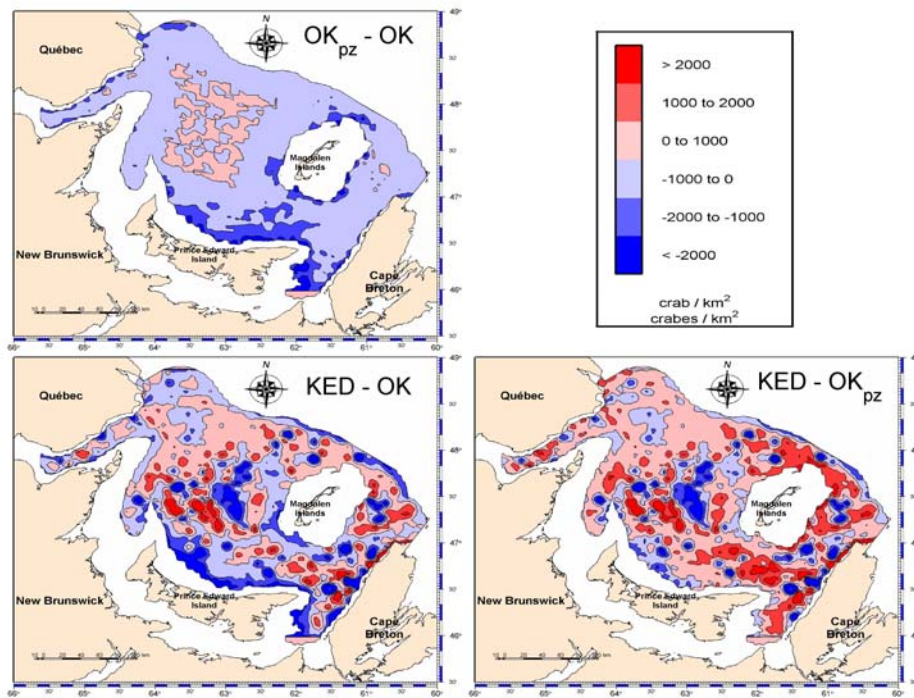
Figure 8: Difference maps for predicted densities for commercial snow crab densities for each pairwise combination of the three kriging methods using the 2003 trawl survey data.
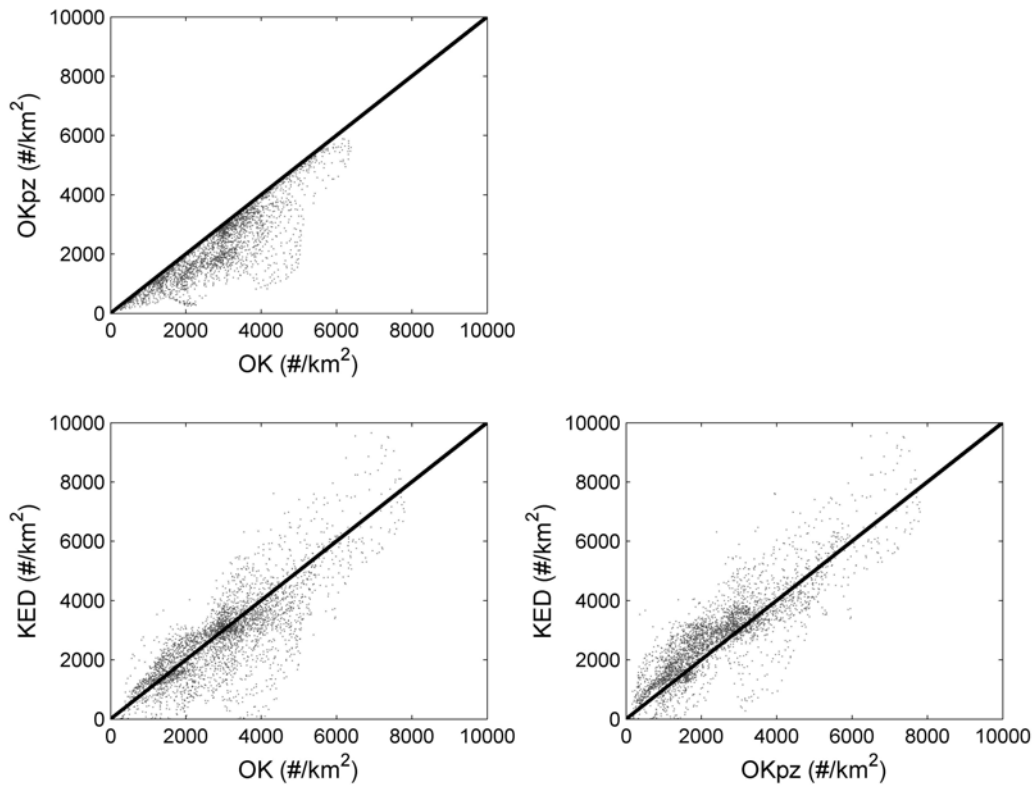
Figure 9: Bivariate plots comparing kriged estimates arising from pairwise combinations of the three kriging methods. A regular lattice of locations was overlain over the study area for these comparisons. For the whole southern Gulf commercial males 2003.
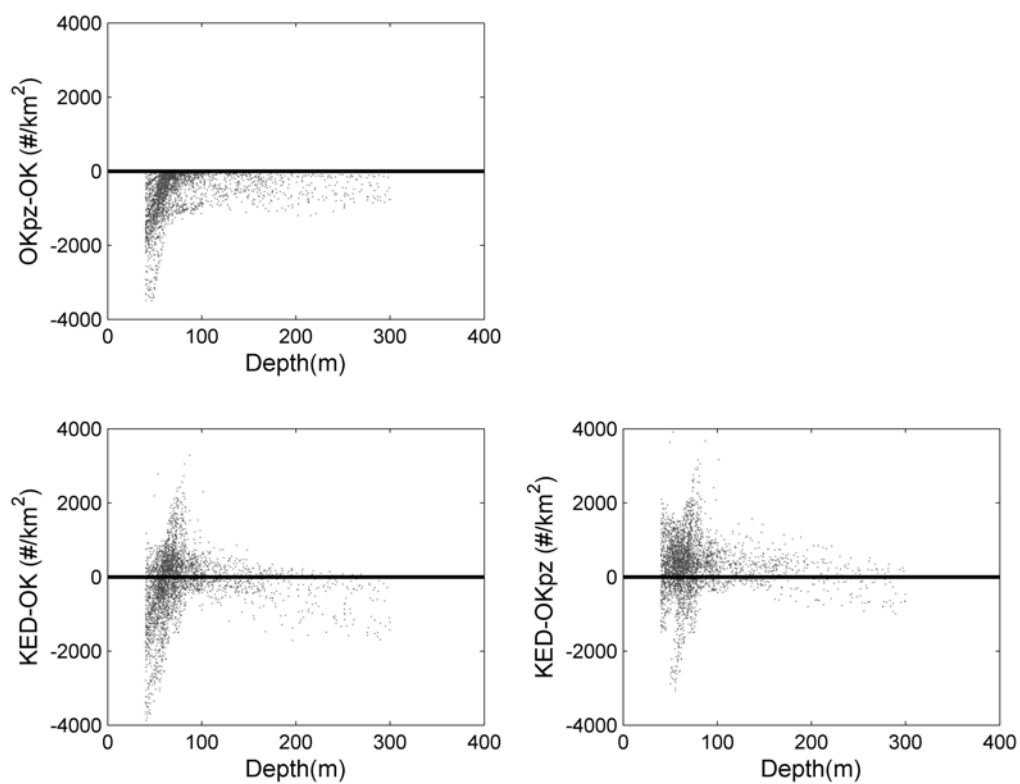
Figure 10: Pairwise differences between kriged estimates from each of the three methods versus water depth. A regular lattice of locations was overlain over the study area for these comparisons. For the whole southern Gulf commercial males 2003.
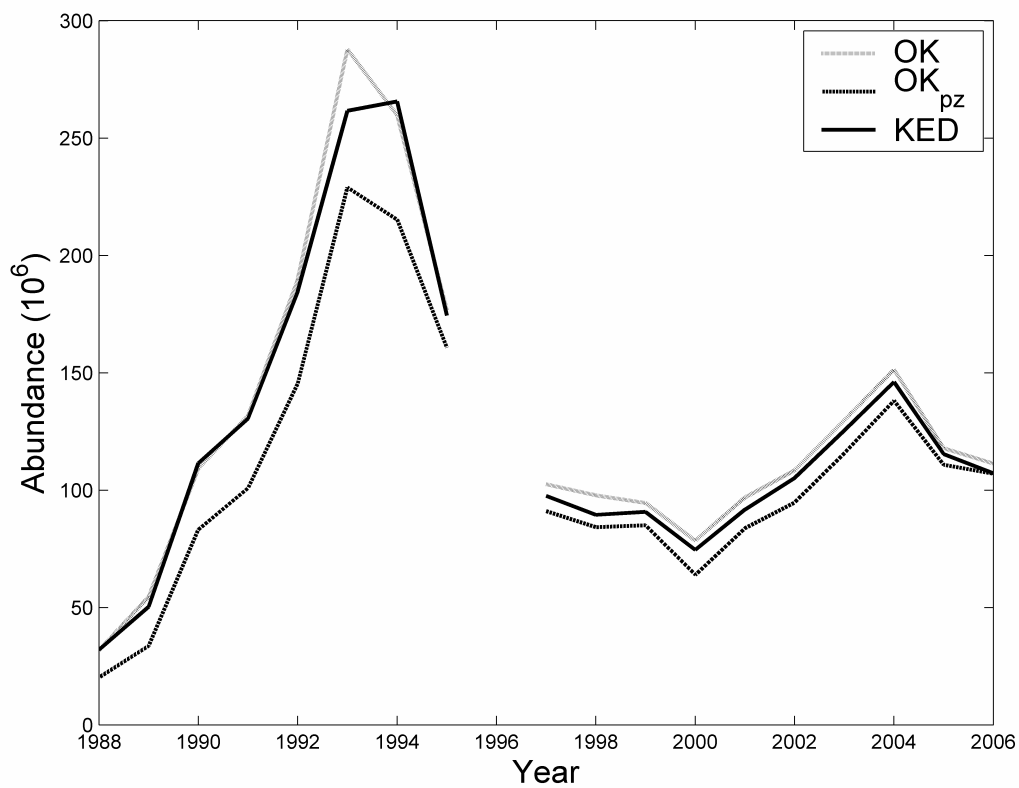
Figure 11: Time series plot of area 12 abundance estimates of the three kriging methods used.