Fisheries and Oceans Canada — Pêches et Océans Canada
Science — Sciences

C S A S — S C C S

Canadian Science Advisory Secretariat — **Secrétariat canadien de consultation scientifique**

Research Document 2005/091 — **Document de recherche 2005/091**

Not to be cited without permission of the authors *

Ne pas citer sans autorisation des auteurs *

Properties of three estimators of recruitment overfishing

Propriétés de trois estimateurs de la surpêche des recrues

Peter A. Shelton

Northwest Atlantic Fisheries Centre
Dept. of Fisheries and Oceans
PO Box 5667, St. John's, NL
A1C 5X1, Canada

* This series documents the scientific basis for the evaluation of fisheries resources in Canada. As such, it addresses the issues of the day in the time frames required and the documents it contains are not intended as definitive statements on the subjects addressed but rather as progress reports on ongoing investigations.

* La présente série documente les bases scientifiques des évaluations des ressources halieutiques du Canada. Elle traite des problèmes courants selon les échéanciers dictés. Les documents qu'elle contient ne doivent pas être considérés comme des énoncés définitifs sur les sujets traités, mais plutôt comme des rapports d'étape sur les études en cours.

Research documents are produced in the official language in which they are provided to the Secretariat.

Les documents de recherche sont publiés dans la langue officielle utilisée dans le manuscrit envoyé au Secrétariat.

This document is available on the Internet at:

Ce document est disponible sur l'Internet à:

http://www.dfo-mpo.gc.ca/csas/

ISSN 1499-3848 (Printed / Imprimé)

Canada

ABSTRACT

Three limit reference point estimators which may indicate recruitment-overfishing are evaluated using stock-recruit data simulated from a Beverton-Holt model with lognormal error. The estimators are the spawner biomass corresponding to 50% of maximum recruitment from Beverton-Holt and changepoint regression model fits to the simulated data, and the spawner biomass corresponding to the intercept of the 50th percentile recruitment value and the 90th percentile of the recruit to spawner biomass ratio (Serebryakov method). The sensitivity of the estimators to a range of data contrast in SSB values and recruitment noise levels is evaluated for data series 20 and 30 years in length. In addition to examining the limit reference points, estimates of the slope of the spawner-recruit relationship near the origin are evaluated for the three methods, since the slope is important in determining population resilience and recovery rates. It is concluded that the LRP estimated from the Beverton-Holt fit to data generated from a Beverton-Holt model is least sensitive to low data contrast and high noise, but that the ability of the nonlinear estimation procedure to find plausible parameter estimates deteriorates with increasing noise and at both low and high levels of data contrast. In comparison, when data are generated from a Beverton-Holt model, both the changepoint regression and Serebryakov methods tend to give risk-prone estimates of the LRP in the sense that the limit reference point is lower than the true spawner biomass corresponding to 50% of maximum recruitment and application in fisheries management may thus not prevent recruitment-overfishing. Estimates of the slope parameter from changepoint regression tend to be lower than the true slope while estimates from Berverton-Holt and Serebryakov methods may be positively biased at high levels of noise. Changepoint regression therefore provides a relatively risk-averse estimate of population resilience.

RÉSUMÉ

Trois estimateurs des points de référence limites susceptibles de révéler une surpêche des recrues sont évalués à l'aide de données stocks-recrues dérivées d'un modèle de Beverton-Holt avec erreur lognormale. Les estimateurs sont, d'une part, la biomasse reproductrice correspondant à 50 % du recrutement maximal indiqué par le modèle de Beverton-Holt avec ajustements apportés aux données ainsi obtenues avec un modèle de régression par points d'inflexion et, d'autre part, la biomasse reproductrice correspondant au point d'intersection de la valeur de recrutement au 50^{e} percentile et au 90^{e} percentile du rapport biomasse reproductrice-recrues (méthode de Serebryakov). La sensibilité des estimateurs à une plage de contrastes de données dans les valeurs de la biomasse du stock reproducteur et à des niveaux de bruits de recrutement est évaluée pour la série sur des périodes de 20 et 30 ans. En plus d'examiner les points de référence limite, on évalue les estimations de la pente du rapport reproducteurs-recrues près de l'origine établies avec les trois méthodes, puisque la pente est importante pour la détermination des taux de résilience et de rétablissement des populations. On conclut que les points de référence limites estimés à partir d'un ajustement de Beverton-Holt apporté aux données produites avec le modèle de Beverton-Holt sont moins sensibles au faible contraste dans les données et au bruit élevé, mais que la capacité de la méthode d'évaluation non-linéaire d'établir des estimations plausibles des paramètres se détériore avec l'augmentation du bruit et à des niveaux faibles et élevés de contraste dans les données. Dans la comparaison, lorsque les données sont produites avec un modèle de Beverton-Holt, la régression par points d'inflexion et les méthodes de Serebryakov ont tendance à donner des estimations plus risquées des points de référence limites en ce sens que le point de référence limite est inférieur à la biomasse reproductrice réelle correspondant à 50 % du recrutement maximal et que, par la suite, l'application de ces estimations à la gestion des pêches risque de ne pas empêcher la surexploitation des recrues. Les estimations du paramètre de pente de la régression par points d'inflexion ont tendance à être inférieures à la pente réelle, tandis que les estimations obtenues avec les méthodes de Berverton-Holt et de Serebryakov peuvent présenter un biais positif à des niveaux de bruit élevés. La régression par points d'inflexion multiple fournit par conséquent une estimation relativement prudente de la résilience des populations.

INTRODUCTION

The Department of Fisheries and Oceans (DFO) Precautionary Approach (PA) framework (Fig. 1) describes three zones with respect to spawning stock biomass (SSB). These zones are separated by an upper or "buffer" reference point (URP or B_{buf}) and a lower limit reference point (LRP or B_{lim}). When the stock is estimated to be in the Healthy Zone it is above B_{buf} and therefore has a low probability of being below B_{lim} at the present time or falling below B_{lim} in the short-term. A typical target fishing mortality (F) in this zone would be $F_{0.1}$, but F should be below F_{lim}, which is typically defined as F_{msy}. When the stock is in the Cautious Zone there is an increased probability of being below B_{lim} at the present time or falling below B_{lim} in the short-term, and F should be reduced, typically to below $F_{0.1}$ according to prescribed harvest control rules. A PA-compliant rebuilding plan may specify rebuilding the stock to above B_{buf} (i.e. to the Healthy Zone) in a specific period of time by reducing F to a predefined level. When the stock is in the Critical Zone it is below B_{lim} and F should be reduced to as close to zero as possible to promote stock rebuilding. Typically this would mean no directed fishing and minimum bycatch. The SSB limit reference point (LPR) or B_{lim} is thus pivotal in the application of the DFO PA Framework. The DFO PA Framework has yet to be applied in fisheries management in Canada.

B_{lim} is generally considered to mark a boundary below which "serious harm" is occurring. In terms of a typical groundfish stock, serious harm would be consistent with severe recruitment overfishing, i.e. a level of spawner biomass depletion which results in much reduced recruitment. In the absence of a depensatory functional response in the stock-recruit relationship, the definition of serious harm or severe recruitment overfishing is somewhat subjective.

A number of candidate SSB LRPs have been suggested (see partial review in Shelton and Rice 2002). One candidate is the SSB corresponding to 50% of the estimated maximum recruitment (Mace 1994, Myers et al. 1994), here termed B_{50}. This can be estimated by fitting a stock-recruit model to the available data. As an alternative to B_{50}, I consider an approach suggested by Serebryakov (1991) and Shepherd (1991) based on percentiles of recruitment (R) and recruits per spawner (R/S). A version of the Serebryakov percentile LRP proposed at the November 2002 DFO Precautionary Approach Workshop in Ottawa, (DFO, 2002) was based on the SSB corresponding to the 50th percentile of R and the 90th percentile of R/S, here termed $B_{50/90}$.

In this research document I attempt to evaluate some of the properties of three B_{lim} estimators through simulation experiments using simulated data generated from a Beverton-Holt model with lognormal error. B_{50} is estimated from the simulated data by fitting a Beverton-Holt model with unknown parameters (B_1) and by changepoint regression using numerical optimization (Julious 2001; B_2). $B_{50/90}$ is estimated from the computation of the appropriate percentiles and determining their point of intersection (B_3). Conclusions are drawn regarding the potential usefulness of these three LRP estimators in any future implementation of the Precautionary Approach in Canada on stocks for which stock-recruit data are available.

The three methods also provide estimates of the slope at low stock size, which is of additional interest (in the case of the Serebryakov method the slope is the 90th percentile *R*/*S* and is taken here to be an estimate of recruitment rate at low stock size). The slope in the *S*-*R* relationship at low stock size is a major determinant of potential recovery rates in depleted stocks and of the extinction threshold or resilience for stocks which continue to be heavily fished at low stock size.

METHODS

Generating fake data

In each simulation experiment, 1000 replicate samples of simulated spawning stock biomass (*S*) and recruit (*R*) data *y* years in length were generated randomly with lognormal error from the predicted *R* in the Beverton-Holt model:

$$R = \exp(\ln(\frac{\alpha S}{1+(S/K)}) + \varepsilon)),$$

where α and K are parameters and ε is drawn randomly from $\mathrm{N}(0,\sigma^2)$. In this formulation of the Beverton-Holt model, maximum recruitment or R_{max} = αK and B_{50} = *K*. Values of S in each experiment were drawn randomly from a uniform distribution U[Min, Max] with Min = 0.1% of the SSB corresponding to $p{*}R_{max}$ and Max = SSB corresponding to $p{*}R_{max}$ where *p* is the proportion of the true R_{max}. Simulated data were generated for experiments which comprised all combinations of the following values of *p*, σ and *y*:

p = 0.4, 0.6, 0.8, 0.9, 0.95
σ = 0.2, 0.4, 0.6, 0.8, 1.0
y = 20, 30

These values broadly bracket commonly experienced values in real data for groundfish stocks. The underlying Beverton-Holt model used to generate the fake data had α =1 and K = 20,000. Note that it is not necessary to explore the sensitivity of B_{lim} estimators over a range of α and K because these parameters merely scale the SSB axis relative to R_{max} (Shelton and Healey 1999). What is important is the range of SSB that is explored relative to the SSB at R_{max}. This is illustrated graphically by superimposing proportions of R_{max} on the plot of the Beverton-Holt model used to generate the fake data (Fig. 2). Note that at *p* = 0.4 the range from which random SSB values are drawn falls entirely to the left of B_{50} and therefore not very informative regarding the asymptote. Conversely, at high values of *p*, a large portion of the data may be drawn from the near-asymptotic portion of the *S-R* curve which is relatively uninformative about the slope near the origin.

Estimating the LRPs

B_1 was estimated by fitting a Beverton-Holt model to the simulated data using maximum likelihood estimation assuming lognormal error (see Myers et al. 1995 for a description of the approach). This estimation method thus matches the data generation method. Parameter estimates were obtained by applying Proc NLP in SAS (SAS/OR © SAS Institute Inc. USA). There is no back-transformation bias adjustment required for the estimate of log(*K*). As σ increases, there is an increasing proportion of runs in which the estimation procedure failed to converge on feasible estimates of the parameters. Replicate *S-R* datasets for which the estimated 50% R_{max}> three times the true 50% R_{max}, estimated B_1 > three times the true B_{50} or B_1 < 5% of the true B_{50}, were discarded. The number of discarded replicates out of the 1000 replicates in each experiment was recorded as an additional indicator of performance of B_1. In practice it would seem that a subjective constraint on R_{max} (i.e. αK) needs to be imposed to obtain reliable estimates, but no formal approach for doing this has been routinely implemented in Canadian Atlantic groundfish assessments.

B_2 was estimated by fitting a changepoint regression model using the numerical optimization approach outlined by Julious (2001), hereafter referred to as the Julious Algorithm. SAS code was written to do the estimation. Changepoint regression, also referred to as segmented regression and "hockey-stick" has been applied to stock-recruit data by Butterworth and Bergh (1993) and Barrowman and Myers (2000), and considered in the context of B_{lim} estimation by O'Brien et al. (2003). The changepoint regression model is

$$\begin{aligned} R &= \alpha S e^{\varepsilon} \text{ when } 0 \le S \le \delta, \text{ and} \\ &= \beta e^{\varepsilon} \text{ when } S \ge \delta. \end{aligned}$$

On the logarithmic scale, this becomes

$$\begin{aligned} \log R &= \log\alpha + \log S + \varepsilon \text{ when } 0 \le S \le \delta, \text{ and} \\ &= \log\beta + \varepsilon \text{ when } S \ge \delta. \end{aligned}$$

α and δ, and hence β, $(\beta = \alpha\delta)$, were estimated by applying the sequence of steps comprising the Julious Algorithm outlined in Fig. 3. $B_2 = 0.5\delta$. The application to data from a Beverton-Holt model is illustrated in Fig. 4. In this example *p*=0.95 and there is no error around the Beverton-Holt model. SSB values are equally spaced between 0 and SSB corresponding to 0.95 of R_{max}. B_2 is much less than B_{50} for this example, with δ being just above B_{50}. The tendency for B_2 to under-estimate B_{50} is explored more fully in the simulation experiments.

B_3 required the computation the appropriate percentiles, 50th percentile R and 90th percentile R/S. The spawner biomass corresponding to the intersection of these two lines demarcates the estimate of B_3. An example application is illustrated in Fig. 5.

In addition to estimation of LRPs, the distributions of estimates of the slope at low stock size from the three methods were also examined, but not as fully as the LRPs. The slope at low stock size has important implications in terms of population resilience and recovery rates. The Beverton-Holt model and changepoint regression estimates of α have to be corrected for bias when back-transformed from the log estimates. The bias correction is $\exp(-\frac{\sigma^2}{2})$. More information on bias correction of back-transformed log estimates in stock-recruit model estimation can be found in MacCall and Ralston (2002). The slope from the Serebryakov percentile method is a simple computation that does not require bias correction for log-transformation. Bias in B_1 arising from non-linearity in the model (Gavaris 1999) is not considered in this analysis.

RESULTS

Examples of single replicates of simulated *S-R* data generated for experiments with y = 30, σ = 0.4 and p = 0.6, 0.8 and 0.95 are given in Figs 6, together with the estimates of the three LRPs based on the data and the true $B_{50.}$ The plot for p = 0.95 is enlarged to reveal the lower left portion of the plot. At p=0.6, all three estimators give values that are lower than the true B_{50}. At p=0.8, the estimate of B_1 is slightly above B_{50}, whereas the other two estimates are below B_{50}. At p=0.95, B_1 and B_2 are slightly below B_{50} in this example replicate, although, as will be shown below, the mean of 1000 replicates of B_1 is slightly above B_{50}. The estimate of B_3 is somewhat higher than B_{50} in this example.

As an example of the differences in the three estimators, distributions for 1000 replicate estimates of the LRP are plotted for y=30, p=0.8 and σ =0.6 (Fig. 7). Clearly the distribution of estimates of B_1 has a long tail to the right of the true value of B_{50} (20,000). The distribution of estimates of B_2 is tighter, but has a mode that is considerably below B_{50}. The distribution of B_3 is the tightest, and has a mode somewhat below B_{50}. These results are explored in more detail below.

The number of successful Beverton-Holt model fits for y =20 and 30, based on the discarding criteria outlined above, are given in Table 1. The number of successful fits decreases with increasing σ and is also lower at the highest and lowest p values. The number of successful fits are somewhat lower for y=20 compared with y=30.

Table 1. Number of successful Beverton-Holt fits out of 1000 based on criteria outlined in the text for experiments at various levels of σ and *p* at *y* = 20 and 30.

20 years

sigma	p=0.4	p=0.6	p=0.8	p=0.9	p=0.95
0.2	969	999	1000	997	986
0.4	833	937	979	968	892
0.6	728	827	922	892	786
0.8	651	788	806	776	697
1	604	670	710	682	608

30 years

sigma	p=0.4	p=0.6	p=0.8	p=0.9	p=0.95
0.2	992	1000	1000	1000	998
0.4	898	972	995	994	972
0.6	790	895	948	959	889
0.8	707	780	888	878	788
1	633	708	787	786	664

For changepoint regression, the number of cases in which δ coincides with the first data point, the last data point or the second last data point is informative regarding evidence of density dependence in the data. If δ coincides with the first data point then the regression is interpreting the data to be from a population in which recruitment does not decline with stock size, i.e. strong density dependence and no evidence that recruitment-overfishing occurs. If δ coincides with the last data point, then change-point regression is interpreting the data to be from a population with no density-dependence in the *S-R* relationship. Similarly, δ coinciding with the second last data point would indicate very weak density dependence.

The incidence of δ coinciding with the smallest SSB data point increased with increasing σ and *p*, being highest at σ =1 and *p*=0.95 (Table 2). The incidence of δ coinciding with the largest or second largest SSB data point increased with increasing σ and decreasing p, being highest at σ =1 and *p*=0.4.

Table 2. Number of changepoint regression cases out of 1000 in which δ coincided with the first (min) largest (max) or second larges (max-1) SSB data points for experiments with y=30 and a range of σ and p.

y=30 min					
Prop	0.4	0.6	0.8	0.9	0.95
sigma					
0.2	0	0	0	1	5
0.4	0	0	1	8	45
0.6	0	2	10	25	60
0.8	1	7	19	54	95
1	1	15	32	79	153

y=30 max-1					
Prop	0.4	0.6	0.8	0.9	0.95
sigma					
0.2	4	0	0	0	0
0.4	43	3	0	0	0
0.6	41	18	1	0	0
0.8	53	32	4	0	1
1	56	30	11	4	1

y=30 max					
Prop	0.4	0.6	0.8	0.9	0.95
sigma					
0.2	10	0	0	0	0
0.4	29	8	0	0	0
0.6	51	22	1	0	0
0.8	55	33	4	1	1
1	67	31	14	1	5

Plots of mean, median and CV for each of the three LRP estimators in each simulation experiment (combination of p, σ and y) against p and σ are given in Figs. 8-13. For y = 20 (Fig. 8), the mean of the replicate estimates of B_1 based on the fit of the Beverton-Holt model, is close to the true B_{50} for experiments at intermediate to high values of p when σ is low, but slightly above B_{50} at low p. B_1 slightly overestimates B_{50} at the highest value of p at all σ levels. The mean of the estimates tends to decrease with increasing σ, particularly at low levels of p. The lowest estimate occurs at the highest σ and lowest p. For changepoint regression, means of the replicate estimates of B_2 are below B_{50} except at high p, and exceed B_{50} at the highest p at intermediate to high σ levels. The means of B_2 increase with increasing p at all values of σ. There appears to be little effect of increasing σ on the mean of the estimates, except at higher values of p. For the Serebryakov percentile method, the means of the replicate estimates of B_3 are generally well below B_{50} but increase abruptly to exceed the true B_{50} at high p. The mean of the estimates generally decreases with increasing σ. Note that, unlike the Beverton-Holt and changepoint estimates, the comparison here is between two different definitions of B_{lim}, i.e. B_3 is not an estimator of B_{50}.

Results are generally similar when y = 30 (Fig. 9). The means of the B_1 and B_2 estimates are somewhat closer to the true B_{50} in general, but not substantially so. There is almost no difference in estimates of B_3 for 30 years of simulated data compared to 20 years.

The medians of replicates for the 3 estimators at y = 20 (Fig. 10) are overall similar to the plots for the means. The medians of the estimates of B_1 are closer to the true B_{50} than the means at low σ, but decrease more with increasing σ at low p compared with the mean. The medians of the estimates of B_2 do not increase as much as the means at high p. The plot of the medians of the estimates of B_3 is very similar to the plot of the means. At y = 30, the medians of B_1 estimates (Fig. 11) tend to be closer to the true B_{50} compared with experiments at y = 20. The medians of the estimates of B_2 are not as close to the true B_{50} at higher values of p at y = 30 as they are at y = 20 at intermediate to high σ levels. Again, there is little difference in the medians of the B_3 estimates at y=30 compared to y=20.

The CVs at y = 20 and y = 30 reflect considerable variability in the estimates of B_1 and B_2 (Figs. 12 and 13). The CVs increase with increasing σ as expected, and in the case of B_2, also increase with increasing p at intermediate to high σ. There is also some increase in B_1 at the lowest levels of σ and highest p. At lower values of σ, there is a slight valley in the surfaces causes by smaller CVs at intermediate values of p for both B_1 and B_2. The reason that the CV increases at the highest values of p is because more of the data points are gathered from the near-asymptotic area of the *S-R* function which means greater uncertainty regarding the slope near the origin and hence greater uncertainty in the estimation of B_{50}. CVs on B_1 and B_2 are somewhat lower at y = 30 compared with y = 20, but not substantially so. The CVs associated with the estimates of B_3 are a lot smaller than CVs on both B_1 and B_2 and also decrease somewhat at y = 30 compared to y = 20.

The mean, median and CV for y = 30 experiments are plotted for each value of p against σ to allow more direct comparison of the estimates from the three methods (Fig. 14-16). Fig. 14 illustrates that the mean B_1 remains close to the true B_{50} with slight under-estimation at low p and high σ, changing to general small over-estimation at higher p. In contrast, the means of B_2 shift steadily higher towards B_{50} with increasing p. The means of B_3 also shift upwards with increasing p, but because of a negative relationship with σ, estimates go from being less than B_{50} at low p to being greater than B_{50} at higher p and low σ, but lower at higher σ. At p=0.95 the means of B_3 are all above B_{50}. As pointed out above, the plots of the medians show generally similar trends (Fig. 15) to those described for the means. CVs all increase with increasing σ as might be expected (Fig. 16) with B_1 having the highest CV at p=0.4 and 0.6, changing to

B_2 having the higher CVs at high σ and high *p*. CVs for B_3 are smaller than for the other two estimators and increase at a slower rate with increasing σ.

The mode for the distribution of estimates of α (bias-corrected) from fitting the Beverton-Holt model to the 1000 simulated data sets at *y*=30, *p*=0.8 and σ=0.6 (Fig. 17) is centered close to the true α, whereas the tail of the distribution extends beyond 3xα. The mode of the distribution of estimates of slope from changepoint regression (bias-corrected) occurs around 0.5, i.e. well below the true slope of the Beverton-Holt model used to generate the data. The distribution is somewhat tighter than for the Beverton-Holt fits. For Serebryakov's percentile approach, the mode of the estimates of the 90th percentile R/S is slightly lower than 1.0 and the distribution is quite compact.

Although equivalent surface plots to those provided for the estimators of B_{50} were not constructed for slope estimates, values are plotted for a range of σ values at *p*=0.8, *y*=30 and for a range of *p* values at σ=0.6, *y*=30 (Fig. 18) to illustrate a subset of the results. With respect to increasing noise at *p*=0.8, means of the estimates of the slopes increase with increasing σ for Beverton-Holt and Serebryakov approaches but remain reasonably constant for changepoint regression. For the Beverton-Holt model, the mean increases from near 1 (true slope value) to about 1.6 with increasing noise. For the Serebryakov method, the increase is from 0.7 to about 1.6. For change point regression the values are relatively constant in the range of 0.6 to 0.7, i.e. well below the true value. At σ=0.6, the mean of the slope estimates from the Serebryakov method are greatest at 1.7 with *p*=0.4, but decrease rapidly to 0.4 with further increase in *p*. Estimates from the Beverton-Holt model are fairly constant at about 1.1-1.2 for *p* between 0.4 and 0.8, but increase to above 1.4 at higher p. The mean of the estimates from changepoint regression are relatively constant between 0.65 and 0.73 except at *p*=0.95 where the mean drops to below 0.6.

DISCUSSION

There is little information in the literature on the evaluation of reference points based on fish stock-recruit data. Myers *et al.* (1994) and Mace (1994) provided some empirical evidence and analytical support for suggesting B_{50} based on the Beverton-Holt model as an LRP. It is the SSB below which the population fails, on average, to produce half of the maximum possible recruitment. An alternative LRP evaluated in this study is Serebryakov's $B_{50/90}$. Although this approach was endorsed by Shepherd (1991), there has been no formal testing of $B_{50/90}$. It is considered to be the point below which the population is unlikely to produce average recruitment under good early life-history stage survival conditions.

Fitting a Beverton-Holt model, carrying out changepoint regression or applying Serebryakov's percentile method provide three approaches for estimating LRPs from *S-R* data. The Beverton-Holt and changepoint regression approaches can be used to estimate B_{50} or SSB corresponding to some other percentage of R_{max}. ICES has considered the change point itself as an LRP (O'Brien *et al*. 2003). It would be difficult to defend the point at which recruitment begins to become impaired (change point) as an LRP under the Canadian precautionary approach (see for example Shelton and Rivard, 2003). Canada defines an LRP as the point at which serious harm is considered to have commenced. The LRP derived from Serebryakov's percentile method, $B_{50/90}$ is similarly arbitrary. In the absence of depensation in the S-R relationship, all LRPs are going have associated with them some degree of arbitrariness.

In this study the true process assumed to be generating *S-R* data is Beverton-Holt with lognormal error. While the Beverton-Holt model is widely applied in fisheries assessments and related analysis, a variety of other generating models could be considered. Lognormal error is also widely applied in modeling *S-R* data but other error structures may be appropriate in some cases. In this study, arbitrary choices for deriving LRPs from the three methods are considered – estimates of B_{50} for the Beverton-Holt and changepoint regression methods and the intersection of the 90th percentile R/S and 50th percentile *R* for the Serebryakov approach. A large number of alternative generating models, error structures and derived LRPs could be considered in a simultaneous analysis. The results of a more thorough analysis of this kind are currently being documented by colleagues (Noel Cadigan and Brian Healey, pers. comm. Science Branch, Dept. of Fisheries and Oceans, PO Box 5667, St. John's NL, Canada A1C 5X1).

Although restricted to a single generating model and error assumption, and to just two LRPs (B_{50} and $B_{50/90}$), the results from this limited study provide some insight into the problems of estimating LRPs from *S-R* data. Estimates of B_{50} obtained by fitting the Beverton-Holt model with lognormal error to the simulated data were overall the most satisfactory, as might be expected given that the simulated data were generated by a Beverton-Holt model with lognormal error. However, as σ increased in the simulated data, the proportion of acceptable fits decreased to as low as 60%, indicating that in practice, with large error around the *S-R* relationship, it might be necessary to constrain the estimation to get acceptable fits. The number of acceptable fits (based on the predefined acceptability criteria described in the methods) decreased at low and high *p* levels as well. Low *p* provides little data contrast for estimating *K* and high *p* provides little information about α. B_1 became negatively biased with respect to B_{50} at high σ and low *p*. This is of some concern because it implies a smaller SSB value for the reference point, the greater the noise in the *S-R* relationship, which is risk-prone (in the sense of setting an LRP lower than it should be, thereby increasing the risk of serious harm in making management decisions). It is some consolation that this effect is greatest at low *p*, and so should not have as large an impact on stocks which have traversed a wide range of spawner biomass levels, provided the *S-R* relationship has been stationary. The tendency in some assessments to restrict the analysis to more recent data would exacerbate the bias. At p=0.4 all the simulated data are to the left of B_{50}. Estimating B_{50} from such data would be expected to lead to biases and should be avoided. However, in practice, with noisy data, it may not be obvious that all the data are from below B_{50}.

In comparison to the LRP based on fitting the Beverton-Holt model, estimating B_{50} from the changepoint regression performed quite poorly. Estimates of B_2 were substantially negatively biased with respect to B_{50} except at the highest data contrast situations (high p). This would imply that if the true process was Beverton-Holt, application of changepoint regression would give estimates of B_{50} that are much lower than they should be – a risk-prone situation. On the positive side, estimates of B_2 appeared to be relatively robust to increasing levels of error around the *S*-*R* relationship.

The Serebryakov percentile approach also performed poorly relative to the true B_{50}. Estimates of B_3 were well below B_{50} for low data contrast situations but were higher than B_{50} at the highest levels of data contrast. In addition, estimates of B_3 decreased with increasing error in the *S*-*R* relationship. A lower estimate of an LRP when the *S*-*R* relationship is noisy, is risk-prone.

The CVs of the LRP estimators increased with increasing noise in the *S*-*R* relationship, as would be expected. Changepoint regression estimates were most sensitive to increasing noise and had particularly high CVs for noisy *S*-*R* data under high data contrast conditions. The simulated data from the Beverton-Holt model at high p is randomly selected over a range of SSB from near the origin to the SSB at which 0.95 of maximum recruitment occurs. In this situation a greater proportion of the data points come from the near-asymptotic portion of the relationship, which is relatively uninformative regarding the slope at the origin. The LRP estimated by the Serebryakov percentile method was most robust to noise in the *S*-*R* relationship. Note that in the case of B_1 the distribution of simulated estimates is trimmed by the application of the discarding rule for extreme low and high estimates.

Although not central to this study, the estimated slope at low stock size from the three LRP estimators has relevance to species-at-risk and stock-rebuilding considerations. Estimates of the slope from a Beverton-Holt model fit were unbiased at low noise levels but became increasingly positively biased with increasing noise in the *S*-*R* relationship. Thus the resilience of a population to overfishing, or the ability of a depleted population to rebuild, when that population has a Beverton-Holt-like *S*-*R* relationship, may be overestimated by a Beverton-Hold model fit to noisy data. This effect was relatively insensitive to the amount of data contrast over the range of p=0.4 to 0.8 but estimates of the slope increased at higher levels of contrast. Changepoint regression, on the other hand, under-estimated the slope. Estimates of the slope decreased slightly with increasing noise and data contrast. Thus changepoint regression estimates would be relatively risk-averse with respect to resilience and recovery. The

Serebryakov percentile method gave estimates that under-estimated the slope at low noise levels and over-estimated it at higher noise levels. Estimates of the slope were highest for low data-contrast situations (most of the data from the lower left portion of the *S-R* relationship), but slope estimates decreased rapidly with increasing data contrast.

O'Brien et al. (2003) evaluated the robustness of the change point as an LRP using simulated data based on model fits to North Sea cod data. They explored a variety of hypotheses regarding the *S-R* relationship (Ricker, Beverton-Holt and recruitment independent of SSB) and data quality (discarding, misreporting and an incorrectly perceived low recruitment for the recent period). They found that there were only small differences in the estimates of the LRP for data generated from the Beverton-Holt and SSB-independent models. Estimates of the LRP from data generated from a Ricker model tended to occur at higher SSB values. There was minimal effect of discards on reference points but both misreporting and a perceived decline in recruitment biased the estimates of the LRP upwards. In comparison, in the present study B_2 under-estimated B_{50} except in the highest data contrast situation. Data contrast and noise effects *per se* were not examined in the O'Brien et al. (2003) study. O'Brien et al. (2003) found that the estimates of the slope (F_{lim}) were more robust. This would appear to be consistent with the relative robustness of the estimates of the slope near the origin from the Beverton-Holt and changepoint regression models in this study.

Barrowman and Myers (2000) compared the fit of the Beverton-Holt and "hockey stick" (changepoint regression) models on 246 *S-R* data sets. They found that the Beverton-Holt model estimated a larger maximum reproductive rate than changepoint regression. This is consistent with the results obtained for estimates of the slope based on simulated data in this study. Barrowman and Myers (1999) also found that the Beverton-Holt model estimated a larger carrying capacity of recruits than changepoint regression. This is also consistent with the estimates of B_1 tending to be larger than B_2 in the present study.

Based on simulated data from a Beverton-Holt model with lognormal error, there does not appear to be a robust method for estimating LRPs in terms of spawner biomass. If the true process is Beverton-Holt with lognormal error, then fitting a Beverton Holt model with lognormal error will give reasonably robust estimates of B_{50} except at highest noise levels and low data contrast. However, the incidence of unacceptable model fits increases with increasing noise and at high and low data contrast which is likely to problematical in practice. Both changepoint regression and the Serebryakov approach are likely to be risk prone (in the sense of providing LRP estimates that are lower than the true B_{50}), except in the highest data contrast situations. Estimates of the slope at the origin from changepoint regression are likely to be risk averse when data are generated by a population displaying Beverton-Holt type behavior and under lognormal error, and can thus be considered to be potentially useful in practice when deciding on population resilience and rebuilding rates at low stock size.

Although the results that are described here represent only a partial evaluation of selected LRPs, some serious concerns are raised. It is commonly assumed that recruitment in groundfish is generated by a Beverton-Holt-like process with lognormal error and LRPs are commonly estimated by fitting a Beverton-Holt model or by changepoint regression. The results show that these methods can lead to biased estimates of a proposed LRP, B_{50}, which would be risk-prone when translated into a fisheries management context. Clearly a more comprehensive evaluation of the precision and accuracy of various LRP estimators under a variety of generating models is warranted. The ultimate aim is to develop LRPs which are robust to the underlying uncertainties.

ACKNOWLEDGEMENTS

Members of the DFO national working group on the PA are thanked for stimulating discussions on the topic of this paper. Dr. Noel Cadigan, Mr. Brian Healey and Dr. Garry Stenson are thanked for useful comments on the draft which led to a number of improvements.

REFERENCES

Barrowman, N.J., and Myers, R.A. 2000. Still more spawner-recruitment curves: the hockey stick and its generalizations. Can. J. Fish. Aquat. Sci. 57:665-676.

Butterworth, D.S. and Bergh, M.O. 1993. The development of a management procedure for the South African anchovy resource. pp. 83-99. In: Smith, S.J., Hunt, J.J. and D. Rivard. eds. Risk Evaluation and Biological Reference Points for Fisheries Management. Can. Spec. Publ. Fish. Aquat. Sci. 120.

DFO. 2002. National Workshop on Reference Points for Gadoids, Ottawa, November 5-8, 2002. DFO Can. Sci. Adv. Sec. Proc. Ser. 2002/033.

Gavaris, S. 1999. Dealing with bias in estimating uncertainty and risk. Proceedings, 5th NMFS NSAW 1999. NOAA Tech. Memo. NMFS-F/SPO-40:46-50.

Julious, S.A. 2001. Inference and estimation in a changepoint regression problem. The Statistician 50:51-61.

MacCall, A.D., and Ralston, S. 2002. Is logarithmic transformation really the best procedure for estimating stock-recruit relationships? N. Am. J. Fish. Man. 22:339-350.

Mace, P.M. 1994. Relationship between common biological reference points used as thresholds and targets for fisheries management strategies. Can. J. Fish. Aquat. Sci. 51:110-122.

Myers, R.A., Rosenberg, A.A., Mace, P.M., Barrowman, N., and Restrepo, V.R. 1994. In search of thresholds for recruitment overfishing. ICES J. mar. Sci. 51:191-205.

Myers, R.A., Bridson, J., and Barrowman, N.J. 1995. Summary of Worldwide Spawner and Recruitment Data. Can. Tech. Rep. Fish. Aquat. Sci. 2024.

O'Brien, C.M., L.T. Kell, L.T., and Smith, M.T. 2003. Evaluation of the use of segmented regression through simulation for a characterisation of the North Sea cod (*Gadus morhua* L.) stock, in order to determine the properties of B_{lim} (the biomass at which recruitment is impaired). ICES CM 2003/Y:10.

Serebryakov, V.P. 1991. Predicting year-class strength under uncertainties related to the survival in the early life history of some North Atlantic commercial fish. NAFO Sci. Coun. Stud. 16:49-56.

Shelton, P.A. and Healey, B.P. 1999. Should depensation be dismissed as a possible explanation for the lack of recovery of the northern cod (*Gadhus morhua*) stock? Can. J. Fish. Aquat. Sci. 56:1521-1524.

Shelton, P.A. and Rice, J.C. 2002. Limits to overfishing: reference points in the context of the Canadian perspective on the precautionary approach. Can. Sci. Adv. Sec. Res. Doc. 2002/084.

Shelton, P.A., and Rivard, D. 2003. Developing a precautionary approach to fisheries management in Canada – the decade following the cod collapses. NAFO SCR Doc. 03/1.

Shepherd, J.G. 1991. Report of NAFO Special Session on Stock Assessment and Uncertainty, NAFO Sci. Coun. Stud. 16:7-12.

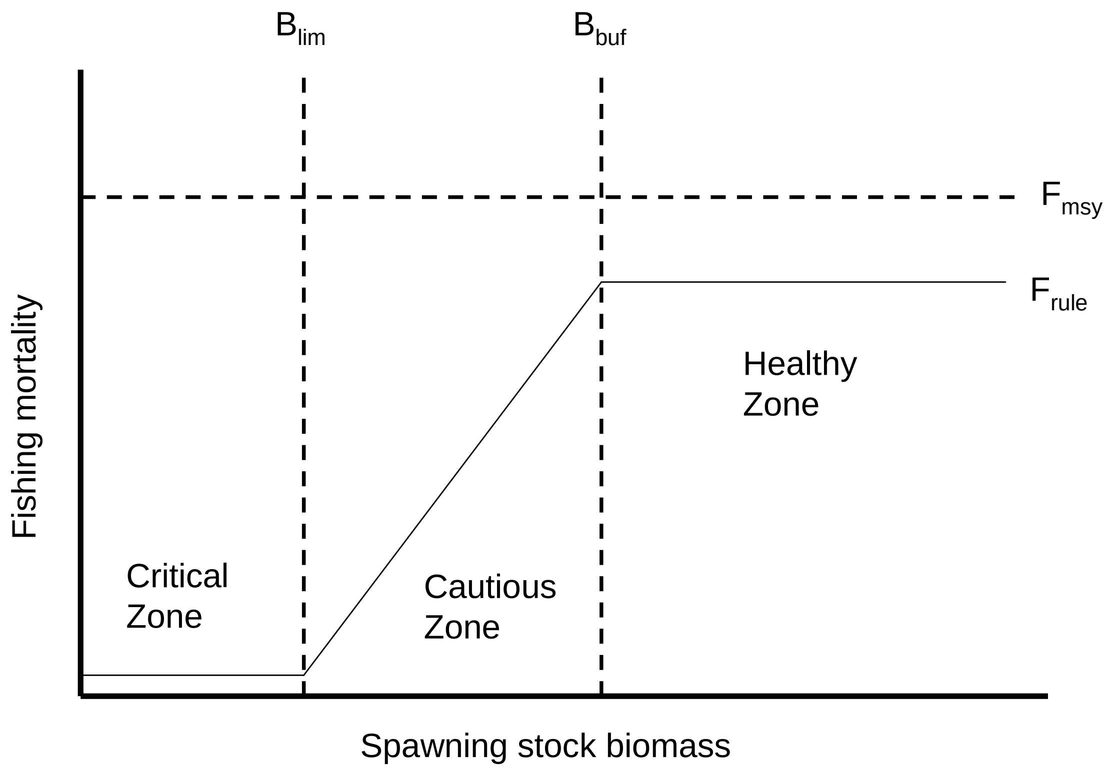

Fig. 1. DFO precautionary approach framework for fisheries management. This paper investigates the properties of three estimators of B_{lim}, corresponding to severe recruitment overfishing.

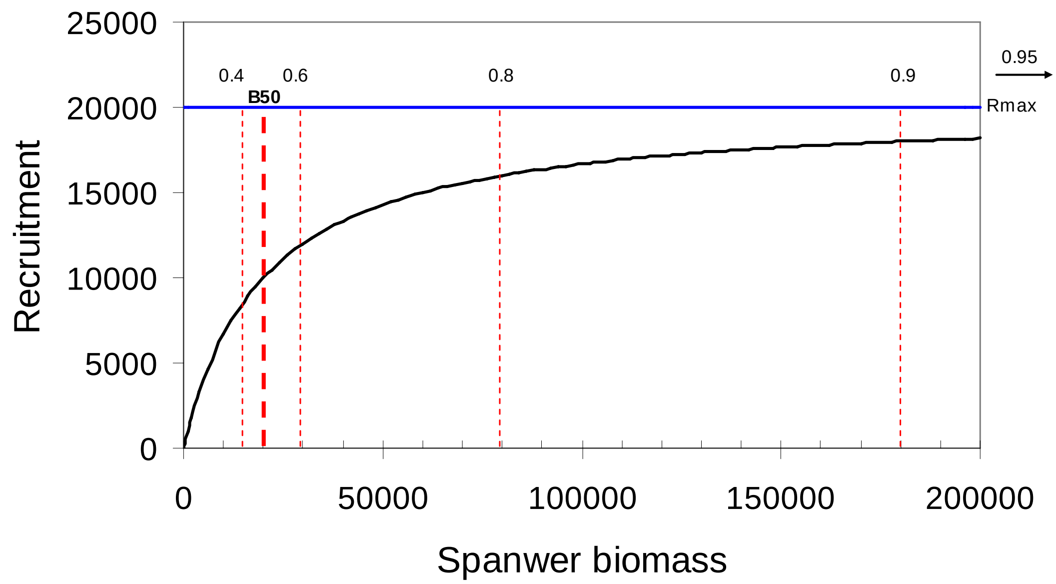

Fig. 2. Beverton-Holt model used to generate simulated data for evaluating the properties of three B_{lim} estimators. B_{50} is denoted by the bold broken vertical line. The remaining vertical lines denote spawner biomass corresponding to proportions *p* of R_{max} used as the upper limit of the ranges of spawner biomass in individual experiments. $p = 0.95$ is off the plot to the right at an SSB of 380,000 t.

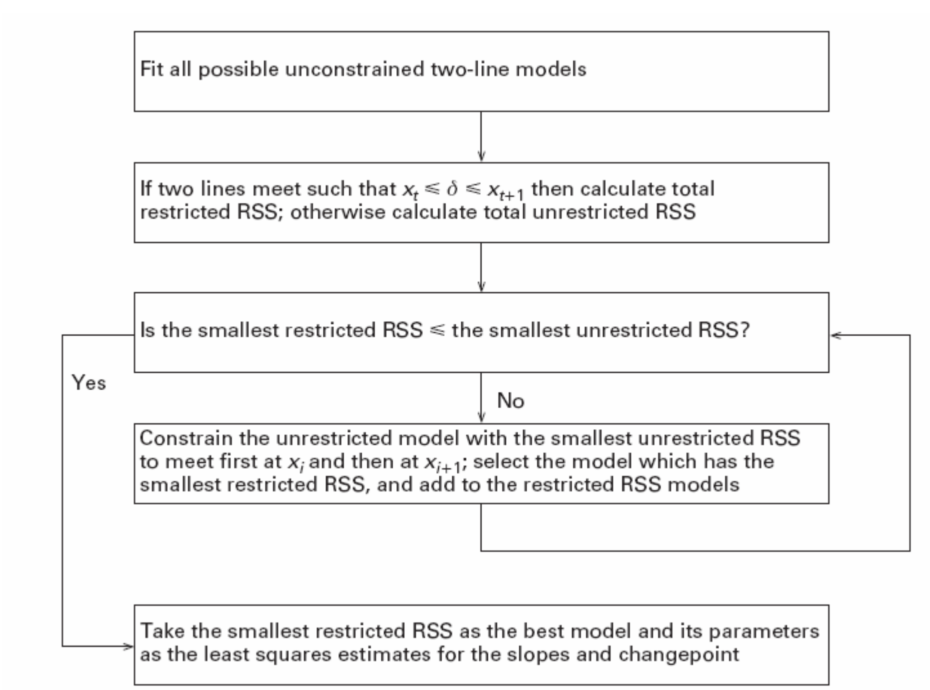

Fig. 3. Julious Algorithm used to obtain an estimate of the changepoint (copied from Julious 2001 from which more details may be obtained).

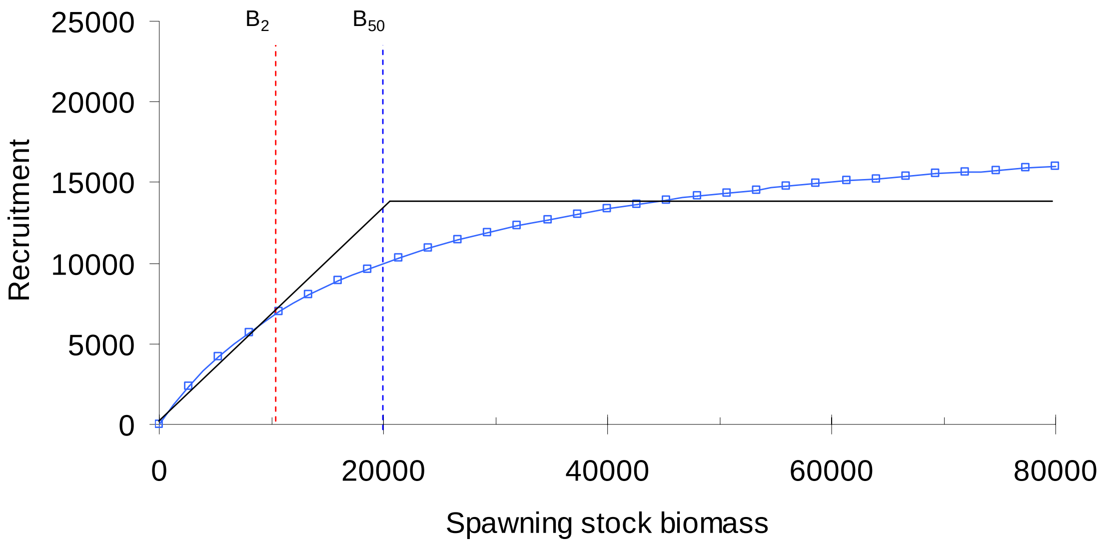

Fig. 4. Example application of changepoint regression (solid lines) to data generated from a Beverton-Holt model with p=0.95 and no error. The squares indicate the data points. The vertical broken line to the right indicates the position of B_{50}., just below the breakpoint estimated in the segmented regression. The estimate of B_2 is well to the left of the true B_{50}.

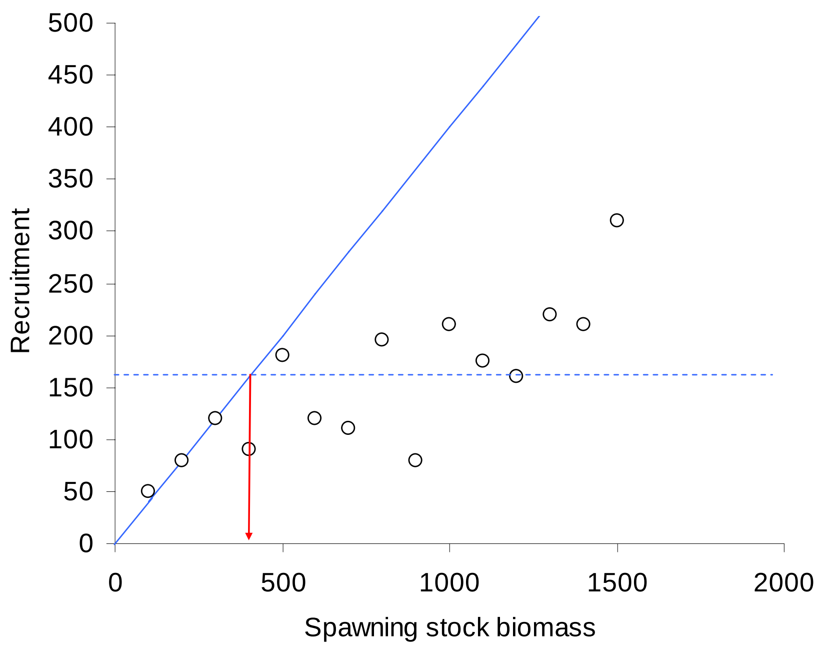

Fig. 5. Schematic illustrating the estimation of the Serebryakov limit reference point $B_{50/90}$, here denoted as B_3. The solid line through the origin is the 90th percentile R/S and the horizontal broken line is the 50th percentile R. The arrow indicates the point on the spawning stock biomass axis corresponding to the intersection of the two percentiles and is the estimate of B_3.

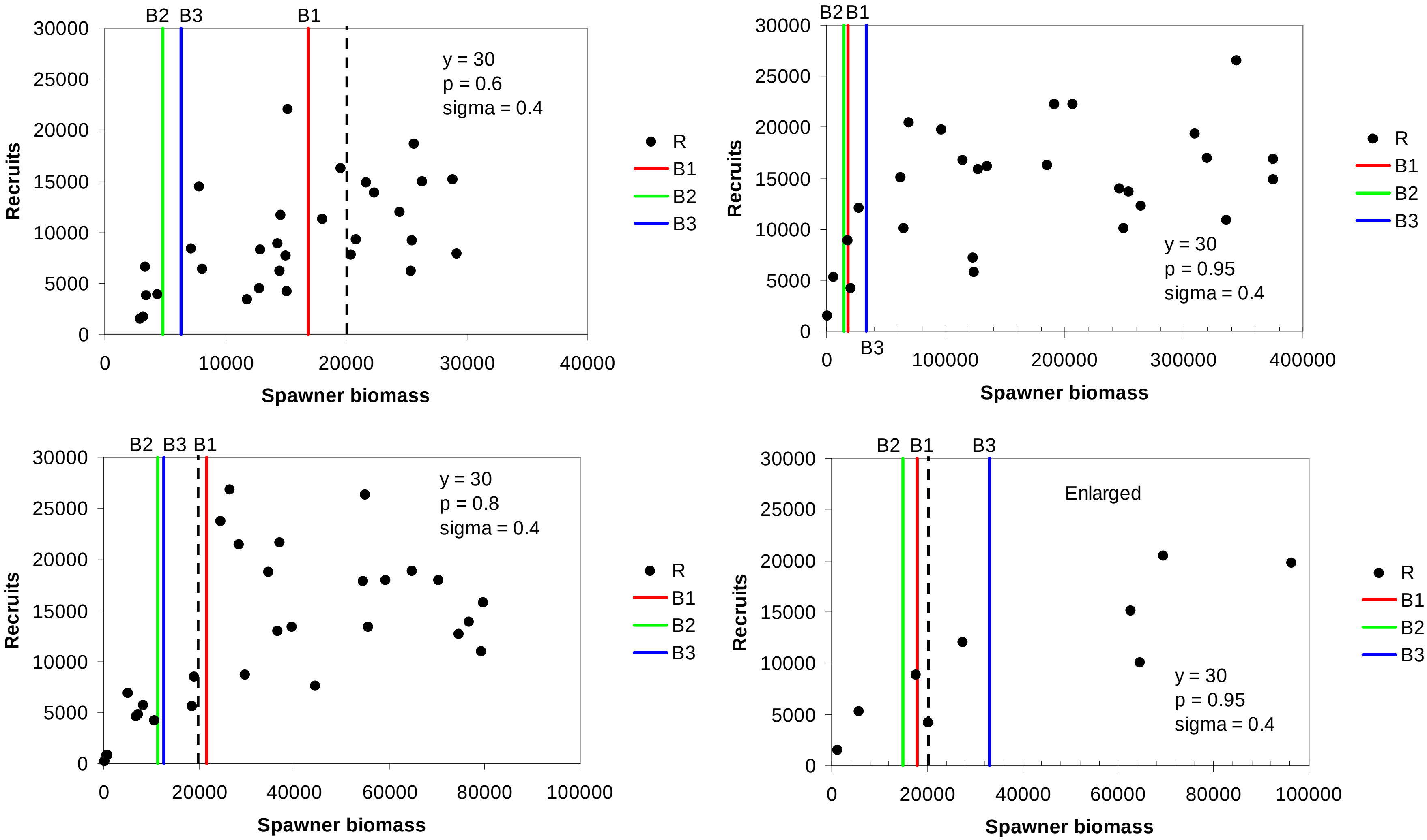

Fig. 6. Example data and estimates for one replicate at *y* = 30, σ = 0.4 and *p* = 0.6, 0.8 and 0.95. The vertical broken line indicates the position of the ture B_{50}. B_1=Beverton-Holt, B_2=changepoint regression and B_3=Serebryakov's percentile method estimates of an LRP.

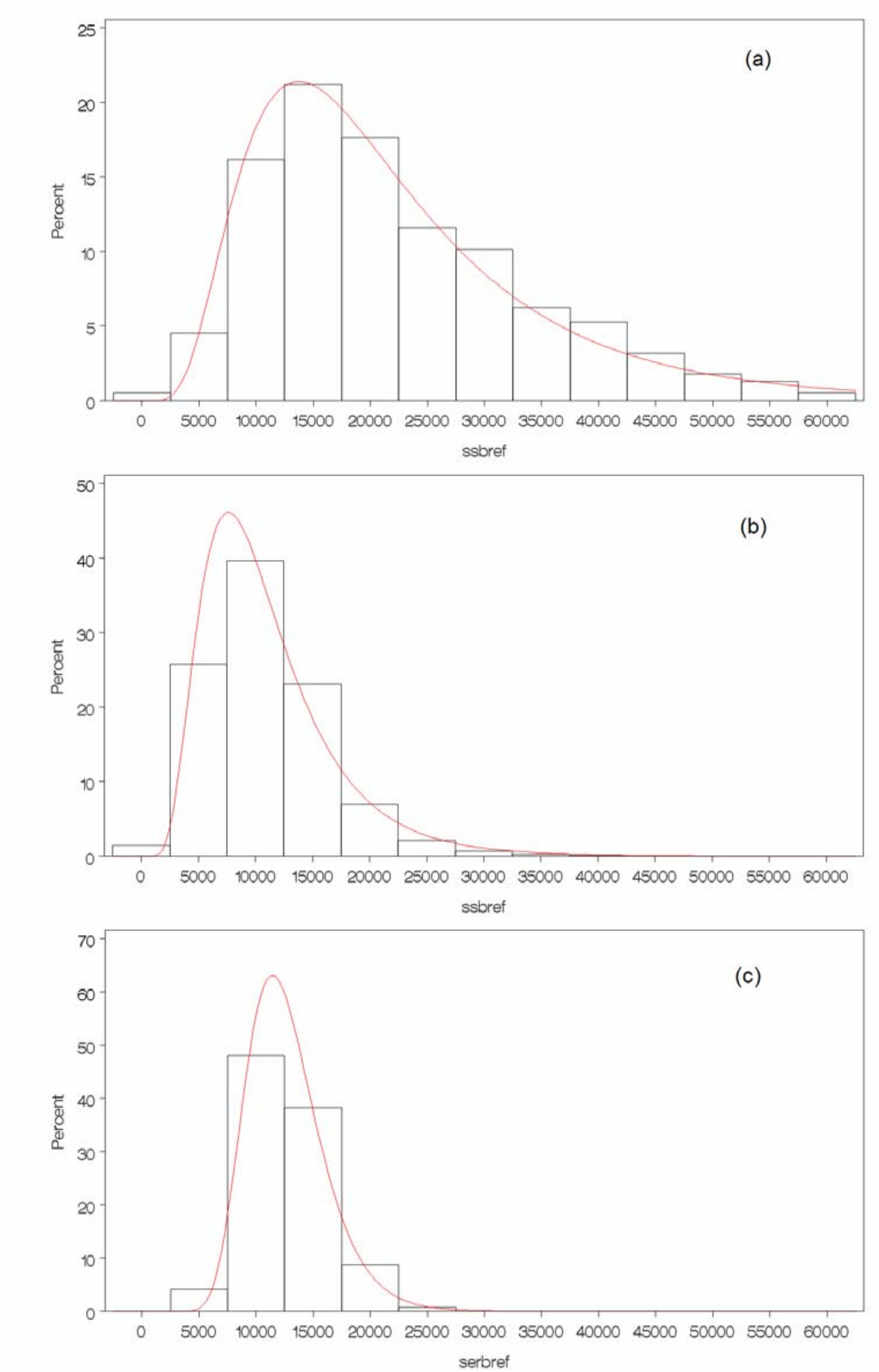

Fig. 7. Histograms of the distributions of the estimates of the LRPs for (a) Beverton-Holt, (b) changepoint regression and (c) Serebryakov's percentile method, applied to simulated data generated from a Beverton-Holt model with y=30, p=0.8 and σ=0.6. Log-normal distributions are fitted to the estimates. The true B_{50} = 20,000.

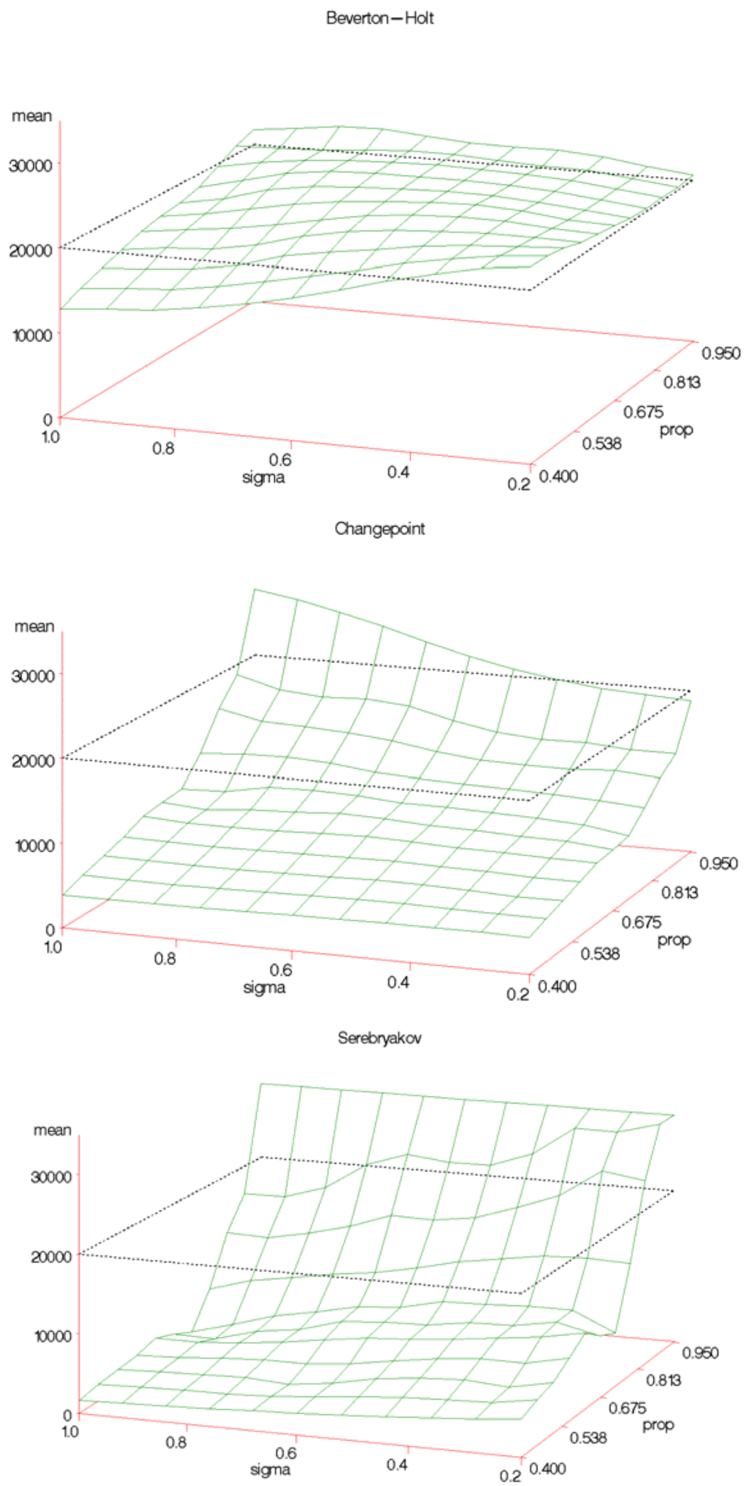

Fig. 8. Comparison of the means of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 20 years in length. The dotted line indicates the true B_{50}. The surface is based on interpolation between experiments. Note that the plot is truncated at 30,000 t , as indicated by the malformed cells in the surface plot for some experiments.

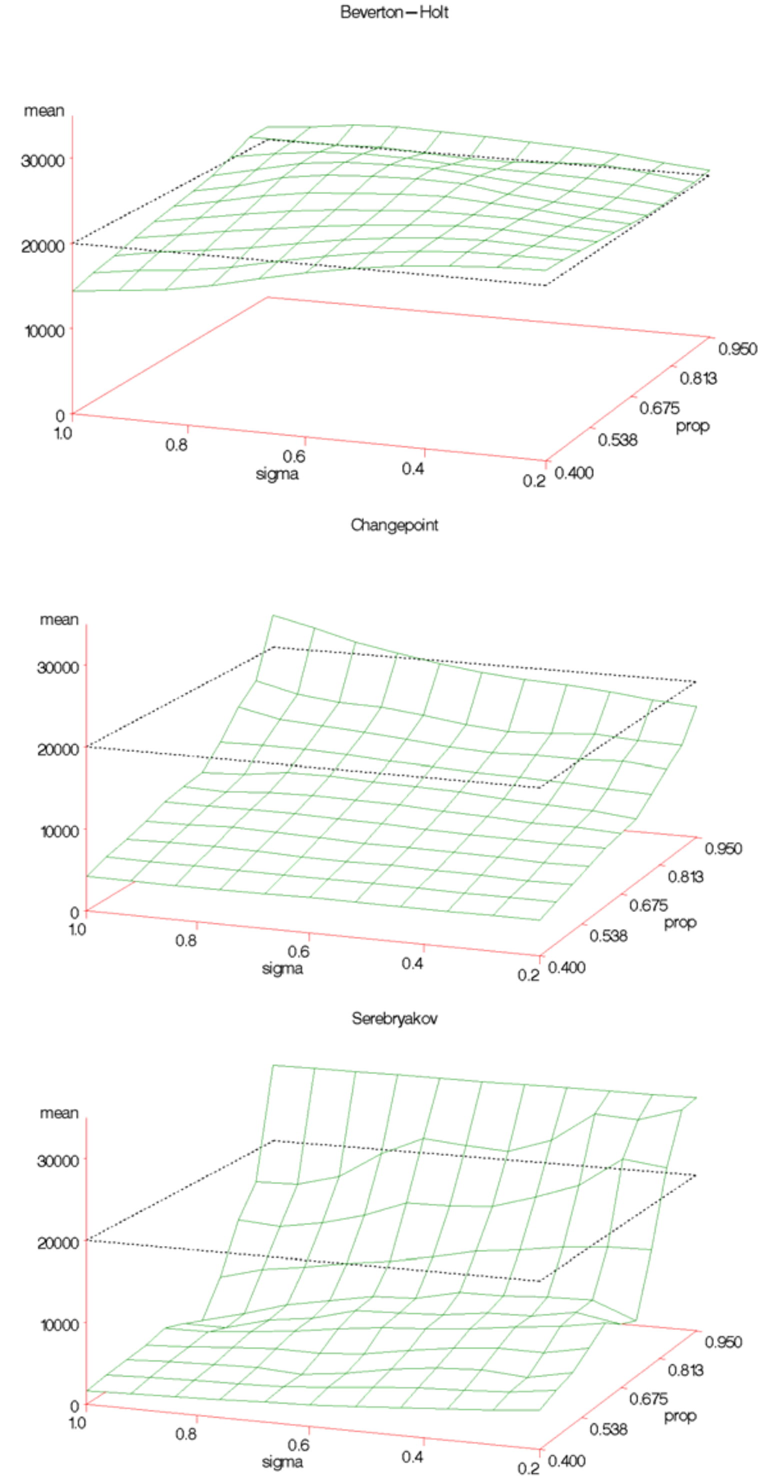

Fig. 9. Comparison of the means of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 30 years in length.

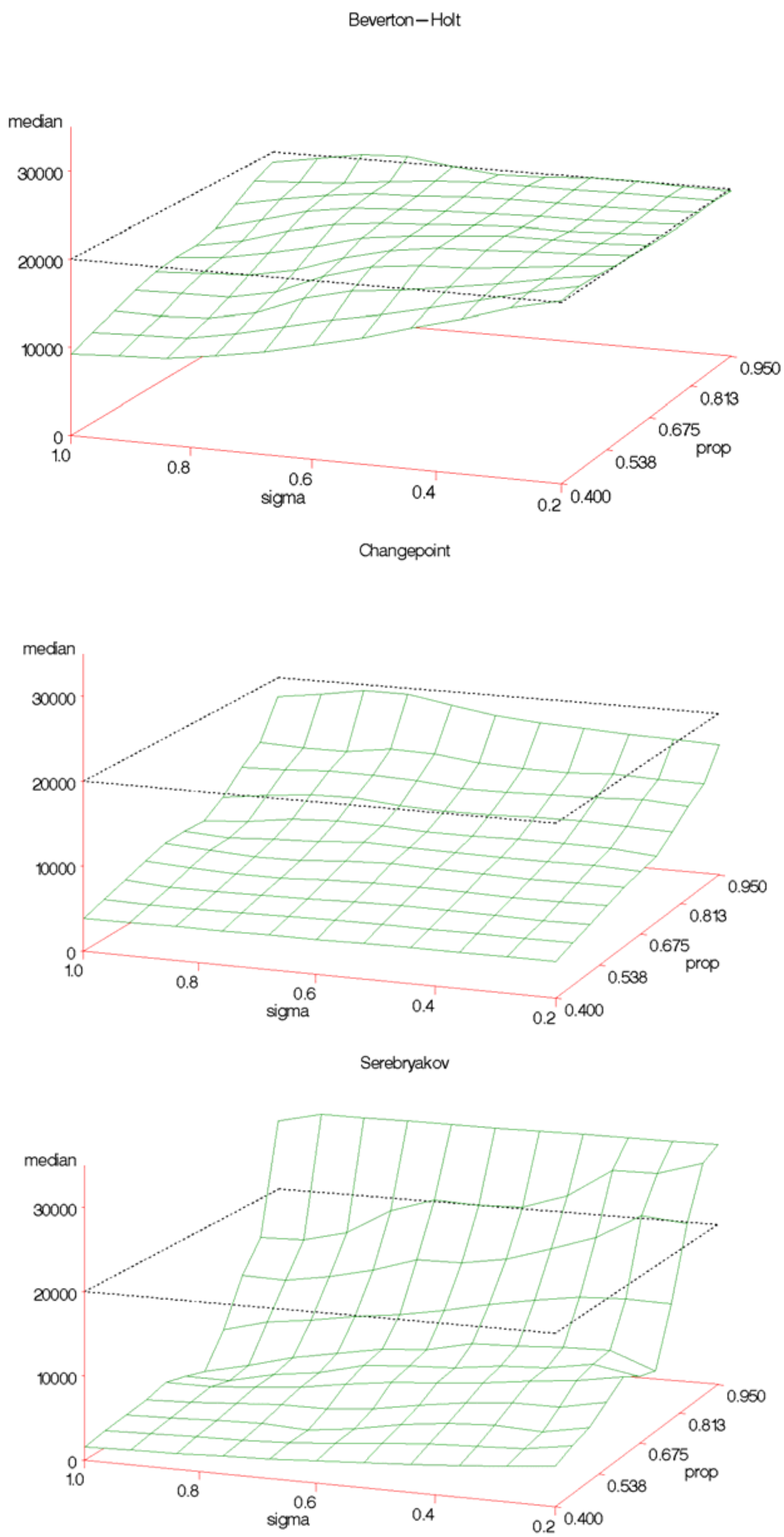

Fig. 10. Comparison of the medians of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 20 years in length.

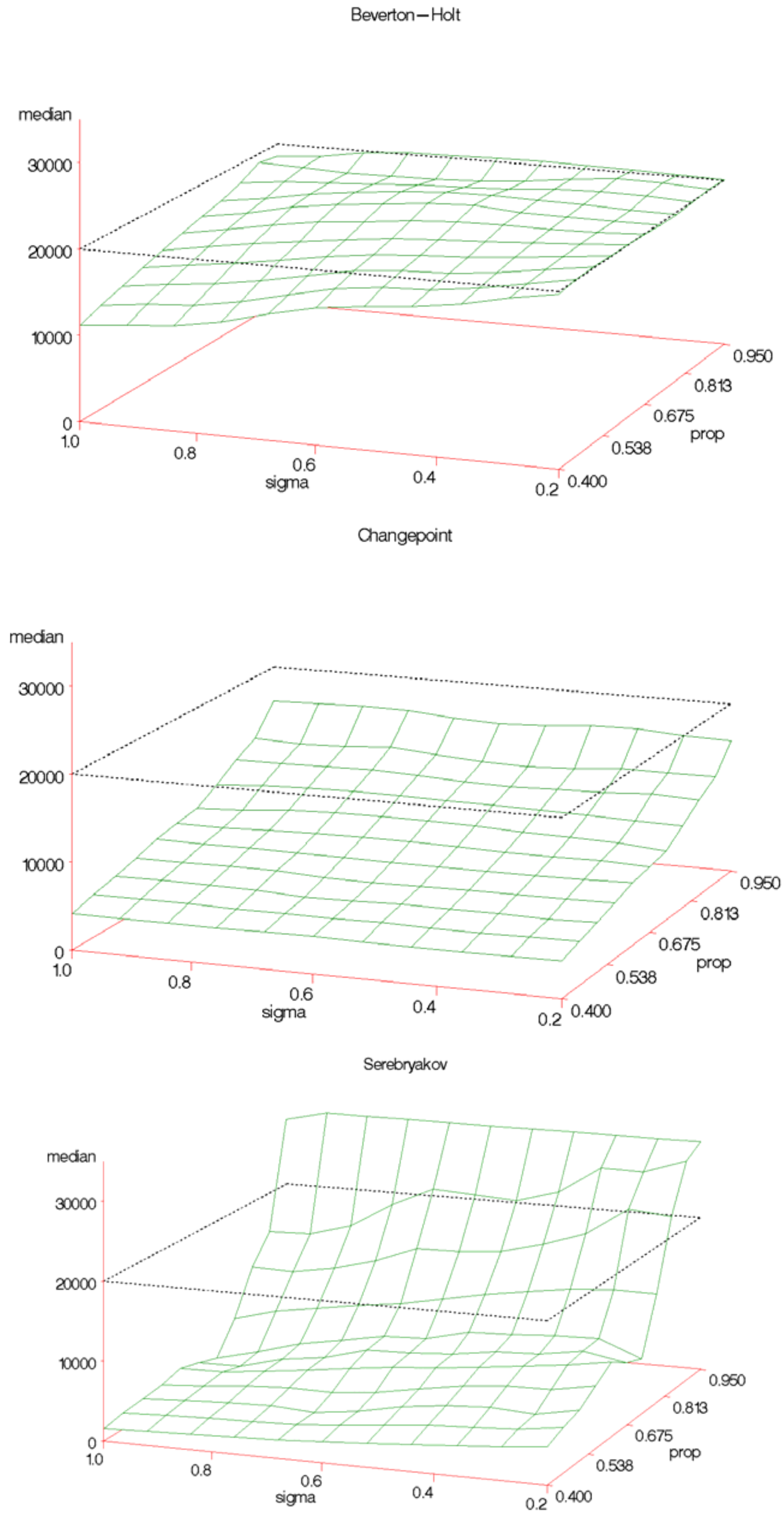

Fig. 11. Comparison of the medians of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 30 years in length.

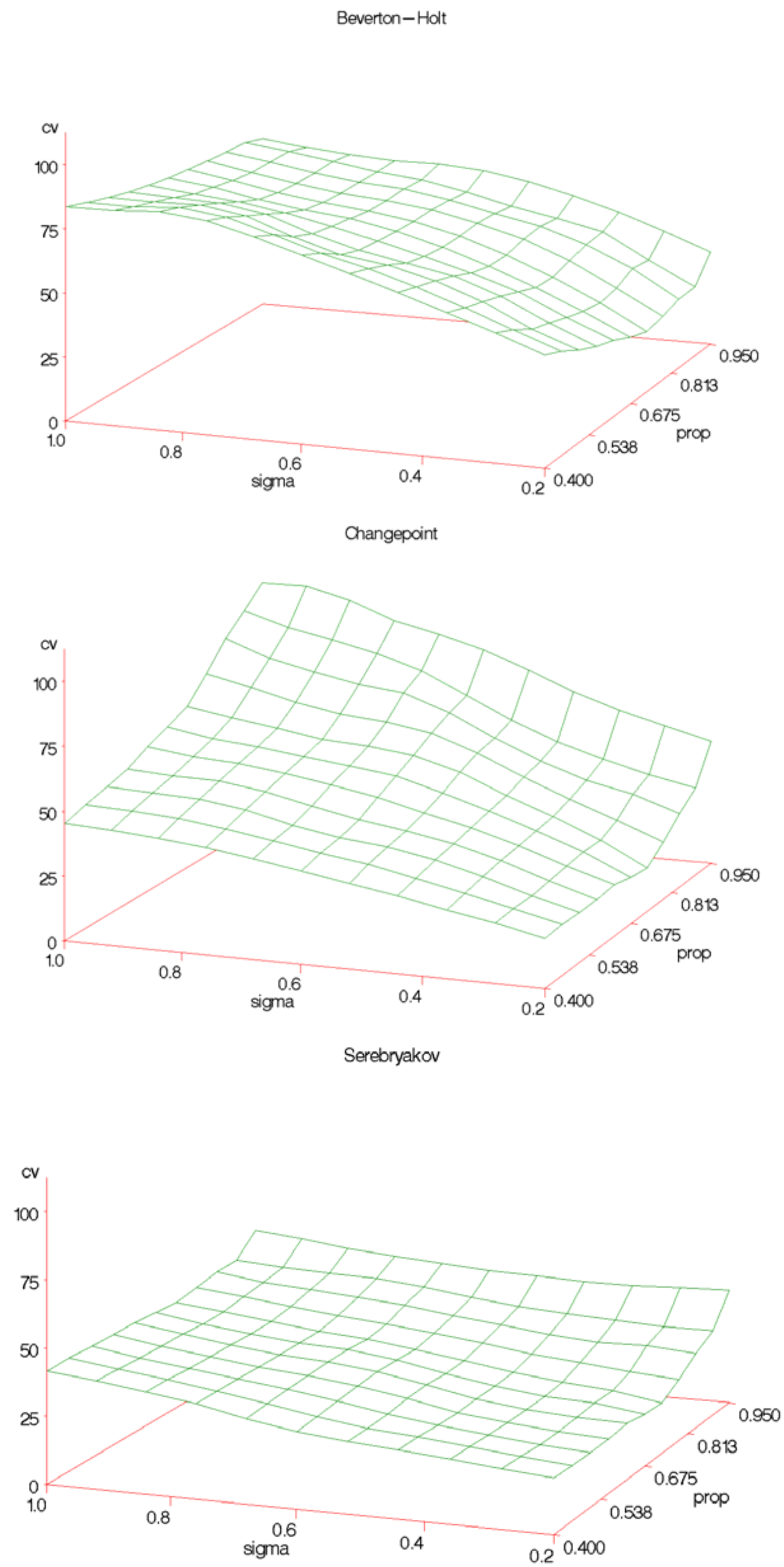

Fig. 12. Comparison of the CVs of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 20 years in length.

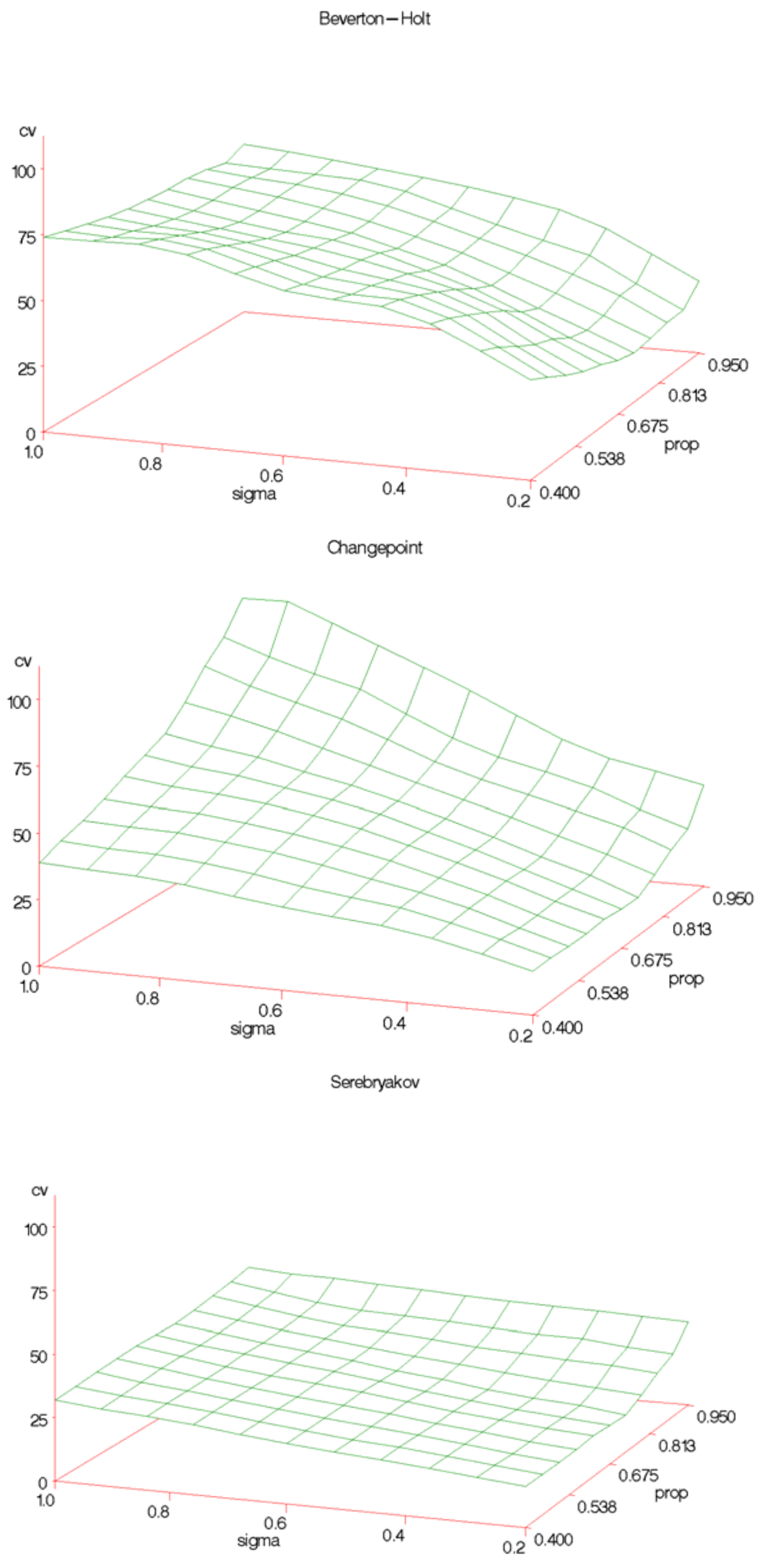

Fig. 13. Comparison of the CVs of the estimates of three B_{lim} estimators applied to simulated stock-recruit data series 30 years in length.

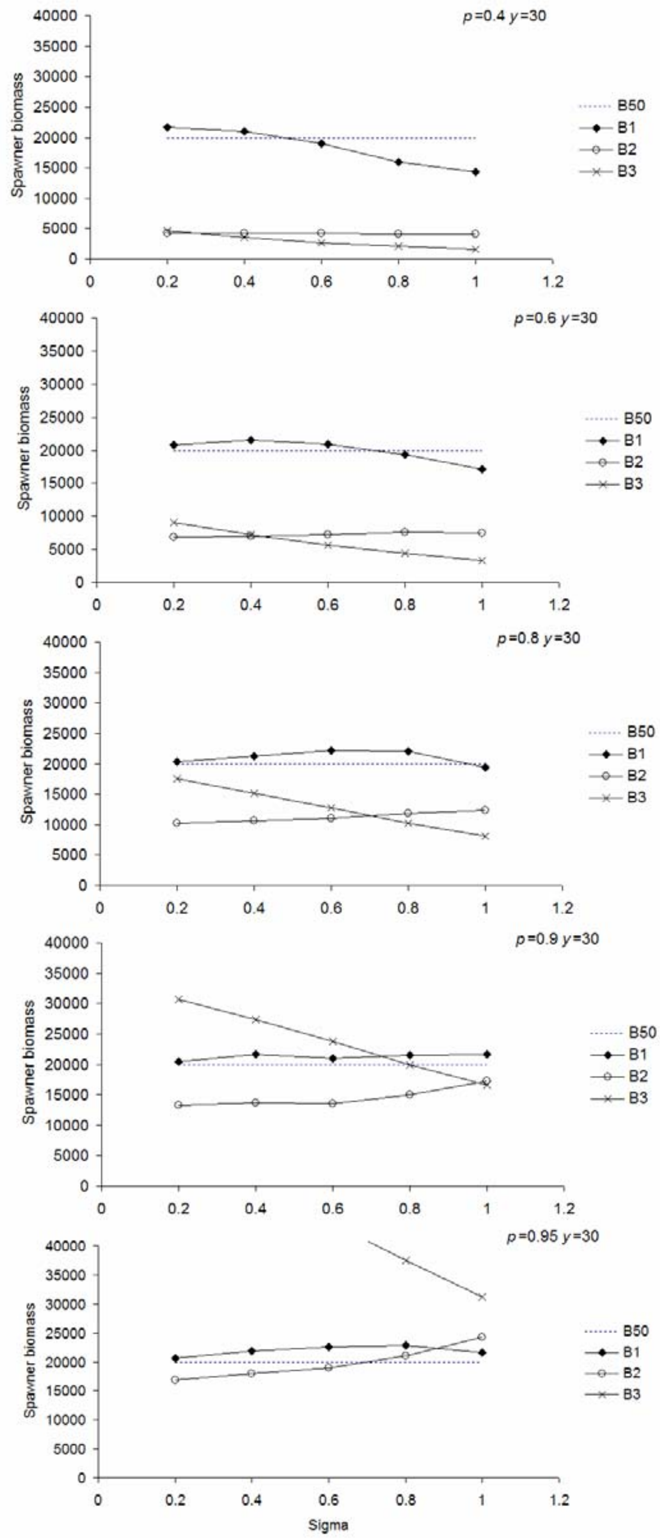

Fig. 14. Plots of the means of estimates for three B_{lim} estimators applied to simulated stock-recruit data. B_1=Beverton-Holt, B_2= changepoint, B_3=Serebryakov's percentile method. B_{50} is indicated by a dotted line.

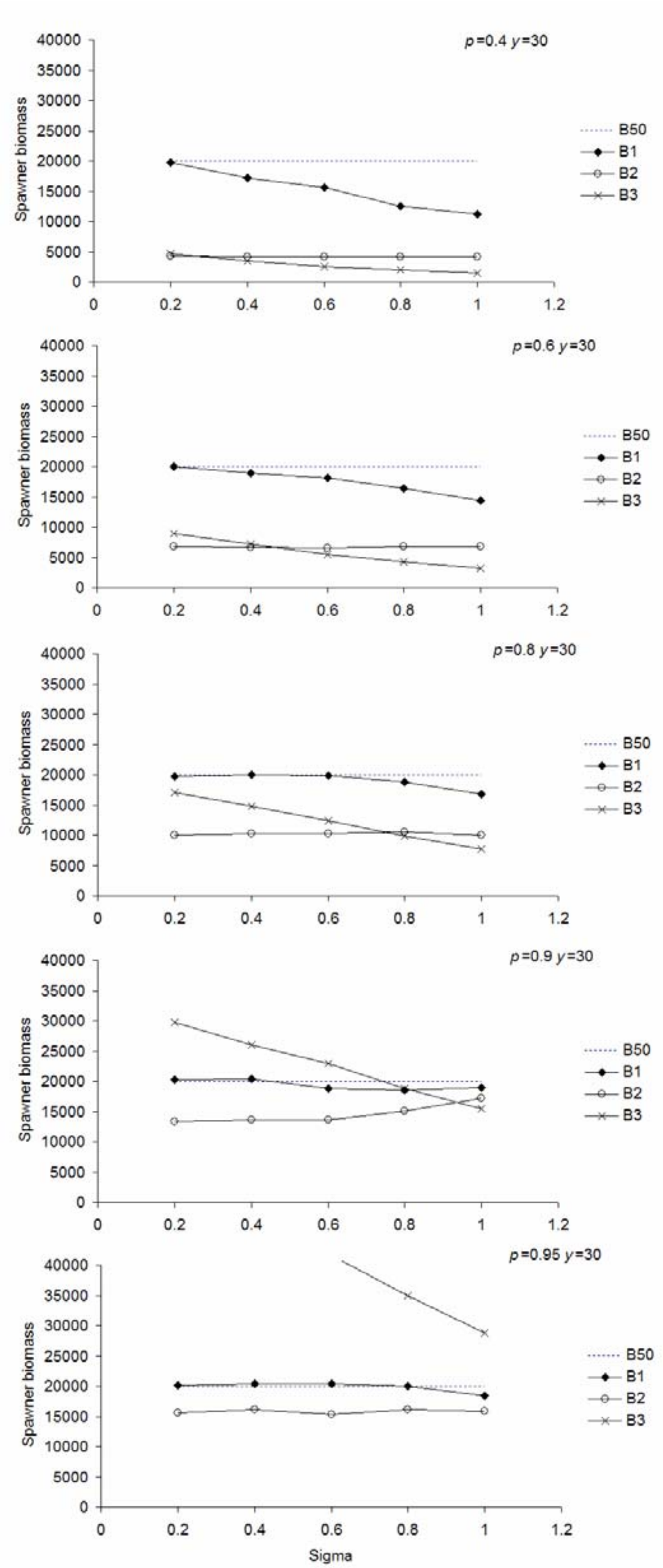

Fig. 15. Plots of medians of estimates for three B_{lim} estimators applied to simulated stock-recruit data. B_1=Beverton-Holt, B_2= changepoint, B_3=Serebryakov's percentile method. B_{50} is indicated by a dotted line.

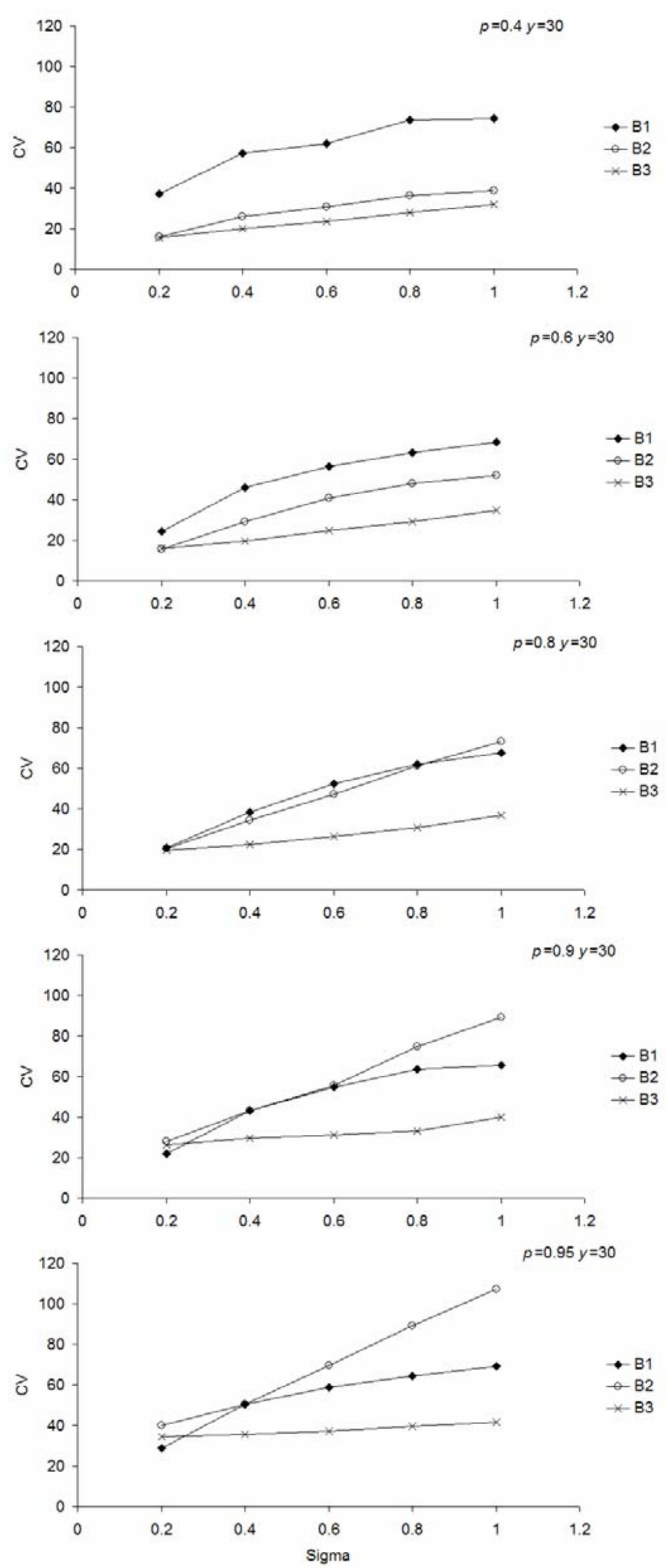

Fig. 16. Plots of CVs of estimates for three B_{lim} estimators applied to simulated stock-recruit data. B_1=Beverton-Holt, B_2=changepoint, B_3=Serebryakov's percentile method.

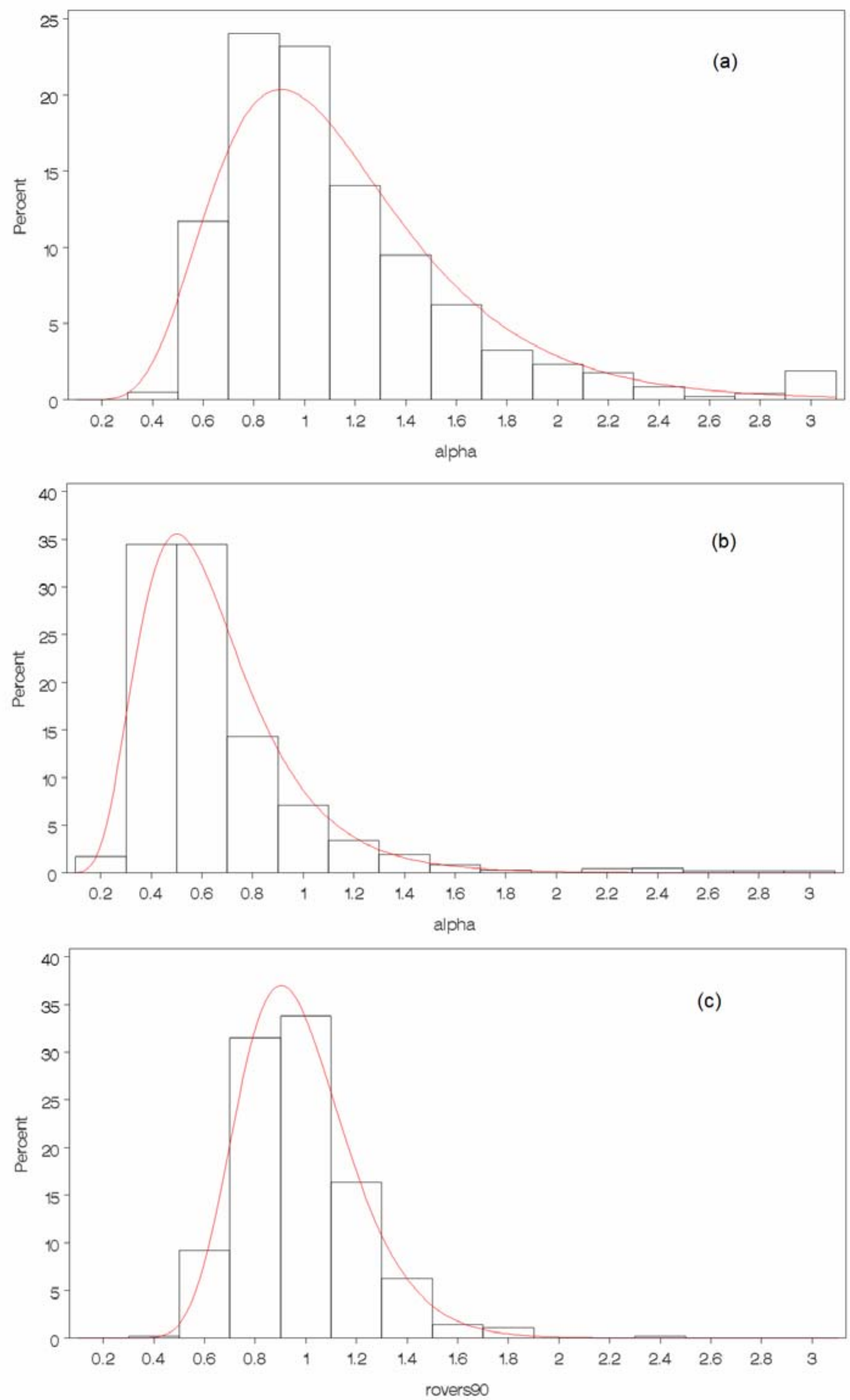

Fig. 17. Histograms of the distributions of the estimates of the slopes for (a) Beverton-Holt, (b) changepoint regression and (c) Serebryakov's percentile method, applied to simulated data generated from a Beverton-Holt model with y=30, p=0.8 and σ=0.6. Log-normal distributions are fitted to the estimates. The true slope near the origin in the Beverton-Holt model used to generate the data is 1.0.

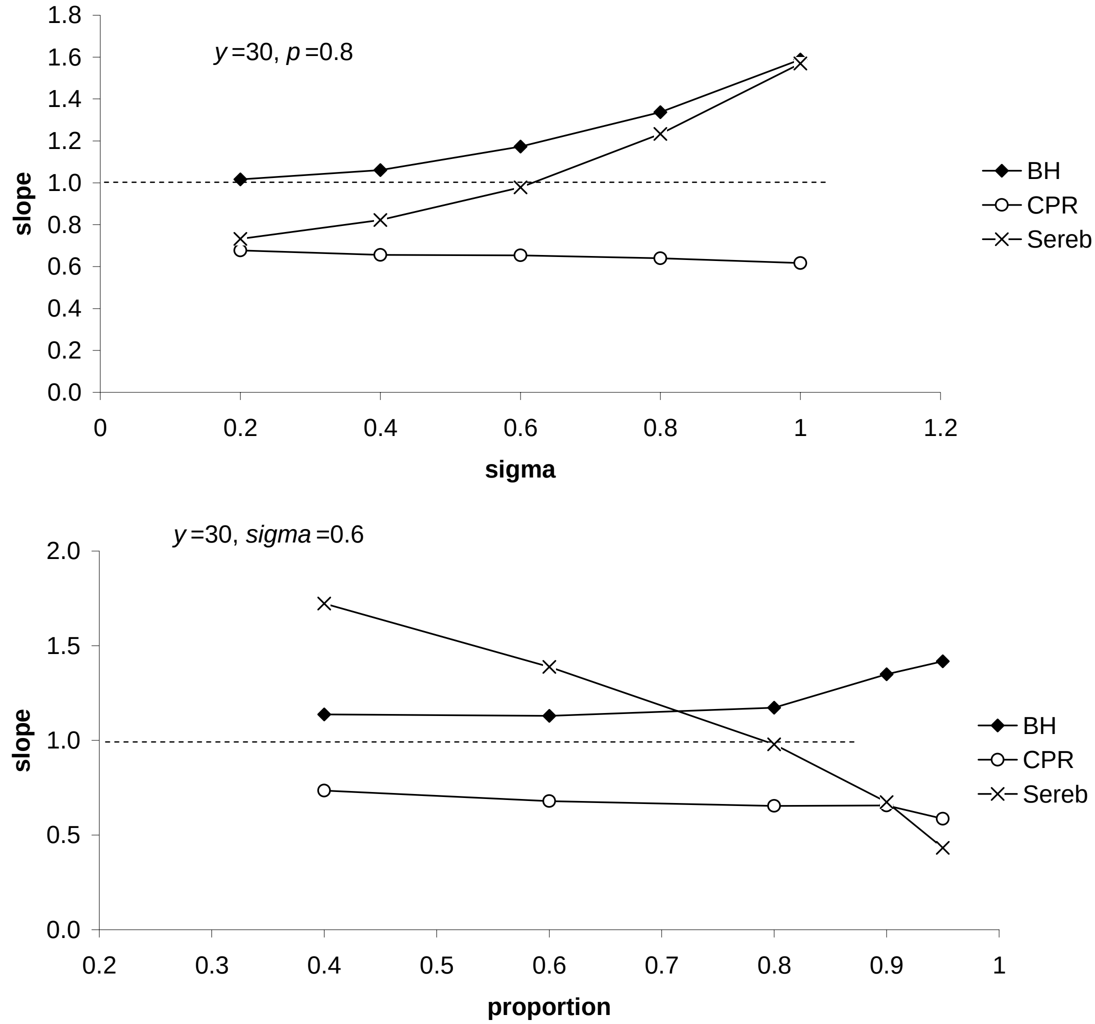

Fig. 18. Illustration of the relationship between the estimated slope at the origin and σ (top) for y=30 and p=0.8, and estimated slope and proportion of the biomass corresponding to R_{max} (bottom) for y=30 and σ=0.6. BH=Beverton-Holt, CPR= changepoint regression, Sereb=Serebryakov's percentile method. The broken line indicates the true value of the slope at the origin. Beverton-Holt and changepoint regression estimates are bias corrected for log-transformation. The largest cell (3) is a plus-group.