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Hecate Strait Pacific cod stock assessments for 1997 and recommended yield options for 1998

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#### Abstract

This document presents stock reconstructions and stock projections, based on catch-atlength analysis for the Hecate Strait Pacific cod stock. The analytical model (MULTIFAN CL) is modified from that used for previous assessments to account for changes in the mesh size regulation for the Hecate Strait trawl fishery. No analyses are conducted for Pacific cod on the west coast of Vancouver Island because of a lack of biological samples from this stock in recent years.

Assessment results indicate that there is conflicting information in the 1997 CPUE and length-frequency data regarding recent recruitment levels for the Hecate Strait stock. The CPUE data indicate a significant increase in stock abundance in 1997, but there are no signs of recent recruitment in the length frequency data. The base case stock reconstruction suggests that the Hecate Strait Pacific cod stock is now at a historic low level. An alternate analysis, with lower weighting on the 1997 length frequency data, is less pessimistic and suggests that stock abundance is increasing. There is no objective basis on which to decide if either the CPUE or the length frequency data is biased.


## Résumé

Ce document présente des reconstructions et des projections de stocks, fondées sur l'analyse des captures selon la longueur, pour le stock de morue du Pacifique du détroit d'Hecate. Le modèle analytique (MULTIFAN CL) a été modifié par rapport à celui utilisé pour les évaluations antérieures afin de tenir compte de la modification du maillage imposée par règlement pour la pêche au chalut dans le détroit d'Hecate. Aucune analyse n'a été effectuée pour la morue du Pacifique de la côte ouest de l'île de Vancouver étant donné l'absence d'échantillons biologiques prélevés pour ce stock au cours des dernières années.

L'évaluation montre une divergence des renseignements tirés du PUE de 1997 et des données de fréquence de longueur en ce qui a trait aux niveaux de recrutement récents du stock du détroit d'Hecate. Les données de PUE indiquent une augmentation appréciable de l'abondance du stock en 1997 tandis que les données des fréquences de longueur ne font état d'aucun indice de recrutement pour les dernières années. La reconstruction de base du stock indique que l'abondance du stock de morue du Pacifique du détroit d'Hecate est à un faible niveau historique. Une autre analyse, faisant appel à une pondération inférieure pour les données des fréquences de longueur de 1997, donne des résultats moins pessimistes et porte à croire à une augmentation de l'abondance. Aucun fondement objectif ne permet de déterminer si les données des PUE ou des fréquences de longueur sont biaisées.

## INTRODUCTION

Pacific cod (Gadus macrocephalus) is a major component of the domestic trawl fishery. In Canadian waters, Pacific cod is close to the southern limit of its commercial abundance and exhibits rapid growth and a short life span. Pacific cod are not aged using hardparts but ages are inferred from length-frequency analysis. Fishery independent abundance indices are not available for the B.C. Pacific cod stocks so assessments have relied on commercial fishery-based catch per unit effort (CPUE) statistics.

Since 1995, a catch-at-length model (MULTIFAN CL) has been used to analyze and reconstruct the stock histories for the Hecate Strait (Major Areas 5C and 5D) and the west coast Vancouver Island (Major Areas 3C and 3D) Pacific cod stocks. However, as noted in the 1996 Pacific cod PSARC document, biological samples from the commercial fisheries have become increasingly sparse for the west coast of Vancouver Island Pacific cod stock (Table 5). Hence, it is not possible to conduct catch-at-length analyses for this stock this year.

In this document we present stock reconstructions and stock projections for the Hecate Strait Pacific cod stock based on catch-at-length analyses. The analytical model is revised from that used last year to account for changes in size selectivity resulting from new mesh size restrictions for the Hecate Strait trawl fishery. One-year ahead stock projections are conducted using stochastic simulation methods. The stochastic element is the size of the first age-class in the population.

## THE FISHERY

In B.C., Pacific cod are caught primarily by trawl gear and this species comprises a significant component of the domestic trawl catch. Landed catches of Pacific cod by region are shown in Table 1 for the 1956-1996 period.

The trawl fishery in B.C. has undergone a number of significant changes in recent years that may influence the quality and comparability of data collected from the fisheries. A brief summary of management initiatives related to the Pacific cod fishery follows.

Prior to 1992 the total catch of Pacific cod by the trawl fleet in B.C. was unrestricted and the main management measures were area/seasonal closures. Total allowable catches (TAC's) were introduced for the management of Pacific cod fisheries in Hecate Strait in 1992 and for fisheries on the west coast of Vancouver Island in 1994 (Table 2). Additionally, trip limits (i.e. limits on the quantity of fish landed per trip) were introduced and these decreased steadily between 1992 and 1995. The Hecate Strait quotas were not achieved between 1993 and 1995 and the west coast Vancouver Island TAC's were not achieved in 1994 and 1995. For the 1996 season, trawl fisheries in both Hecate Strait and on the west coast of Vancouver Island were restricted to bycatch only for Pacific cod because of stock concerns. In 1997, an individual vessel quota system (IVQ) was introduced for the B.C. trawl fishery, and coincidentally the fishing season was changed from a calendar year to an April-March season.

Beginning with the 1991 Pacific Groundfish Trawl Management Plan, it was suggested that fishermen voluntarily adopt a 140 mm minimum cod-end mesh size for bottom-trawl gear operating in Hecate Strait (the coast-wide regulation was a 76 mm minimum). This suggestion was continued in later management plans until 1995 when the 140 mm minimum was legislated for the Hecate Strait region.

## CATCH-EFFORT STATISTICS

It is generally recognized that catch-per-unit-effort (CPUE) statistics calculated from fisheries data can be unreliable indices of stock abundance because of factors such as technology improvements and the non-random distribution of both fishing effort and fish. Beyond these concems, the collection of catch and effort data from the commercial trawl fishery in B.C. has undergone changes, which may effect the comparability of the data over time.

Prior to 1991, catch and effort data were obtained through a voluntary log book program. Data were reported for each trip made, and estimates of the total effort and species catch were reported by location and depth stratum for each area fished during the trip. We refer to this data as "trip-based". Since 1991, the maintenance of logbook data records is mandatory in the trawl fishery and the detail of information reported in logbooks has increased. Species catch, effort, and location/depth information is recorded for each tow made. We refer to this data as "towbased". The groundfish data base system was modified to generate data records that summarized the tow-based data to a form more consistent with the trip-based data, so for the 1991-1995 period the data can be analyzed either as tow-based or trip-based. Since 1996, a mandatory observer program was instituted in the B.C. trawl fishery and observers report tow-based data on species catch (landed and discarded), effort and location/depth. Fishermen continue to maintain logbook records, but this information is not being computerized. At this time, the observer data is not available in a trip-based form. In summary, for years prior to 1991 data is only available as "trip-based" summaries and for 1996 and 1997 the data is only available as "tow-based" records. For the 1991-1995 period, the catch-effort data can be used either way.

Fig. 1 shows the frequency distribution of Pacific cod catch per tow for all Hecate Strait tows with a reported catch of cod for 1991 through 1997. For the 1996 and 1997 observer-collected data the proportion of tows with small catches of Pacific cod (i.e. less that 50 or 100 lbs .) is significantly greater than for the 1991-1995 data. It is likely that fishermen did not record small catches which observers now report. Of course, given the restrictions on Pacific cod catches in Hecate Strait, particularly in 1996, it is not clear to what extent the increase in the number of tows with small Pacific cod catches reflects the fishermen avoiding this species. The proportion of tows with no Pacific cod catch reported was lower in 1996 and 1997 than for the 1991-1995 period (Table 3), supporting the premise that observers report small catches which fishermen would not have reported. The proportion of tows where Pacific cod comprised less than $10 \%$ of the total catch is only slightly lower for the 1997 data than for the 1991-1995 data ( $38.8 \%$ versus $42.6 \%$, Table 3), although it is significantly lower for the 1996 data (20.6\%).

For last years Hecate Strait Pacific cod stock assessment, CPUE indices were calculated as the sum of catch divided by the sum of effort over all "qualified" data records. "Qualified" data
records were those where Pacific cod comprised at least $10 \%$ of the total catch, and "trip-based" data records were used. The $10 \%$ qualification criteria may eliminate some of the potential problems with different data reporting styles between observers and fishermen, however applying this criteria to the tow-based data is not equivalent to applying it to the trip-based data. Fig. 2 shows the frequency distribution of CPUE per "qualified" data record for the 1991-1997 period. For the tow-based data records it appears that the $10 \%$ qualification level eliminates some of the differences in the left side of the frequency distribution between the observer and fisherman reported data. However, the distribution of trip-based observations for the 1991-1995 data is shifted relative to the tow-based observations. The CPUE indices calculated by year and quarter for 1991-1995 from both trip-based and tow-based data records are plotted in Fig. 3. The tow-based indices are consistently higher, and the linear regression fit between the two is:

$$
\text { trip-based CPUE }=16.65+0.807 \text { (tow-based CPUE) } \quad \mathrm{R}^{2}=.985
$$

For this years Hecate Strait stock assessment, effort indices calculated from two alternate CPUE indices are evaluated. Both CPUE indices are calculated as the sum of catch divided by the sum of effort for all data qualified at the $10 \%$ level. Both indices use the trip-based data for 19561990. The first index uses tow-based data for 1991-1997 (effort 1). The second uses trip-based data for 1991-1995 and converts the tow-based data for 1996 and 1997 using the regression equation shown above (effort 2). The two series of CPUE indices are shown in Fig. 4.

## CATCH-AT-LENGTH ANALYSIS

A catch-at-length model, MULTIFAN CL has been used for analysis of Pacific cod fisheries data for the past two assessments (Haist and Fournier 1995, Haist and Fournier 1996). MULTIFAN CL integrates length-frequency analysis with catch-age analysis so that growth parameters and catch equation parameters are estimated simultaneously, rather than through a step-wise procedure. The formulation of the model was modified this year to account for changes in size selectivity resulting from changes in mesh size regulations for the Hecate Strait fishery. The model is described in Appendix "A".

The procedure used for model development and selection of the most appropriate model formulation is the same as that developed for MULTIFAN analysis (Fournier et al. 1990, Fournier et al. 1991). That is, for each model formulation the data is fit at a range of initial K estimates (von Bertalanffy growth coefficient), M estimates (natural mortality rate), and number of age-classes. For each formulation the best fit across K, M, and age-classes is selected based on likelihood ratio tests. Similarly, a more complex model formulation (i.e. more parameters) is selected over a simpler formulation when the likelihood ratio test suggests there is significant improvement in model fit for the more complex formulation.

For the current assessment only a limited set of model runs were conducted. Published estimates of the instantaneous natural mortality rate for Pacific cod in Hecate Strait range from 0.38 to 0.99 (Westrheim 1996). Previous analyses using the MULTIFAN CL model for B.C. cod stocks showed that better fits were obtained with $\mathrm{M}=0.65$ than with $\mathrm{M}=0.40$ (Haist and Fournier 1995). Also, results showed that model fits were relatively insensitive to the initial K estimate that 4
age-classes were the appropriate number to model. Therefore, analyses this year were limited to runs with an initial K estimates of $0.30, \mathrm{M}=0.65$, and 4 age-classes. Specific model structure that is evaluated in a step-wise procedure is; first age-class fishing mortality deviations, seasonal growth, first age-class length bias, length-dependent standard deviations, and cohort-strength dependent growth.

Data input for the analysis includes catch estimates (in numbers), effort indices, and lengthfrequency data. The data were compiled and analyzed by quarter (Q1-Q4) for the period January 1956 to October 1997. The sample sizes for the length-frequency data is shown in Table 4. Analyses were conducted using two effort indices based on the CPUE calculations described above.

The length frequencies of Pacific cod sampled from the commercial fisheries show decreased proportions of smaller fish through the early 1990s (Fig. 5), apparently as a result of the change in mesh size regulation. The change in mesh size was first suggested in 1991 and was regulated in 1995. Many fishermen changed their nets prior to 1995. We model a change in selectivity-atlength beginning in 1993. The length frequency data also indicate that a change in the proportion of small Pacific cod landed occurred around 1972, and a third selectivity period is modeled to account for this observed change. The following table shows the twelve fisheries that are modeled and the common catchability $(\mathbb{q})$ and selectivity parameters between them.

| Time-period | Q 3 | Q 4 | Q 1 | Q 2 |
| :---: | :---: | :---: | :---: | :---: |
| 1956-1971 | $\begin{gathered} \text { Fishery - } 1 \\ \text { Sel - } 1 \\ \text { Q-1 } \end{gathered}$ | $\begin{gathered} \text { fishery -2 } \\ \text { sel -1 } \\ \text { q-2 } \end{gathered}$ | $\begin{gathered} \text { fishery - } 3 \\ \text { sel -1 } \\ \text { q-3 } \end{gathered}$ | $\begin{gathered} \text { fishery - } 4 \\ \text { sel }-1 \\ q-4 \end{gathered}$ |
| 1972-1992 | $\begin{gathered} \text { Fishery -5 } \\ \text { Sel - } 2 \\ Q-1 \end{gathered}$ | $\begin{gathered} \text { fishery - } 6 \\ \text { sel - } 2 \\ q-2 \end{gathered}$ | $\begin{gathered} \text { fishery - } 7 \\ \text { sel }-2 \\ q-3 \end{gathered}$ | $\begin{gathered} \text { fishery - } 8 \\ \text { sel }-2 \\ q-4 \end{gathered}$ |
| 1993-1997 | $\begin{gathered} \text { Fishery - } 9 \\ \text { Sel - } \\ \text { Q-1 } \end{gathered}$ | $\begin{gathered} \text { fishery }-10 \\ \text { sel }-3 \\ q-2 \end{gathered}$ | $\begin{gathered} \text { fishery - } 11 \\ \text { sel }-3 \\ q-3 \end{gathered}$ | $\begin{gathered} \text { fishery - } 12 \\ \operatorname{sel}-3 \\ q-4 \end{gathered}$ |

Values of the objective function for the alternate model formulations are shown in Table 6. For both effort indices, as the model complexity increases, each additional component of the model structure significantly improves the model fit. The model fit to the data observations is slightly better for the analysis using the first effort indices. The following discussion refers to the results from the most complex model formulation using the effort 1 series.

The estimates of the mean selectivity-at-age indicate a decrease in selectivity of smaller fish for the 1993-1997 period. The estimates for the twelve fisheries are:

| Fishery | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}+$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 1 | 0.016 | 0.442 | 0.967 | 1.000 |
| 2 | 0.045 | 0.651 | 0.994 | 1.000 |
| 3 | 0.059 | 0.821 | 1.000 | 1.000 |
| 4 | 0.179 | 0.920 | 1.000 | 1.000 |
|  |  |  |  |  |
| 5 | 0.160 | 0.534 | 0.972 | 1.000 |
| 6 | 0.260 | 0.709 | 0.995 | 1.000 |
| 7 | 0.171 | 0.851 | 1.000 | 1.000 |
| 8 | 0.430 | 0.934 | 1.000 | 1.000 |
|  |  |  |  |  |
| 9 | 0.135 | 0.472 | 0.969 | 1.000 |
| 10 | 0.099 | 0.669 | 0.995 | 1.000 |
| 11 | 0.072 | 0.830 | 1.000 | 1.000 |
| 12 | 0.172 | 0.925 | 1.000 | 1.000 |
|  |  |  |  |  |

Estimates of the numbers-at-age, the fully recruited fishing mortality rates, and spawning stock biomass are presented in Table 7. The stock reconstruction suggests that the spawning stock biomass decreased steadily from 1988 to the present and is currently at historic low levels. The estimates of the year-classes recruiting in 1995 through 1997 are the smallest ever. -These results differ from those obtained from the assessment conducted in the fall of 1996 (Haist and Fournier 1996a) where the year-class recruiting in 1996 was estimated as average in size. That result was based on significant quantities of small fish occurring in the length-frequency samples obtained during the second and third quarter of 1996 (April-September, Fig. 5) and an increase in the CPUE statistics for 1996 from the Hecate Strait Pacific cod survey. The length-frequency samples obtained in 1997 do not indicate significant numbers of the year-class that recruited in 1996.

The time-series of effort residuals for each quarter are shown in Fig. 6. The residuals for the 1st, 2nd and 3rd quarter of 1997 are the largest ever and indicate that the stock reconstruction is not adhering to the observed increase in CPUE. Clearly, there are inconsistencies in the 1997 data. CPUE statistics indicate an increase in stock abundance while the length frequency data do not support the presence of younger year-classes in the fisheries catch. Given the changes in the fishery which have occurred in recent years, either of the two data types could be biased. Changes in the cod-end mesh size, observer reporting of catch and effort statistics, and the introduction of an IVQ system may influence both the CPUE statistics and the length frequency samples collected from the fisheries.

Because the stock reconstruction appears to fit the 1997 length frequency data at the expense of fitting the CPUE data, we have looked for potential biases in the length frequency samples collected in 1997. In particular we have looked at the time, depth, and locations of samples collected in 1997 relative to those collected in earlier years (Table 8 and 9). Samples collected in 1997 were pre-dominantly taken in March and April, and hence the mean sample dates for the first quarter are later, and for the second quarter earlier, than for most other years. Table 10 compares the cumulative length frequency of fish in samples taken in January and February versus those taken in March, and Table 11 shows similar data for samples taken in April versus those taken in May and June. For the January/February versus March comparison the differences in the cumulative length frequencies are generally small, and where there are major differences the March samples tend to have greater proportions of smaller fish (e.g. 1990). For the April versus May/June comparison the number of years with significantly more small fish in the May/June samples than in the April samples (1984, 1985, 1989, 1991, and 1995). It is possible that the distribution of sampling effort in 1997 created biased length frequency samples relative to those obtained in other years.

To evaluate the potential impact of bias in the 1997 length-frequency data, we have conducted a series of alternative stock reconstructions using the MULTIFAN CL model where the sample sizes for the 1997 length frequency data were weighted at $0.0,0.25$, and 0.5 times the actual sample sizes. Estimates of the recruitment time series for the 0.25 weighting and the spawning stock biomass estimates for all alternative weightings are compared with those for the base case reconstruction (effort 1) in Fig. 8. With lower weighting of the 1997 length frequency data, the estimated size of the year classes recruiting in 1996 and in 1997 increase significantly, although the uncertainty in these estimates are larger as well. The fit to the CPUE data improves with the alternate stock reconstruction, and the 1997 effort residuals are not as extreme as for the base case analysis (Fig. 7).

## HARVEST POLICY AND STOCK PROJECTIONS

In the 1995 Pacific cod PSARC document, analyses related to harvest dynamics were presented (Haist and Fournier 1995). The recommendations arising from those analyses were for a target fishing mortality rate (fully recruited F) of 0.30 , and for threshold spawning stock biomass (SSB) levels below which cessation of fisheries would be recommended. The threshold SSB levels was set at $25 \%$ of the median unfished SSB, and the threshold estimate for Hecate Strait is 4570 t .

Stock abundance is projected to April 1998 using the October 1997 estimates of numbers-at-age from the MULTIFAN CL stock reconstruction. For the April 1, 1997 - Sept. 30, 1997 period, 623 t of Pacific cod were landed in the Hecate Strait trawl fishery, and we assume that the remainder of the 1997/98 quota ( 1000 t .) will be taken between October 1/97 and April 1/98. A stochastic element is included in the projections through uncertainty in the size of the first ageclass. 1000 stock projections are generated by re-sampling from the distribution of 1997 recruits estimated by the MULTIFAN CL analysis, assuming a log-normal distribution for recruitment.

For both the base case analysis and the analysis with a 0.25 weighting on the 1997 lengthfrequency data, 1000 stock projections were made. Various quantiles of the projected exploitable biomass (EB), catch, and spawning stock biomass (SSB) are:

|  | Base case |  |  | 0.25 length-frequency weight |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Quantile | 1998 | 1998 | 1998 | 1998 | 1998 | 1998 |
|  | SSB (t.) | EB (t.) | Catch (t.) | SSB (t.) | EB (t.) | Catch (t.) |
| 5 | 1922 | 1357 | 353 | 7276 | 6461 | 1684 |
| 10 | 1929 | 1366 | 355 | 7340 | 6567 | 1713 |
| 25 | 1952 | 1401 | 365 | 7600 | 7000 | 1833 |
| 50 | 2021 | 1504 | 393 | 8476 | 8463 | 2238 |
| 75 | 2203 | 1781 | 469 | 11130 | 12931 | 3479 |
| 90 | 2713 | 2586 | 690 | 20057 | 28062 | 7690 |
| 95 | 3192 | 3364 | 905 | 29705 | 44449 | 12254 |

The probability that the 1997 spawning stock biomass will be below the specified minimum level ( 4570 t ) is 0.983 for the base case scenario and 0 for the 0.25 length frequency weighting scenario. Stock projections are presented for both scenarios to demonstrate the current level of uncertainty in the estimates of abundance for the Hecate Strait Pacific cod stock. There is no objective basis on which to decide if either the 1997 CPUE or length frequency data is seriously biased.

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Table 1. Annual Pacific cod landed catch (tonnes) estimates for the Strait of Georgia (SoG), west coast of Vancouver Island (WCVI), Queen Charlotte Sound (QSD) and Hecate Strait (HS) for the period 1956-1996.

| Year | SoG | WCVI | QSD | HS | Coastwide |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1956 | 578. | 1468. | 1753. | 1046. | 2679. |
| 1957 | 607. | 1814. | 2744. | 1106. | 4027. |
| 1958 | 650. | 850. | 1178. | 3058. | 4722. |
| 1959 | 1047. | 907. | 946. | 2203. | 4284. |
| 1960 | 744. | 635. | 618. | 2360. | 3119. |
| 1961 | 415. | 420. | 240. | 1616. | 2083. |
| 1962 | 478. | 633. | 422. | 1690. | 2722. |
| 1963 | 675. | 1231. | 677. | 2927. | 4107. |
| 1964 | 713 | 1221. | 1275. | 5228. | 7279. |
| 1965 | 484 | 2768. | 1940. | 9119. | 11224. |
| 1966 | 297. | 3136. | 1811. | 9519. | 12276. |
| 1967 | 472. | 1941. | 1501. | 5112. | 6778. |
| 1968 | 349. | 1425. | 960. | 5165. | 6741. |
| 1969 | 388. | 1092. | 699. | 2987. | 4445. |
| 1970 | 502. | 1095. | 299. | 1315. | 2878. |
| 1971 | 740. | 3328. | 928. | 1477. | 5004. |
| 1972 | 630. | 5629. | 2320. | 2696. | 8639. |
| 1973 | 441. | 3712. | 1914. | 3996. | 7467. |
| 1974 | 681. | 3474. | 2292. | 4766. | 8886. |
| 1975 | 991. | 4000. | 2444. | 5036. | 10311. |
| 1976 | 927. | 3797. | 2271. | 4993. | 10082. |
| 1977 | 1148. | 2948. | 1268. | 3510. | 7650. |
| 1978 | 1373. | 1998. | 1959. | 2103. | 6674. |
| 1979 | 1202. | 1861. | 1904. | 4699. | 9549. |
| 1980 | 1611. | 1126. | 1383. | 4542. | 8703. |
| 1981 | 1749. | 896. | 853. | 3190. | 6694. |
| 1982 | 1012. | 1123. | 596. | 2066. | 4798. |
| 1983 | 904. | 694. | 183. | 2715. | 4497. |
| 1984 | 652. | 675. | 383. | 1748. | 3461. |
| 1985 | 463. | 492. | 299. | 1064. | 2329. |
| 1986 | 804. | 498. | 241. | 2099. | 3651. |
| 1987 | 1015. | 809. | 3243. | 8870. | 13941. |
| 1988 | 1223. | 1807. | 1849. | 6199. | 11095. |
| 1989 | 604. | 2991. | 763. | 4788. | 9152. |
| 1990 | 114. | 1953. | 772. | 3607. | 6455. |
| 1991 | 68. | 2177. | 2018. | 7655. | 11921. |
| 1992 | 412. | 2773. | 2043. | 5103. | 10340. |
| 1993 | 158. | 2527. | 1449. | 3965. | 8105. |
| 1994 | 90. | 1211. | 679. | 1561. | 3547. |
| 1995 | 24. | 652. | 345. | 1322. | 2346. |
| 1996 | 11. | 109. | 176. | 403. | 710. |

Table 2. The recommended yields, TAC's, and landed catches (tonnes) for the Hecate Strait and the west coast Vancouver Island (WCVI) Pacific cod stocks, 1992-1997.

|  | Hecate Strait |  |  | WCVI |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Recommended <br> Yield | TAC | Landed <br> Catch | Recommended <br> Yield | TAC | Landed <br> Catch |
| $1997 / 98$ | L: 1075 <br> H: 2165 | 1620 | NA | 0 | 694 | N/A |
| 1996 | 0 | by-catch <br> only | 403 | L: 694 <br> H: 916 | by-catch <br> only | 109 |
| 1995 | L: 1870 <br> M: 3040 <br> H: 5520 | 1870 | 1322 | L: 1300 <br> M: 2200 <br> H: 5330 | 1300 | 652 |
| 1994 | L: 1670 <br> M: 3850 <br> H: 7790 | 3850 | 1561 | L: 650 <br> M: 2170 <br> H: 5880 | 2170 | 1211 |
| 1993 | L: 3200 <br> H: 6500 | 5100 | 3965 | no <br> advice | no <br> quota | 2527 |
| 1992 | L: 600 <br> M: 2800 <br> H: 3800 | 3400 | 5103 | no <br> advice | no <br> quota | 2773 |

Table 3. Summary statistics for Hecate Strait catch-effort data from the groundfish database, 1956-1997. The 1997 data is from April to October only.

|  | No. of data <br> records/year | Prop. of data <br> records with zero <br> P. cod catch | Prop. of data <br> records qualified at <br> 10\% level |
| :--- | :---: | :---: | :---: |
| 1997 data - observer tow | 2247 | 32.6 | 38.8 |
| 1996 data - observer tow | 4801 | 43.2 | 20.8 |
| 1991-95 - logbook tow | 9156 | 47.5 | 42.6 |
| 1991-95 - logbook trip | 2665 | 47.8 | 38.2 |
| 1980-90 - logbook trip | 852 | 39.6 | 48.6 |
| 1968-79 - logbook trip | 695 | 29.8 | 54.2 |
| 1956-67 - logbook trip | 422 | 27.7 | 64.3 |

Table 4. The number of measured fish and the number of samples (in brackets) used in the MULTIFAN CL analysis of the Hecate Strait stock by year and quarter.

| Year | Q1 |  | Q2 |  | Q3 |  |  | Q4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1956 | 481 | (4) | 560 | (4) | 296 | (2) |  | 0 | (0) |
| 1957 | 426 | (3) | 461 | (3) | 0 | (0) |  | 227 | (1) |
| 1958 | 2314 | (13) | 1209 | (6) | 664 | (3) |  | 2033 | (10) |
| 1959 | 4213 | (20) | 1949 | (11) | 3623 | (16) | -- | 404 | (2) |
| 1960 | 1851 | (9) | 1840 | (6) | 3604 | (15) |  | 2031 | (12) |
| 1961 | 2778 | (12) | 4175 | (16) | 2743 | (9) |  | 1330 | (6) |
| 1962 | 4972 | (18) | 1488 | (6) | 1215 | (4) |  | 1093 | (4) |
| 1963 | 4607 | (19) | 3121 | (11) | 1403 | (5) |  | 1629 | (6) |
| 1964 | 4077 | (19) | 6332 | (25) | 3767 | (14) |  | 1649 | (7) |
| 1965 | 5993 | (25) | 5732 | (21) | 4040 | (16) |  | 2524 | (11) |
| 1966 | 3528 | (14) | 7459 | (30) | 3709 | (15) |  | 2131 | (10) |
| 1967 | 4341 | (16) | 2424 | (9) | 3580 | (14) |  | 3085 | (15) |
| 1968 | 3196 | (14) | 4701 | (22) | 2062 | (9) |  | 858 | (4) |
| 1969 | 2017 | (10) | 3561 | (15) | 1391 | (7) |  | 196 | (1) |
| 1970 | 1012 | (5) | 1145 | (6) | 713 | (3) |  | 172 | (1) |
| 1971 | 1692 | (9) | 1723 | (9) | 135 | (1) |  | 0 | (0) |
| 1972 | 458 | (2) | 804 | (3) | 548 | (2) |  | 1228 | (6) |
| 1973 | 682 | (3) | 2854 | (11) | 2727 | (13) |  | 1595 | (10) |
| 1974 | 451 | (2) | 2097 | (10) | 2151 | (11) |  | 2133 | (10) |
| 1975 | 2443 | (14) | 3206 | (14) | 120 | (1) |  | 884 | (5) |
| 1976 | 1590 | (12) | 1845 | (15) | 1051 | (9) |  | 457 | (4) |
| 1977 | 770 | (6) | 1793 | (14) | 2372 | (20) |  | 960 | (8) |
| 1978 | 816 | (7) | 2694 | (21) | 1316 | (11) |  | 797 | (8) |
| 1979 | 1656 | (13) | 3639 | (23) | 2500 | (17) |  | 634 | (5) |
| 1980 | 3774 | (26) | 2191 | (16) | 596 | (5) |  | 120 | (1) |
| 1981 | 0 | (0) | 120 | (1) | 478 | (4) |  | 240 | (2) |
| 1982 | 1576 | (9) | 2333 | (10) | 2192 | (10) |  | 228 | (1) |
| 1983 | 2807 | (15) | 3888 | (20) | 923 | (4) |  | 0 | (0) |
| 1984 | 1874 | (8) | 2170 | (9) | 1402 | (6) |  | 1259 | (5) |
| 1985 | 1723 | (8) | 1174 | (5) | 907 | (4) |  | 0 | (0) |
| 1986 | 1844 | (8) | 4120 | (17) | 416 | (2) |  | 236 | (1) |
| 1987 | 5497 | (14) | 2846 | (7) | 1406 | (3) |  | 540 | (2) |
| 1988 | 1689 | (5) | 1464 | (4) | 368 | (1) |  | 350 | (1) |
| 1989 | 752 | (2) | 731 | (2) | 0 | (0) |  | 400 | (1) |
| 1990 | 2583 | (8) | 231 | (1) | 912 | (2) |  | 789 | (4) |
| 1991 | 955 | (6) | 2475 | (14) | 756 | (4) |  | 147 | (1) |
| 1992 | 1697 | (11) | 1604 | (10) | 292 | (2) |  | 0 | (0) |
| 1993 | 873 | (7) | 1643 | (13) | 276 | (2) |  | 0 | (0) |
| 1994 | 945 | (8) | 348 | (3) | 116 | (1) |  | 0 | (0) |
| 1995 | 558 | (5) | 558 | (5) | 123 | (1) |  | 0 | (0) |
| 1996 | 0 | (0) | 404 | (3) | 569 | (4) |  | 0 | (0) |
| 1997 | 782 | (8) | 355 | (3) | 130 | (1) |  |  |  |

Table 5. The number of fish measured and number of samples (in brackets) collected for the west coast Vancouver Island stock by year and quarter.

| Year | Q1 |  | Q2 |  | Q3 |  | Q4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1956 | 62 | (1) | 1869 | (12) | 528 | (4) | 291 | (2) |
| 1957 | 0 | (0) | 2577 | (11) | 1678 | (8) | 352 | (1) |
| 1958 | 0 | (0) | 1119 | (6) | 1348 | (8) | 965 | (4) |
| 1959 | 170 | (1) | 2470 | (8) | 1310 | (6) | 0 | (0) |
| 1960 | 0 | (0) | 942 | (4) | 1345 | (5) | 138 | (1) |
| 1961 | 0 | (0) | 2305 | (8) | 542 | (2) | 281 | (2) |
| 1962 | 0 | (0) | 2498 | (9) | 1554 | (7) | 0 | (0) |
| 1963 | 350 | (1) | 2746 | (10) | 671 | (2) | 524 | (2) |
| 1964 | 1163 | (6) | 2234 | (9) | 1238 | (5) | 184 | (1) |
| 1965 | 444 | (2) | 3239 | (13) | 1014 | (4) | 566 | (2) |
| 1966 | 2565 | (10) | 4453 | (19) | 2239 | (11) | 661 | (3) |
| 1967 | 1742 | (7) | 1949 | (9) | 0 | (0) | 411 | (2) |
| 1968 | 1832 | (9) | 594 | (3) | 0 | (0) | 745 | (4) |
| 1969 | 1337 | (5) | 1058 | (4) | 1056 | (6) | 1539 | (8) |
| 1970 | 1759 | (9) | 3313 | (13) | 2326 | (9) | 993 | (4) |
| 1971 | 1980 | (9) | 2950 | (13) | 2950 | (13) | 1292 | (6) |
| 1972 | 1671 | (8) | 2372 | (9) | 2312 | (10) | 867 | (4) |
| 1973 | 1338 | (7) | 2367 | (11) | 1277 | (6) | 179 | (2) |
| 1974 | 2211 | (15) | 2209 | (10) | 2434 | (11) | 1238 | (6) |
| 1975 | 2179 | (13) | 1533 | (7) | 793 | (4) | 344 | (2) |
| 1976 | 3267 | (25) | 706 | (5) | 657 | (6) | 0 | (0) |
| 1977 | 2766 | (22) | 586 | (5) | 840 | (7) | 236 | (2) |
| 1978 | 1713 | (13) | 944 | (6) | 120 | (1) | 120 | (1) |
| 1979 | 448 | (3) | 1676 | (9) | 839 | (4) | 0 | (0) |
| 1980 | 0 | (0) | 465 | (4) | 539 | (4) | 0 | (0) |
| 1981 | 0 | (0) | 342 | (2) | 120 | (1) | 0 | (0) |
| 1982 | 2248 | (10) | 498 | (2) | 269 | (1) | 158 | (1) |
| 1983 | 1122 | (5) | 186 | (1) | 0 | (0) | 0 | (0) |
| 1984 | 604 | (3) | 200 | (1) | 0 | (0) | 0 | (0) |
| 1985 | 0 | (0) | 561 | (2) | 0 | (0) | 254 | (1) |
| 1986 | 0 | (0) | 629 | (3) | 0 | (0) | 0 | (0) |
| 1987 | 400 | (1) | 400 | (2) | 0 | (0) | 0 | (0) |
| 1988 | 698 | (2) | 281 | (1) | 546 | (1) | 0 | (0) |
| 1989 | 1136 | (3) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1990 | 373 | (1) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1991 | 881 | (5) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1992 | 1365 | (6) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1993 | 1025 | (7) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1994 | 1233 | (4) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1995 | 200 | (1) | 0 | (0) | 0 | (0) | 0 | (0) |
| 1996 | 0 | (0) | 171 | (1) | 0 | (0) | 0 | (0) |
| 1997 | 0 | (0) | 0 | (0) | 0 | (0) |  |  |

Table 6. MULTIFAN CL results for Hecate Strait Pacific cod analyses using two effort indices. Model formulation is progressive in that successive runs include all model structures from previous runs. Results shown are log-likelihood function value (1), average fully-recruited fishing mortality rate (f), and growth model K estimate.

|  | Model using effort series 1 |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of <br> parameters | 1 | f | K | 1 | f | K K using effort series 2 |
| Model formulation | 231 | -6742.8 | 0.64 | 0.11 | -6738.7 | 0.64 | 0.11 |
| Baseline | 232 | -6821.7 | 0.50 | 0.25 | -6818.7 | 0.49 | 0.25 |
| First age-class length bias | 233 | -6865.5 | 0.38 | 0.32 | -6863.4 | 0.37 | 0.32 |
| Length-dep. st. devs. | 387 | -7321.5 | 0.36 | 0.33 | -7319.2 | 0.36 | 0.33 |
| Fishing mortality deviations | 389 | -7366.2 | 0.35 | 0.36 | -7363.4 | 0.34 | 0.36 |
| Seasonal growth | 390 | -7377.1 | 0.36 | 0.35 | -7374.5 | 0.35 | 0.36 |

Table 7. Estimated number-at-age, total abundance, full-recruited fishing mortality (f), and spawning stock biomass (SSB) from catch-at-length analysis for the Hecate Strait Pacific cod stock.

| Season | Numbers-at-age (x10-3) |  |  |  |  | f | SSB (t.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5+ | Total |  |  |
| 1955/56 | 5161. | 557. | 702. | 702. | 7122. | $0.19{ }^{1}$ | 6443. |
| 1956/57 | 8643. | 2639. | 246. | 608. | 12135. | 0.16 | 8932. |
| 1957/58 | 2481. | 4452. | 1213. | 379. | 8525. | 0.37 | 11322. |
| 1958/59 | 6278. | 1225. | 1704. | 574. | 9781. | 0.31 | 9415. |
| 1959/60 | 4625. | 3208. | 501. | 869. | 9202. | 0.35 | 9469. |
| 1960/61 | 3525. | 2345. | 1290. | 503. | 7663. | 0.35 | 8632. |
| 1961/62 | 8181. | 1788. | 957. | 663. | 11589. | 0.23 | 9342. |
| 1962/63 | 10305. | 4221. | 786. | 674. | 15985. | 0.34 | 12857. |
| 1963/64 | 26025. | 5229. | 1710. | 545. | 33510. | 0.27 | 20890. |
| 1964/65 | 8961. | 12850. | 2223. | 902. | 24937. | 0.37 | 28185. |
| 1965/66 | 8204. | 4517. | 5090. | 1133. | 18944. | 0.77 | 21684. |
| 1966/67 | 12511. | 4031. | 1308. | 1517. | 19368. | 0.46 | 15692. |
| 1967/68 | 1595. | 6430. | 1545. | 935. | 10505. | 0.68 | 14407. |
| 1968/69 | 5958. | 787. | 2011. | 657. | 9414. | 0.49 | 9462. |
| 1969/70 | 1034. | 2973. | 279. | 855. | 5140. | 0.29 | 7531. |
| 1970/71 | 3541. | 521. | 1270. | 442. | 5774. | 0.29 | 6198. |
| 1971/72 | 14381. | 1697. | 217. | 671. | 16966. | 0.21 | 9194. |
| 1972/73 | 4613. | 7080. | 764. | 377. | 12833. | 0.29 | 13841. |
| 1973/74 | 9676. | 1900. | 2970. | 449. | 14994. | 0.29 | 13408. |
| 1974/75 | 8546. | 4382. | 795. | 1333. | 15055. | 0.53 | 13216. |
| 1975/76 | 6192. | 3795. | 1532. | 656. | 12175. | 0.45 | 12210. |
| 1976/77 | 9135. | 2884. | 1392. | 733. | 14144. | 0.44 | 10892. |
| 1977/78 | 3612. | 4453. | 1103. | 715. | 9883. | 0.34 | 11098. |
| 1978/79 | 8723. | 1424. | 1823. | 676. | 12645. | 0.36 | 10930. |
| 1979/80 | 6557. | 3910. | 553. | 915. | 11935. | 0.63 | 10396. |
| 1980/81 | 9799. | 2884. | 1258. | 408. | 14350. | 0.50 | 10506. |
| 1981/82 | 6889. | 4749. | 1027. | 530. | 13195. | 0.21 | 12754. |
| 1982/83 | 2513. | 3335. | 2099. | 657. | 8604. | 0.31 | 12540. |
| 1983/84 | 6483. | 1166. | 1348. | 1057. | 10055. | 0.19 | 10299. |
| 1984/85 | 1871. | 3280. | 521. | 1044. | 6717. | 0.19 | 9548. |
| 1985/86 | 2827. | 912. | 1482. | 677. | 5898. | 0.38 | 8015. |
| 1986/87 | 31379. | 1395. | 340. | 771. | 33885. | 0.71 | 14651. |
| 1987/88 | 11329. | 14363. | 391. | 287. | 26369. | 0.37 | 24452. |
| 1988/89 | 3466. | 4556. | 5551. | 243. | 13817. | 0.33 | 21834. |
| 1989/90 | 10106. | 1645. | 1800. | 2169. | 15720. | 0.25 | 16694. |
| 1990/91 | 16081. | 4911. | 696. | 1621. | 23309. | 0.47 | 18365. |
| 1991/92 | 2546. | 7483. | 1696. | 757. | 12482. | 0.43 | 16937. |
| 1992/93 | 3366. | 982. | 2743. | 831. | 7921. | 0.49 | 11389. |
| 1993/94 | 1042. | 1667. | 339. | 1143. | 4191. | 0.40 | 6547. |
| 1994/95 | 2941. | 514. | 642. | 518. | 4615. | 0.38 | 4794. |
| 1995/96 | 881. | 1484. | 195. | 416. | 2975. | 0.11 | 4109. |
| 1996/97 | 740. | 443. | 721. | 286. | 2189. | 0.42 | 3444. |
| 1997/98 | 234. | 359. | 163. | 346. | 1102. | $0.08{ }^{2}$ |  |

[^1]Table 8. Summary information for Pacific cod samples collected in Hecate Strait, 1997.

| Cumulative percentage of fish in length group (cm) or smaller |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Quarter | Date | Major- minor- locality | Depth (m) | $<45$ | <48 | <51 | <54 | $<57$ | <60 | <63 | <66 | <69 | $<72$ | Total number |
| Q1 | 1/27 | 8-4-0 | 50 | 2.7 | 5.5 | 6.4 | 9.1 | 14.5 | 25.5 | 38.2 | 57.3 | 71.8 | 84.5 | 110 |
|  | 3/05 | 8-5-1 | 60 | 0.0 | 0.0 | 0.0 | 0.0 | 4.8 | 12.0 | 16.9 | 28.9 | 45.8 | 68.7 | 83 |
|  | 3/17 | 8-0-0 | 49 | 0.0 | 0.9 | 0.9 | 2.7 | 2.7 | 9.7 | 12.4 | 28.3 | 46.0 | 63.7 | 113 |
|  | 3/17 | 8-5-1 | 57 | 0.0 | 0.0 | 1.0 | 4.1 | 8.2 | 12.2 | 23.5 | 33.7 | 50.0 | 72.4 | 98 |
|  | 3/19 | 8-5-2 | 65 | 0.0 | 0.0 | 0.0 | 3.3 | 4.3 | 12.0 | 26.1 | 42.4 | 53.3 | 75.0 | 92 |
|  | 3/19 | 8-5-2 | 68 | 0.0 | 0.0 | 1.0 | 2.0 | 4.0 | 9.9 | 16.8 | 26.7 | 49.5 | 67.3 | 101 |
|  | 3/26 | 8-5-2 | 62 | 0.0 | 2.8 | 2.8 | 5.6 | 10.3 | 19.6 | 33.6 | 48.6 | 70.1 | 85.0 | 107 |
|  | 3/27 | 8-5-1 | 62 | 0.0 | 0.0 | 1.3 | 2.6 | 2.6 | 6.4 | 15.4 | 26.9 | 41.0 | 55.1 | 78 |
| Q2 | 4/01 | 8-5-0 | 63 | 0.0 | 0.8 | 1.5 | 6.2 | 16.2 | 29.2 | 43.1 | 59.2 | 73.8 | 90.0 | 130 |
|  | 4/02 | 8-5-1 | 61 | 0.8 | 0.8 | 4.2 | 5.1 | 8.5 | 19.5 | 31.4 | 50.8 | 68.6 | 79.7 | 118 |
|  | 4/29 | 8-0-0 | 50 | 0.0 | 0.0 | 1.9 | 4.7 | 10.3 | 17.8 | $34.6$ | 54.2 | 69.2 | 82.2 | 107 |
| Q3 | 7/10 | 8-4-2 | 35 | 0.8 | 3.8 | 5.4 | 10.0 | 14.6 | 26.2 | 34.6 | 55.4 | 71.5 | 87.7 | 130 |

Table 9. Summary information for Hecate Strait Pacific cod samples, 1980-1997.

|  | Avg. julian date per quarter |  |  |  | Avg. depth (fm) per quarter |  |  |  | Number of samples by quarter |  |  |  | Number of samples by minor area |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | Q3 | Q4 | 2/6 | 3 | 4 | 5 |
| 80 | 44 | 128 | 205 | 325 | 53 | 33 | 40 | 57 | 26 | 16 | 5 | 1 | 18 | 0 | 12 | 18 |
| 81 | 0 | 135 | 238 | 307 | 0 | 12 | 52 | 53 | 0 | 1 | 4 | 2 | 0 | 0 | 4 | 3 |
| 82 | 39 | 146 | 218 | 310 | 59 | 32 | 41 | 61 | 9 | 11 | 10 | 1 | 11 | 0 | 13 | 7 |
| 83 | 59 | 129 | 189 | 0 | 56 | 38 | 40 | 0 | 15 | 19 | 4 | 0 | 5 | 0 | 19 | 14 |
| 84 | 67 | 133 | 206 | 325 | 55 | 55 | 58 | 62 | 8 | 9 | 7 | 5 | 4 | 0 | 10 | 15 |
| 85 | 53 | 130 | 228 | - | 63 | 55 | 63 | - | 8 | 4 | 4 | 0 | 4 | 0 | 6 | 6 |
| 86 | 58 | 134 | 195 | 304 | 75 | 41 | 60 | 53 | 8 | 16 | 2 | 1 | 5 | 2 | 13 | 7 |
| 87 | 43 | 152 | 224 | 318 | 62 | 45 | 42 | 56 | 14 | 7 | 3 | 2 | 7 | 0 | 7 | 12 |
| 88 | 37 | 134 | 236 | 304 | 60 | 39 | 51 | 65 | 5 | 4 | 2 | 2 | 7 | 0 | 2 | 4 |
| 89 | 44 | 127 | - | 345 | 64 | 48 | - | 49 | 8 | 3 | 0 | 1 | 2 | 0 | 1 | 9 |
| 90 | 44 | 123 | 231 | 328 | 62 | 59 | 52 | 62 | 9 | 1 | 2 | 4 | 7 | 0 | 1 | 8 |
| 91 | 71 | 116 | 237 | 328 | 46 | 33 | 46 | 67 | 6 | 12 | 4 | 1 | 6 | 6 | 2 | 9 |
| 92 | 41 | 132 | 209 | - | 49 | 34 | 40 | - | 8 | 9 | 2 | 0 | 1 | 4 | 6 | 8 |
| 93 | 62 | 126 | 231 | 323 | 50 | 47 | 57 | 73 | 8 | 10 | 2 | 1 | 5 | 1 | 5 | 10 |
| 94 | 60 | 136 | 199 | - | 70 | 36 | 55 | - | 8 | 3 | 1 | 0 | 5 | 0 | 3 | 4 |
| 95 | 59 | 130 | 190 | - | 54 | 46 | 36 | - | 5 | 5 | 1 | 0 | 3 | 1 | 7 | 0 |
| 96 | - | 121 | 210 | - | - | 54 | 42 | - | 0 | 2 | 4 | 0 | 0 | 0 | 5 | 1 |
| 97 | 72 | 101 | 190 | - | 59 | 58 | 35 | - | 8 | 3 | 1 | - | 0 | 0 | 2 | 8 |

Table 10. Comparison of cumulative length-frequency distributions for Hecate Strait Pacific cod samples collected in January/February versus March, 1980-1996.

|  |  | Cumulative percentage of fish in length group (cm) or smaller |  |  |  |  |  |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | $<45$ | $<48$ | <51 | <54 | $<57$ | $<60$ | <63 | <66 | <69 | $<72$ | number |
| 80 | Jan./Feb. | 7.3 | 15.9 | 28.2 | 43.4 | 57.8 | 70.2 | 80.4 | 86.7 | 91.2 | 94.5 | 2923 |
| 80 | March | 7.8 | 15.5 | 23.4 | 32.8 | 45.2 | 61.6 | 72.6 | 80.8 | 86.8 | 91.5 | 851 |
| 82 | Jan./Feb. | 6.4 | 14.4 | 28.1 | 46.3 | 64.1 | 74.9 | 82.0 | 87.3 | 91.9 | 95.6 | 1169 |
| 82 | March | 0.2 | 5.4 | 18.9 | 38.8 | 56.8 | 70.3 | 81.6 | 88.7 | 91.2 | 94.3 | 407 |
| 83 | Jan./Feb. | 0.7 | 2.3 | 6.0 | 10.6 | 20.7 | 35.1 | 51.7 | 69.2 | 82.1 | 90.6 | 1287 |
| 83 | March | 0.9 | 2.2 | 6.3 | 10.6 | 20.1 | 37.1 | 56.0 | 72.5 | 85.3 | 92.0 | 1520 |
| 84 | Jan./Feb. | 1.0 | 2.5 | 5.2 | 8.1 | 13.6 | 23.0 | 39.0 | 53.4 | 70.2 | 80.9 | 764 |
| 84 | March | 0.4 | 2.2 | 4.9 | 8.4 | 15.9 | 27.3 | 45.0 | 59.9 | 75.5 | 87.4 | 1110 |
| 85 | Jan./Feb. | 1.4 | 4.1 | 10.9 | 18.9 | 33.3 | 45.1 | 58.1 | 66.9 | 79.9 | 90.1 | 1061 |
| 85 | March | 0.5 | 2.4 | 6.9 | 13.9 | 28.5 | 48.3 | 64.5 | 75.1 | 84.9 | 93.2 | 662 |
| 86 | Jan./Feb. | 0.2 | 2.5 | 7.1 | 10.0 | 13.7 | 25.7 | 38.4 | 53.3 | 82.1 | 93.5 | 649 |
| 86 | March | 0.2 | 2.6 | 5.9 | 7.7 | 12.6 | 22.6 | 37.2 | 52.3 | 76.5 | 92.1 | 1195 |
| 87 | Jan./Feb. | 10.3 | 22.9 | 35.6 | 41.9 | 45.8 | 51.1 | 59.1 | 68.6 | 77.6 | 85.6 | 4198 |
| 87 | March | 18.5 | 36.7 | 58.4 | 74.8 | 82.8 | 86.5 | 90.0 | 92.8 | 95.6 | 97.2 | 1853 |
| 88 | Jan./Feb. | 3.2 | 11.7 | 26.2 | 43.2 | 62.4 | 78.4 | 89.7 | 95.0 | 98.2 | 98.5 | 873 |
| 88 | March | 3.8 | 11.3 | 26.4 | 44.2 | 64.8 | 79.5 | 90.2 | 95.6 | 97.7 | 99.0 | 815 |
| 89 | Jan./Feb. | 4.0 | 6.8 | 8.6 | 11.3 | 17.4 | 28.4 | 46.9 | 65.0 | 82.3 | 91.9 | 1913 |
| 89 | March | 0.6 | 2.8 | 6.0 | 10.2 | 18.1 | 32.9 | 57.6 | 76.0 | 88.0 | 94.9 | 1145 |
| 90 | Jan./Feb. | 1.2 | 4.1 | 8.3 | 11.2 | 15.6 | 24.1 | 36.1 | 48.9 | 62.7 | 75.2 | 2585 |
| 90 | March | 48.1 | 52.1 | 55.2 | 57.9 | 60.9 | 65.5 | 70.0 | 76.4 | 82.7 | 88.4 | 750 |
| 91 | Jan./Feb. | 3.1 | 14.9 | 23.0 | 29.8 | 36.0 | 46.6 | 59.6 | 69.6 | 78.3 | 86.3 | 161 |
| 91 | March | 5.3 | 12.2 | 19.3 | 23.8 | 29.8 | 37.4 | 49.9 | 63.7 | 76.6 | 86.1 | 794 |
| 92 | Jan./Feb. | 1.4 | 4.2 | 7.2 | 11.6 | 22.5 | 35.5 | 58.4 | 76.7 | 87.8 | 93.8 | 898 |
| 92 | March | 0.6 | 3.4 | 6.5 | 14.1 | 26.0 | 43.1 | 59.7 | 77.1 | 87.0 | 92.9 | 799 |
| 93 | Jan./Feb. | 0.5 | 0.5 | 1.5 | 2.6 | 4.1 | 8.7 | 18.5 | 31.8 | 52.8 | 71.3 | 195 |
| 93 | March | 0.0 | 2.1 | 5.7 | 7.5 | 10.4 | 19.3 | 35.0 | 57.0 | 74.5 | 87.6 | 703 |
| 94 | Jan./Feb. | 0.6 | 1.9 | 4.3 | 12.4 | 22.3 | 39.5 | 54.5 | 60.9 | 69.3 | 80.0 | 466 |
| 94 | March | 0.0 | 1.0 | 2.5 | 7.5 | 15.2 | 26.5 | 41.9 | 54.0 | 67.1 | 82.3 | 480 |
| 95 | Jan./Feb. | 0.5 | 0.5 | 1.5 | 1.5 | 2.5 | 6.1 | 13.7 | 34.0 | 48.7 | 69.0 | 197 |
| 95 | March | 2.2 | 3.9 | 5.8 | 8.0 | 15.8 | 30.5 | 45.4 | 60.1 | 74.0 | 83.9 | 361 |

Table 11. Comparison of cumulative length-frequency distributions for Hecate Strait Pacific cod samples collected in April versus May/June, 1980-1996.

|  |  | Cumulative percentage of fish in length group (cm) or smaller |  |  |  |  |  |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year |  | $<45$ | $<48$ | $<51$ | $<54$ | $<57$ | $<60$ | $<63$ | $<66$ | $<69$ | $<72$ | Number |
| 8080 | April | 2.9 | 7.6 | 16.4 | 29.4 | 47.2 | 64.7 | 79.2 | 87.5 | 92.8 | 96.7 | 1054 |
|  | May/June | 2.2 | 8.4 | 17.5 | 28.8 | 43.4 | 59.3 | 74.1 | 85.0 | 91.5 | 95.6 | 1137 |
| 8282 | April | 2.5 | 5.5 | 11.0 | 35.5 | 53.0 | 71.0 | 84.0 | 87.0 | 91.0 | 95.5 | 200 |
|  | May/June | 3.0 | 6.5 | 12.6 | 21.5 | 35.6 | 52.3 | 68.9 | 82.4 | 91.6 | 96.1 | 2299 |
| 8383 | April | 1.5 | 5.3 | 13.6 | 20.1 | 28.9 | 44.5 | 63.6 | 79.9 | 89.9 | 95.2 | 2036 |
|  | May/June | 0.8 | 5.9 | 17.4 | 27.8 | 34.2 | 42.9 | 57.5 | 71.3 | 82.0 | 90.3 | 1852 |
| 8484 | April | 1.3 | 5.1 | 10.4 | 14.1 | 20.8 | 32.6 | 48.0 | 63.4 | 79.1 | 90.4 | 1004 |
|  | May/June | 6.3 | 19.3 | 40.1 | 50.7 | 59.4 | 67.3 | 80.2 | 86.0 | 91.3 | 96.4 | 1166 |
| 85 | April | 0.0 | 1.7 | 4.7 | 9.4 | 16.3 | 27.4 | 43.7 | 56.3 | 70.4 | 87.8 | 533 |
|  | May/June | 0.3 | 3.0 | 8.7 | 20.3 | 38.4 | 60.5 | 79.1 | 84:2 | 89.2 | 93.3 | 641 |
| 86 | April | 0.4 | 2.0 | 4.9 | 8.4 | 16.1 | 26.2 | 42.8 | 62.1 | 78.3 | 88.1 | 1632 |
|  | May/June | 0.9 | 2.3 | 5.5 | 10.4 | 18.2 | 24.8 | 35.7 | 54.2 | 77.6 | 89.2 | 2214 |
| 8787 | April | 6.8 | 22.8 | 43.3 | 66.3 | 86.8 | 94.8 | 98.0 | 98.3 | 99.0 | 99.3 | 400 |
|  | May/June | 11.5 | 27.5 | 46.7 | 64.7 | 79.1 | 88.8 | 93.1 | 95.6 | 97.4 | 98.4 | 2447 |
| 888 | April | 0.5 | 4.9 | 13.4 | 30.0 | 52.6 | 73.6 | 86.8 | 94.8 | 98.9 | 99.8 | 614 |
|  | May/June | 0.2 | 3.5 | 14.8 | 35.9 | 58.0 | 76.0 | 88.5 | 95.8 | 98.7 | 99.5 | 850 |
| 89 | April | 0.6 | 1.8 | 3.9 | 11.8 | 19.6 | 33.5 | 48.0 | 66.8 | 82.5 | 91.2 | 331 |
|  | May/June | 2.1 | 9.2 | 20.5 | 29.6 | 40.9 | 56.8 | 74.0 | 89.1 | 94.8 | 97.3 | 477 |
| 91 | April | 2.1 | 7.2 | 14.8 | 22.8 | 29.8 | 41.4 | 57.1 | 74.9 | 85.4 | 92.2 | 1090 |
|  | May/June | 2.6 | 13.8 | 33.1 | 50.3 | 63.9 | 70.8 | 77.8 | 85.6 | 91.3 | 95.2 | 1164 |
| 9292 | April | 1.4 | 4.6 | 11.5 | 17.9 | 29.8 | 47.3 | 65.5 | 79.0 | 89.5 | 95.7 | 715 |
|  | May/June | 1.0 | 5.2 | 13.3 | 22.5 | 30.3 | 40.3 | 54.9 | 69.7 | 82.8 | 91.2 | 1061 |
| $\begin{aligned} & 93 \\ & 93 \end{aligned}$ | April | 0.3 | 1.7 | 3.0 | 4.2 | 7.8 | 14.9 | 28.8 | 46.7 | 66.7 | 83.0 | 664 |
|  | May/June | 0.6 | 2.9 | 6.7 | 9.8 | 14.0 | 18.9 | 29.1 | 45.2 | 65.6 | 82.2 | 986 |
| 9595 | April | 0.9 | 0.9 | 1.4 | 2.8 | 5.1 | 10.2 | 21.3 | 34.3 | 55.6 | 72.2 | 216 |
|  | May/June | 2.6 | 9.4 | 17.8 | 21.3 | 28.1 | 35.7 | 45.6 | 59.1 | 70.5 | 82.7 | 342 |
| 9696 | April | 2.6 | 5.2 | 9.5 | 12.9 | 23.3 | 44.8 | 57.8 | 67.2 | 73.3 | 79.3 | 116 |
|  | May/June | 0.0 | 3.2 | 7.1 | 15.9 | 38.1 | 59.5 | 74.6 | 81.0 | 85.7 | 88.9 | 126 |



Fig. 1. Distribution of Hecate Strait catch (lbs.) per tow from log book (1991-1995) and observer collected (1996; 1997) data.


Fig. 2. Distribution of Hecate Strait CPUE ( $\mathrm{kg} / \mathrm{hr}$ ) from log book (1991-1995) and observer collected (1996; 1997) data.


Fig. 3. Comparison of tow-based and trip-based CPUE indices (kg/hr) for Hecate Strait, 19911995. Line indicates $1: 1$ relationship.




Fig. 4 CPUE (kg/hr) indices for Hecate Strait Pacific cod by quarter, 1956-1997. The solid line shows the tow-based indices and the dashed line the trip-based indices for 19911997.


Fig. 5 Proportion-at-length data for Hecate Strait by quarter, 1956-1997. Dot area is proportional to relative frequency.


Fig. 5 Cont'd.


Fig. 6 Effort residuals ( $\ln$ (predicted)-ln(observed)) for the MULTIFAN CL analysis of Hecate Strait Pacific cod data by quarter for 1956-1997.


Fig. 7 Effort residuals ( $\ln$ (predicted)-ln(observed)) for the MULTIFAN CL analysis with a 0.25 weighting on the 1997 length-frequency data, by quarter for 1956-1997.


Fig. 8 Recruitment and spawning stock biomass estimates from MULTIFAN CL analysis, 1955-1997. The different lines show results from analyses with alternative weights on the 1997 length-frequency data. The vertical lines on the top figure show the estimated $95 \%$ confidence limits for the recruitment estimates. The recruitment figure is truncated, and the upper $95 \%$ limit for the estimated 1997 recruitment from the alternate analysis is 109,000 .

## APPENDIX "A"

The description of the model, MULTIFAN-CL, in the following pages is extracted from a paper that describes aspects of the model and its application to South Pacific Albacore [David A Fournier, John Hampton and John R. Sibert. (in press) MULTIFAN-CL: A length-based, agestructured model for fisheries stock assessment, with application to South Pacific Albacore Thunnus Alalunga]. As such, certain aspects of the model structure that are described are not used in the B.C. Pacific cod stock assessment and there are additional components of the model structure in this assessments that are not described in the document.

Aspects of the Pacific cod implementation which are not consistent with the model description are:

- only a single region is modeled (therefore, no movement parameters are estimated)
- natural mortality is constant over years and ages
- catchability (q) is time-invariant, however we estimate separate catchability parameters for each quarter of the year
- we estimate separate selectivity parameters for three time periods, but assume a common selectivity for fish greater than 61 cm .

In this paper, we describe an age-structured model that extends the MULTIFAN method of estimating catch age composition from length composition (Fournier et al. 1990). An important feature of our model, which we call MULTIFAN-CL (Catch at Length), is that it is fully integrated - growth and catch age structure are estimated simultaneously with recruitment, selectivity, catchability, natural mortality, and other parameters. Estimated confidence intervals are therefore conditional not on catch-at-age, but catch-at-length data. The statistical framework of the model is amenable to the formulation and testing of various hypotheses regarding the dynamics of the stock. Some of the model hypotheses that we formulate and test in this paper include the number of significant age classes in the catch, spatial structuring of the population and fisheries, density-dependent growth, time-series trends and seasonal cycles in catchability, age-dependent rates of natural mortality, and age-dependent fish movement.

## Data Structures

The fundamental data structure of the model is based on the notion of a fishery, which is defined as a collection of fishing units operating in a particular geographical region, and which have similar catchability and selectivity characteristics with respect to the target species. Each occurrence of a fishery at a particular time is termed a fishing incident. In reality, fishing is more or less continuous, so the data for each fishery need to be aggregated over appropriate time intervals.

## The Catch Equations

It is assumed for simplicity of notation in this description that there is only one fishery operating in each region and that there is only one fishing incident per fishery per year. The model is designed to accommodate a variable number of different fisheries per region and fishing incidents per fishery per year; the equations that follow could easily be generalized in this way.

The catch equations relate the numbers of fish in the population to the numbers of fish in the catch of the fisheries. The form of the catch equations used in the model is described by the following relationships:

$$
\begin{align*}
& C_{i j k}=\frac{F_{i j k}}{Z_{i j k}}\left[1-\exp \left(-Z_{i j k}\right)\right] N_{i j k} \quad \text { for } \quad 1 \leq i \leq n, \quad 1 \leq j \leq a, \quad 1 \leq k \leq r  \tag{1}\\
& T_{i+1, j+1, k}=\exp \left(-Z_{i j k}\right) N_{i j k} \text { for } 1 \leq i \leq n, \quad 1 \leq j<a, \quad 1 \leq k \leq r  \tag{2}\\
& T_{i+1, a k}=\exp \left(-Z_{i, a-1, k}\right) N_{i, a-1, k}+\exp \left(-Z_{i a k}\right) N_{i a k} \quad \text { for } \quad 1 \leq i<n, \quad 1 \leq k \leq r  \tag{3}\\
& T_{i l k}=\gamma_{k} R_{i} \text { for } 1 \leq i<n, \quad 1 \leq k \leq r \quad \text { where } \quad \sum_{k} \gamma_{k}=1 \quad \text { and } \quad \gamma_{k} \geq 0  \tag{4}\\
& N_{i j k}=\sum_{l} \beta_{j k k} T_{i j l} \text { for } 1 \leq i \leq n, \quad 1 \leq j \leq a, \quad 1 \leq k \leq r, \quad 1 \leq l \leq r  \tag{5}\\
& Z_{i j k}=F_{i j k}+M_{i j k} \text { for } 1 \leq i \leq n, \quad 1 \leq j \leq a, \quad 1 \leq k \leq r  \tag{6}\\
& C_{i, k}=\sum_{j} C_{i j k} \text { for } 1 \leq i \leq n, \quad 1 \leq j \leq a, \quad 1 \leq k \leq r \tag{7}
\end{align*}
$$

where
$i$ indexes year,
$j$ indexes age class,
$k$ indexes region,
$n$ is the number of years of fishing,
$a$ is the number of age classes in the population,
$r$ is the number of regions,
$C_{i j k}$ is the catch (in number of fish) of age class $j$ fish in region $k$ in year $i$,
$C_{i . k}$ is the total catch observed in region $k$ in year $i$,
$F_{i j k} \quad$ is the instantaneous fishing mortality rate for age class $j$ fish in region $k$ in year $i$,
$M_{i j k} \quad$ is the instantaneous natural mortality rate for age class $j$ fish in region $k$ in year $i$,
$Z_{i j k}$ is the instantaneous total mortality rate for age class $j$ fish in region $k$ in year $i$,
$T_{i j k} \quad$ is the number of age class $j$ fish in the population in region $k$ at the beginning of year $i$ before movement has taken place,
$N_{i j k} \quad$ is the number of age class $j$ fish in the population in region $k$ at the beginning of year $i$ after movement has taken place,
$R_{i}$ is the recruitment at the beginning of year $i$,
$\gamma_{k}$ is the proportion of recruitment occurring in region $k$, and
$\beta_{j k l} \quad$ is a $k$ by $k$ diffusion matrix $\mathbf{B}_{j}$ for age class $j$ fish.

Note that the last ( $a$ th) age class consists of all the older fish in the population, which is useful when, as often occurs, the aging estimates are especially inaccurate for the older age classes (Fournier and Archibald 1982). It is also useful when analyzing catch-at-length data to group the older age classes together after the fish reach an age where they essentially stop growing (Fournier et al. 1991).

## Movement Hypothesis

For each $j$, the elements of $\mathbf{B}_{j}$ must be specified. In the case of South Pacific albacore, we use a one-dimensional diffusion model operating in three regions ( $k=3$ ). In this case, the elements of $\mathbf{B}_{j}$ are given by

$$
\left[\begin{array}{ccc}
1+\theta_{j} & -d_{2} \theta_{j} & 0  \tag{8}\\
-\theta_{j} & 1+2 d_{2} \theta_{j} & -d_{3} \theta_{j} \\
0 & -d_{2} \theta_{j} & 1+d_{3} \theta_{j}
\end{array}\right]^{-1}
$$

where $1, d_{2}$ and $d_{3}\left(d_{2}>0\right.$ and $\left.d_{3}>0\right)$ specify the relative distribution of cohort abundance among regions at equilibrium and $\theta_{j}$ is the age-dependent diffusion rate. We employ a flexible
parameterization of $\theta_{j}$ which can result in increasing, decreasing or constant diffusion rate with increasing age:

$$
\begin{align*}
& \theta_{j}=\phi_{0} \exp \left\{\phi_{1}\left[-\left(-\kappa_{j}\right)^{\phi_{3}}\right]\right\} \quad \text { where } \quad \phi_{0} \geq 0, \phi_{1} \geq 0 \text { and } \kappa_{j}<0  \tag{9}\\
& \theta_{j}=\phi_{0} \exp \left\{\phi_{1} \kappa_{j}^{\phi_{3}}\right\} \quad \text { where } \quad \phi_{0} \geq 0, \phi_{1} \geq 0 \text { and } \kappa_{j} \geq 0
\end{align*}
$$

where $\kappa_{j}=\frac{2(j-1)}{a-1}-1$, which expresses age scaled between -1 and 1.
A one-dimensional movement hypothesis was considered appropriate for South Pacific albacore on the basis of tagging data (Labelle 1993) and the variation in albacore size with latitude (smallest in the south, increasing towards the equator). Other movement hypotheses and/or spatial configurations (including homogeneity) could easily be incorporated into the model, as warranted by the particular case being studied.

## Assumptions Regarding Constraints On Natural And Fishing Mortality Rates

A fundamental characteristic of statistical age-structured models is that they constrain the variation of mortality rates by age and time in a regular fashion. The objective of such constraints is to create degrees of freedom that enable a statistical estimation of parameters to proceed. Constraints are normally placed separately on the variability of natural and fishing mortality rates.

## Natural mortality

In the South Pacific albacore application, we assumed that the instantaneous natural mortality rate is independent of year and region, but may vary with age. Later, we show that this age dependency is justified on statistical grounds. For a given application, a range of more and less restrictive constraints on natural mortality can be tested.

## Fishing mortality

We restrict the variation in the instantaneous fishing mortality rates $F_{i j k}$ according to the "separability" assumption. Consider for simplicity an individual fishery (i.e. drop the $k$ subscript). We assume that

$$
\begin{equation*}
\log _{e}\left(F_{i j}\right)=\log _{e}\left(s_{j}\right)+\log _{e}\left(q_{i}\right)+\log _{e}\left(E_{i}\right)+\varepsilon_{i} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
\log _{e}\left(q_{i+1}\right)=\log _{e}\left(q_{i}\right)+\eta_{i} \tag{11}
\end{equation*}
$$

where
$s_{j} \quad$ is the selectivity for age class $j$ (assumed constant over time),
$q_{i} \quad$ is the catchability in year $i$,
$E_{i}$ is the observed fishing effort in year $i$,
$\varepsilon_{i} \quad$ are normally distributed random variables representing large transient deviations in the effort-fishing mortality relationship, and
$\eta_{i} \quad$ are normally distributed random variables representing small permanent changes in catchability.

The notion, as implied in equation (10), that fishing mortality consists of a "separable" agedependent effect (selectivity) and a time-dependent effect (catchability) was first introduced by Doubleday (1976) and later elaborated upon by Paloheimo (1980) and Fournier and Archibald (1982).

## Selectivity

It is possible to model selectivity as a function of age class, for example using a gamma function (Deriso et al. 1985). We have preferred to allow the $s_{j}$ to be separate parameters but have applied a transformation that essentially makes selectivity a length-based rather than age-based concept. The transformation is as follows:
$s_{j}=\sum_{k=-2}^{2} \omega_{k}\left\{t\left[\psi_{1}\left(\mu_{j}+k \sigma_{j}\right)\right]+\psi_{2}\left(\mu_{j}+k \sigma_{j}\right)\left(t\left[\psi_{1}\left(\mu_{j}+k \sigma_{j}\right)+1\right]-t\left[\psi_{1}\left(\mu_{j}+k \sigma_{j}\right)\right]\right)\right\}$
where
$\omega_{k} \quad$ are weights determined from the normal distribution of length-at-age $k$ standard deviations from the mean,
$\psi_{1}$ is the integer part of the age class number corresponding to length $\mu_{j}+k \sigma_{j}$,
$\psi_{2}$ is the fractional part of the age class number corresponding to length

$$
\mu_{j}+k \sigma_{j}
$$

$\mu_{j} \quad$ is the mean length of age class $j$ fish,
$\sigma_{j}$ is the standard deviation of length of age class $j$ fish, and
$t$ is an estimated parameter.
This transformation effectively ensures relatively small differences in $s_{j}$ between adjacent age classes having large overlap of their length distributions, as would be expected where selectivity is fundamentally length-based.

## Catchability

Catchability is allowed to vary slowly over time. We assume that the $q_{i}$ have the simple time series structure of a random walk (equation 11), which is the simplest statistical model of a slowly varying random quantity. The assumption that catchability has a time series structure was introduced by Gudmundsson (1994) for the analysis of catch-at-age data. Gudmundsson also included trend components in his time series formulation.

We make the prior assumption that the variance of $\eta_{i}$ is small compared to $\varepsilon_{i}$, i.e. the $\varepsilon_{i}$ represent relatively large transient effects (noise) while the $\eta_{i}$ represent relatively small permanent changes in the catchability.

In the simple case of annual fishing incidents, $\eta_{i}$ modifies catchability at each successive fishing incident. In general, each step of the random walk can be taken less frequently, as might be appropriate when multiple fishing incidents by one fishery occur within a year. In the albacore analysis (where the frequency of fishing incidents is quarterly for the longline fisheries and monthly for the surface fisheries), random walk steps are taken annually for all fisheries.

Where the frequency of fishing incidents is quarterly or more, we may allow catchability within a year to vary with a regular seasonal pattern. Equation (10) then becomes

$$
\begin{equation*}
\log _{e}\left(F_{i j}\right)=\log _{e}\left(s_{j}\right)+\log _{e}\left(q_{i}\right)+\log _{e}\left(E_{i}\right)+c_{1} \sin \left[24 \pi\left(m-c_{2}\right)\right]+\varepsilon_{i} \tag{13}
\end{equation*}
$$

where $m$ is the month in which the fishing incident occurred and $c_{1}$ and $c_{2}$ are the seasonality parameters.

## Assumptions Regarding Length-At-Age

MULTIFAN-CL uses length data to estimate age structure and therefore makes assumptions concerning the length distribution of the fish that are very similar to the assumptions used in Fournier et al., 1990:

1. The lengths of the fish in each age class are normally distributed (see equation 14).
2. The mean lengths-at-age lie on (or near) a von Bertalanffy growth curve (see equation 16) modified to include, where appropriate, density-dependent growth (see equation 18).
3. The standard deviations of the lengths for each age class are a simple function of the mean length-at-age (see equation 19).

The following symbols are used in the mathematical expression of these assumptions:
$\alpha$ subscript indexing the length frequency intervals.
$N_{I}$ the number of length intervals in each length frequency data set.
$S_{i}$ the number of fish in the $i$ th length frequency data set.
$f_{a i}$ the number of fish whose lengths lie in the $\alpha$ th length interval in the $i$ th length frequency data set.
$p_{i j \alpha}$ the probability that an age class $j$ fish picked at random from the fish which were sampled to get the $i$ th length frequency data set has a length lying in length interval $\alpha$.
$Q_{a i}$ the probability that an animal picked at random from the fish which composed the $i$ th length frequency data set has a length lying in length interval $\alpha$.
the observed proportion of fish in the $i$ th length frequency data set having a length lying in length interval $\alpha$.
$\mu_{i j} \quad$ the mean length of the age class $j$ fish in the $i$ th length frequency data set.
$\sigma_{i j}$ the standard deviation of the length distribution of the age class $j$ fish in the $i$ th length frequency data set.
$x_{i} \quad$ the midpoint of the $i$ th length frequency interval.
$w$ the width of the length frequency intervals.
$L_{1} \quad$ the mean length of the first age class on the von Bertalanffy curve in month 1.
$L_{r} \quad$ the mean length of the last age class on the von Bertalanffy curve in month 1.
$K$ the von Bertalanffy $K$ parameter.
$\rho$ the Brody growth coefficient $\left(K=-\log _{e}(\rho)\right)$.
$\lambda_{1}, \lambda_{2}$ parameters determining the standard deviations $\sigma_{j \alpha}$.
$\xi_{i a}$ parameters determining the relative variances of the sampling errors within the $i$ th length frequency data set.
$\tau$ parameter determining the overall variance of the sampling errors in all the length frequency data sets.

## Assumption 1: Normal distribution of length-at-age

If the lengths of the age class $j$ fish in the $\alpha$ th length frequency data set are normally distributed around their mean $\mu_{j \alpha}$ with standard deviations $\sigma_{j \alpha}$, the $p_{i j \alpha}$ can be expressed in terms of $\mu_{j \alpha}$ and $\sigma_{j \alpha}$ by

$$
\begin{equation*}
p_{i j \alpha}\left(\mu_{j \alpha}, \sigma_{j \alpha}\right)=\frac{1}{\sqrt{2 \pi \sigma_{j \alpha}}} \int_{x_{i}-w / 2}^{x_{i}+w / 2} \exp \left\{\frac{-\left(x-\mu_{j \alpha}\right)^{2}}{2 \sigma_{j \alpha}^{2}}\right\} d x \tag{14}
\end{equation*}
$$

As long as $\sigma_{j \alpha}>w$, the integral can be approximated sufficiently well by setting

$$
\begin{equation*}
p_{i j \alpha}\left(\mu_{j a}, \sigma_{j a}\right)=\frac{w}{\sqrt{2 \pi \sigma_{j a}}} \exp \left\{\frac{-\left(x-\mu_{j \alpha}\right)^{2}}{2 \sigma_{j a}^{2}}\right\} . \tag{15}
\end{equation*}
$$

## Assumption 2: Relationship of length to age

## Parameterization of von Bertalanffy growth

If the mean lengths $\mu_{j \alpha}$ lie on a von Bertalanffy curve, then, using the parameterization given by Schnute and Fournier (1980)

$$
\begin{equation*}
\mu_{j \alpha}=L_{1}+\left(L_{N,}-L_{1}\right)\left[\frac{1-\rho^{j-1+(m(\alpha)-1) / 12}}{1-\rho^{N_{J-1}}}\right] \tag{16}
\end{equation*}
$$

where $L_{1}$, the mean length of the first age class, $L_{N_{N}}$, the mean length of the last age class, and $\rho$, the Brody growth coefficient, are the three parameters that determine the form of the von Bertalanffy curve, and $m(\alpha)-1$ is the number of months after the presumed birth month of the fish in the $a$ th length frequency data set.

## Density-dependent growth

For many species it is suspected that individuals of small (in numbers of fish) cohorts may grow more quickly than those of large cohorts (i.e. density-dependent growth). If true, this phenomenon could have a large effect on the conclusions drawn from a length-based stock assessment. To test for evidence of the existence of the dependence of the mean length-at-age on cohort strength we have incorporated density-dependent growth into the model in the following fashion.

Consider a cohort $k$ at age $j$ in year $i$. If we denote recruitment as occurring at age 1 , the strength of cohort $k$ is $N_{k 1}$, where $k=i-j+1$. Let $A=\frac{1}{n} \sum_{k} N_{k 1} \quad$ be the average recruitment. The normalized relative cohort strength is given by

$$
\begin{equation*}
R_{k}=\frac{\left(N_{k 1}-A\right)}{\sqrt{\sum_{k} N_{k 1}^{2}}} \tag{17}
\end{equation*}
$$

The changes in mean length are effected by changing the apparent age of the fish before the length-at-age is calculated. If the age class is $j$ the apparent age $a$ is

$$
\begin{equation*}
a=j+1.9\left[\frac{1}{1+\exp \left(-d R_{k}\right)}-0.5\right] \tag{18}
\end{equation*}
$$

where $d$ determines the amount of density-dependent growth; if $d=0, a=j$. Since the standard deviation of the $R_{k}$ has been normalized to 1 , the "generic" variation in the $R_{k}$ will be about -2 to 2. Thus the difference in $a$ between the largest and smallest cohorts of any given age class will be approximately $1.9\left[\frac{1}{1+\exp (-2 d)}-\frac{1}{1+\exp (2 d)}\right]$. For $d=-1.08$ (which is the estimate for the albacore data) this yields a generic variation of about -1.5 years, i.e. the apparent age of the largest cohort is about 1.5 years more than that of the smallest cohort.

Assumption 3: Relationship of standard deviations in length-at-age to mean length-at-age
The standard deviations $\sigma_{j \alpha}$ are parameterized as a simple function of length involving two parameters $\lambda_{1}$ and $\lambda_{2}$ :

$$
\begin{equation*}
\sigma_{j a}=\lambda_{1} \exp \left\{\lambda_{2}\left[-1+2\left(\frac{1-\rho^{j-1+(m(\alpha)-1) / 12}}{1-\rho^{N_{J-1}}}\right)\right]\right\} \tag{19}
\end{equation*}
$$

where the term enclosed in square brackets expresses the length dependency of the standard deviations independently of the numerical values of the parameters $L_{1}$ and $L_{N_{j}}$ (cf. equation 16). The two coefficients, $\lambda_{1}$ and $\lambda_{2}$, transform the re-scaled length to the standard deviations. $\lambda_{1}$ determines the magnitude of the standard deviations, and $\lambda_{2}$ determines the length-dependent trend in the standard deviations. If $\lambda_{2}=0$, the standard deviations are length-independent.

## Maximum Likelihood Estimation

The parameters of the model are estimated by maximizing the log-likelihood function (or more generally by maximizing the sum of the log-likelihood function and the log of the density of the Bayesian prior distribution). The log-likelihood function consists of the sum of several components, the most important of which correspond to the length frequency data and the total catch estimates.

## The log-likelihood contribution for the length frequency data

Due to the large variability in the length samples that often occurs for length frequency data, we employ a robust maximum likelihood estimation procedure. The motivation for using this procedure and the technicalities behind the procedure are described in Fournier et al. (1990). We
shall not repeat this discussion here, but for convenient reference we briefly describe the form of the log-likelihood function employed.

If the $\widetilde{Q}_{c i}$ are derived from a random sample of size $S_{i}$, they would be random variables with means $Q_{\alpha i}$ and variances $\left(1-Q_{\alpha i}\right) Q_{\alpha i} / S_{i}$. Two modifications have been made to this formula. If $Q_{\alpha i}=0$ the formula implies that the variance of $\tilde{Q}_{\alpha i}=0$. To decrease the influence of areas where no observations are expected, we add a small number to the variance formula in such cases. To reduce the influence of very large sample sizes we have assumed that sample sizes $>1,000$ are no more accurate than sample sizes of 1,000 . Set $\xi_{\alpha}=\left(1-Q_{\alpha i}\right) Q_{\alpha i}$ and set $\tau_{i}^{2}=1 / \min \left(S_{i}, 1000\right)$. Assume the variance of $\tilde{Q}_{\alpha i}$ is given by $\left(\xi_{i \alpha}+.1 / N_{t}\right) \tau_{i}^{2}$. The likelihood function contribution for the length frequency data is then

$$
\begin{equation*}
\prod_{a=1}^{N_{A}} \prod_{i=1}^{N_{1}}\left[\frac{1}{\sqrt{2 \pi\left(\xi_{i \alpha}+.1 / N_{t}\right) \tau}}\left(\exp \left\{-\frac{\left(\widetilde{Q}_{i \alpha}-Q_{i \alpha}\right)^{2}}{2\left(\xi_{i \alpha}+.1 / N_{t}\right) \tau^{2}}\right\}+.01\right)\right] \tag{20}
\end{equation*}
$$

Taking the logarithm of expression (20) we obtain the log-likelihood function for the length frequency data:

$$
\begin{align*}
& -1 / 2 \sum_{\alpha=1}^{N_{A}} \sum_{i=1}^{N_{I}} \log _{e}\left[2 \pi\left(\xi_{i \alpha}+.1 / N_{I}\right)\right] \\
& \quad-\sum_{\alpha=1}^{N_{A}} N_{I} \log _{e}(\tau)  \tag{21}\\
& +\sum_{\alpha=1}^{N_{A}} \sum_{i=1}^{N_{I}} \log _{e}\left[\exp \left\{\frac{-\left(\widetilde{Q}_{i \alpha}-Q_{i \alpha}\right)^{2}}{2\left(\xi_{i \alpha}+.1 / N_{l}\right) \tau^{2}}\right\}+0.01\right]
\end{align*}
$$

The log-likelihood contribution for the observed total catches
Assuming for simplicity that there is only one fishery per year, the log-likelihood contribution for the observed total catches is given by

$$
\begin{equation*}
p_{c} \sum_{i}\left(\log \left(C_{i .}^{o b s}\right)-\log \left(C_{i .}\right)\right)^{2} \tag{22}
\end{equation*}
$$

where $p_{c}$ is determined by the prior assumption made about the accuracy of the observed catch data. For the albacore analysis, we assumed $p_{c}=200$, which is consistent with a coefficient of variation of about 0.07 .

The log-likelihood contribution for the Bayesian priors on the effort-fishing mortality relationship

Given the random walk structure assumed to operate for time-series changes in catchability, it follows that the prior distribution for the $\eta_{i}$ is normal. However, the prior distribution for $\varepsilon_{i}$ is
assumed to be a robustified normal distribution, i.e. the probability of events at the tails of the distribution is increased relative to a standard normal distribution. Then, the log-likelihood contribution for the Bayesian priors on the $\eta_{i}$ and $\varepsilon_{i}$, (see equations 10 and 11) is given by

$$
\begin{equation*}
p_{\eta} \sum_{i} \eta_{i}^{2}-\sum_{i} \log _{e}\left[\exp \left(-p_{\varepsilon} \varepsilon_{i}^{2}\right)+0.01\right] \tag{23}
\end{equation*}
$$

The size of the constants $p_{\eta}$ and $p_{\varepsilon}$ are adjusted to reflect prior assumptions about the variances of these random variables. For the albacore analysis, we assumed $p_{\eta}=25$ and $p_{\varepsilon}=10$, which is equivalent to assuming that the coefficients of variation of $\eta_{i}$ and $\varepsilon_{i}$ are 0.14 and 0.22 , respectively. Note that the second term of equation (23) is a component of the log-likelihood function that corresponds to an improper density. As a result, the variance corresponding to the weight $p_{\varepsilon}$ cannot be estimated and must be specified.

## Nonlinear optimization

The parameters of the model are estimated by maximizing the log-likelihood function (or posterior density in the Bayesian framework) as described above. The maximization was performed using the nonlinear modeling package AD Model Builder, which employs an efficient optimization using exact derivatives with respect to the model parameters. The derivatives were calculated using an extension of the technique known as automatic differentiation (Griewank and Corliss 1991). This approach is especially useful for models with large numbers of parameters. It also provides quick and accurate estimates of the Hessian matrix at the maximum, which can be used to obtain estimates of the covariance matrix and confidence limits for the parameters of interest (see later).

## Hypothesis Testing

It is frequently of interest in statistical modeling to add model structure in the form of one or more hypotheses concerning some process(es) of interest, and to observe the resulting change in model performance. Two approaches are taken to the addition and testing of hypotheses - a frequentist approach and a Bayesian approach.

With the frequentist approach to hypothesis testing, parameters representing a more complex model are added to the simpler model and the resulting improvement in fit is calculated. If this improvement in fit is large enough, the more complicated model is accepted. Otherwise the more complicated model is rejected and the simpler model is accepted as providing an adequate description of the data. Various more complicated models may be investigated in this fashion. There are various statistical criteria that might be used to decide whether to accept or reject a more complex model, such as likelihood-ratio tests (e.g. as applied in Fournier et al. 1990) or the Akaike Information Criterion (e.g. as applied in Bigelow et al. 1995). Note that such tests are approximate and that the strict statistical conditions assumed rarely hold in practice. However, in the case of likelihood-ratio tests, simulations have indicated that such tests still provide a useful method of model screening even when the strict statistical conditions are not met (Hastie and Tibshirani 1990). This is the approach adopted in this paper.

Some hypotheses that are useful in length-based stock assessment cannot be well represented in a frequentist context. An example is the existence of a time-series (random walk) trend in catchability for a fishery. For such hypotheses, the results of the analysis are not as clear-cut as they are for the frequentist approach. We neither accept nor reject the existence of a trend in catchability. Instead, the analysis will produce a probability distribution for quantities of interest. For example, we can obtain an approximate probability distribution for the ratio of the catchability for the first year of a fishery to the catchability for the last year of the fishery. This can be used to produce, for example, an estimate of the probability that the catchability has increased by $30 \%$ or more.

## Estimation Of Confidence Intervals

A great advantage of an integrated model such as this is that the estimates of the uncertainty in the parameter estimates automatically take into account the effect of all of the model's assumptions, such as the uncertainty in the age at length, the possibility of trends in catchability, effects caused by variability in the length frequency data and errors in the estimates of fishing effort.

Confidence limits for the parameter estimates are calculated by employing the usual second order approximation to the posterior distribution at its mode. Let $\theta_{1}, \ldots, \theta_{n}$ denote a minimal set of $n$ model parameters from which all model parameters can be calculated, and let $p\left(\theta_{1}, \ldots, \theta_{n}\right)$ be some parameter of interest, while $L\left(\theta_{1}, \ldots, \theta_{n}\right)$ is the logarithm of the posterior distribution. Then the estimated standard deviation $p_{\sigma}$ for $p$ is given by the square root of $\sum_{i j} \partial / \partial \theta_{i} \partial / \partial \theta_{j} \Lambda_{i j}$ where $\Lambda=\left(\partial^{2} L / \partial \theta_{i} \partial \theta_{j}\right)^{-1}$ and the calculations are carried out at the mode of the posterior distribution. Then, 0.95 confidence limits for the $p$ are given by $\left[p-1.96 p_{\sigma}, p+1.96 p_{\sigma}\right]$. These confidence limits are not invariant under reparameterization. To compensate somewhat for this, the confidence limits for parameters which must be positive, such as estimates of biomass, are calculated by computing the confidence limits for the logarithms of these parameters and then transforming the confidence limits. This yields the confidence limits $\left[p \exp \left(-1.96 p_{\sigma} / p\right), p \exp \left(1.96 p_{\sigma} / p\right)\right]$.

The above procedure provides approximate confidence intervals for the model parameters (initial cohort size, selectivity and catchability coefficients, natural mortality rates, growth parameters, etc). For stock assessment purposes, it may be desirable to have confidence intervals for quantities of interest, such as adult biomass, that are functions of the model parameters. The variances (and hence confidence intervals) for such quantities may be determined using the delta method.

Note that confidence intervals derived as described above are conditional on the model structure used. It may be possible to define the best model from a finite range of alternatives for a particular set of data on the basis of the maximum likelihood criterion. However, there is never any guarantee that any given model is the best of all possible models. Uncertainty regarding what is the best model is not incorporated into the confidence intervals; therefore such confidence
intervals will tend to understate the true uncertainty in the model parameters and other quantities of interest.

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