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Semi-parametric inferences about fish stock size using sequential population analysis (SPA) and quasi-likelihood theory.

by

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Abstract

In this paper we present Quasi-likelihood methods for stock size inferences based on an SPA and estimation with relative abundance indices. These methods are applicable to a variety of types of indices. These methods are semi-parametric in that they only require assumptions about the mean and variance of the indices, and resulting inferences have a degree of distributional robusteness. Profile quasi-likelihood methods are used to stochastically evaluate the impact of a commercial fishing quota on future stock size. These methods are applied to a cod stock off the east coast of Canada.

Résumé

Nous présentons ici des méthodes de quasi-vraisemblance permettant des inférences sur la taille des stocks à partir d'une ASP et d'une estimation avec indices de l'abondance relative. Ces méthodes, qui s'appliquent à divers types d'indices, sont semi-paramétriques car elles ne demandent que des hypothèses sur la moyenne et la variance des indices, et les inférences obtenues possèdent un certain degré de robustesse sur le plan de la distribution. Les méthodes de profil par quasi-vraisemblance servent à évaluer stochastiquement l'impact d'un quota de pêche commerciale sur la taille future d'un stock. Ces méthodes sont appliquées à un stock de morue sur la côte est du Canada.

1 Introduction

For most fish stocks it is impossible to get direct estimates of the size of the population. The data usually available only give estimates of the size of some component or subgroup of the stock. However, when a stock is commercially exploited by an extensive fishery whose landings are accurately monitored then it is often possible to reconstruct historical stock abundance for extinct cohorts using time series of commercial catches. A cohort model is used for the reconstruction, which essentially involves adding up annual catches from a cohort to give a minimum estimate of the population size, when first exploited by the fishery, that must have existed so that subsequent catches could be observed. If a relative index of population abundance is also available then this information can be used to estimate current stock size based on the relationship between historical stock size and the abundance index.

We develop semi-parametric estimators of current stock size in this paper. The estimators are semi-parametric in that they do not require completely specified stochastic models for the relative abundance indices used in estimation. These data are usually composites from complex surveys, and assumed distributions such as the lognormal are tenuous. As such it is desirable to have an inferential procedure that is robust to the exact stochastic nature of the abundance data. The methods in this paper require specification only of the mean and variance functions for the abundance indices. The variance function can easily be specified to accommodate Normal, Poisson, and Gamma/Lognormal types of variation. Estimation and inference is based on the quasi-likelihood method (see McCullagh and Nelder, 1989).

When the stock abundance when first exploited by the fishery is known then the complete exploited abundance can be constructed using a cohort model. This is called a sequential population analysis (SPA). The model considered here is based on the assumption that the commercial catches are removed at the middle of the year. Let N_{ay} denote the unknown beginning of year stocks numbers at age a in year y, an let C_{ay} denote the commercial catch from N_{ay} during the year. The cohort model is

$$N_{a+1\,y+1} = N_{ay}e^{-M_{ay}} - C_{ay}e^{-M_{ay}/2}, \tag{1}$$

where M_{ay} is the known natural mortality. In this model stock numbers decline for the first half of the year, with total mortality equal to $\exp(-M_{ay}/2)$. Commercial catches are removed at mid-year from the surviving population. The population surviving fishing then further declines due to natural mortality, with total mortality again equal to $\exp(-M_{ay}/2)$. Ages and years are standardized so that $a = 1, ..., A$, and $y = 1, ...Y$. Model (1) is commonly used in fisheries, and provides very reasonable results even if the fishery is prosecuted at times other than the mid-year (see Mertz and Myers, 1996).

Of course N_{1y}'s, the stock abundance first exploited by the fishery, are rarely known and usually must be estimated using additional population assumptions or auxiliary stock status information. Alternatively, the numbers at age for the final exploited stock stages can be estimated using (1) backwards. This is a common approach, and is followed here.

We assume that one or more indices of stock size are available for estimation purposes, and we also assume a relationship between the indices and the size of some stock component. Let R_{say} denote the value of the sth index for age a and year y. The basic relationship we consider is $R_{say} \approx q_{sa}N_{ay}$. The q_{sa} term is referred to as the catchability of the index and is typically unknown. We assume that the index represents only a portion of the stock, but that portion is the same in each year. Stock numbers at age and catchabilities are commonly estimated (e.g. Gavaris, 1988; Myers and Cadigan, 1995) using nonlinear least squares with (1) and a stochastic observation model

$$\log(R_{say}) = \log(q_{sa}) + (1-t)\log(N_{ay}) + t\log(N_{a+1\,y+1}) + \varepsilon_{say}, \tag{2}$$

where ε_{say} is a random error term with mean zero and constant, but unknown, variance. The variance term includes both measurement variability in the indices and process error in (1). Model (2) is based on an index that is observed at a fraction t since the beginning of the year.

When the ε_{say}'s are independent then standard asymptotic inferential procedures can be employed that require relatively minor additional assumptions about the distribution of the errors (see Seber and Wild, 1989), and so the above procedure has a degree of distributional robustness.

In this paper we present quasi-likelihood methods that retain the same degree of distributional robustness but allow more arbitrary models for the variance of the indices. An overview of quasi-likelihood methods is presented in **Section 2.1**, and specific procedures for estimating stock abundance are developed in **Section 2.2**. Inferential procedures for forecasts of stock size remaining after a commercial fishery are presented in **Section 3**. In **Section 4** the methods are applied to a cod data set. Finally, a discussion of the methods and results is presented in **Section 5**.

2 Quasi-likelihood estimators of stock abundance

In this section a brief review of quasi-likelihood estimation theory is first presented, followed by specific procedures for estimating stock abundance.

2.1 Overview of quasi-likelihood theory

Quasi-likelihood estimators are based only on assumptions about the mean and variance of a random variable. The mean and variance functions we assume for stock abundance indices are

$$\begin{aligned} E(R_{say}) &= \mu_{say} = q_{say} N_{ay}(t), \\ Var(R_{say}) &= \phi_s \mu_{say}^{\theta}, \end{aligned}$$

where $N_{ay}(t) = N_{ay}^{1-t} N_{a+1\,y+1}^{t}$, ϕ_s is the dispersion parameter, and θ is a fixed parameter used to model heterogeneity in the random variables, as a function of the mean. A wide variety of data can be efficiently analyzed using this model. For example, normally distributed indices can be analyzed by setting $\theta = 0$; poisson distributed indices can be analyzed with $\theta = 1$; gamma or lognormally distributed indices can be analyzed with $\theta = 2$. The latter specification for θ leads to a constant coefficient of variation model and is particularly useful for fisheries data. In practise θ is often chosen through trial and error with the aid of residual plots.

Quasi-likelihood estimators are defined in terms of an estimating equation from which a fit function, or quasi-likelihood, can often be developed. This quasi-likelihood is useful for inferences, and is also useful for deciding among estimates when multiple roots of the estimating equation exist. The quasi-likelihood has many of the same properties that a regular likelihood function has. Note that in the case of independent random variables a quasi-likelihood can always be constructed, which is the case here. For a single set of indices the estimating equation is

$$\sum_{a,y} \frac{r_{ay} - \mu_{ay}}{\phi \mu_{ay}^{\theta}} \dot{\mu}_{ay}, \tag{3}$$

where $\dot{\mu}_{ay}$ is the derivative of μ_{ay} with respect to the unknown parameters. The lower case r denotes an observed index. The unknown N_{ay}'s in (1) and the q's in (2) are estimated as the solution of setting (3) equal to zero. Equation (3) is optimal among all equations that are linear in r (see McCullagh and Nelder, 1989). The quasi-likelihood function associated with this model is

$$Q(r,\mu) = \int_r^\mu \frac{r-t}{\phi t^\theta} dt.$$

The deviance function

$$D(r,\mu) = -2\phi \left\{Q(r,\mu) - Q(r,r)\right\}$$

is a measure of the discrepancy between r and μ. The deviance for the power of the mean variance model is

$$D(r,\mu) = \begin{cases} 2[r\log(r/\mu) - (r-\mu)], & \text{if } \theta = 1, \\ 2[r/\mu - \log(r/\mu) - 1], & \text{if } \theta = 2, \\ 2\left[\frac{y^{2-\theta}-(2-\theta)y\mu^{1-\theta}+(1-\theta)\mu^{2-\theta}}{(1-\theta)(2-\theta)}\right], & \text{if } \theta \neq 1,2. \end{cases}$$

The total deviance for all ages and years is $D = \sum_{a,y} D(r_{ay}, \mu_{ay})$.

If the variance functions for indices differ, as will often be the case, then the deviances can no longer be added together. Variance functions may differ if, for example, we use different stock size indices, i.e. from different surveys, and we do not wish to assume that the dispersion parameters (ϕ_s's) are the same for each set of indices. For this case we use a df-adjusted extended quasi-likelihood function (see eq. 10.6 in McCullagh and Nelder, 1989):

$$Q^+(r_s,\mu) = -\frac{1}{2}\log\left\{2\pi\phi_s V_s(r_s)\right\} - \frac{\nu_s}{2} D_s(r_s,\mu)/\phi_s, \tag{4}$$

where $\nu_s = (n_s - n_p)/n_s$, n_s is the total number of indices in the sth set, and n_p is the number of unknown parameters to estimate. The combined extended quasi-likelihood for all sets of indices is $Q^+ = \sum_s \sum_{a,y} Q^+(r_{say}, \mu_{say})$.

2.2 Concentrated quasi-likelihood estimators of survivors

In this section a concentrated quasi-likelihood function for inferences about stock abundance is developed in which the only unknowns are the survivors, $N_{1Y}, ..., N_{AY}$. Recall that these are required for the backwards reconstruction of historical stock abundance using (1). This is a substantial reduction in the number of potential unknowns, which are $N_{A1}, ..., N_{AY-1}, N_{1Y}, ..., N_{AY}$, $\{q_{say}; a = 1, ..., A_s,\ y = 1, .., Y\}$, and ϕ_s for each set s of indices. The s subscript for A in q is used because the cohort model often extends beyond the last age in a set of indices, so that $A_s < A$. Many of the unknowns are removed using constraints.

2.2.1 Parameter constraints

Usually the q_{say}'s are constrained to be equal for all years. In the approach taken here the q's are merely assume to be equal for d_s disjoint sets of ages and years, denoted as $C_{s1}, ..., C_{sd_s}$.

The unknown numbers at age A in years $1, \ldots Y-1$ are also constrained as follows. Define the fishing mortality for N_{ay} as

$$F_{ay} = \log(\frac{N_{ay}}{N_{a+1\,y+1}}) - M_{ay}. \tag{5}$$

The N_{Ay}'s are constrained so that their fishing mortalities equal the average for some range of younger ages. In effect,

$$N_{Ay} = \frac{C_{Ay}e^{M_{Ay}/2}}{1-e^{-F_{\mathrm{ave}\;y}}}.$$

Note that $F_{\mathrm{ave}\;y}$ is a function of $N_{a\,y+1}$'s, so the result of the F constraint is that N_{Ay}'s for $y < Y$ are constrained to be functions of unknown survivors. Hence, the only unknowns in the cohort model are the N_{aY}'s.

2.2.2 Concentrated quasi-likelihood

The concentrated quasi-likelihood is obtained by finding closed form expressions for the catchabilities and the ϕ's in terms of the survivors and other data, and substituting these expressions for the respective parameters in the quasi-likelihood. This reduces the number of parameters to iteratively estimate to just the survivors. Also, inference about population numbers based on the concentrated quasi-likelihood automatically incorporates uncertainty resulting from the unknown q's and ϕ's.

The quasi-likelihood estimator of catchability is

$$\hat{q}_{sd} = \frac{\sum_{a,y\in C_{sd}} r_{ay}N_{ay}^{1-\theta}(t)}{\sum_{a,y\in C_{sd}} N_{ay}^{2-\theta}(t)},$$

under the assumption that θ is constant for all $a, y \in C_{sd}$. If this assumption is invalid then it does not seem possible to "concentrate-out" catchabilities from the quasi-likelihood. Recall that $\mu_{say} = q_{sd}N_{ay}(t)$. It is not difficult to show that $E(\hat{q}_{sd}) = q_{sd}$, and $Var(\hat{q}_{sd}) = \phi_s q_{sd}^{\theta} / \sum_{a,y\in C_{sd}} N_{ay}^{2-\theta}(t)$. An estimator of ϕ_s is

$$\hat{\phi}_s = \frac{\sum_{a,y} D_{say}(r_{say}, \mu_{say})}{\nu_s n_s}.$$

Substituting the $\hat{q}_{sd}$'s and $\hat{\phi}_s$'s for the q_{sd}'s and ϕ_s's in (4) gives the concentrated df-adjusted extended quasi-likelihood function (Q_c^+), which apart from a constant term is

$$\Lambda = -2Q_c^+ = \sum_s (n_s - n_p) \log\left\{\sum_{a,y} D_{say}(r_{say}, \mu_{say}^c)\right\}, \tag{6}$$

where $\mu_{say}^c = \hat{q}_{say}N_{ay}(t)$. The only unknowns in (6) are the survivors because $\hat{q}_{say} = \hat{q}_{say}(N_{1Y}, \ldots, N_{AY})$. Λ is the fit function used for stock size inferences. The elements of the gradient of Λ are given by

$$\frac{\partial \Lambda}{\partial N_{iY}} = \sum_s \frac{(n_s - n_p)\sum_{a,y}(r_{say} - \mu_{say}^c)\mu_{say}^{c-\theta_{syy}}(\partial \mu_{say}^c / \partial N_{iY})}{\sum_{a,y} D_{say}(r_{say}, \mu_{say}^c)},\ i = 1, \ldots, A$$

and are weighted sums of (3) for each set of indices. The weights are just $1/\hat{\phi}_s$. The survivor estimators are defined as the solution of

$$\left.\frac{\partial \Lambda}{\partial N_{aY}}\right|_{\hat{N}_{ay}} = 0.$$

They are analogous to non-nested mixed-effects linear regression estimators.

2.2.3 Inferences

In this section we present methods for estimating standard errors and confidence intervals for estimators obtained using (6). The asymptotic distribution of $\hat{\mathbf{N}}_Y$ using linear approximation theory is

$$\hat{\mathbf{N}}_Y \sim \text{Normal}(\mathbf{N}_Y, \mathbf{\Sigma}_{N_Y}),$$

where

$$\mathbf{\Sigma}_{N_Y}^{-1} = \sum_s \sum_{a,y} \frac{\partial \mu_{say}}{\partial \mathbf{N}_Y} \frac{\partial \mu_{say}}{\partial \mathbf{N}_Y'} / (\mu_{say}^{\theta_{say}} \phi_s).$$

This covariance matrix can be consistently estimated by $\hat{\mathbf{\Sigma}} = \mathbf{\Sigma}_{\hat{\mathbf{N}}_Y}$. The basic condition for this result is $\lambda_{\min}(\mathbf{\Sigma}_{N_Y}) \overset{a,y}{\to} 0$, where $\lambda_{\min}(\cdot)$ is the smallest eigenvalue of $(\cdot)$. The number of catchabilities estimated will increase $\lambda_{\min}$, and reduce the accuracy of the distributional approximation.

Inferences about survivors can also be based on inverting the asymptotic distribution of (6). This procedure is particularly useful when only a subset of survivors, or a function of survivors, are of interest. We use this procedure in **Section 3** for inferences about stock projections. This approach is similar to profile likelihood inference (see Ch. 9 in Cox and Hinkley, 1979; for a more detailed discussion see Cox and Barndorff-Nielsen, 1994). For a fixed N_{aY},

$$\Lambda(N_{aY}) - \hat{\Lambda} \overset{asy}{\sim} \chi_1^2,$$

where $\Lambda(N_{aY})$ is the value of (6) obtained with the other survivors equal to their quasi-likelihood estimators obtained for the fixed N_{aY}. A $(1-\alpha)100\%$ confidence interval for N_{aY} may be computed as the two values of N_{aY} that solve $\Lambda(N_{aY}) - \hat{\Lambda} = \chi_{1,1-\alpha}^2$, where $\chi_{1,1-\alpha}^2$ is the $1-\alpha$ percentile of a χ_1^2 distribution. Such an interval is usually called a profile quasi-likelihood interval (PQL). PQL intervals are invariant to nonsingular transformations of parameters. Examples of these procedures are presented in Nelder and Pregibon (1987). These authors show that a PQL interval is similar to a bootstrap result for one example they considered.

3 Forecasting stock size remaining after a fishery

In this section we describe a procedure for determining the probability that an estimator of next year's stock size surpasses a reference point, given a total allowable catch (TAC) by the commercial fishery. We first describe a procedure for projecting stock size based on a TAC and a known population, and then we describe methods for stochastically projecting stock size based on an estimated population.

3.1 Forecasting stock size for a fixed TAC option

The basic procedure used when evaluating the impact of a TAC on future stock size is to forecast stock abundance at age following the removal of a TAC by the commercial fishery. We assume that only data up to year Y is available for forecasting, and we consider only a TAC option for the year $Y+1$; that is, next year. In previous sections we have shown how N_{aY}'s may be estimated. Using (1) and known commercial catches in year Y we can estimate stock abundance at the beginning of $Y+1$ for all ages, except $a=1$. A common procedure is to estimate $N_{1\,Y+1}$ using the geometric mean of $N_{1\,Y-2}$, $N_{1\,Y-1}$, and N_{1Y}, and we use this procedure here. The TAC is removed from the $N_{a\,Y+1}$'s. This involves partitioning the TAC into catch numbers at age (C_{ay}) and then using (1) again.

The partitioning of the TAC is based on an assumed pattern of fishing mortalities relevant for the commercial fleet sector likely to operate in $Y+1$. Once we know the $F_{a\,Y+1}$'s we can estimate stock size remaining after the fishery in $Y+1$, i.e. $N_{a\,Y+2}$'s, using (5) for $a>1$. We again use the geometric mean of $N_{1\,Y-1}$, $N_{1\,Y-2}$, and N_{1Y+1} to forecast $N_{1\,Y+2}$. The first step in fixing the $F_{a\,Y+1}$'s is to assume that

$$F_{aY+1} = F\mathcal{P}_a, \quad 0 \leq \mathcal{P}_a \leq 1,$$

where $\mathcal{P}_a$ is the known partial recruitment of age a fish to the fishery, and F is the unknown fully recruited fishing mortality. The $\mathcal{P}_a$'s are usually based on historic information about the relevant commercial fleet sector. The second step in fixing the $F_{a\,Y+1}$'s is to express F in terms of the TAC. This is obtained using the forecasted commercial catches in $Y+1$:

$$C_{a\,Y+1} = N_{a\,Y+1}e^{-M_{a\,Y-1}/2}(1-e^{-F_{a\,Y+1}}).$$

This equation is obtained by solving for C_{ay} in (1), and using (5). The TAC is usually expressed in weight, so the $C_{a\,Y+1}$'s are multiplied by average commercial catch weights at age, w_a^t's, and summed to give the forecasted TAC given F; that is,

$$TAC = \sum_a w_{a\,Y+1}^t C_{a\,Y+1} = \sum_a w_{a\,Y+1}^t N_{a\,Y+1}(1-e^{-F\mathcal{P}_a})e^{-M_{a\,Y+1}/2}.$$

This equation can be used to solve for F. The projection is straightforward once a value for F is available.

3.2 Stochastic description of future stock size relative to historic reference points.

We use profile quasi-likelihoods to describe the distribution of the forecasted stock size relative to a reference point. More specifically, we compute $\Pr(B_{m\,Y+2} \leq \delta B_{m\,Y+1})$ where $B_{m\,Y}$ is the beginning of year spawner (mature) biomass in the year Y, and $\delta = 1, 1.1$, and 1.2. We also considered $\Pr(F \geq F_{0.1})$, where $F_{0.1}$ is a reference level of fishing mortality.

These probabilities can be formulated within a general framework. Define

$$\mathcal{R} = \frac{\sum_{y=y_o}^{Y+2} \sum_a w_{ay}^n N_{ay}}{K + \sum_{y=y_o}^{Y+2} \sum_a w_{ay}^d N_{ay}}.$$

The above probabilities can be specified as $\Pr(\mathcal{R} \leq 1)$ using appropriate definitions of w_{ay}^n, w_{ay}^d, and K. For example, if $K = 0$ and

$$\begin{aligned} w_{ay}^n &= \begin{cases} p_{ay} w_{ay}^b, & \text{for } y = Y+2, \\ 0, & \text{otherwise,} \end{cases} \\ \text{and } w_{ay}^d &= \begin{cases} p_{ay} w_{ay}^b / \delta, & \text{for } y = Y+1, \\ 0, & \text{otherwise,} \end{cases} \end{aligned}$$

where w_{ay}^b are the beginning of year weights at age and p_{ay} are the proportions mature at age, then $\Pr(\mathcal{R} \leq 1)$ is equivalent to $\Pr(B_{m\,Y+2} \leq \delta B_{m\,Y+1})$. The probability of exceeding $F_{0.1}$ can also be formulated within this framework by noting that $F \geq F_{0.1} \Rightarrow N_{a+1\,Y+2} \leq N_{a\,Y+1} e^{-F_{0.1}\mathcal{P}_a - M_{aY+1}}$, so for any age a' set

$$\begin{aligned} w_{ay}^n &= \begin{cases} 1, & \text{for } y = Y+2 \text{ and } a = a' + 1, \\ 0, & \text{otherwise,} \end{cases} \\ \text{and } w_{ay}^d &= \begin{cases} e^{-F_{0.1}\mathcal{P}_a - M_{aY}}, & \text{for } y = Y+1 \text{ and } a = a', \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

The constant K is included in the generalization to accommodate fixed reference points; however, these are not considered in this paper. The generalized framework also facilitates the use of historic reference points; however, there is considerable debate about what these points should be, and they are also not considered here.

3.3 Profile quasi-likelihood probabilities

In this section we describe the procedure we use for computing $\Pr(\mathcal{R} \leq 1)$. Let $\Lambda(r)$ denote the value of (6) evaluated when $\mathcal{R} = r$ is fixed, and let $\hat{\Lambda}$ denote the unrestricted minimum value for Λ. Usually $r = 1$, but it is easier to understand the procedure for an arbitrary r. The asymptotic distribution of the quasi-loglikelihood difference

$$\Delta_r = \Lambda(r) - \hat{\Lambda}$$

is chi-square with one degree of freedom, because the survivors have one nonlinear parameter constraint given by $\mathcal{R} = r$. Let $\mathcal{R}_{QL}$ denote the unrestricted quasi-likelihood estimate of $\mathcal{R}$. If $r \leq \mathcal{R}_{QL}$ and $\alpha = \Pr\left[\chi_1^2 > \Delta_r\right]$ then r is the lower bound on a $1 - \alpha$ confidence interval for

$\mathcal{R}$. There is another value r_u such that $r_u \geq \mathcal{R}_{QL}$ and $\Pr[\chi^2_1 > \Delta_{r_u}] = \alpha$. As such we cannot not directly infer $\Pr(\mathcal{R}_{QL} \leq r)$ unless we know the two tail area probabilities of the confidence interval. We assume they are equal, which for $r = 1$ gives

$$\Pr(\mathcal{R}_{QL} \leq 1) = \alpha/2, \text{ if } \mathcal{R}_{QL} \geq 1.$$

Similarly,

$$\Pr(\mathcal{R}_{QL} \leq 1) = 1 - \alpha/2, \text{ if } \mathcal{R}_{QL} \leq 1.$$

The difficult part of computing probabilities in this manner is finding $\Lambda(1)$. This involves a constrained optimization. The algorithm we employ involves finding the roots of the Lagrangian equation associated with minimizing Λ subject to the constraint $\mathcal{R} = 1$ (see eq. 3.6.7 in Bard, 1974). For an SPA this only seems feasible if an analytic gradient for Λ is available, and computing this gradient is not a simple task. Convergence when $|\mathcal{R}_{QL} - 1|$ is large is also a problem, although in this case the probabilities may be set to zero or one with sufficient accuracy.

4 Results: St. Pierre Bank Cod

An example is presented that involves cod off the south coast of Newfoundland in NAFO division 3PS. This stock has historically provided a substantial and important fishery. By the early 1990's this stock declined dramatically in size, and a moratorium on commercial fishing was established in 1993. In 1997 a 10000 tonne quota was permitted. The results from SPA's play an important role in establishing stock quota's; therefore, a reliable framework for inferences about stock size is necessary.

The core data available for the assessment of this stock are

1. Canadian research survey during the winter and spring from 1983-1997 (see Table 1 in the **Appendix**),
2. French research survey during the winter from 1980-1991, and
3. landings from the commercial fishery from 1959-1997.

The research surveys involve stratified random sampling with a survey trawl. Strata-size weighted averages of the number of cod caught in a standardized tow of a trawl are used as indices of stock abundance. Details about the survey design are available in Doubleday (1981). The construction of stock size indices is an important topic but beyond the scope of this paper. The commercial catch data consists of total numbers at age caught by most fishers operating in 3PS. The catch has been subdivided into offshore and inshore components since 1977, and the offshore data is presented in Table 2 in the **Appendix**.

The SPA we investigate involves only offshore catches because that is the region covered by the research survey, and there is a concern that trends in the survey indices do not reflect trends in the inshore components of 3PS cod (see Shelton *et. al.,* 1996). The cohort model includes ages 3 to 14. The F_{Ay}'s are set equal to the average F's for ages 7-10. Catchabilities are estimated separately for each age and survey. Also, because of evidence that seasonal timing affects catchability, we estimate different catchabilities for Canadian surveys that occur in the winter and those that occur in the spring of the year. Surveys were conducted in the winter (February and March) in 1985-92.

Estimates of survivors are presented in Table 3 in the **Appendix**, along with coefficients of variation (CV's) and linear approximation confidence intervals. Note that $\log_e$ survivors were actually estimated, and the confidence intervals in Table 3 are exponentiated log survivor intervals. The rationale for this procedure is that the local linear approximation is more valid for $\log_e(N_{aY})$ than N_{aY}. The estimates of ϕ are 0.971 and 0.570 for the Canadian and French surveys respectively. Estimates of catchabilities and their standard errors are presented in Table 4. The estimates generally increase with age. The catchabilities for winter surveys appear higher than for spring surveys for ages ≤ 8, but lower otherwise.

We can test the statistical significance of differences in catchabilities between winter and spring using the procedures outlined in **Section 2.2.3**. The PQL statistic for this hypothesis is 60.07, with 10 degrees of freedom. The degrees of freedom in this case equal the number of ages in the Canadian survey; that is, the number of catchability constraints in the null hypothesis. The p-value for this test is < 0.0001, based on a chi-square distribution; hence, modelling winter and spring catchabilities separately significantly improves the model fit.

Estimates of population numbers, biomass, and fishing mortalities are presented in Tables 5-7. These abundances and biomass are also plotted in Figures 1-2 for several age groups. The results suggest a decline in stock abundance and biomass since the mid 1980's, followed by an increase in abundance since the 1991, and in total biomass since 1993. Estimated fishing mortalities (see Table 7) increased relatively sharply in 1990-1992, but have declined rapidly since the fishing moratorium. The increase in estimated 3+ abundance is not caused by an increase in recruitment; that is, the estimated number of age 3 fish has not increased substantially in the 1990's. The increase seems more related to the survival of strong year classes first seen in 1992 and 1993; that is, 1989 and 1990 year classes. The rather dramatic increases in biomass reflect the large portion of seven and eight year old fish in the stock. This proportion, 0.31, is the by far highest since 1980. These trends will not continue in future years unless recruitment increases.

Predicted and observed indices are plotted in Figures 3a-b. Standardized Pearson residuals, $(r_{say} - \hat{\mu}_{say})/SE(\hat{\mu}_{say})$, and unstandardized residuals are presented in Tables 8-9. Residuals are plotted in Figures 4a-b. These plots are useful in assessing the adequacy of the assumed model. Year effects are apparent in the survey indices; however, this is not pursued further for reasons given in the **Discussion**. The residuals appear homogeneous in terms of age. There is some evidence in the lower panel of Figure 4a that the constant CV assumption may not be correct. This will be investigated in future research.

The residuals in Table 9 are often quite large, and indicate substantial discrepancies between the observed and predicted stock indices. This is illustrated in Figures 5a-b, in which the observed indices are plotted versus their predicted values. While the correlations between r_{ay} and $\hat{R}_{ay}$ can be large, the scatter of points quite often departs substantially from a one-to-one relationship. Note that the large residuals are not apparent in Table 8 because of the large variance associated with the unstandardized residuals.

Estimated stock size trajections computed with Y, the last year in the cohort model, equal to $1990, ..., 1997$ are presented in Figures 6a-c. These are commonly referred to as retrospective plots. A retrospective pattern is apparent for many ages and years; that is, for a fixed year y', $\hat{N}_{ay'}$ decreases as Y increases. For example, based on data up to 1995, the estimated spawner biomass in 1995 was over 100000 tonnes (see Figure 6c). Based on 1997 data we estimate the 1995 biomass to be just less than 52000 tonnes. This represents a 50% change in our 1995 population inference. It seems likely that our 1997 and 1998 estimates will be revised downwards in future years. The retrospective problem is endemic within fisheries, and is clearly present in this application.

Stochastic descriptions of 1999 stock size arising from 7 TAC options are presented in Figure 7. Most notable in this figure are:

1. The estimated increase in spawner biomass in 1999 is, if there is no fishing, about 12500 tonnes.

2. The probability that, with no fishing, the spawner biomass will not increase is about 0.05.

3. There is a 90% chance that the spawner biomass will decrease with a 30000 tonne quota in the offshore of 3PS.

4. The probability that the stock will not increase very much between 1998 and 1999, even with no fishing, is large; that is, there is a 50% chance the stock will not grow by more than 10%, and a 75% chance the stock will not grow by more than 20%.

5. There is about a 10% chance that fishing mortality will exceed $F_{0.1} = 0.24$ with a 12500 tonne quota. This probability increases to 0.8 for a 30000 tonne quota.

5 Discussion and Conclusions

In this paper a semi-parametric and practical methodology has been developed for stock size inferences based on an SPA and estimation with relative abundance indices. The methods are applicable to a variety of types of indices and distributional assumptions. The methods are semi-parametric in that the exact distribution for abundance indices is not required.

If the distribution is within the exponential family then the estimators proposed here are maximum likelihood estimators (MLE's), and will have high efficiency. Otherwise, the loss of efficiency is usually not great. Firth (1987) studied the asymptotic relative efficiency (ARE), or the ratio of asymptotic variances, of quasi-likelihood estimators compared with MLE's. For the lognormal and inverse gaussian distributions (constant CV models) he found that $ARE \approx 0.7$

when $\phi = 1$ and $ARE \approx 0.9$ when $\phi = 0.2$. Note that the quasi-deviance for a constant CV model is identical to the deviance for the gamma distribution. Firth (1988) found that the $ARE_{LN}(G) > ARE_G(LN)$, where $ARE_{LN}(G)$ is the ARE of gamma MLE's when the population distribution is lognormal, and likewise for $ARE_G(LN)$; however, the differences were small when ϕ was small.

The 3PS application is reasonably valid apart from the large variability in the estimates. A likely model misspecification concerns the independence of survey indices within years, which the methods developed in this paper can not address. Myers and Cadigan (1995) consider an SPA that accounted for random years effects. The type of model was not used here for three reasons.

1. Foremost, the year effects in Figures 4a may not be completely random, and little is known about the consequences of using a misspecified random effects model.

2. Little is known about the ability of such a model to detect trends in stock abundance.

3. The extension of quasi-likelihood theory to correlated observations is not trivial because a unique quasi-likelihood function no longer necessarily exists. This complicates the basis for inference.

Further research into these issues is required.

The assumption about the variance structure may have a significant effect on current stock size inferences (unreported results). It would be desirable to have methods that reliably discriminate the correct variance structure, which may be more complicated than the power model considered in this paper. This will likely involve developing inferential procedures for the variance parameter(s) rather than simply assuming a value. This will be considered in future research.

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Appendix

Table 1. 3PS survey stratified mean numbers per tow from the Canadian (CAN) and French (Frn) reserach surveys.

		Age									
	Year	3	4	5	6	7	8	9	10	11	12
	1983	6.52	1.14	3.72	1.62	0.48	0.89	1.61	0.75	0.36	0.14
	1984	2.33	1.55	0.63	2.11	0.77	0.37	0.46	0.71	0.18	0.15
	1985	14.88	12.57	9.96	3.28	2.66	0.79	0.48	0.42	0.42	0.49
	1986	5.65	6.48	7.95	6.33	2.13	1.47	0.84	0.29	0.24	0.29
	1987	5.67	4.97	13.82	8.31	3.35	1.29	0.69	0.28	0.23	0.16
	1988	5.93	2.96	2.84	6.50	5.84	3.65	1.49	0.84	0.74	0.35
Can	1989	4.66	3.17	1.51	1.16	2.15	1.21	0.67	0.37	0.41	0.13
	1990	9.82	14.49	10.89	5.67	3.84	3.14	1.15	0.71	0.32	0.16
	1991	5.03	10.00	11.24	5.75	2.84	1.58	1.19	0.74	0.56	0.22
	1992	6.95	2.11	4.15	2.03	1.03	0.53	0.26	0.24	0.08	0.04
	1993	1.99	4.04	1.49	1.35	0.47	0.10	0.04	0.03	0.04	0.01
	1994	1.46	4.31	6.10	1.73	1.62	0.50	0.08	0.04	0.03	0.02
	1995	1.19	1.54	12.04	18.08	4.05	5.29	2.01	0.23	0.18	0.01
	1996	3.52	3.74	1.26	2.56	2.77	0.51	0.44	0.09	0.09	0.02
	1997	2.33	1.04	0.50	0.28	0.30	0.24	0.14	0.05	0.02	0.00
	Year	3	4	5	6	7	8	9	10	11	12
	1980	1.72	0.50	2.67	4.52	1.66	0.67	0.29	0.22	0.18	0.11
	1981	4.91	4.94	5.14	7.45	5.64	1.60	1.19	0.47	0.15	0.14
	1982	1.96	8.32	7.97	6.06	4.55	5.30	1.58	0.87	0.42	0.15
	1983	5.40	2.98	7.21	6.11	4.55	2.77	2.08	0.75	0.25	0.19
	1984	7.64	15.07	8.74	18.97	5.59	2.13	3.09	2.21	0.61	0.16
Frn	1985	14.49	7.47	3.93	1.06	1.95	1.14	0.78	0.86	1.09	1.32
	1986	4.21	15.19	26.47	21.66	9.12	6.97	3.85	0.79	0.59	0.72
	1987	11.51	2.83	8.30	12.49	8.32	2.95	1.94	0.95	0.20	0.36
	1988	14.89	9.22	3.62	6.53	4.69	1.60	0.78	0.35	0.35	0.16
	1989	16.02	8.20	5.81	3.48	4.43	2.03	1.01	0.27	0.13	0.06
	1990	18.26	20.11	7.66	2.46	0.73	1.00	0.44	0.26	0.11	0.09
	1991	7.08	12.96	12.68	7.56	2.42	1.07	0.91	0.62	0.06	0.15

Table 2. 3PS offshore commercial catches (1000's) for ages 3-14 during 1980-1997.

	Age												
year	3	4	5	6	7	8	9	10	11	12	13	14	3+
1980	137	280	1038	1933	1018	259	51	9	2	3	3	1	4734
1981	159	505	765	1429	1841	500	110	31	12	11	4	1	5368
1982	46	1229	1448	1047	1057	918	201	70	28	10	4	4	6062
1983	127	620	3008	1606	706	454	323	61	25	5	2	2	6939
1984	92	1982	1776	2211	1066	258	275	123	45	15	3	1	7847
1985	97	2623	5537	2787	2257	656	249	215	176	66	8	7	14678
1986	32	1843	6706	7042	2791	1095	375	111	89	70	38	8	20200
1987	101	766	5397	6145	3248	991	649	206	98	27	55	7	17690
1988	696	2399	2904	3273	3177	1089	427	136	51	48	25	11	14236
1989	567	3233	2577	1083	827	534	284	151	61	13	14	7	9351
1990	555	3347	3639	1388	611	855	467	220	85	60	24	13	11264
1991	436	2210	4557	3050	921	458	410	323	56	59	29	15	12524
1992	136	1000	1616	2149	900	260	133	135	113	53	23	26	6544
1993	4	305	457	752	313	128	32	20	30	9	5	2	2057
1994	1	9	19	8	6	3	1	0	0	0	0	1	48
1995	0	1	6	15	6	5	2	1	0	0	0	1	37
1996	1	4	7	16	27	7	7	4	1	0	0	1	75
1997	0	27	52	30	126	130	17	1	6	0	0	0	389

Table 3. 1997 estimates of 3PS offshore population abundance (N_{ay}'s in 1000's), with coefficients of variation (CV's), and 95% confidence intervals (95% L, 95% U).

Age	3	4	5	6	7	8	9	10	11	12	13	14
N_{ay}	18529	14803	9259	4418	11939	11339	2442	2720	197	15	20	5
CV	0.92	0.66	0.54	0.50	0.39	0.33	0.34	0.31	0.43	0.55	0.75	1.09
95% L	3065	4087	3219	1665	5545	5882	1261	1469	85	5	5	1
95% U	112000	53621	26634	11722	25707	21860	4726	5038	452	43	87	43

Table 4. Estimated 3PS catchabilities (q's), with standard errors (SE's). Values are in 1000's.

	Canadian and French surveys.										
	Age	3	4	5	6	7	8	9	10	11	12
Canada	q	0.133	0.167	0.277	0.369	0.429	0.701	1.854	2.394	2.912	1.366
Spring	SE	0.050	0.062	0.103	0.137	0.160	0.261	0.690	0.892	1.085	0.509
Canada	q	0.416	0.469	0.677	0.838	1.141	1.186	1.045	1.011	1.697	1.971
Winter	SE	0.145	0.163	0.236	0.292	0.397	0.413	0.364	0.352	0.591	0.687
	Age	3	4	5	6	7	8	9	10	11	12
France	q	0.494	0.563	0.782	1.222	1.323	1.463	2.016	2.392	2.657	4.233
	SE	0.108	0.123	0.170	0.266	0.288	0.319	0.439	0.521	0.579	0.922

Table 5. Estimated 3PS offshore population abundance (1000's) for ages 3-14 during 1980-1997.

	Age												
year	3	4	5	6	7	8	9	10	11	12	13	14	3+
1980	14164	7342	7798	8250	2763	731	214	56	38	26	7	3	41390
1981	24417	11472	5757	5445	5006	1341	364	129	37	29	19	3	54019
1982	22594	19847	8936	4022	3165	2432	645	198	77	20	14	12	61962
1983	40970	18457	15137	6006	2345	1635	1161	346	99	38	7	8	86209
1984	41033	33428	14550	9672	3464	1281	928	658	228	58	27	4	105332
1985	26541	33512	25575	10306	5918	1872	816	511	428	146	34	19	105677
1986	11541	21642	25064	15929	5916	2803	939	442	224	191	60	21	84771
1987	13706	9420	16052	14453	6670	2318	1304	429	262	102	93	15	64824
1988	19975	11130	7019	8259	6273	2522	1001	480	165	126	59	26	57036
1989	18763	15724	6942	3119	3800	2261	1079	433	270	89	59	26	52567
1990	18862	14849	9949	3352	1574	2363	1368	627	218	166	61	36	53424
1991	10743	14941	9129	4853	1488	736	1161	697	314	102	82	28	44274
1992	31513	8402	10233	3351	1213	385	188	580	279	206	30	41	56420
1993	26630	25678	5974	6916	799	179	80	33	352	126	121	4	66892
1994	8064	21800	20747	4477	4982	371	31	37	9	261	95	95	60968
1995	13818	6601	17840	16969	3659	4073	301	24	30	8	214	78	63615
1996	18082	11313	5404	14601	13880	2990	3330	244	19	25	6	175	70069
1997	18529	14803	9259	4418	11939	11339	2442	2720	197	15	20	5	75686
1998	16667	15170	12095	7534	3590	9661	9166	1984	2226	155	12	16	78277

Table 6. Estimated 3PS offshore biomass (tonnes) for ages 3-14 during 1980-1997.

	Age												
year	3	4	5	6	7	8	9	10	11	12	13	14	3+
1980	5977	3987	6682	10684	5589	2214	953	304	259	202	62	33	36946
1981	9254	7354	5614	7764	9781	3818	1441	714	268	236	158	29	46431
1982	7433	12067	8587	6165	6523	6261	2307	952	459	158	122	114	51148
1983	17740	11351	15319	9165	5026	4535	3825	1538	583	275	66	78	69500
1984	23881	25974	15772	15659	7939	3996	3650	3013	1257	450	259	41	101893
1985	15314	25101	28926	16314	13925	5641	3548	2728	2492	961	323	207	115479
1986	5216	14868	25089	23958	12340	8339	3610	2325	1363	1393	456	225	99184
1987	6346	6076	15297	20046	13753	6279	4816	2012	1529	674	730	121	77680
1988	11106	7546	6429	11744	11799	6550	3292	2231	884	804	429	209	63023
1989	10113	11227	6769	4158	7364	6114	3739	1866	1513	569	425	210	54068
1990	9620	10929	10088	4911	3144	6139	5159	2867	1251	1149	476	323	56054
1991	5995	9861	9156	7216	3117	1964	3862	2947	1784	710	662	254	47528
1992	11881	5419	9025	4527	2388	1009	652	2621	1453	1454	267	412	41106
1993	6232	14354	5167	8569	1455	449	284	141	1795	874	886	33	40239
1994	4233	11728	19523	6335	8688	896	98	160	48	1576	677	704	54668
1995	5223	4779	20195	27592	7840	9735	927	95	130	39	1410	613	78578
1996	10560	8100	6068	26179	31424	8058	9985	913	86	110	34	1304	102821
1997	9468	11517	10491	7364	27067	32442	7801	9181	845	81	128	45	116429
1998	8517	11803	13704	12559	8138	27641	29286	6695	9573	861	76	145	128997

Table 7. Estimated 3PS offshore fishing mortalities for ages 3-14 during 1980-1997.

	Age											
year	3	4	5	6	7	8	9	10	11	12	13	14
1980	0.011	0.043	0.159	0.300	0.523	0.497	0.306	0.197	0.060	0.136	0.630	0.381
1981	0.007	0.050	0.159	0.343	0.522	0.531	0.407	0.309	0.439	0.543	0.271	0.442
1982	0.002	0.071	0.197	0.339	0.461	0.540	0.422	0.494	0.510	0.823	0.386	0.479
1983	0.003	0.038	0.248	0.350	0.405	0.367	0.367	0.216	0.327	0.157	0.374	0.339
1984	0.002	0.068	0.145	0.291	0.416	0.252	0.397	0.231	0.246	0.334	0.133	0.324
1985	0.004	0.090	0.273	0.355	0.547	0.490	0.412	0.626	0.607	0.690	0.298	0.519
1986	0.003	0.099	0.351	0.671	0.737	0.565	0.583	0.325	0.580	0.520	1.202	0.552
1987	0.008	0.094	0.465	0.635	0.773	0.640	0.799	0.756	0.534	0.344	1.062	0.742
1988	0.039	0.272	0.611	0.576	0.820	0.649	0.638	0.375	0.418	0.548	0.625	0.620
1989	0.034	0.258	0.528	0.484	0.275	0.302	0.344	0.486	0.287	0.176	0.301	0.352
1990	0.033	0.286	0.518	0.612	0.560	0.511	0.474	0.491	0.563	0.509	0.570	0.509
1991	0.046	0.178	0.802	1.186	1.152	1.165	0.495	0.717	0.219	1.026	0.498	0.882
1992	0.005	0.141	0.192	1.234	1.714	1.370	1.525	0.298	0.594	0.334	1.912	1.227
1993	0.000	0.013	0.088	0.128	0.568	1.563	0.582	1.080	0.099	0.082	0.047	0.948
1994	0.000	0.000	0.001	0.002	0.001	0.009	0.037	0.000	0.000	0.000	0.000	0.012
1995	0.000	0.000	0.000	0.001	0.002	0.001	0.007	0.047	0.000	0.000	0.000	0.014
1996	0.000	0.000	0.001	0.001	0.002	0.003	0.002	0.018	0.060	0.000	0.000	0.006
1997	0.000	0.002	0.006	0.008	0.012	0.013	0.008	0.000	0.034	0.000	0.000	0.000

Table 8. 3PS standardized residuals. Root mean square residual follows the country code.

		Age									
	Year	3	4	5	6	7	8	9	10	11	12
	1983	0.41	-0.71	-0.00	-0.21	-0.37	0.08	0.15	0.46	0.76	1.68
	1984	-0.71	-0.91	-1.07	-0.45	-0.35	-0.48	-0.67	-0.31	-0.61	1.06
	1985	0.61	-0.23	-0.54	-0.77	-0.67	-0.66	-0.35	-0.04	-0.30	0.98
	1986	0.32	-0.46	-0.71	-0.59	-0.77	-0.55	0.04	-0.27	-0.22	-0.08
	1987	0.06	0.24	0.66	-0.24	-0.58	-0.50	-0.41	-0.22	-0.38	-0.10
	1988	-0.37	-0.54	-0.45	0.08	-0.02	0.50	0.64	0.81	1.73	0.50
Can	1989	-0.53	-0.76	-0.84	-0.60	-0.55	-0.60	-0.38	-0.09	-0.03	-0.18
0.87	1990	0.42	1.64	1.05	1.58	1.75	0.34	-0.12	0.21	-0.03	-0.43
	1991	0.22	0.67	1.42	0.90	1.25	1.27	0.10	0.16	0.13	0.24
	1992	-0.67	-0.56	-0.50	-0.12	0.03	0.47	0.60	-0.57	-0.79	-0.86
	1993	-0.50	0.06	-0.06	-0.56	0.80	0.37	-0.47	-0.18	-0.99	-0.80
	1994	0.51	0.40	0.12	0.05	-0.13	1.20	0.64	-0.25	0.31	-0.88
	1995	-0.35	0.60	1.99	2.55	2.28	1.50	3.68	3.59	1.33	0.06
	1996	0.71	1.40	-0.15	-0.70	-0.60	-0.82	-1.19	-0.74	0.88	-0.22
	1997	0.03	-0.64	-0.97	-0.96	-1.21	-1.32	-1.21	-1.29	-0.93	-0.65
	Year	3	4	5	6	7	8	9	10	11	12
	1980	-1.13	-1.26	-0.79	-0.81	-0.67	-0.35	-0.26	0.86	0.97	0.11
	1981	-0.93	-0.29	0.36	0.40	-0.01	-0.08	1.01	0.84	0.79	0.36
	1982	-1.33	-0.34	0.40	0.58	0.37	1.03	0.51	1.38	1.56	1.16
	1983	-1.26	-1.14	-0.54	-0.12	0.89	0.36	-0.00	0.05	0.14	0.30
	1984	-1.05	-0.28	-0.28	1.25	0.53	0.29	1.13	0.79	0.22	-0.26
Frn	1985	0.30	-1.02	-1.38	-1.50	-1.11	-0.73	-0.59	-0.17	0.24	1.77
0.83	1986	-0.32	0.52	0.87	0.50	0.57	1.31	1.76	-0.15	0.27	0.06
	1987	1.24	-0.66	-0.43	-0.34	0.15	-0.03	-0.18	0.17	-0.74	-0.06
	1988	0.99	0.89	-0.36	-0.43	-0.52	-0.72	-0.72	-0.78	-0.04	-0.71
	1989	1.37	0.01	0.34	0.05	-0.05	-0.47	-0.64	-0.82	-0.90	-0.88
	1990	1.79	2.61	0.24	-0.41	-0.77	-0.93	-1.10	-0.98	-0.84	-0.96
	1991	0.65	1.02	1.73	0.92	0.72	0.29	-0.74	-0.68	-1.07	-0.59

Table 9. 3PS unstandardized residuals. Root mean square residual follows the country code.

		Age									
	Year	3	4	5	6	7	8	9	10	11	12
	1983	1.53	-1.58	-0.00	-0.35	-0.26	0.06	0.18	0.24	0.17	0.10
	1984	-2.71	-3.38	-3.13	-1.19	-0.35	-0.32	-0.70	-0.28	-0.29	0.09
	1985	4.42	-2.22	-5.51	-4.10	-2.65	-1.03	-0.25	-0.02	-0.18	0.25
	1986	1.10	-3.05	-6.97	-4.35	-2.97	-1.21	0.03	-0.11	-0.07	-0.03
	1987	0.24	0.79	4.33	-1.66	-2.50	-0.94	-0.41	-0.09	-0.15	-0.02
	1988	-2.09	-2.02	-1.48	0.36	-0.09	1.05	0.56	0.38	0.48	0.12
Can	1989	-2.86	-3.84	-2.77	-1.17	-1.63	-1.19	-0.36	-0.04	-0.01	-0.04
2.35	1990	2.25	7.89	4.74	3.20	2.32	0.69	-0.14	0.12	-0.01	-0.14
	1991	0.73	3.28	5.77	2.39	1.49	0.87	0.10	0.10	0.06	0.05
	1992	-5.69	-1.66	-2.36	-0.22	0.03	0.17	0.11	-0.31	-0.34	-0.33
	1993	-1.29	0.18	-0.09	-1.14	0.22	0.03	-0.06	-0.01	-0.72	-0.14
	1994	0.47	1.03	0.51	0.07	-0.20	0.29	0.04	-0.02	0.01	-0.29
	1995	-0.51	0.55	7.23	11.78	2.71	2.94	1.59	0.19	0.11	0.00
	1996	1.30	2.04	-0.19	-2.84	-2.28	-1.21	-4.21	-0.30	0.05	-0.01
	1997	0.05	-1.18	-1.99	-1.36	-4.05	-6.29	-3.27	-4.33	-0.41	-0.02
	Year	3	4	5	6	7	8	9	10	11	12
	1980	-4.79	-3.43	-2.92	-4.55	-1.49	-0.27	-0.09	0.11	0.09	0.01
	1981	-6.30	-1.18	1.03	1.54	-0.03	-0.10	0.56	0.21	0.07	0.04
	1982	-8.40	-2.19	1.65	1.71	0.94	2.24	0.46	0.49	0.26	0.09
	1983	-13.6	-6.96	-3.58	-0.52	1.78	0.59	-0.00	0.03	0.03	0.05
	1984	-11.3	-2.85	-1.81	8.19	1.50	0.39	1.44	0.85	0.10	-0.06
Frn	1985	2.19	-10.5	-14.3	-10.4	-4.93	-1.32	-0.68	-0.14	0.18	0.81
3.84	1986	-1.14	3.62	8.83	4.87	2.45	3.33	2.22	-0.12	0.11	0.03
	1987	5.14	-2.23	-2.92	-3.04	0.71	-0.07	-0.29	0.12	-0.37	-0.02
	1988	5.67	3.41	-1.15	-2.36	-2.35	-1.66	-0.96	-0.63	-0.01	-0.29
	1989	7.39	0.04	1.09	0.11	-0.16	-1.02	-0.93	-0.59	-0.47	-0.28
	1990	9.65	12.56	1.04	-0.99	-1.04	-2.01	-1.91	-0.96	-0.34	-0.49
	1991	2.15	5.10	6.75	2.90	0.87	0.21	-1.13	-0.71	-0.65	-0.18

Figure 1. Estimated 3PS offshore population abundance.

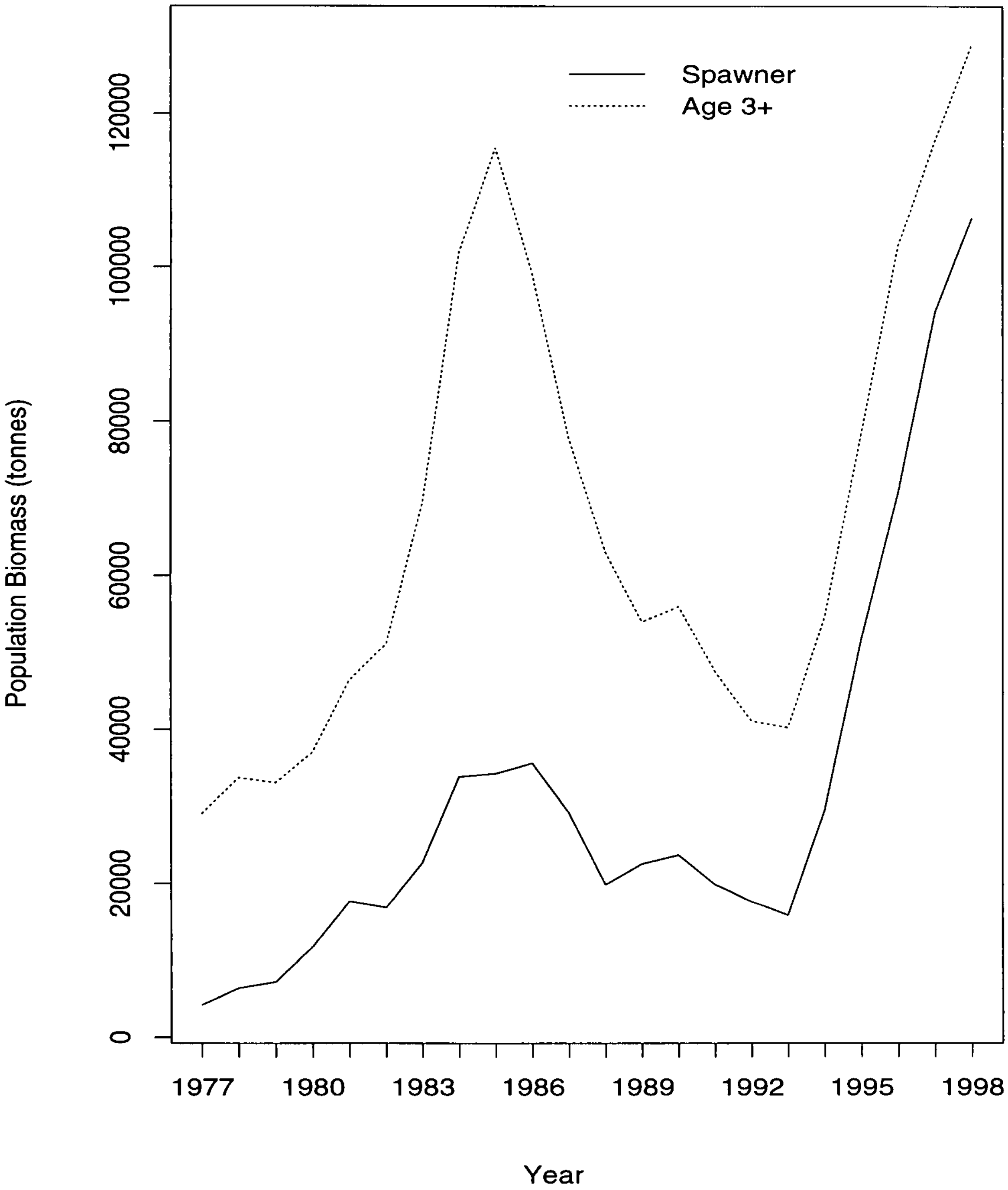

Figure 2. Estimated 3PS offshore population biomass.

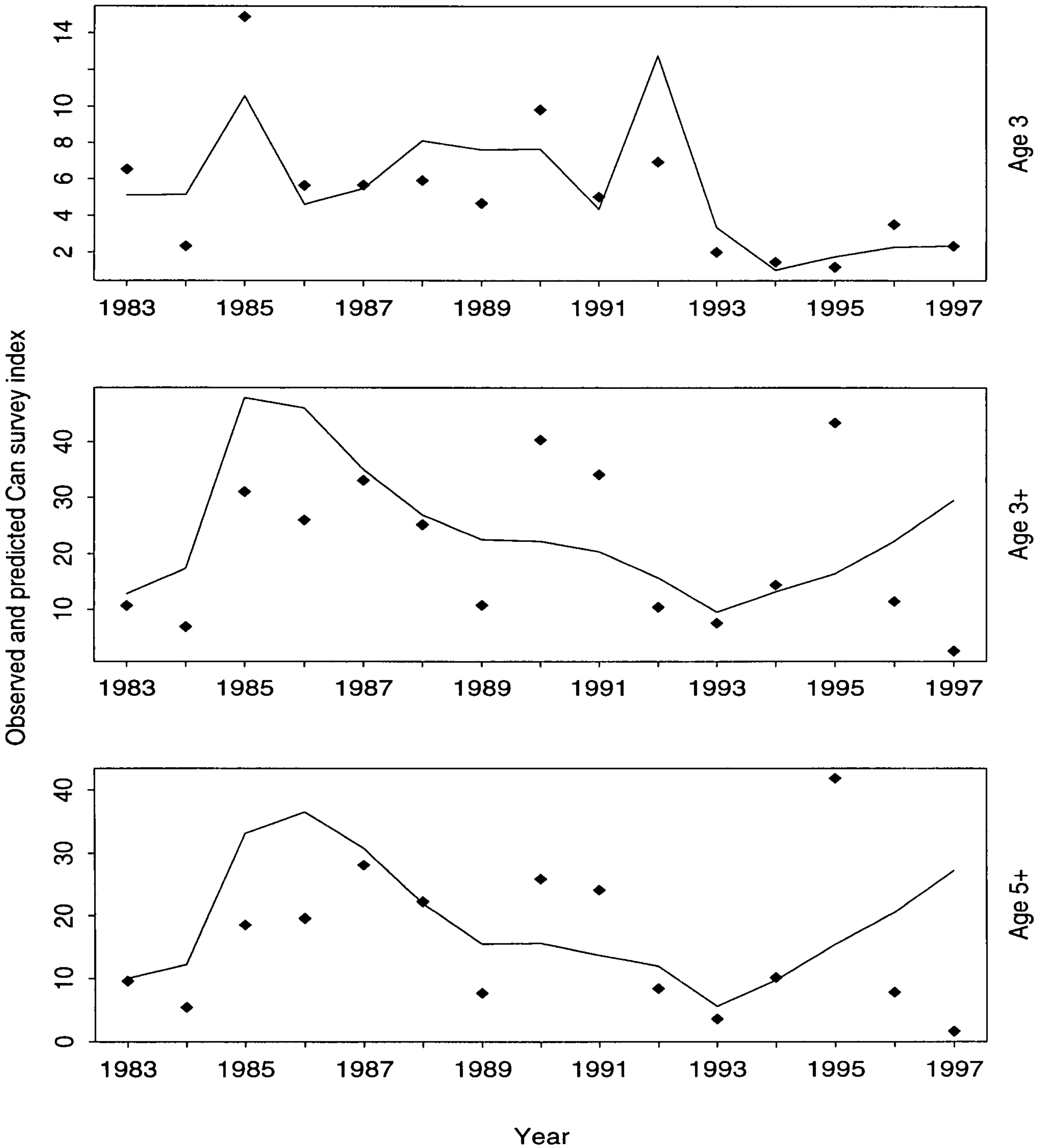

Figure 3a. Observed (points) and predicted (lines) Canadian 3PS survey indices.

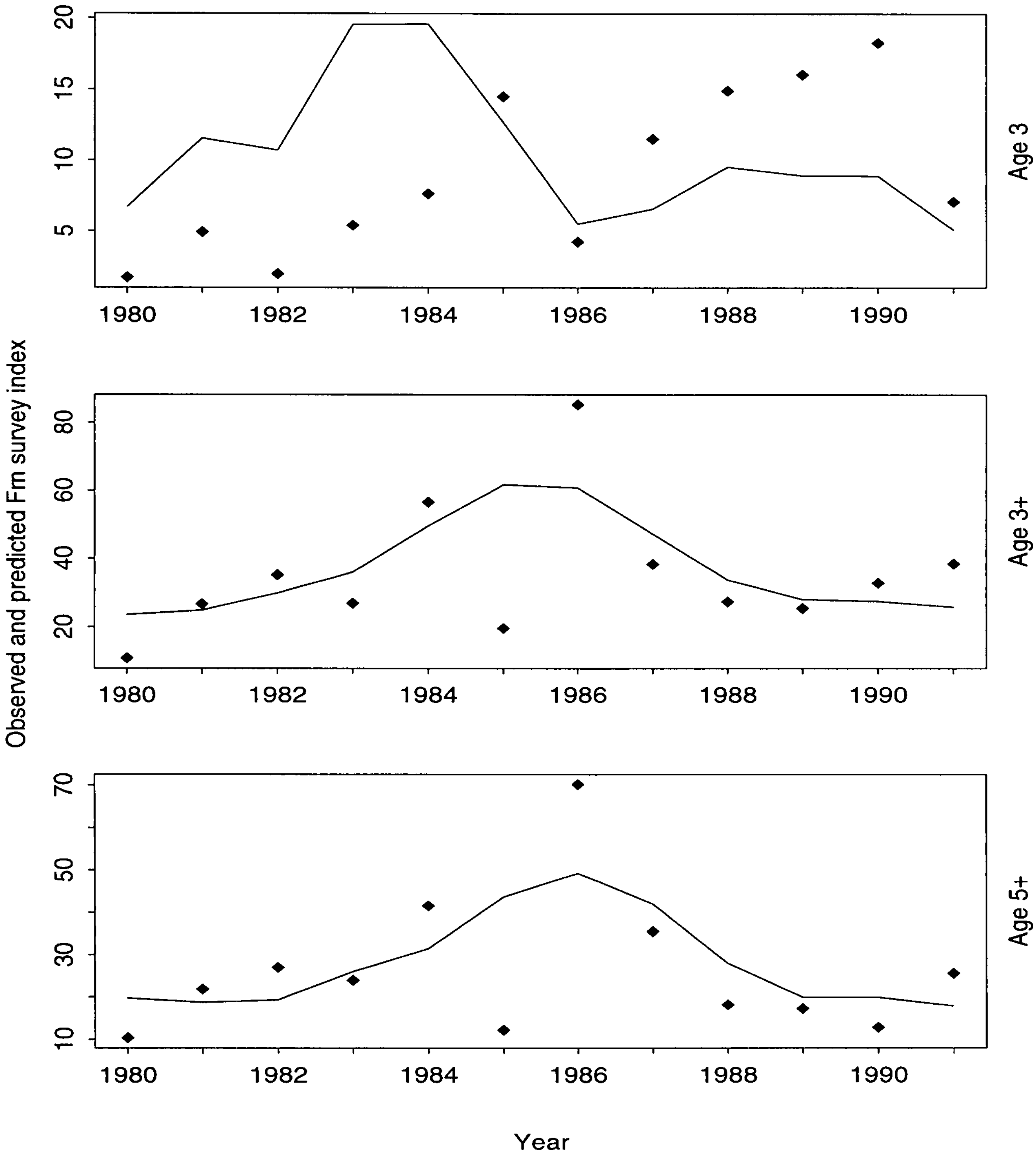

Figure 3b. Observed (points) and predicted (lines) French 3PS survey indices.

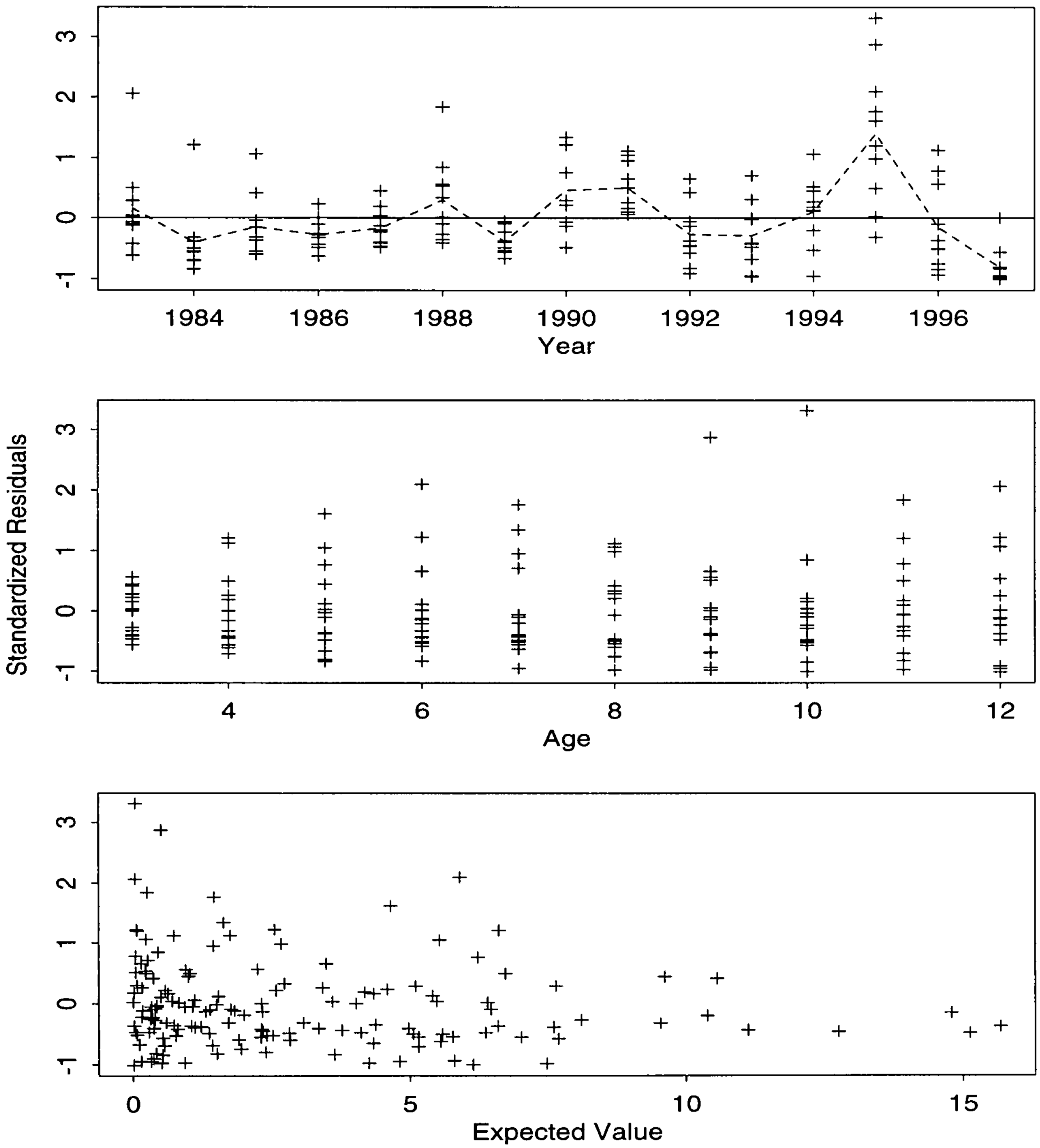

Figure 4a. Standardized residuals for the Canadian 3PS survey indices. Upper panel: residuals versus year. Middle panel: residuals versus age. Lower panel: residuals versus the model estimate of R_{ay}.

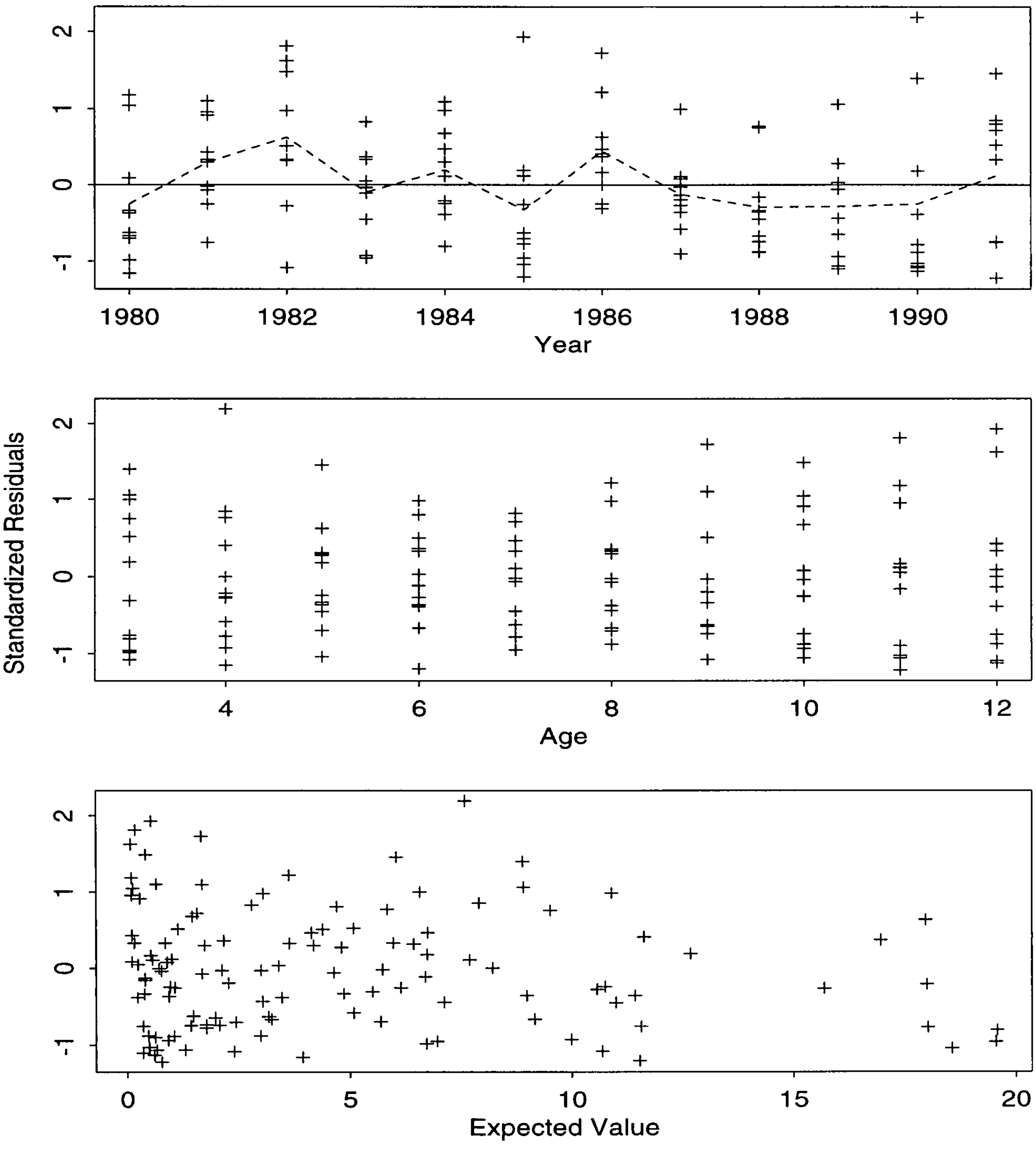

Figure 4b. Standardized residuals for the French 3PS survey indices. Upper panel: residuals versus year. Middle panel: residuals versus age. Lower panel: residuals versus the model estimate of R_{ay}.

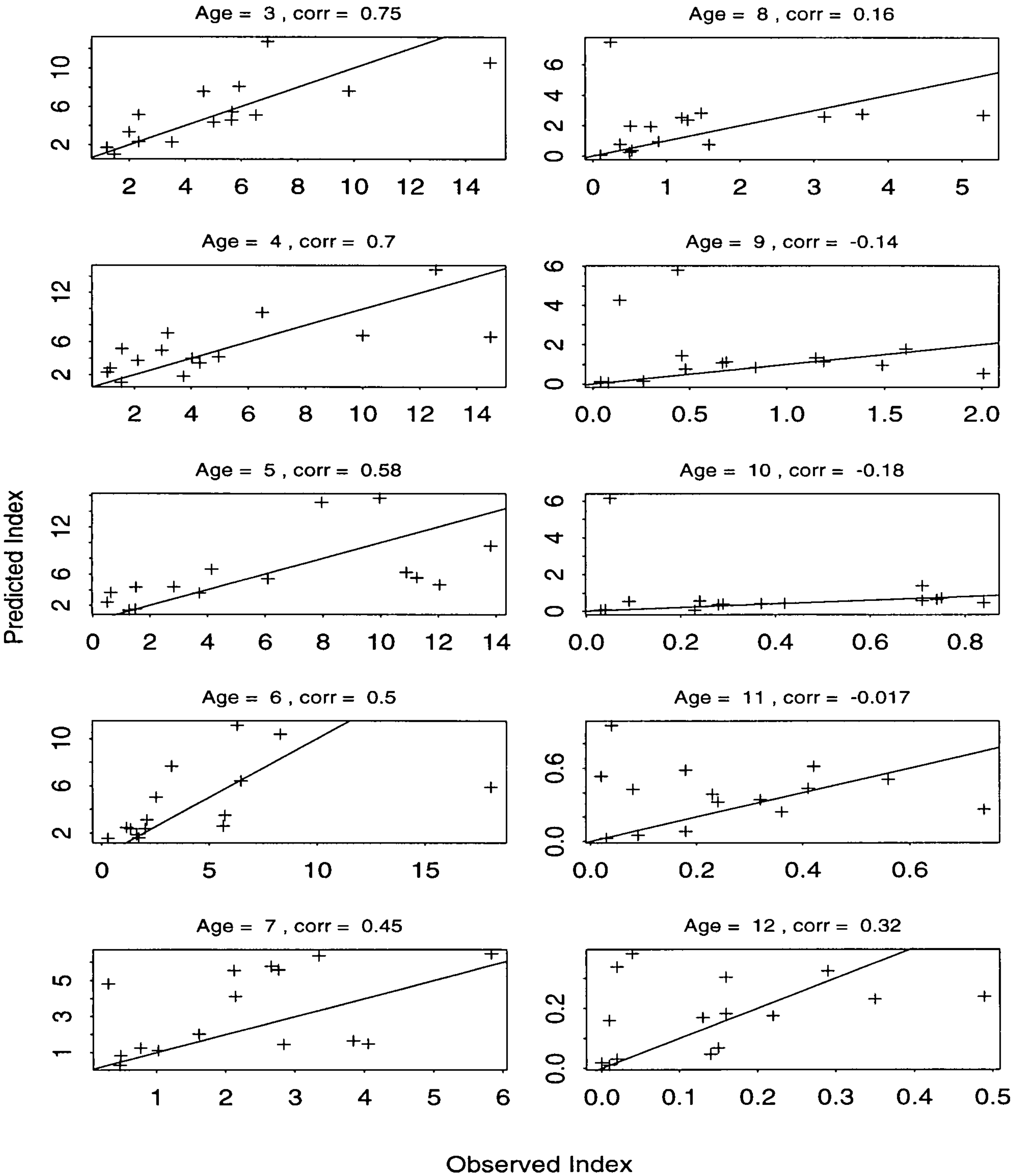

Figure 5a. Age disaggregated Canadian 3PS survey index versus its predicted value. The correlation between r_{ay} and $\hat{R}_{ay}$, for $y = 1, ..., Y$ is presented at the top of each panel.

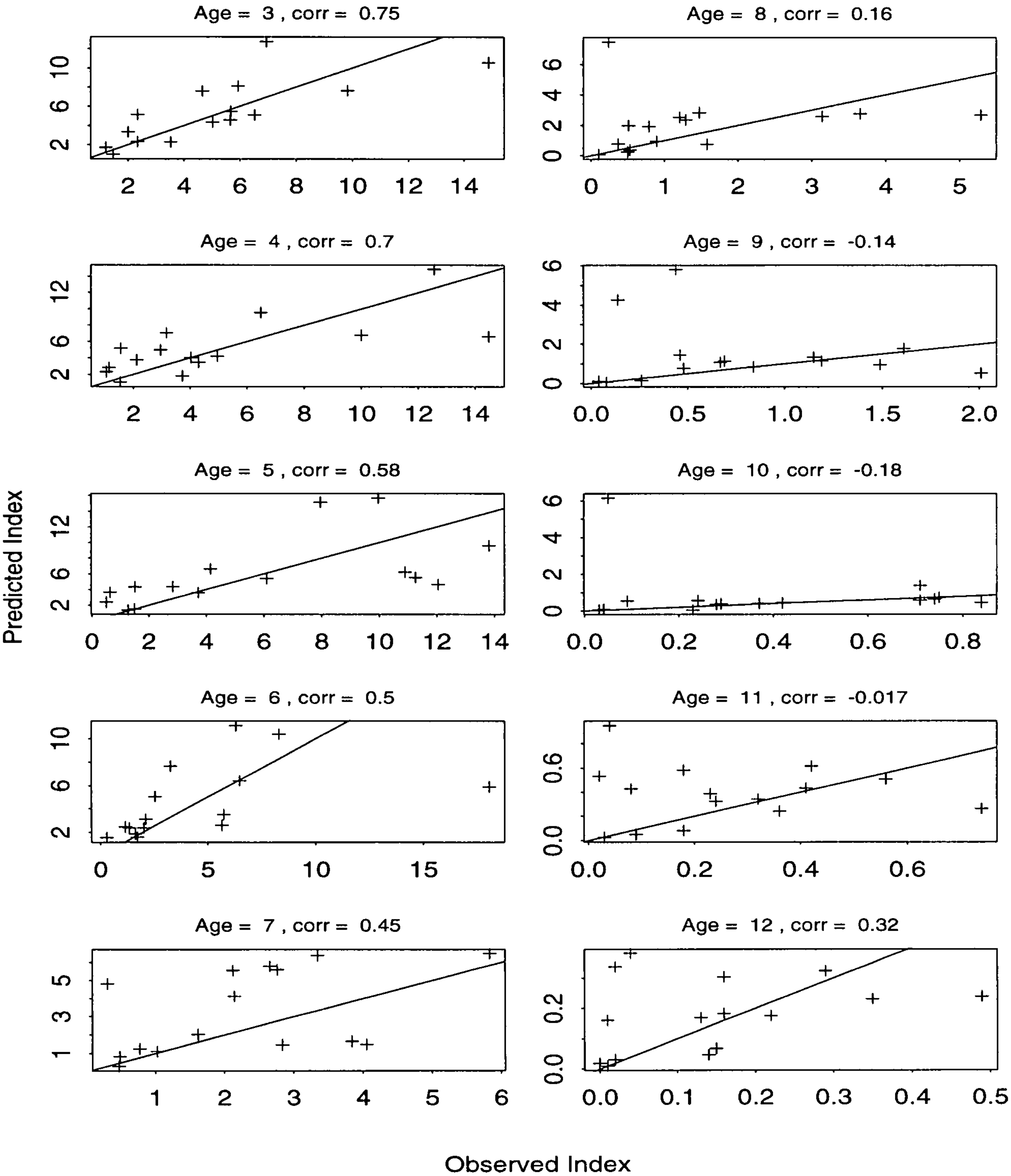

Figure 5b. Age disaggregated French 3PS survey index versus its predicted value. The correlation between r_{ay} and $\hat{R}_{ay}$, for $y = 1, ..., Y$ is presented at the top of each panel.

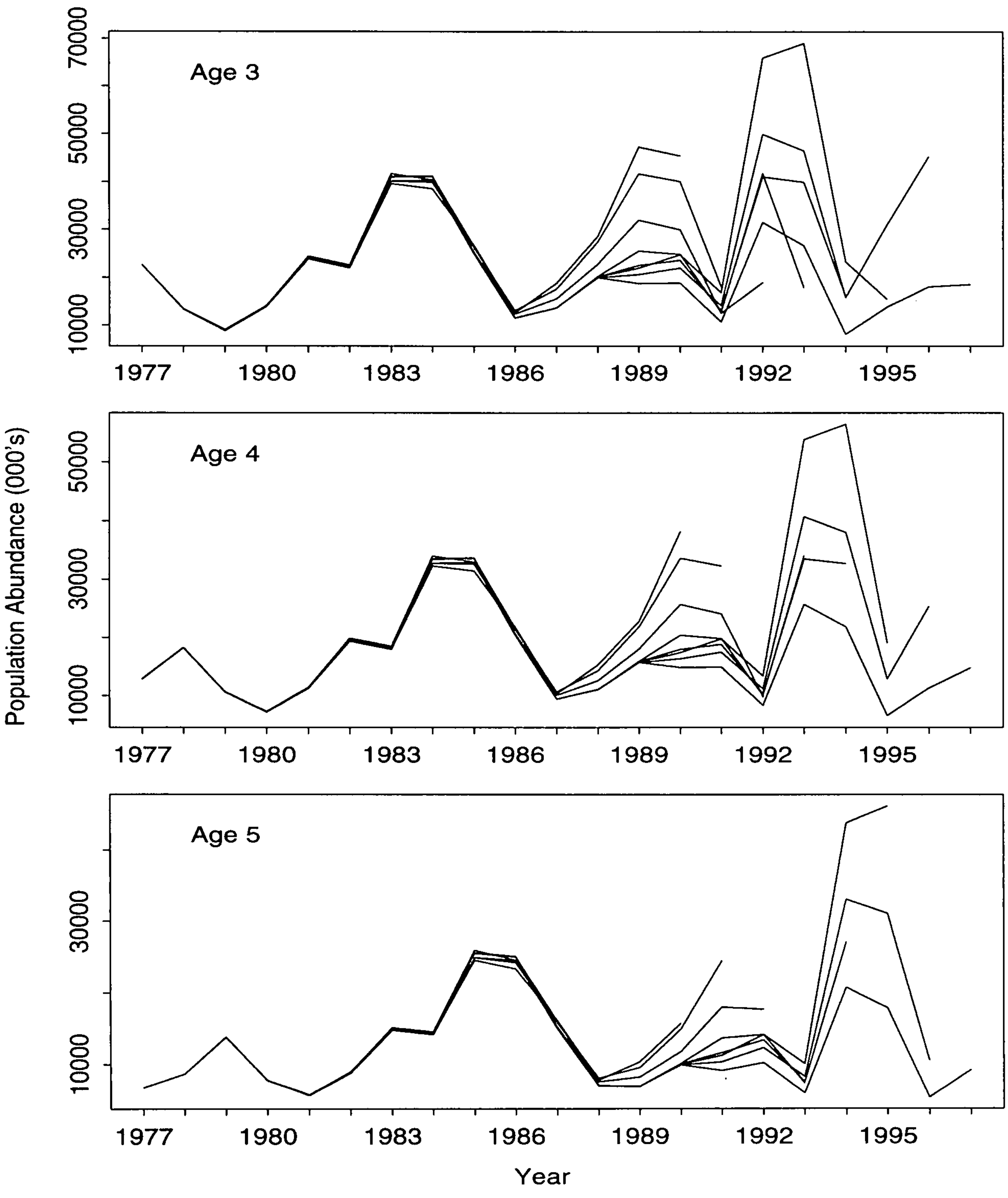

Figure 6a. Eight retrospective estimates, $Y = 1990, ..., 1997$, of stock components from the 3PS offshore SPA.

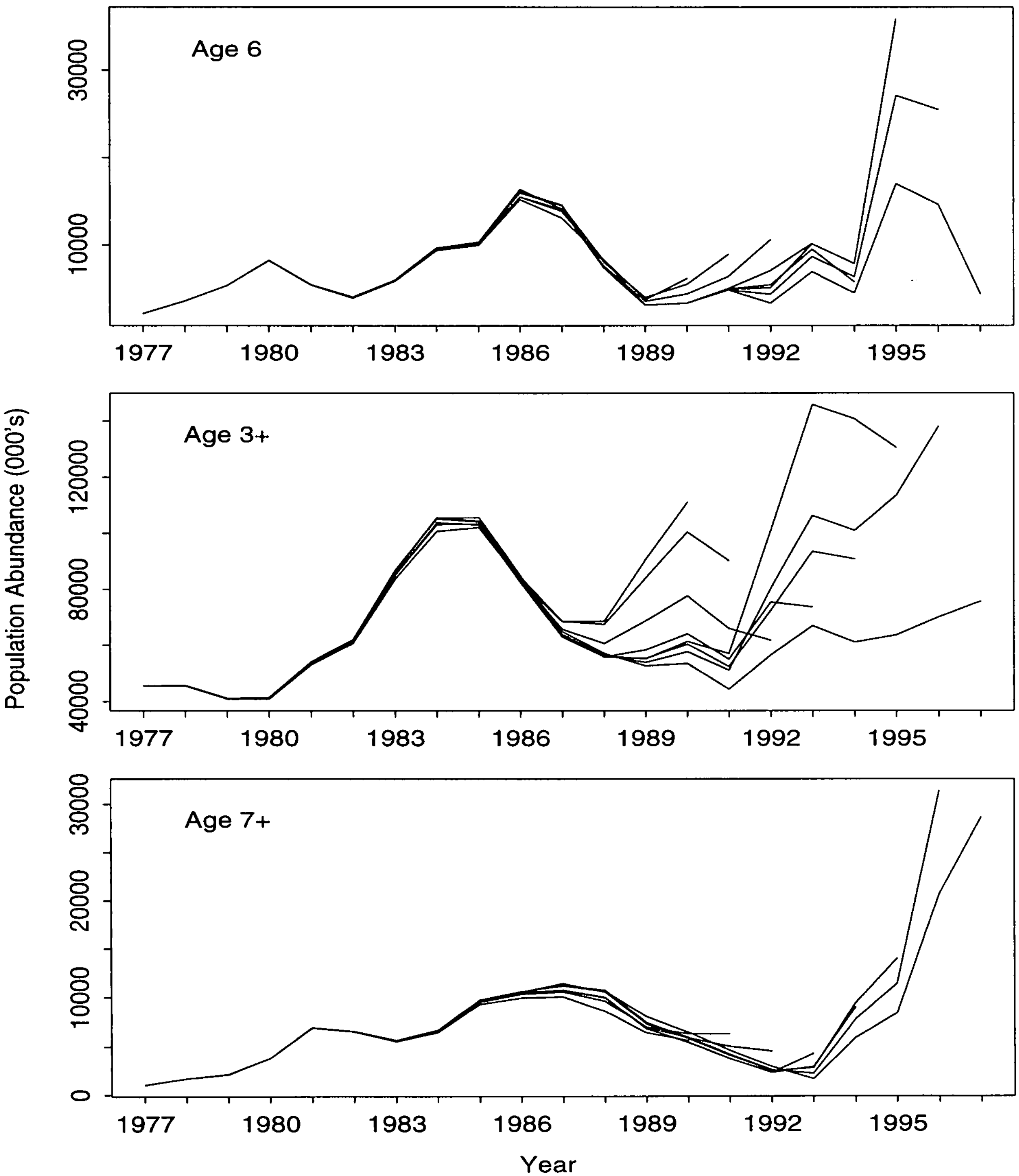

Figure 6b. Eight retrospective estimates, $Y = 1990, ..., 1997$, of stock components from the 3PS offshore SPA.

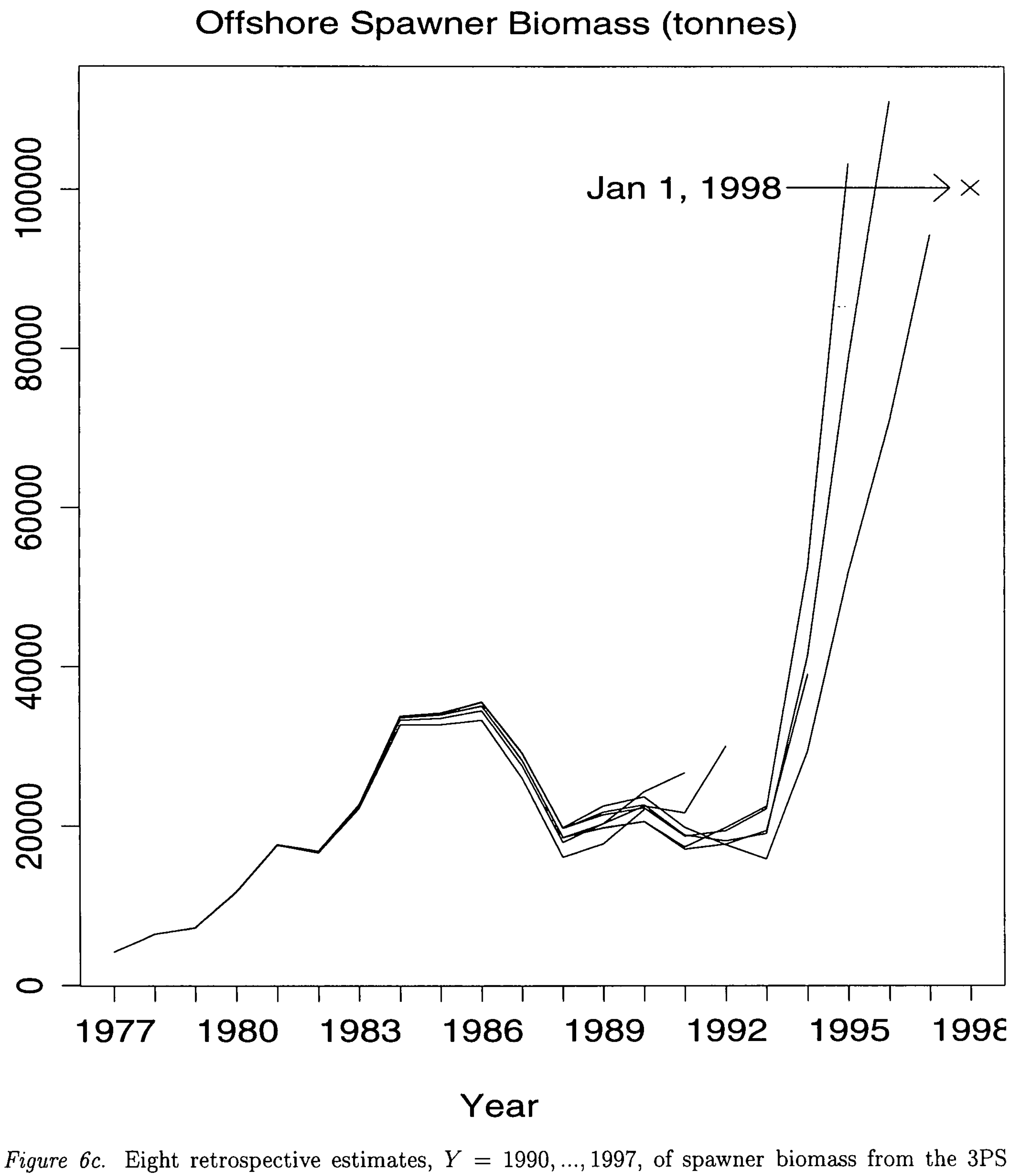

Figure 6c. Eight retrospective estimates, $Y = 1990, ..., 1997$, of spawner biomass from the 3PS offshore SPA.

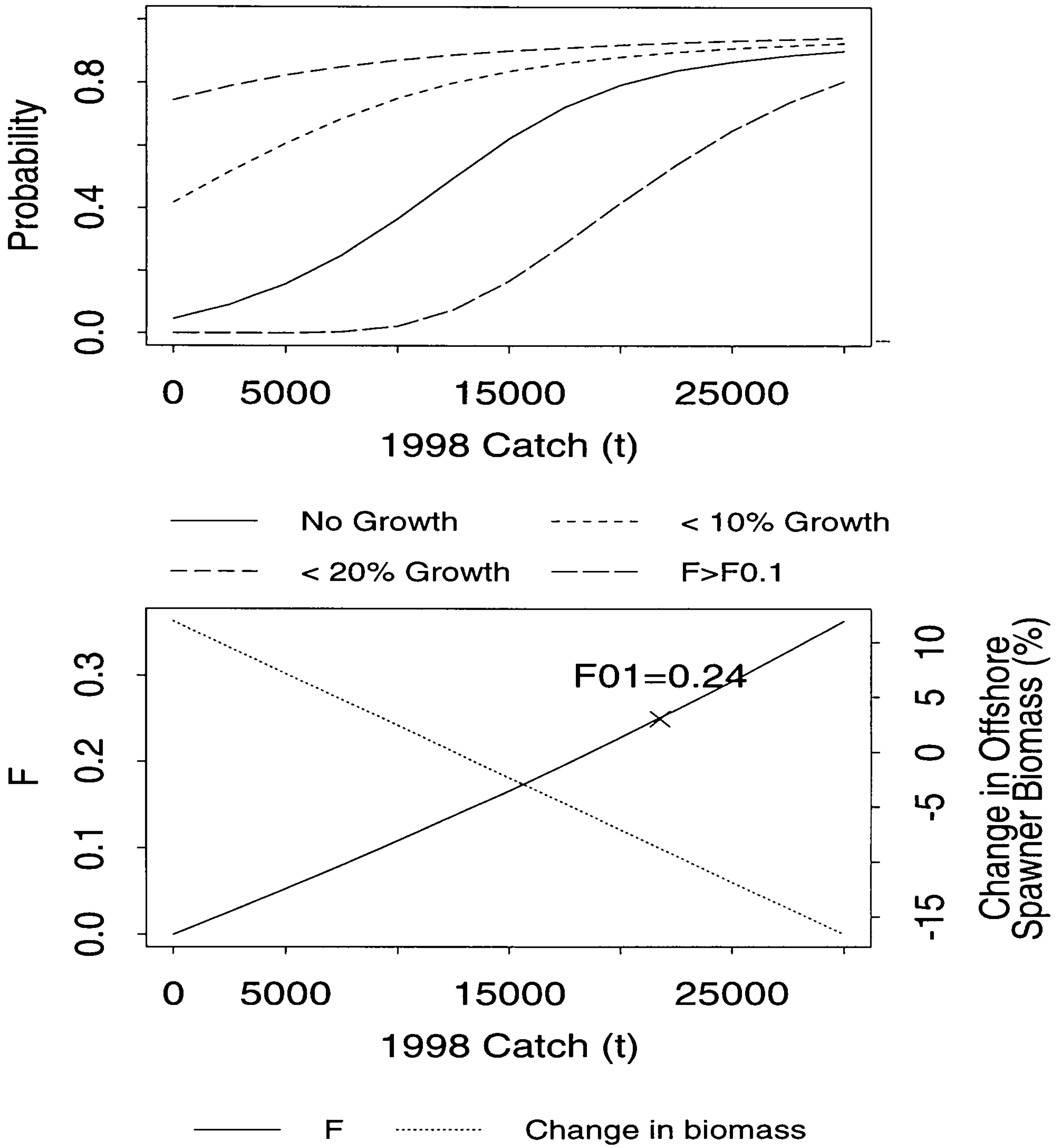

Figure 7. Profile quasi-likelihood based stochastic descriptions of the change in offshore 3PS spawner biomass (SB) and fishing mortality (F) for 7 TAC options. The top panel presents the probabilities that 1999 SB will not grow by more than 0, 10, and 20% of 1998 SB, along with the probability of exceeding $F_{0.1}$, which is taken as 0.24 for this stock. The bottom panel presents the estimated change in SB and F for the various TAC's.