What does purse seine catch per unit effort measure? A simple fishery model

Howard Powles<br>Fisheries Sciences Branch<br>Québec, Québec


#### Abstract

A model of a purse seine fishery on a small pelagic species was constructed to examine the catch-per-unit-effort/biomass relationship. In the model, schools are concentrated at a given time in a subregion of the stock distribution area; vessels initially search the stock distribution area to locate the subregion and later search the subregion to locate and capture schools; and vessels cormunicate. Results of the model suggest that the degree to which CPUE is sensitive to abundance will depend on the relationship between abundance, area over which schools are distributed, and density of schools within this area, a relationship presently unquantified. CPUE varies nonlinearly with abundance, the shape of the curve depending on the abundance/ density relationship; CPUE varies little over abundance variations of several orders of magnitude. Variables such as fleet size, learning factors (which affect the size of the area searched initially for the subregion of school concentration), and length of time spent on schools once the subregion is located produce larger variations in CPUE than abundance variations. These results are in general agreement with those of other studies of aimed mobilegear fisheries on schooling species and suggest that CPUE from such fisheries alone can not be used to measure abundance trends. Biological information (the abundance/school density and abundance/school size relationships in particular) is presently inadequate to permit correction of CPUE trends or development of new abundance indices from purse seine fisheries.


RESUME

On a construit un modèle d'une pêche à la senne bourse sur une espèce de petits poissons pélagiques, afin d'examiner la relation entre les captures par unité d'effort (CUE) et l'abondance. Dans le modèle, les bancs sont concentrés à tout mament donné dans une sous-région de leur aire totale de distribution; les bateaux font des recherches dans une phase initiale afin de trouver la sous-région et dans une deuxième phase recherchent et capturent des bancs; et les bateaux communiquent entre eux. Les résultats du modèle suggèrent que la sensitivité des CUE à l'abondance dépendra de la relation entre l'abondance, la superficie de la région sur laquelle les bancs sont distribués, et la densité des bancs à l'intérieur de cette région. Cette relation n'a pas été étudiée. La relation CUE/abondance est courbée, et la forme de la courbe dépend de la relation entre l'abondance et la densité des bancs. CUE ne varie que peu même quand les variations d'abondance sont de quelques ordres de grandeur. D'autres variables, telles que le nombre de bateaux dans la flotte, des facteurs d'apprentissage (qui déterminent la superficie sur laquelle la flotte fait ses recherches initiales pour la sousrégion), et la durée de la pêche dans la sous-région une fois qu'elle est localisée, produisent des variations de CUE plus importantes que les variations d'abondance. Ces résultats concordent avec ceux d'autres études des pêches par engins mobiles sur des espèces qui se tiennent en bancs, et ils suggèrent qu'on ne peut pas utiliser CUE seule corme mesure de tendances d'abondance. Les informations biologiques (la relation abondance/densité des bancs et la relation abondance/taille des bancs) ne sont pas suffisantes pour permettre la correction de tendances de CUE ni le développement de nouvelles indices d'abondance à partir de pêches à la senne bourse.

Paloheimo and Dickie (1964) observed that the assumptions underlying the concept of constant catchability coefficient (random distribution of fish and fishing, individual effort units acting independently) were rarely satisfied in real fisheries, and provided evidence that catchability coefficient would vary inversely with stock abundance in situations where fishermen could concentrate on aggregations of fish. Purse seine fisheries represent a situation where these assumptions are seriously violated since a) fish are available only when schooled b) schools tend to congregate in areas whose positions become highly predictable to fishermen with experience and $c$ ) vessels may act as fleets rather than independently.

Several studies, both of fishery data and of fishery models, have suggested that purse seine CPUE may not measure stock abundance, and that severe overexploitation could result from estimating stock abundance trends from CPUE trends. Pope (1978) and Ulltang (1978) reviewed CPUE trends from purse seine and trawl fisheries on European herring stocks, showing that CPUE's did not vary consistently over large abundance fluctuations. Ulltang (1978) and MacCall (1976) suggested that CPUE was related to abundance by


$$
\text { Where } \begin{aligned}
C & =\text { catch } \\
F & =\text { effort } \\
N & =\text { abundance } \\
K_{1} B & =\text { constants }
\end{aligned}
$$

Values of B of 0.3 - 0.8 were calculated for Pacific sardine and European herring fisheries; at these values, the CPUE/ N relation is curved, increasing to an asymptote. Clark and Mangel (1980), from a model study of eastern Pacific tuna fisheries, suggested that CPUE might overestimate or underestimate abundance declines, depending on biological parameters (the school size/abundance relationship and the rate of formation of schools from unschooled fish) which are unquantified for most pelagic species. The authors of all these studies indicate that extreme caution should be used in interpreting CPUE trends from purse seine fisheries.

The present study represents an attempt to model a purse seine fishery on a small pelagic species, using several basic features of such fisheries, in order to:

1) Investigate the relationship between biomass and CPUE (effort measured in time, including searching time).
2) Investigate effects on CPUE of changes in fishery variables (such as number of boats in fleet, fishery duration, learning) which are independent of biomass.
3) Identify information needed to better understand dynamics of such a fishery.

The features used in the model are:

1) Schools are concentrated at any given time within a subregion of the total stock distribution area.
2) At the opening of the fishery, vessels search the stock distribution area for the subarea.
3) Once the subarea is located, vessels search for and capture schools within the subarea.
4) There is communication between vessels.

## DESCRIPTION OF THE MODEL

Fish always occur in schools of mass $S$ (tons) and radius $r(k m)$. The fish inhabit a given region (the stock distribution area) of area $\mathrm{X}\left(\mathrm{km}^{2}\right)$, but migrate during the year so that at any given time all are found in a subregion, of area $\mathrm{x}\left(\mathrm{km}^{2}\right)$. For convenience, both regions are taken to be circular (Fig. 1).

Vessels search for fish schools as a fleet. Searching is divided into two phases: Phase I, where the fleet searches the stock distribution area for the subregion in which schools are concentrated, and Phase II where vessels search within the subregion for individual schools (Fig. l). At the end of Phase I, when one vessel has found a school, other vessels transit from their positions at the end of the initial search to the subregion. The model fishery runs in cycles, each cycle being the period required for all boats to locate and capture a school. Once the subregion of school concentration has been located as a result of a Phase I Search, the fleet may continue to fish in the subregion for several cycles. Total effort is expressed as the total time expended in locating and capturing schools over a fishery period. Total catch is expressed as the product of school mass, fleet size, and number of cycles in the fishery.

Phase I Search. At the start of the fishery, a fleet of B vessels begins searching from randomly distributed positions on the circumference of the circular stock distribution area. Vessels search at velocity V and can detect schools over a distance $\ell(\mathrm{km})$. Searching is assumed to be random. Searching continues until one vessel has found a school within the subregion.
(1)

The expression for time taken to locate the first school is:

Where $\quad t_{1}^{\prime}=$ time to locate first school (hr)
$X=$ area of stock distribution region ( $\mathrm{km}^{2}$ )
$x=$ area of subregion ( $\mathrm{km}^{2}$ )
$B=$ number of boats in fleet
$V=$ searching velocity ( $\mathrm{km} / \mathrm{hr}$ )
l = width over which schools can be detected (km)

Details of the derivation of this expression are in Appendix 1.

Transit. Once one vessel locates a school, other vessels transit to the subregion (strictly, to the first boat's position or the nearest school on a straight line to it).
(2)

The expression for mean transit time is:

$$
\begin{equation*}
\bar{T}=\frac{\sqrt{\frac{\left(R^{\prime}\right)^{2}+2 R^{2}}{2}}-R}{V} \tag{1}
\end{equation*}
$$

```
Where \(\bar{T}=\) mean transit time ( hr )
    \(R^{\prime}=\) radius of stock distribution region (km)
    \(R=\) radius of subregion (km)
    \(V=\) boat velocity ( \(\mathrm{km} / \mathrm{hr}\) )
```

Details of the derivation of this expression are in Appendix 1.

Phase II Search. Once in the subregion, each vessel searches until it locates a school. The expression for time taken to locate a school within the subregion is:

$$
\begin{equation*}
t_{2}^{\prime}=\frac{1}{2 \lambda_{2} v(B-1)(l+r)} \tag{3}
\end{equation*}
$$

$$
\text { Where } \begin{aligned}
& \quad t_{2}^{\prime}=\text { time required to locate a school (hr) } \\
& \lambda_{2}=\text { density of schools within subregion }\left(\mathrm{km}^{-2}\right) \\
& r=\text { radius of schools (km) } \\
& \ell=\text { width over which schools can be detected (km) } \\
& V=\text { boat velocity }(\mathrm{km} / \mathrm{hr}) \\
& B=\text { number of boats in fleet }
\end{aligned}
$$

Derivation of this expression is in Appendix 1.

Capture. Capture time ( $\tau$ ) is assumed constant for this preliminary version of the model

Total effort. Total effort is the sum of all time expended by the fleet in locating and capturing fish.

$$
\begin{equation*}
E=B t_{1}^{\prime}+(B-1) \tau+(B-1) t_{2}^{\prime}+B(\tau) \tag{4}
\end{equation*}
$$

Where $E=$ total effort
$\overline{\mathcal{T}}$ and $t_{2}^{\prime}$ are multiplied by ( $B-1$ ) rather than $B$ since one boat has already found a school at the end of Phase I.

Both $B t_{1}^{\prime}$ and $(B-1) t_{2}^{\prime}$, total search times for Phase I and Phase II, are independent of fleet size, since:

$$
B t_{1}^{\prime}=B\left(\frac{x-x}{2 B v l}\right)=\frac{x-x}{2 v l}
$$

and

$$
(B-1) t_{2}^{\prime}=(B-1)\left(\frac{1}{2 \lambda_{2} \vee(B-1)(l+r)}\right)=\frac{1}{2 \lambda_{2} \vee(l+r)}
$$

In essence, the time required for the first boat to locate a school is inversely related to the number of boats in the fleet (the more boats in the fleet, the sooner one will find a school). This length of time is then multiplied by the number of boats (since all will record this searching time in their logs). Intuitively, it appears easy to visualize the situation demonstrated by these equations: one vessel on its own will take approximately the same amount of time to find a school (or an aggregation of schools) as the total time required by several boats.

Total catch. Total catch is expressed as

$$
\begin{equation*}
\mathrm{C}=\mathrm{BSD} \tag{5}
\end{equation*}
$$

Where $\quad C=$ total catch
$D=$ number of cycles in fishery
$\mathrm{S}=$ school biomass (see next section)

School biomass: Few published observations on school size in small pelagics exist. For the purposes of this preliminary model, school biomass is taken to be constant. This assumption should not affect the results of the model if school size is independent of stock biomass. It could be argued that schools exist for biological reasons (reduction of predation, hydrodynamic advantages) which are independent of stock biomass, and in some species school size has been observed to be independent of stock abundance (northern anchovy, California sardine, Radovich 1979). Incorporation of a school size/biomass relationship would significantly refine the model. The present version of the model also assumes a constant school biomass/school diameter relationship, which is probably highly variable in nature.

Catch per unit effort. CPUE is expressed as C/E.
Relationship between biomass, school density, and area of subregion

Variations in biomass may lead to variations in density of schools within the subregion where these occur $\left(\lambda_{2}\right)$ and in the area of the subregion (x) (since school size is assumed constant, this is not affected by biomass variations). Few data apparently exist on the relationships between these variables in real stocks. Neyman (1949) assumed that density of schools increased directly with stock biomass for a purse seine fishery model. Since
data are lacking, it would be advantageous to study the effects of different relationships between these variables on the model fishery. To do this, the relationship between biomass (A) and school density ( $\lambda_{2}$ ) is expressed as:

$$
\begin{equation*}
\log A=a \log \lambda_{2}+b \tag{6}
\end{equation*}
$$

where a and b are constants. In the model, biomass is input, density is calculated using assumed values of $a$ and $b$, and area of the subregion occupied by schools is then calculated from biomass and density.

A flowchart for the model is given in Fig. 2.

## RESULTS

The model has been run using values for the variables which somewhat resemble those characterising the Gulf of St.Lawrence herring fishery. Results are presented in two parts, the first showing the effects of individual variables on catch per unit effort, the second showing catch per unit effort trends in simulated fisheries.

Effect of individual variables on catch per unit effort

Biomass. The model predicts that the relationship between biomass and catch per unit effort will be curved rather than linear, and that (except under highly unrealistic conditions) catch per unit effort will vary little over biomass variations of several orders of magnitude (Fig. 3). The biomass/CPUE relationship was determined for 4 sets of values for $a$ and $b$ in equation (6); corresponding to situations in which
(1) density of schools remains constant with increasing biomass
(2) density of schools increases by a factor of 1.5 with a 10 x increase in biomass
(3) school density increases 2.7 x with a biomass increase of 10 x
(4) school density increases $10 x$ with a biomass increase of $10 x$.

Under situation (1), CPUE is almost constant at biomass values $10^{3}-10^{5}$ but increases sharply at higher biomass values. Under situation (4), CPUE is essentially constant at biomass values $10^{3}-10^{6}$, and under the other situations CPUE increases slightly at high biomass values ( $10^{5}-10^{6}$ ).

Fleet size. CPUE increases with increasing number of vessels in the fleet (Fig. 4). The relationship appears to increase toward an asymptote but with "realistic" numbers of vessels in the fleet (1-40) the relationship approximates a straight line.

This result of the model is somewhat surprising, although once put forward it appears intuitively reasonable; it is a result of fleet operations rather than individual vessels' acting independently. As noted (page 7) total search time in both Phase I and Phase II remains constant independent of fleet size; thus the only amounts of time which vary with fleet size are transit time and capture time. Since total effort (time) does not vary directly with fleet size, while total catch does, CPUE increases with fleet size.

Duration of fishery. CPUE increases quasi-linearly with the number of cycles during which the fleet fishes in the subregion of school concentration once this has been located as the result of a Phase I search (Fig. 5). In a real fishery, the length of time over which a fleet can exploit a concentration of schools may depend on such factors as weather, mobility of the fish, and market conditions (saturation).

Stock distribution area - learning factor. Although the area initially searched by the seiner fleet in Phase I has been defined as the stock distribution area, it could also be considered as a hypothetical area in which the seiners know the concentration of schools to be and which they must search to find the schools (i.e. a subarea of the true stock distribution area). A decrease in this area could represent one component of a learning factor in the fishery. As skippers gain experience with the stock over time, they should become capable of predicting where schools will be at a given time with increasing precision, thus decreasing the area which must be searched at the start of the fishery to locate schools. For example, early in the history of the Gulf of St.Lawrence seine fishery shippers probably had to search widely for herring in springtime; more recently, after many years of experience, skippers are capable of going directly to the "Edge" area and finding herring after a shorter period of searching.

Learning factor thus expressed has a marked effect on CPUE, which increases dramatically (quasi-exponentially?) with a decrease in the area to be searched in Phase I (Fig. 6).

## Fishery simulations

Three simple simulations have been run, combining conditions of biomass variations, learning factors, changes in fleet size and fishery duration in order to examine the relative importance of each factor in determining CPUE trends. All simulations incorporate declining biomass such as might occur in a stock in which recruitment is weak and total mortality is
high; biomass declines are expressed in an equivalent total mortality ( $Z=0.4-0.5$ in the simulations). Simulations are run until the fleet's capacity exceeds total biomass and the CPUE figure for this final "year" represents the final point on the graph.

1. Declining biomass, constant learning factor, variable duration. Biomass declines at the equivalent of $Z=0.5$ from a start value of $1 \sigma^{6} t$. Learning decreases the area to be searched during Phase I by $10 \% / \mathrm{yr}$. Weather conditions are bad in two years, good in two years (chosen randomly), normal in other years. Normal weather permits 5 cycles on the fish following a Phase I search, bad weather 2 cycles, good weather 10 cycles. Other values for variables are given in Fig. 7.

Over the 12 years of the fishery, biomass declines by 2 orders of magnitude but CPUE increases by a factor of 2 from year 1 to year 11 (Fig. 7). In effect, the CPUE trend is caused almost entirely by the trend in area to be searched during Phase I, thus by learning by the skippers; in the absence of this factor, CPUE would have remained essentially constant despite the biomass decline. In year 12, CPUE decreases slightly when fleet capacity exceeds biomass and the stock is "extinguished". Variable weather conditions add considerable noise to the trend and it is possible that no trend would be detected on examination of such a series of data points. $R^{2}$ for the regression of CPUE on year is 0.13 , while for the regression of CPUE on biomass $R^{2}$ is 0.10 . Weather noise has not been incorporated into further simulations.
2. Declining biomass, learning factor declining with time. One would expect learning in a real fishery to be high in early years and to decline with time. In this simulation, learning decreases the stock distribution area by $20 \% / \mathrm{yr}$. in years 2 and 3, 10\%/yr. in years 4 and $5,5 \% / \mathrm{yr}$. in years 6 and 7, and 0 thereafter. Biomass declines at the equivalent of $Z=0.5$. Other variable values are given in Fig. 8.

The CPUE trend here appears (as in the first example) to be determined almost entirely by the learning factor trend, although biomass declines by 2-3 orders of magnitude (Fig. 8). CPUE increases to a maximum at year 7 when learning is complete and declines slightly thereafter to year 12. In year 13, when fleet capacity exceeds stock biomass, there is only a slight decline in CPUE despite the fact that the population has essentially been wiped out.
3. Declining biomass, changing fleet size, declining learning factor. Effort may vary with market conditions and catch rates. In this simulation, number of vessels in the fleet increases by 5/yr. for the first five years and decreases by $3 / \mathrm{yr}$. for the next 5 yr . Biomass declines at the equivalent of $z=0.4$ from a start value of $10^{6} t$. Learning decreases area searched at $10 \% / \mathrm{yr}$. in years $2-5$, after which area searched is held constant. Other variable values are given in Fig. 9.

The CPUE trend parallels the fleet size trend (Fig. 9 ), despite a biomass decline of almost 2 orders of magnitude. As with learning factor, this factor unrelated to fish abundance influences CPUE more than abundance.

## DISCUSSION

The results of this preliminary version of a purse seine fishery model suggest that CPUE in purse seine fisheries may be far more sensitive to such variables as fleet size, weather and market conditions, and skippers' experience than to stock biomass. The model incorporates several major characteristics of such fisheries: schooling and migration of fish, searching in two phases, and operation of purse seiners as a fleet rather than independent units. The latter two characteristics have not been investigated in earlier model studies. The model can be used to simulate fishery trends under conditions observed in real fisheries: entrainment of effort into successful fisheries, decline in effort due to market conditions or fishery regulation, learning, varying periods on fish concentrations due to weather, market conditions or movement of schools, and stock biomass variations.

Studies of several schooling pelagic species support the conclusion of this study that purse seine CPUE (as defined here, i.e. total time spent in searching and fishing) is not a good measure of stock abundance. Pope (1978) and Ulltang (1976), (1978) have presented evidence from purse seine and trawl fisheries on European herring stocks to the effect that CPUE in these fisheries has varied little with over order-of-magnitude changes in stock abundance. Ulltang (1978), working on herring, and McCall (1976), working on California sardine, have suggested that catchability coefficient is a function of stock size, and that CPUE is related to stock size by

$$
\frac{C}{F}=K N^{1-B}
$$

where $K$ and $B$ are constants. Values of $B$ in these studies have been calculated to range from 0.3 to 0.8 . As B approaches $1, C / f$ approaches a constant value, thus becoming meaningless as a measure of stock size. Clark and Mangel (1980) have developed a model of eastern Pacific tuna fisheries the results of which suggest that, depending on the relationship of school size to stock size and the rate of school formation from unschooled fish, CPUE may vary little as stock size declines to zero. Ulltang (1978) has concluded, because of the problem of variation in catchability with stock abundance and the resultant sensitivity of schooling pelagic stocks to exploitation, that mobile gear CPUE should not be used as an abundance index for such stocks and that every effort should be used to develop other abundance indices (fixed-gear CPUE, acoustic surveys, larval surveys).

Other approaches to the use of purse seine fishery data to provide abundance indices may prove valuable. Neyman (1949) has suggested that time taken to locate the first school at the start of a fishery might be a valid abundance index in such a fishery. Such an index would have to be corrected for fleet size and possibly for learning factor, based on the results of the present model. Correction for learning, duration on school concentrations, and fleet size might provide usable CPUE figures from a mobile-gear fishery but a considerable logbook data base would be required and in any case this model's results suggest that even a corrected CPUE series would not measure abundance accurately. Sinclair et al. (1979) have used CPUE's corrected for learning as abundance indices in the 4WX herring fishery and consider this index an improvement over uncorrected CPUE's.

Use of purse seine fishery information other than CPUE for abundance measurement would appear to require information on biological parameters which is presently unavailable. Information on school mass distributions appears to be almost completely lacking in the literature for any pelagic species, and has been identified in all published studies as well as in the present study as necessary to an understanding of pelagic fishery dynamics. It might be argued (as by Radovich, 1979) that school size is a biological feature which has evolved in response to specific selection pressures (e.s. predator avoidance, hydrodynamic advantages) and which should thus be at least somewhat independent of abundance. If this is so, CPUE and abundance will bear little relationship. Radovich (1979) has presented limited information that school size has varied little with large biomass variations in California sardine and northern anchovy stocks. The relationship between stock biomass, density of schools, and area occupied by schools is also little known for pelagic stocks and should be investigated in order to interpret fishery information in terms of abundance. Untested assumptions about these parameters have been used in past model studies; Neyman (1949) assumed school density to vary directly with abundance, while Clark (1974) assumed that school size was proportional to abundance.

## ACKNOWLEDGEMENTS

My thanks to Lynn Cleary and Serge Labonté for their ideas during the development of this model and to Denis Rivard for a careful and constructive reading of the manuscript.

## IITERATURE CITED

Clark, C.W. 1974. Possible effects of schooling on the dynamics of exploited fish populations. J. Cons. Int. Explor. Mer 36: 7-14.

Clark, C.W. and M. Mangel. 1980. Aggregation and fishery dynamics: a theoretical study of schooling and the purse seine tuna fisheries. Fish. Bull. 77: 317-337.

MacCall, A.D. 1976. Density dependence of catchability coefficient in the California sardine, Sardinops sagax caerulea, purse seine fishery. Cal. Coop. Oceanic Fish. Invest. Rep. 18: 136-148.

Neyman, J. 1949. On the problem of estimating the number of schools of fish. Univ. Calif. Publ. Stat. 1: 21-36.

Paloheimo, J.E. and L.M. Dickie. 1964. Abundance and fishing success. ICES Rapp. Proc.-verb. 155: 152-163.

Pope, J.G. 1978. Some consequences for fisheries management of aspects of the behaviour of pelagic fish. ICES Symp. Biol. Basis Pel. Fish. Stock Management, Pap. 12: 1-27.

Radovich, J. 1979. Managing pelagic schooling prey species. pp. 365-375 in Clepper, H. ed. Predator-Prey Systems in Fisheries Management. Sports Fishing Institute, Washington, DC.

Sinclair, M. and T.D. Iles. 1980. 1979 4WX Herring assessment. CAFSAC Res. Doc. 80/47: 47 pp .

Ulltang, O. 1976. Catch per unit of effort in the Norwegian purse seine fishery for Atlanto-Scandian (Norwegian spring spawning) herring. FAO Fish. Tech. Pap. 155: 91-101.
1978. Factors of pelagic fish stocks which affect their reaction to exploitation and require a new approach to their assessment and management. ICES Symp. Biol. Basis Pel. Fish. Stock Management Pap. 34:1-38

Table 1. Variables incorporated in purse seine model.

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| A | Stock biomass | tons |
| S | School biomass | tons |
| $N$ | Number of schools ( $=\mathrm{A} / \mathrm{S}$ ) |  |
| $\checkmark$ | Boat velocity | $\mathrm{km} / \mathrm{hr}$. |
| $B$ | Number of boats in fleet |  |
| $\ell$ | Total width over which boats can detect schools | km |
| $x$ | Area of stock distribution region | knt ${ }^{2}$ |
| $R^{\prime}$ | Radius of stock distribution area | km |
| $\psi$ | Area of subregion of X in which schools are present at a given time | $k n{ }^{2}$ |
| $R$ | Radius of subregion x | km |
| $\lambda_{2}$ | Density of schools within subregion ( $=\mathrm{N} / \mathrm{x}$ ) | $\mathrm{km}^{-2}$ |
| $r$ | Radius of school | km |
| $a, b$ | Constants in $\log \lambda_{2}=a \log \mathrm{~A}+\mathrm{b}$ |  |
| $t_{1}^{\prime}$ | Mean time required per boat to locate first school within $X$ (Phase I Search) | hr. |

Table 1. Variables incorporated in purse seine model (contd).

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $t_{2}^{\prime}$ | Mean time required to locate subsequent schools within $x$ (Phase II Search) | hr. |
| T | Transit time from vessel position at end of Phase I Search to nearest school | hr. |
| $\tau$ | Time required to capture a school once located | hr. |
| (1) | Number of fishery cycles associated with one Phase I Search | hr. |
| $E$ | Total effort expended in fishery | hr. |
| $c$ | Total catch ( $=$ BDS ) | tons |

## APPENDIX

The equations used in this version of the model are not exact. However, they approximate exact equations, incorporate the relevant variables, and behave properly (going to 0 or maxima when they should). Further refinement of this model will depend on refinement of the base equations.

1) Derivation of expression for $t_{1}^{\prime}$

Vessels begin searching from randamly-distributed points on the circumference of circle of area X, for a smaller circle of area $x$. Each vessel searches an area $v \ell$ per unit time ( $v=$ vessel velocity, $l=$ detection width); the fleet searches area $B v \ell$ per unit time ( $B=$ fleet size). On average, it is necessary to search $\frac{1}{2}$ the difference between the areas of the two circles to find the smaller circle. This difference can be equated to the area searched by the fleet in a given time:

$$
\frac{x-x}{2}=B r l t_{1}^{\prime}
$$

Solving for $t_{1}^{\prime}$ gives:

$$
t_{1}^{\prime}=\frac{x-x}{2 B v l}
$$

This expression would only be exact if vessels began searching from randomly-distributed positions in the area between $X$ and $\psi$ (rather than from the circumference), and if searching were random. It has been used in this preliminary version of the model pending development of an exact expression. It has the advantages of incorporating the variables involved and of "behaving" correctly (going to 0 as $\notin$ goes to $X$, e.g.).
2) Derivation of expression for $T$


At the end of the initial phase, vessels are randomly distributed over the area between the circumference of the large circle (here identified as X) and the circumference of the small circle ( $火$ ). Their mean position is thus on an intermediate circle ( $X^{\prime}$ ), whose dimensions are such that the area between its circumference and the circumference of $X$ is equal to the area between its circumference and the circumference of $\psi$. So:

$$
\begin{aligned}
& x^{\prime}-x=x-\left(x^{\prime}-x\right) \\
& 2 x^{\prime}=x+2 x \\
& 2 \pi\left(r^{\prime}\right)^{2}=\pi R^{2}+2 \pi r^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \left(r^{\prime}\right)^{2}=\frac{R^{2}+2 r^{2}}{2} \\
& r^{\prime}=\sqrt{\frac{R^{2}+2 r^{2}}{2}}
\end{aligned}
$$

Mean transit time $\bar{T}$ is the difference between $r^{\prime}$ and $r$ divided by vessel velocity:

$$
\begin{aligned}
\bar{T} & =\frac{r^{\prime}-r}{V} \\
& =\frac{\sqrt{\frac{R^{2}+2 r^{2}}{2}-R}}{V}
\end{aligned}
$$

The expression for $\overline{\mathrm{T}}$ is exact only when 4 and X are concentric; however, trial and error have shown $\bar{T}$ to be underestimated only by about $20 \%$ when $\%$ and X are tangential so the expression has been used pending development of an exact one.
3) Derivation of expression for $t_{2}^{\prime}$

The derivation of this expression follows Paloheimo and Vickie's (1964) derivation of their catch equation (p. 157). If school centers are randomly distributed and fishing proceeds for a short enough period so as not to appreciably reduce the population, searching can be expressed as a search for the school centers, with school radius added to the effective range of detection. One vessel searches $2(r+l) v$ area per unit time where $r=$ school radius, $\ell=$ detection range, $v=$ vessel velocity.

The fleet (if randomly distributed) searches $2 \mathrm{Bv}(\mathrm{r}+\boldsymbol{l}$ ) per unit time where $B=$ fleet size. In time $t_{z}^{\prime}$ the fleet will locate one school, when the area covered in time $t_{2}^{\prime}$ is equal to that occupied (on average) by a school.

$$
2 B v(r+l) t_{2}^{\prime}=\frac{1}{\lambda_{2}}
$$

and solving for $t_{2}^{\prime}$

$$
t_{2}^{\prime}=\frac{1}{2 B v \lambda_{2}(r+l)}
$$

This expression is exact insofar as
a) distribution of schools and boats is random within the area where schools are present;
b) searching is random;
c) fishing goes on over a short enough period that density of schools is not appreciably reduced by fishing.


Figure 1. Purse seine fishery model concept.

Input Biomass (A)
most parameter values $(r, v, \ell, x$,
Calculate $\lambda_{2}$ $B, a, b$ etc)
from $\log \lambda_{2}=a \log A+b$
Calculate $R$

$$
=\sqrt{\frac{N}{\pi \lambda_{2}}}
$$

Calculate $t_{1}^{\prime}$

Calculate $\bar{T}$


Calculate $E=$

$$
B t_{1}^{\prime}+(B-1) \bar{T}+(B-1) t_{2}^{\prime}+B \tau
$$

Calculate $C$

$$
=B D S
$$



Output $\mathrm{c} / \mathrm{E}$
(Time for Phase I search)
(Mean transit time)
(Time for Phase II search)
(Capture time - constant here)
Repeat for as many fishery cycles (D) as desired

Catch per unit effort $(t / h r)$.

Figure 2. Purse seine fishery model flowchart.


| $S=100 \mathrm{t}$ | $\mathrm{r}=0.01 \mathrm{~km}$ |
| :--- | :--- |
| $\mathrm{~V}=8 \mathrm{~km} / \mathrm{hr}$ | $\mathrm{t}=1 \mathrm{hr}$ |
| $\mathrm{B}=10 \mathrm{boats}$ | $\mathrm{d}=2 \mathrm{cyc} 1 \mathrm{es}$ |
| $I=1 \mathrm{~km}$ |  |
| $X=10,000 \mathrm{~km}^{2}$ |  |


| Line | a | b | Ratio $A=\lambda_{2}$ |
| :---: | :--- | :--- | :---: |
| 1 | -0.097 | 0 | $10: 1 \times$ |
| 2 | -0.81 | 0.18 | $10: 1.5$ |
| 3 | -1.82 | 0.43 | $10: 2.7$ |
| 4 | -4.10 | 1.00 | $10: 10$ |

Figure 3. Relationship between biomass and catch-per-unit-effort under different assumptions of the biomass-school density relationship.


FLEET SIZE (boats)
$S=100 \mathrm{t}$
$\mathrm{V}=8 \mathrm{~km} / \mathrm{hr}$
1 = 1 km
$X=10,000 \mathrm{~km}^{2}$
$\mathrm{r}: 0.01 \mathrm{~km}$
$a=-1.82\}$
b: 0.43$\}$
$t=1 \mathrm{hr}$
D : 3 cycles

Figure 4. Relationship between number of vessels in fleet and CPUE.


| $A=100,000 \mathrm{t}$ | $\mathrm{r}=0.01 \mathrm{~km}$ |
| :--- | :--- |
| $\mathrm{~S}=100 \mathrm{t}$ | $\mathrm{a}=-1.82$ |
| $\mathrm{~V}=8 \mathrm{~km} / \mathrm{hr}$ | $\mathrm{b}=0.43$ |
| $\mathrm{~B}=10 \mathrm{boats} \mathrm{s}^{2}$ | $\mathrm{t}=1 \mathrm{hr}$ |
| $\mathrm{X}=10,000 \mathrm{~km}^{2}$ | $1=1 \mathrm{~km}$ |

Figure 5. Relationship between number of cycles over which the fleet can exploit a concentration of schools and CPUE.


Figure 6. Relationship between stock distribution area (over which searching is distributed in Phase I) and CPUE.


```
S = 100 t
r = 0.01 km
V = 8 km/hr
a=-1.82
B= 10 boats
b=0.43
I = 1 km
t = 1 hr
X = 10,000 km}\mp@subsup{}{2}{2}\textrm{yr 1;
    decreases 10%/yr
d = 10 cycles yrs 3 & 7
    2 in years 6 & 8
    5 in other yrs.
```

Figure 7. CPUE trend in a simulated fishery with declining biomass, learning factor acting to decrease stock area by $10 \% / \mathrm{yr} .$, bad weather in years 6 and 8, good weather in years 3 and 7, nomal weather in other years.


| $S=100 \mathrm{t}$ | $\mathrm{a}=-1.82$ |
| :--- | :--- |
| $\mathrm{~V}=8 \mathrm{~km} / \mathrm{hr}$ | $\mathrm{b}=0.43$ |
| $\mathrm{I}=1 \mathrm{~km}$ | $\mathrm{t}=1 \mathrm{hr}$ |
| $\mathrm{B}=10$ boats | $\mathrm{D}=3$ cycles |
| $\mathrm{r}=0.01 \mathrm{~km}$ |  |

Figure 8. CPUE trend in a simulated fishery where biomass declines, and learning decreases with time.


```
\(S: 100 \mathrm{t}\)
\(V=8 \mathrm{~km} / \mathrm{hr}\)
1. 1 km
\(X=10,000 \mathrm{~km}^{2}\)
\(r=0.01 \mathrm{~km}\)
```

a $=-1.82$
$b=0.43$
$t=1 \mathrm{hr}$
D $=3$ cycles

Figure 9. CPUE trend in a simulated fishery where biomass declines, learning decreases with time, and fleet size increases then decreases.

