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# Model estimates of harp seal numbers at age for the Northwest Atlantic 

> by

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#### Abstract

Survey estimates of pup production in six years, estimates of annual pregnancy rates, and annual catch-at-age data are used to fit a two parameter age structured population model which describes trajectories of numbers at age for the northwest Atlantic harp seal population for the period 1955 to 1994. Two formulations are considered - one in which the pup mortality rate is equal to that of the $1+$ population and another in which the pup mortality rate is 3 times the $1+$ mortality rate. The uncertainty associated with the population trajectory from the first formulation, and related quantities including replacement harvest, are explored by randomly resampling from the joint probability distribution of the estimated model parameters. The total population is estimated to be about 4.8 million in 1994 and to be growing at about $5 \%$ per year. The estimated replacement harvest at current population size is about 286,700 animals, exceeding the current TAC of 186,000 animals.


## RÉSUMÉ

On s'est servi de l'évaluation de la production des petits en six ans, du taux annuel de gravidité et des données annuelles de prises selon l'âge dans un modèle de population à deux paramètres en fonction de l'âge qui décrit les trajectoires de l'effectif en fonction de l'âge de la population atlantique du phoque du Groenland pour la période de 1955 à 1994. Deux formules sont considérées: l'une dans laquelle le taux de mortalité des petits est égal à celle de la population d'âge $1+$ et une autre dans laquelle le taux de mortalité est égal à trois fois le taux de celle de la population $1+$. On étudie l'incertitude liée à la trajectoire de la population à partir de la première formule, ainsi que des paramètre connexes comme le niveau d'exploitation de remplacement, en prenant un autre échantillonnage au hasard à partir de la distribution de la probabilité commune des paramètres estimés dans le cadre du modèle. La population totale est estimée à environ 4,8 millions en 1994, et elle croît au rythme d'environ $5 \%$ l'an. Le niveau d'exploitation de remplacement estimé pour l'effectif actuel est d'environ 286700 animaux, ce qui est supérieur au TAC actuel qui est de 186000 animaux.

## Introduction

Various approaches to estimating the size of the harp seal population in the northwest Atlantic have been explored in the past. Those methods based primarily on interpreting age composition data fall into two categories - the survival index approach (SI) and virtual population analysis (VPA). Alternative methods have depended on fitting various forms of a two parameter population model (variation of a Leslie model) to independent field estimates of pup production for several years here termed the population model (PM) approach.

The SI method was originally formulated by Sergeant (1971) and then applied by Sergeant (1975), Benjaminsen and Øritsland (1975) and Winters (1978) with minor variations. It provides a single estimate of pup production for a period of years for which pup production is assumed to be constant. Cooke (1985) reviewed the method and concluded that it was unreliable because of its poor mathematical formulation. He provided a modified SI formulation in which the age sample is viewed as a Poisson distributed random variable. By taking into account selectivity-cumcumulative survival to age he developed a multinomial for the age composition for which parameters could be estimated by maximum likelihood. The method performed well on simulated data. He applied the method to the pup kill and age composition sample data for 2 to 8 year old animals taken in the large vessel hunt, tabled in Bowen (1982), to get pup production estimates for two 10 year periods for which pup production was assumed to be constant (1958-67 and 196877). However, in order to obtain a trajectory of population size, the assumption of constant pup production must be replaced with a model in which pup production is linked to the mature population via pregnancy rates. This approach was applied to the northwest Atlantic harp seal population by Cooke et al. (1985) using assumed pregnancy rates.

VPA (more correctly Pope's (1972) approximation to VPA, called cohort analysis) has been applied to harp seal catch-at-age data up to the mid to late 1970s by Lett and Benjaminsen (1977) and Winters (1978). Both VPAs provided very similar trajectories (Stenson et al. 1993), despite the fact that their methods for obtaining terminal fishing mortality differed. Although illustrative of the general relative trends of populations at the time, these applications predate the development of "calibrated" VPAs and therefore are unscaled with respect to absolute population size.

The PM approach involving fitting a population model to independent estimates of pup production was first applied to the northwest Atlantic harp seal population by Roff and Bowen (1983). They suggested that their approach was similar to that of Beddington and Williams (1980). However, Beddington and Williams (1980) fit their population model to catch-at-age data and is therefore more similar to the approach of Cook et al. (1985), whereas Roff and Bowen (1983) fit their model to survey estimates of pup production, using age composition data only in the estimation of initial pup production and in subsequent updating of numbers at age from one year and age to the next.

The PM approach adopted by Cadigan and Shelton (1993) and used here to estimate the trajectory of the number of pups and total population size in each year up to 1994 is very similar to that of Roff and Bowen (1983), but benefits from a more objective method for obtaining initial pup production and an improved method for parameter estimation. The model is applied to six welldocumented independent survey (mark-recapture and aerial) estimates of pup production. Two formulations are considered, Formulation 1 in which the natural mortality rate on pups is the same as that on the $1+$ population, and Formulation 2 in which the natural mortality on the pups is 3 times the mortality on the $1+$ population. Estimates of replacement harvest are made for both formulations. For formulation 1 the probability distribution of total population size, population growth rate, replacement harvest, replacement population size and replacement exploitation rate are
calculated, taking into account only the uncertainty in the population model parameter estimates.

## Methods

The model developed by Cadigan and Shelton (1993) consists of a population dynamics model and a statistical model.

The population dynamics model is

$$
n_{a, t}=\left(n_{a-1, t-1} e^{-\frac{m}{2}}-c_{a-1, t-1}\right) e^{-\frac{m}{2}}
$$

for $0<\mathrm{a}<\mathrm{A}$,

$$
n_{A, t}=\left(n_{A-1, t-1} e^{-\frac{m}{2}}-c_{A-1, t-1}\right) e^{-\frac{m}{2}}
$$

for $\mathrm{a}=\mathrm{A}$, where $\mathrm{A}-1$ is taken as ages $\mathrm{A}-1$ and greater, and

$$
\mathrm{n}_{\mathrm{a}, \mathrm{t}}=\sum_{\mathrm{a}=1}^{\mathrm{A}} \mathrm{n}_{\mathrm{a}, \mathrm{t}} \mathrm{p}_{\mathrm{a}, \mathrm{t}}
$$

for $\mathrm{a}=0$;
where $\mathrm{n}_{\mathrm{a}, \mathrm{t}}=$ population numbers at age a in year t , $c_{a, t}=$ the numbers caught at age $a$ in year $t$,
$p_{a, t}=$ per capita pregnancy rate of age a parents in year $t$, assuming a $1: 1$ sex ratio
$\mathrm{m}=$ instantaneous rate of natural mortality.
A = the "plus" age class (i.e. older ages are lumped into this age class and not dealt with separately, taken as age 12 in this analysis).

In order to estimate numbers at age for years prior to the first year for which continuous pregnancy data are available, it was assumed that the annual pup catch is a constant proportion $s$ of the number of pups born ( $s=(1 /$ exploitation rate $)$ ). Thus, for years prior to the first year for which pregnancy data are available ( $\mathrm{t}_{0}$ )

$$
n_{a, t o-1}=s e^{-m a} c_{0, t} \sigma_{\sigma^{-a}-1}-\sum_{i=1}^{a} e^{-m\left(i-\frac{1}{2}\right)} c_{a-i, t} t_{0^{-i-1}}
$$

for $\mathrm{a}=1$ to $\AA$, where $\AA$ is a terminal (rather than a plus) age ( $=25$ years in the formulations that follow). This equation is applied iteratively to go back in time and fill in the numbers-at-age matrix. The numbers at age for the initial years do not have a large influence on model estimates beyond the mid-1970s but do influence perceptions about the decline and recovery of the population.

The statistical model is

$$
\tilde{\mathrm{n}}_{0, \mathrm{t}_{\mathrm{i}}} \sim \mathrm{~N}\left(\mathrm{n}_{0, \mathrm{t}_{\mathrm{i}}}, \widetilde{\sigma}_{\mathrm{t}_{\mathrm{i}}}^{2}\right)
$$

where $\tilde{\mathrm{n}}_{0, \mathrm{t}_{\mathrm{i}}}$ is the ith survey estimate of $\mathrm{n}_{0, \mathrm{t}_{\mathrm{i}}}$ and $\widetilde{\sigma}_{\mathrm{t}_{\mathrm{i}}}$ is its estimated variance.

The model is rewritten in matrix notation and transformed into a standard nonlinear regression model (Cadigan and Shelton 1993). Maximum likelihood (or equivalently least-squares) estimates of the parameters $m$ and $s$ are obtained using PROC NLIN in SAS applying the Newton iterative method. Following the statistical model given above, the survey estimates of pup production are given weights that are inversely proportional to their variance.

The uncertainty in the population trajectory for Formulation 1 is illustrated by randomly sampling 50 pairs of parameter values ( s and m ) from a bivariate normal distribution defined by the parameter estimates, their standard errors and the correlation between the parameter estimates, and plotting the corresponding population trajectories. While perhaps useful for illustration purposes, many more samples are required to provide an adequate representation of the uncertainty associated with the parameter estimates. The frequency distribution and cumulative probability distribution of estimates of, population size in 1994, population growth rate (total 1994 population divided by total 1993 population), replacement harvest, replacement population size and replacement exploitation rate was estimated from 1000 random samples of pairs of parameter values. This provides only a partial exploration of the uncertainty associated with the estimates. It is conditional on assumptions that the pregnancy rates and catch-at-age estimates are known precisely and that the model structure is correct (e.g. catches taken in the middle of the year, pup mortality is equal to the mortality on the $1+$ population, and for replacement calculations, that the age composition of the catch and the pregnancy rates remain unchanged from recent estimated values). The uncertainty is therefore underestimated in this analysis.

To calculate replacement harvest, the estimated numbers at age up until 1994 were projected to year 2064 using the 1994 estimates of pregnancy-at-age (see below). Catch was removed by applying the 1993 estimated proportions at age in the catch (only the total catch for 1994, i.e. aggregated by age, is presently available). A constant annual total catch for the period 1995 to 2064 was varied until a constant population size was attained.

The total annual catch-at-age up to 1993 (Sjare and Stenson, unpublished data) is given in Table 1 and illustrated in Fig. 1. The data up to 1990 are described in Shelton et al. (1992). The pregnancy-at-age sample data are given in Table 2.

The pregnancy-rate data is characterized by highly variable sample sizes; for example, for the $7+$ age class, the sample size ranges from 1 in 1985 to 164 in 1969. The data are also suggestive of changes in pregnancy rates over time. Rather than use the overall average (by age) or the individual-year estimates, many of which are subject to relatively large sampling error, our objective was to find the most parsimonious representation of pregnancy rates consistent with the data.
"Harmonising" the pregnancy data was accomplished as follows. For a given age class, let $n_{i}$ denote the number of seals examined in year $i$ and let $x_{i}$ denote the number of these determined to be pregnant. We start by forming the 2 times 2 contingency table

$$
\begin{array}{lll}
x_{1} & n_{1}-x_{1} & \ln \\
x_{2} & n_{2}-x_{2} & \mid n_{2} \\
\hdashline x . & n-x . & \ln .
\end{array}
$$

where $\mathrm{x}_{\mathrm{i}}=\mathrm{x}_{1}+\mathrm{x}_{2}$, etc. The conventional $\chi^{2}$ statistic, on 1 d.f., was calculated for this table and if the null hypothesis (of common pregnancy rate) was accepted (at the $5 \%$ level), these data were
pooled and a new 2 times 2 table formed by including the next year's data, namely

where $\mathrm{x}_{.}=\mathrm{x} .+\mathrm{x}_{3}$, etc. This procedure was continued as long the the successive $\chi^{2}$ values remained non significant. When a significant $\chi^{2}$ was encountered, the sequence was terminated, and a new sequence begun, starting with the year for which a (significant) change in pregnancy rate was indicated.

Although the method is as objective as possible, given the data, some minimal amount of subjectivity was nevertheless required:
(i) For Age 3, the procedure grouped 1978 with 1954-70 for an estimate of 0.0192, and an estimate of 0.1017 for 1979-1994. For consistency with Ages 4 and 5, it seems preferable to place 1978 with 1979-94. Also, although not significant, there is a drop in the rate after 1988 which is incorporated in the above table for greater consistency with the estimates for the remaining ages.
(ii) For Age 4, the procedure indicated the break to be between 1988 and 1989 rather than between 1987 and 1988. However the sample ratio for 1988 seems more consistent with the ratios of the following years. Further, placing the break between 1987 and 1988 results in a slightly greater likelihood value for all data combined.
(iii) For Age 5, the procedure indicates a drop in rate for 1993, followed, in 1994, by a return to more or less the rate prior to 1993. There is a reluctance to have different rates for isolated individual years and, since the sample size in 1993 is only 4, it seems reasonable to combine 1993 with 1985-92 and 1994.
(iv) For the same reason, for Age 6, 1986 is included in the set 1967-89. The increase in rate between 1966 and 1967 appears genuine.
(v) For Age 7+, the ratios in 1989 and 1994 are high in relation to their neighbouring years. Again to avoid having different rates for individual isolated years and to obtain the most consistency, 1989 has been included with 1954-58 and 1994 with 1990-93.
(vi) On viewing the overall estimates given above, it was found that the estimates for Age 5 would be more compatible with those for the other ages if the second and third groups were taken as 1978-88 and 1989-94 with estimates 0.8043 and 0.4048 , respectively. Although this departs from the sequential procedure as described, the overall likelihood for this grouping is slightly greater.
(vii) The test statistics used are asymptotic and may be suspect for small sample sizes. Exact tests can be performed. However, since the transitions in the data show up as relatively sharp, it seems unlikely that exact tests will result in any consequential changes.
(viii) The sequential $\chi^{2}$ tests have been carried out moving forwards in time. While this seems logical, from the purely statistical viewpoint, they might equally well have been carried out moving backwards in time from 1994. Again, because the transitions in the data show up as relatively
sharp, moving backwards should give essentially the same outcome.
For years with missing data, the gap was filled by averaging the value within an age class for the year before and after the gap and assuming this average value pertained to the entire period for which no data are available. Alternative methods, such as linear interpolation could have been applied but would have been equally arbitrary.

In the model fits reported here, mark-recapture estimates for 1978, 1979, 1980 and 1980, and the aerial survey estimates for 1990 and 1994 are used. The mark-recapture estimates are critically reviewed in Warren (1991) and all but the 1994 estimate are discussed in Stenson et al. (1993). The 1994 estimate is given in Stenson et al. (1995). The model was applied to estimated pregnancy rates back to 1955 and the catch-at-age data back to 1952 . Thus the pup exploitation rate parameter s was estimated from pup harvests for the three year period 1952 to 1954. Ages 12 and older were lumped into a "plus" age class in the analysis.

## Results and Discussion

The estimates of pregnancy-at-age are given in Table 3 and illustrated in Fig. 2. The estimates suggest that the proportion of pregnant females aged 3 to $7+$ increased in the early 1970s and then decreased abruptly in the late 1980s. These changes are, to some extent, consistent with a density dependent response by the population, if the estimated population trajectory (see below) is accurate. However, because the population trajectory estimated here depends on the pregnancy rates, caution must be used in following this line of reasoning.

Estimates of pup production and total population size for the two formulations are given in Table 4 and illustrated in Figs. 3 and 4. Parameter estimates, estimates of population growth rate, replacement population size, replacement harvest and replacement exploitation rate for both formulations are given in Table 5. A random sample of 50 population trajectories for Formulation 1 is illustrated in Fig. 5. The frequency distribution and cumulative probability plots of estimates of population size in 1994, population growth rate, replacement harvest, replacement population size and replacement exploitation rate estimated from 1000 random samples from the joint probability distribution of the model parameters are given in Figs 6-10.

Pup production trajectories estimated from the two formulations are very similar (Fig. 3). Estimates of pup production from the Winters' (1978) VPA are also plotted for comparison. The overall trend in the VPA and PM estimates are similar, however the VPA pup production in the late 1970s is substantially lower (by about 100,000 pups) than the mark-recapture estimates of the late 1970s and early 1980s. The drop in pup production in 1990 coincides with the abrupt decline in pregnancy rates.

Total population size trajectories for the two formulations are also similar. The VPA estimates at the start of the period are close to those from the PM, but diverge by as much as 500,000 animals in the 1970s (Fig. 4). Overall, the model illustrates a declining population over the 1960s, reaching a minimum in the early 1970s, and then rapidly increasing to the present. The rate at which the population is growing is estimated to have slowed slightly in recent years as a consequence of the decline in the pregnancy rate.

The trajectories of pup production and total population size estimated here are not substantially different from those estimated up to 1980 by Roff and Bowen (1983). Cooke et al. (1985) provide several different trajectories based on using different subsets of the catch data. The pup production
trajectory based on the "large vessel" age samples, as an example, is not very different from the trajectory estimated here. However, pup production trajectories in Winters (1978), Roff and Bowen (1983) and Cooke et al. (1985) all gave numbers below 400,000 for the trough in pup production in the 1970s, whereas the present estimates are just above.

As indicated above, parameter estimates for the two formulations are similar (Table 5). In Formulation 1, the instantaneous rate of natural mortality (all ages), m , is 0.107 , corresponding to an annual survival rate as a result of natural causes of about $90 \%$. Lett and Benjaminsen (1977) and Winters (1978) estimated m from age composition samples from the molting patch to be 0.114 and between 0.08 and 0.109 respectively. The VPA estimates illustrated in Figs. 3 and 4 are for m $=0.1$. In Formulation $2 \mathrm{m0}=0.2695$ and $\mathrm{ml}+=0.0898$. Roff and Bowen (1983) estimated $\mathrm{m}=$ 0.075 and for their separable m formulation, $\mathrm{m} 0=0.2175$ and $\mathrm{ml}+=0.0725$. The estimates of m for Formulations 1 and 2 are somewhat higher than those of Roff and Bowen (1983) but the Formulation 1 estimate is similar to those by Lett and Benjaminsen (1977) and Winters (1978). Note that the separable m formulation (Formulation 2) gives a very similar outcome to the ageindependent $m$ formulation (Formulation 1), confirming the finding of Roff and Bowen (1983) that models of this form are relatively insensitive to this assumption.

Calculations of replacement harvests and equivalent equilibrium population size for the two formulations are quite similar (Table 5). It is noteworthy that, assuming the 1993 age composition of the catch, population growth rate is halted at a relatively low exploitation rate ( $6 \%$ ), although, given current population size, the equilibrium harvest $(286,700)$ is substantially higher than the current TAC of 186,000 animals. Although harvests in the 1950s were as high as 400,000 and averaged about 300,000 over the 1960s (Table 1) the population was declining over this period.

The frequency distributions and cumulative probability plots for the population trajectory and related quantities, including replacement harvest, are underestimates of the uncertainty - they only include the variance in the model parameter estimates and are conditional on all the associated assumptions that have been made. As indicated by the 50 randomly sampled trajectories illustrated in Fig. 5, but shown more clearly in the 1000 realizations in Fig. 6, there is a range of feasible population trajectories. Ignoring the limitations of this analysis it could be considered unlikely that the present population size is below 3.5 million or above 5.1 million. Current population growth rate estimates range from $3.4 \%$ to $5 \%$ (Fig. 7). Growth rate was somewhat higher ( $\pm 8.5 \%$ ) before the recent drop in pregnancy rates. Although replacement harvest may be as low as 170,000 animals, the present analysis suggests that it is around 280,000 but not higher than 300,000 animals, given the assumptions that have been made (Fig. 8). Replacement population size ranges from 3.4 million to 5.0 million (Fig. 9). Replacement exploitation rate is estimated to be about $6 \%$ but may be as low as $4.6 \%$ or has high as $7.2 \%$ (Fig. 10). It is important to note that the replacement harvest and associated exploitation rate will be quite sensitive to any changes in pregnancy rate from the assumed (1994) values.

In the two formulations considered in this working paper, only six mark-recapture and aerial survey estimates are used for fitting the model. Stenson et al. (1993) provide a composite of pup production estimates which includes estimates from VPA (Lett and Benjaminsen 1977, Winters 1978) and modified SI estimates (Cooke 1985). There are also other SI estimates available (e.g Sergeant 1975). Should all these estimates be used to fit the harp seal model? In VPA the instantaneous rate of natural mortality must be provided in order to estimate numbers at age (including pup production) from catch-at-age data and in the SI approaches a quantity or vector of quantities related to mortality are estimated. We maintain that to use estimates of pup production derived from VPA or SI methods in a model in which the natural mortality rate is estimated would be circular and illogical.

There are essentially three kinds of information that relate to estimating the population size of harp seals in the northwest Atlantic: (i) age composition samples (from commercial harvests, research on molting patches and research on pregnant females); (ii) samples of pregnancy at age; and (iii) survey estimates of pup production. Future research into combining these sources of information in a single estimation procedure needs to be considered. The best approach may be an extension of the methods of Cooke (1985) and Cooke et al. (1985) in that likelihood equations are developed to describe the combined probability of all sampled values for different estimates of a few parameters that, when used in a population model, describe the trajectory of the population. In using the age composition of the catch, the variability in the selectivities of 1 -year old and 2 -year old animals (Roff and Bowen 1985, Cook et al. 1985), problems with respect to missing catch data and the very low overall selectivity on $1+$ animals (Shelton et al. 1992) will be important considerations. Research samples of the age composition of seals on the molting patch for the period 1967 to 1983 (Roff and Bowen 1986) and 1992/95 (Stenson and Sjare, unpublished data) have a broader age composition than the commercial hunt data and are likely to be of most value, despite the bias in these kinds of samples documented by Roff and Bowen (1986).

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| Year | Age 0 | Age 1 | Age 2 | Age 3 | Age 4 | Age 5 | Age 6 | Age 7 | Age 8 | Age 9 | Age 10 | Age 11 | Age 12 | Age 13 | Age | Age 15 | Age 16 | Age 17 | Age 18 | Age 191 | Age 201 | Age 21 | Age 22 | Age 23 | Age 24 | Age 25 | To |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1952 | 207800 | 7939 | 12105 | 8279 | 6321 | 7143 | 12212 | 8400 | 8177 | 5910 | 6770 | 6457 | 1743 | 1638 | 2411 | 4205 | 1544 | 2349 | 1892 | 988 | 5363 | 992 | 74 | 487 | 1336 | 1873 | 32440 |
| 1953 | 207712 | 23415 | 753 | 641 | 439 | 3960 | 323 | 2855 | 2915 | 276 | 235 | 353 | 61 | 121 | 972 | 190 | 1899 | 1380 | 880 | 656 | 789 | 1627 | 915 | 642 | 467 | 374 | 28842 |
| 1954 | 186253 | 35151 | 1414 | 5539 | 6033 | 352 | 4080 | 3440 | 3377 | 2125 | 3121 | 2211 | 619 | 2479 | 300 | 1736 | 2245 | 1019 | 340 | 1124 | 626 | 323 | 309 | 707 | 152 | 1102 | 285086 |
| 1955 | 261471 | 23964 | 913 | 6327 | 4948 | 4070 | 3956 | 337 | 3510 | 3020 | 329 | 3503 | 2206 | 1707 | 378 | 235 | 2278 | 149 | 910 | 932 | 257 | 131 | 808 | 669 | 589 | 857 | 350 |
| 1956 | 34 | 13991 | 5305 | 3777 | 3024 | 2393 | 2303 | 2020 | 2027 | 1736 | 1928 | 2246 | 1284 | 1057 | 855 | 1407 | 1307 | 864 | 497 | 518 | 1420 | 75 | 475 | 357 | 323 | 524 | 400270 |
| 1957 | 173121 | 23875 | 8853 | 6680 | 588 | 479 | 4679 | 3658 | 3737 | 3109 | 3468 | 362 | 22 | 1871 | 1391 | 2377 | 2180 | 1471 | 804 | 838 | 2475 | 1306 | 796 | 671 | 582 | 103 | 2655 |
| 1958 | 150916 | 27050 | 10962 | 11685 | 11 | 9828 | 6378 | 6086 | 499 | 4887 | 9733 | 6376 | 6958 | 4001 | 3104 | 8904 | 5905 | 2998 | 545 | 1530 | 5377 | 292 | 2430 | 994 | 194 | 3420 | 311292 |
| 1959 | 244112 | 23 | 9549 | 7664 | 5052 | 4176 | 4223 | 3439 | 325 | 278 | 3198 | 3167 | 2126 | 670 | 273 | 2339 | 220 | 1429 | 769 | 803 | 246 | 1321 | 801 | 634 | 54 | 813 | 333428 |
| 1960 | 165 | 35044 | 13243 | 9350 | 7714 | 5882 | 586 | 4887 | 4801 | 4191 | 4693 | 4763 | 3133 | 2522 | 1801 | 34 | 32 | 20 | 11 | 11 | 364 | 194 | 1163 | 951 | 806 | 1250 | 294 |
| 1961 | 17 | 70 | 264 | 23 | 2632 | 1622 | 1137 | 1252 | 971 | 789 | 1127 | 1224 | 323 | 355 | 433 | 435 | 381 | 294 | 18 | 201 | 243 | 44 | 179 | 85 | 40 | 195 | 202176 |
| 1962 | 212 | 31029 | 34823 | 10145 | 8941 | 6208 | 092 | 2461 | 2565 | 2554 | 33 | 1762 | 1959 | 958 | 1418 | 1856 | 734 | 1747 | 757 | 651 | 1338 | 223 | 617 | 98 | 195 | 714 | 330173 |
| 190 | 27 | 10 | 8751 | 7326 | 4274 | 3489 | 3869 | 3873 | 3590 | 3277 | 3769 | 4191 | 2615 | 2855 | 2802 | 2103 | 2635 | 1807 | 11 | 961 | 930 | 850 | 606 | 463 | 39 | 589 | 3845 |
| 1964 | 271745 | 6500 | 5836 | 6067 | 7317 | 4912 | 6995 | 765 | 3075 | 2742 | 4172 | 2770 | 2174 | 10 | 1639 | 2658 | 092 | 2520 | 3892 | 2035 | 88 | 1979 | 989 | 983 | 147 | 2477 | 203 |
| 1965 | 188184 | 12952 | 6501 | 5317 | 5139 | 6248 | 5921 | 2471 | 984 | 867 | 1435 | 1131 | 1612 | 193 | 1208 | 1046 | 622 | 336 | 658 | 699 | 287 | 455 | 259 | 33 | 18 | 728 | 245 |
| 1966 | 25587 | 14385 | 11 | 518 | 4849 | 5206 | 513 | 493 | 3384 | 1783 | 1987 | 279 | 1745 | 150 | 1745 | 148 | 1382 | 1021 | 1613 | 996 | 1088 | 787 | 317 | 638 | 402 | 105 | 33256 |
| 1967 | 280 | 14683 | 6826 | 2992 | 2452 | 2931 | 3784 | 3232 | 2438 | 1553 | 1465 | 2108 | 1334 | 1002 | 1438 | 1541 | 1064 | 130 | 1362 | 140 | 906 | 585 | 462 | 50 | 283 | 943 | 33885 |
| 19 | 160 | 753 | 4865 | 3590 | 2371 | 222 | 1766 | 257 | 25 | 181 | 187 | 1898 | 102 | 100 | 120 | 1092 | 98 | 818 | 115 | 982 | 738 | 464 | 567 | 316 | 245 | 567 | 204 |
| 19 | 2371 | 21346 | 390 | 3422 | 2722 | 3099 | 2200 | 224 | 2980 | 2397 | 2117 | 2107 | 1181 | 1286 | 1300 | 1419 | 944 | 1402 | 911 | 1055 | 958 | 605 | 394 | 475 | 243 | 829 | 541 |
| 19 | 221 | 9399 | 7603 | 2865 | 2345 | 2204 | 1352 | 1394 | 13 | 1074 | 1566 | 1392 | 994 | 716 | 630 | 721 | 810 | 399 | 453 | 444 | 383 | 305 | 23 | 172 | 111 | 203 | 26015 |
| 1971 | 21285 | 8281 | 309 | 2068 | 1328 | 1011 | 745 | 608 | 485 | 648 | 648 | 897 | 521 | 315 | 26 | 281 | 218 | 184 | 225 | 172 | 231 | 152 | 13 | 90 | 51 | 30 | 23581 |
| 1972 | 120263 | 4862 | 2798 | 1745 | 1475 | 746 | 657 | 664 | 417 | 364 | 402 | 814 | 457 | 329 | 313 | 352 | 276 | 190 | 165 | 103 | 151 | 63 | 176 | 16 | 18 |  | 37860 |
| 1973 | 103435 | 7060 | 4875 | 3264 | 2575 | 358 | 1845 | 1129 | 1460 | 782 | 819 | 1306 | 57 | 570 | 400 | 48 | 452 | 28. | 179 | 222 | 225 | 103 | 85 | 71 | 54 | 27 |  |
| 1974 | 119 | 131 | 7783 | 3370 | 2556 | 2407 | 2771 | 1358 | 1051 | 1002 | 755 | 881 | 680 | 700 | 524 | 421 | 382 | 316 | 307 | 193 | 116 | 85 | 94 | 79 | 130 | 432 | 160998 |
| 1975 | 144449 | 14 | 6247 | 3276 | 1886 | 1371 | 1282 | 1104 | 674 | 580 | 925 | 558 | 456 | 37 | 265 | 389 | 301 | 25 | 161 | 218 | 158 | 114 | 92 | 94 | 96 | 87 | 析 |
| 1976 | 136 | 15565 | 769 | 4166 | 256 | 743 | 395 | 410 | 419 | 182 | 15 | 400 | 232 | 124 | 172 | 144 | 118 | 79 | 75 | 49 | 78 | 34 | 47 | 48 | 33 | 35 | 170 |
| 1977 | 134893 | 9222 | 6831 | 6580 | 5066 | 3075 | 1702 | 1081 | 681 | 407 | 507 | 537 | 291 | 233 | 448 | 463 | 271 | 120 | 81 | 83 | 84 | 45 | 36 | 41 | 38 | 98 | 172914 |
| 1978 | 121058 | 18409 | 11010 | 5958 | 3938 | 25 | 18 | 663 | 82 | 290 | 815 | 224 | 24 | 279 | 122 | 286 | 107 | 143 | 182 | 98 | 152 | 123 | 118 | 47 | 41 | 93 | 169600 |
| 1979 | 139200 | 16161 | 7580 | 4345 | 2691 | 2009 | 1459 | 1058 | 729 | 515 | 796 | 268 | 260 | 179 | 294 | 391 | 208 | 195 | 72 | 84 | 20) | 120 | 110 | 44 | 47 | 625 | 179640 |
| 19 | 13 | 18205 | 9770 | 6269 | 4249 | 3305 | 2243 | 1690 | 1117 | 756 | 410 | -665 | 699 | 470 | 576 | 656 | 393 | 413 | 184 | 255 | 295 | 135 | 108 | 100 | 104 | 152 | 940 |
| 1981 | 18459 | 916 | 5038 | 3830 | 2409 | 1887 | 1748 | 1027 | 845 | 482 | 417 | 575 | 398 | 269 | 282 | 382 | 304 | 407 | 266 | 245 | 207 | 238 | 22 | 11 | 91 | 24 | 2156 |
| 1982 | 153096 | 14996 | 7195 | 3444 | 1727 | 1307 | 715 | 733 | 425 | 351 | 346 | 302 | 227 | 166 | 173 | 336 | 155 | 234 | 115 | 182 | 233 | 124 | 87 | 108 | 74 | 193 | 18704 |
| 1983 | 58544 | 7608 | 576 | 14 | 416 | 115 | 943 | 679 | 535 | 350 | 409 | 289 | 295 | 164 | 216 | 375 | 82 | 217 | 177 | 156 | 212 | 111 | 76 | 101 | 95 | 169 | 81759 |
| 198 | $\underline{31850}$ | 590 | 5315 | 280 | 1729 | 921 | 722 | 582 | 420 | 336 | 386 | 264 | 256 | 145 | 155 | 275 | 130 | 175 | 11 | 12 | 180 | 84 | 100 | - 64 | 71 | 105 | 5321 |
| 1985 | 2169 | 6725 | 13 | 2517 | 1222 | 747 | 591 | 536 | 388 | 281 | 34 | 213 | 215 | 147 | 22 | 359 | 166 | 171 | 144 | 152 | 189 | 85 | 71 | 82 | 71 | 119 | 4236 |
| 1986 | 2824 | 4747 | 336 | 2412 | 121 | 662 | 562 | 419 | 328 | 263 | 310 | 226 | 229 | 136 | 168 | 264 | 131 | 161 | 121 | 122 | 182 | 104 | 78 | 75 | 82 | 149 | 74 |
| 1987 | 40951 | 568 | 4139 | 3369 | 2234 | 1171 | 1012 | 806 | 591 | 536 | 479 | 441 | 308 | 240 | 244 | 377 | 238 | 249 | 177 | 173 | 282 | 161 | 95 | 155 | 161 | 694 | 64 |
| 1988 | 74244 | 10722 | 9602 | 5489 | 3500 | 1737 | 1689 | 805 | 776 | 534 | 390 | 308 | 342 | 382 | 310 | 443 | 380 | 304 | 363 | 456 | 50 | 171 | 116 | 15 | 29. | 770 | 114980 |
| 198 | 6117 | 675 | 512 | 3377 | 2167 | 1514 | 1068 | 481 | 352 | 286 | 389 | 307 | 295 | 20 | 163 | 309 | 203 | 157 | 245 | 135 | 236 | 142 | 275 | 153 | 159 | 329 | 86008 |
| 1990 | 40482 | 8905 | 5952 | 5141 | 4128 | 515 | 184 | 106 | 432 | 644 | 581 | 844 | 695 | 675 | 342 | 639 | 255 | 406 | 308 | 106 | 498 | 84 | 211 | 275 | 73 | 105 | 7915 |
| 1991 | 49673 | 6005 | 3395 | 2740 | 2527 | 1994 | 1251 | 642 | 452 | 386 | 461 | 537 | 485 | 395 | 252 | 421 | 157 | 246 | 218 | 186 | 190 | 84 | 152 | 67 | 15 | 431 | 73499 |
| 1992 | 51155 | 10763 | 7125 | 4024 | 3173 | 2257 | 2040 | 1549 | 819 | 1033 | 676 | 526 | 370 | 270 | 289 | 305 | 414 | 487 | 318 | 539 | 224 | 157 | 79 | 210 | 147 | 59 | 8954 |
| 1993 | 23695 | 7095 | 4659 | 2673 | 1833 | 125 | 116 | 867 | 513 | 572 | 459 | 348 | 273 |  |  | 253 | 242 | 283 | 192 | 293 | 180 | 116 | 73 |  | 103 |  |  |

Table 1. Catch at age for the northwest Atlantic harp seal.

Table 2. Proportion of females pregnant at age from samples together with sample sizes.

| Year |  | Age 3 |  | Age 4 |  | Age 5 |  | Age 6 |  | Age 7+ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Prop | N | Prop | N | Prop | N | Prop | N | Prop | N |
| 1954 | 0.00 | 4 | 0.33 | 3 | 0.67 | 3 | 0.75 | 16 | 0.88 | 33 |
| 1955 |  |  |  |  |  |  |  |  |  |  |
| 1956 |  |  |  |  |  |  |  |  |  |  |
| 1957 |  |  |  |  |  |  |  |  |  |  |
| 1958 |  |  |  |  |  |  |  |  |  |  |
| 1959 |  |  |  |  |  |  |  |  |  |  |
| 1960 |  |  |  |  |  |  |  |  |  |  |
| 1961 |  |  |  |  |  |  |  |  |  |  |
| 1962 |  |  |  |  |  |  |  |  |  |  |
| 1963 |  |  |  |  |  |  |  |  |  |  |
| 1964 |  |  |  |  |  |  |  |  |  |  |
| 1965 | 0.03 | 30 | 0.11 | 44 | 0.54 | 37 | 0.71 | 38 | 0.88 | 109 |
| 1966 | 0.00 | 7 | 0.11 | 9 | 0.35 | 17 | 0.73 | 11 | 0.88 | 49 |
| 1967 | 0.00 | 10 | 0.21 | 19 | 0.61 | 33 | 0.97 | 29 | 0.89 | 123 |
| 1968 | 0.00 | 27 | 0.32 | 19 | 0.70 | 20 | 0.92 | 12 | 0.87 | 55 |
| 1969 | 0.04 | 25 | 0.16 | 25 | 0.44 | 16 | 0.82 | 28 | 0.88 | 164 |
| 1970 | 0.00 | 13 | 0.23 | 13 | 0.50 | 12 | 0.90 | 10 | 0.86 | 107 |
| 1971 |  |  |  |  |  |  |  |  |  |  |
| 1972 |  |  |  |  |  |  |  |  |  |  |
| 1973 |  |  |  |  |  |  |  |  |  |  |
| 1974 |  |  |  |  |  |  |  |  |  |  |
| 1975 |  |  |  |  |  |  |  |  |  |  |
| 1976 |  |  |  |  |  |  |  |  |  |  |
| 1977 |  |  |  |  |  |  |  |  |  |  |
| 1978 | 0.03 | 40 | 0.61 | 38 | 0.90 | 20 | 0.67 | 9 | 0.85 | 41 |
| 1979 | 0.33 | 9 | 0.67 | 9 | 1.00 | 3 | 1.00 | 4 | 0.91 | 11 |
| 1980 | 0.00 | 2 | 0.50 | 2 | 1.00 | 1 |  |  | 0.83 | 12 |
| 1981 | 0.20 | 5 | 0.50 | 4 | 0.50 | 2 | 0.86 | 7 | 0.78 | 18 |
| 1982 | 0.00 | 4 | 0.40 | 5 | 1.00 | 1 | 0.75 | 4 | 0.33 | 3 |
| 1983 |  |  |  |  |  |  |  |  |  |  |
| 1984 |  |  |  |  |  |  |  |  |  |  |
| 1985 | 0.00 | 4 | 0.33 | 3 | 0.40 | 5 | 1.00 | 3 | 1.00 | 1 |
| 1986 | 1.00 | 1 |  |  | 0.50 | 2 | 0.00 | 1 | 1.00 | 7 |
| 1987 | 0.17 | 12 | 0.38 | 8 | 0.78 | 9 | 1.00 | 4 | 0.63 | 24 |
| 1988 | 0.06 | 16 | 0.17 | 6 | 1.00 | 3 |  |  | 0.74 | 19 |
| 1989 | 0.00 | 8 | 0.00 | 9 | 0.33 | 6 | 0.67 | 3 | 0.95 | 22 |
| 1990 | 0.00 | 8 | 0.14 | 7 | 0.33 | 3 | 0.00 | 1 | 0.60 | 10 |
| 1991 | 0.09 | 11 | 0.18 | 11 | 0.57 | 7 | 0.33 | 3 | 0.61 | 28 |
| 1992 | 0.20 | 10 | 0.27 | 11 | 0.44 | 9 | 0.75 | 8 | 0.66 | 32 |
| 1993 | 0.00 | 8 | 0.12 | 17 | 0.00 | 4 | 0.75 | 8 | 0.39 | 23 |
| 1994 | 0.05 | 20 | 0.14 | 14 | 0.46 | 13 | 0.50 | 6 | 0.83 | 30 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

Table 3. Estimates of pregnancy at age (see text for method used).

| Year | Age 3 | Age 4 | Age 5 | Age 6 | Age 7+ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1955 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1956 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1957 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1958 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1959 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1960 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1961 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1962 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1963 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1964 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1965 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1966 | 0.0172 | 0.1818 | 0.5435 | 0.7231 | 0.8648 |
| 1967 | 0.0172 | 0.1818 | 0.5435 | 0.8684 | 0.8648 |
| 1968 | 0.0172 | 0.1818 | 0.5435 | 0.8684 | 0.8648 |
| 1969 | 0.0172 | 0.1818 | 0.5435 | 0.8684 | 0.8648 |
| 1970 | 0.0172 | 0.1818 | 0.5435 | 0.8684 | 0.8648 |
| 1971 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1972 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1973 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1974 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1975 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1976 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1977 | 0.057 | 0.36625 | 0.7162 | 0.8684 | 0.8648 |
| 1978 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1979 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1980 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1981 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1982 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1983 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1984 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1985 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1986 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1987 | 0.0968 | 0.5507 | 0.8043 | 0.8684 | 0.8648 |
| 1988 | 0.0968 | 0.1467 | 0.8043 | 0.8684 | 0.8648 |
| 1989 | 0.0615 | 0.1467 | 0.4048 | 0.8684 | 0.8648 |
| 1990 | 0.0615 | 0.1467 | 0.4048 | 0.6154 | 0.6341 |
| 1991 | 0.0615 | 0.1467 | 0.4048 | 0.6154 | 0.6341 |
| 1992 | 0.0615 | 0.1467 | 0.4048 | 0.6154 | 0.6341 |
| 1993 | 0.0615 | 0.1467 | 0.4048 | 0.6154 | 0.6341 |

Table 4. Pup production and total population size estimates for the period 1955 to 1995 for model Formulation 1 (mortality on pups = mortality on the $1+$ population) and Formulation 2 (mortality on pups $=3$ times the mortality on the $1+$ population).

|  | Formulation 1 |  | Formulation 2 |  |
| :---: | :---: | :---: | :---: | :---: |
| Year | Pups | Total population | Pups | Total population |
|  |  |  |  |  |
| 1955 | 509184.23 | 2804495 | 496789.91 | 2624143.7 |
| 1956 | 522981.19 | 2709660.3 | 512220.12 | 2542104.8 |
| 1957 | 540463.69 | 2594927.3 | 531350.92 | 2450064.9 |
| 1958 | 543818.95 | 2622906.4 | 536528.73 | 2469549.5 |
| 1959 | 513605.73 | 2574474.6 | 508150.32 | 2410979.7 |
| 1960 | 493130.28 | 2489518.7 | 489250.1 | 2336302.3 |
| 1961 | 461990.88 | 2419095.3 | 458762.61 | 2265405.4 |
| 1962 | 470566.86 | 2451785.7 | 465879.22 | 2302048 |
| 1963 | 471671.04 | 2360945.4 | 465923.32 | 2217834.9 |
| 1964 | 464005.04 | 2249252 | 458021.85 | 2120411.2 |
| 1965 | 452062.83 | 2138164.1 | 446053.2 | 2021050.4 |
| 1966 | 447384.18 | 2135397.2 | 442061.05 | 2017477.7 |
| 1967 | 441952.23 | 2044777.1 | 436939.46 | 1936905.1 |
| 1968 | 426807.11 | 1942276.8 | 423722.2 | 1848629.4 |
| 1969 | 412931.79 | 1963366.3 | 411328.36 | 1866860.4 |
| 1970 | 401861.47 | 1882322.1 | 401038.57 | 1797432.3 |
| 1971 | 414091.75 | 1858237.9 | 411773.82 | 1780531 |
| 1972 | 411913.35 | 1857490.2 | 410647.66 | 1784314.6 |
| 1973 | 411835.09 | 1949575.5 | 411856.12 | 1868241.5 |
| 1974 | 405049.72 | 2027164.3 | 406954.57 | 1938906.3 |
| 1975 | 400543.91 | 2068756.7 | 403909.96 | 1979906 |
| 1976 | 410603.35 | 2098544.7 | 413904.83 | 2013988.6 |
| 1977 | 432298.38 | 2155207.3 | 434716.4 | 2071560.6 |
| 1978 | 473797.75 | 2245725.5 | 473145.45 | 2157266.9 |
| 1979 | 482037.37 | 2338409.1 | 482464.92 | 2240143.8 |
| 1980 | 490971 | 2421075.8 | 492490.61 | 2317779.2 |
| 1981 | 501574.76 | 2496679.8 | 503292.45 | 2388232.4 |
| 1982 | 523566.57 | 2561667 | 524187.58 | 2454386.2 |
| 1983 | 549680.56 | 2673298.2 | 549373.13 | 2559331.6 |
| 1984 | 575235.63 | 2898902.4 | 574748.72 | 2762639.9 |
| 1985 | 595699.88 | 3149056.2 | 595943.19 | 2988952.9 |
| 1986 | 625364.15 | 3413694.8 | 625345.33 | 3230817.9 |
| 1987 | 677094.89 | 3700862.3 | 674556.07 | 3495443 |
| 1988 | 672270.76 | 3934806.6 | 673714.78 | 3711758.4 |
| 1989 | 667203.48 | 4092470.6 | 672347.11 | 3865670.6 |
| 1990 | 560521.69 | 4154860.1 | 560154.43 | 3920017 |
| 1991 | 603707.33 | 4260577.8 | 602308.86 | 4032016.1 |
| 1992 | 647392.34 | 4404579.6 | 645462.53 | 4178062.4 |
| 1993 | 683227.17 | 4554551.2 | 681963.65 | 4326467.8 |
| 1994 | 714525.13 | 4759984.9 | 715017.01 | 4525148.4 |

Table 5. Comparison of estimates from model Formulations 1 and 2.

| Model estimates | Model 1 | Model 2 |
| :---: | :---: | :---: |
|  | $\mathrm{MO}=\mathrm{M} 1+$ | M0 $=3$ * ${ }^{\text {1 }}+$ |
| Instantaneous mortality rate | 0.107364154 | 0.089826031 |
| Pups |  | 0.269478093 |
| Proportion survival rate | 0.898198531 | 0.914090195 |
| Pups |  | 0.763778011 |
| 1/Exploitation rate on pups (1952-54) | 2.912800865 | 2.92833602 |
| Exploitation rate (pups) | 0.343312175 | 0.341490865 |
| Total population size |  |  |
| 1993 | 4554551.2 | 4326467.8 |
| 1994 | 4759984.9 | 4525148.4 |
| Growth rate | 1.045105147 | 1.045922126 |
| Number of pups in 1994 | 714525.13 | 715017.01 |
| Approximate replacement |  |  |
| Replacement population size | 5030000 | 4648000 |
| Replacement harvest | 286700 | 274450 |
| Exploitation rate | 0.06 | 0.06 |



Fig.1. Total annual catch for the period 1952 to 1994.

Estimates of pregnancy ages 3 to $7+$


Fig. 2. Estimates of proportion of females pregnant-at-age used in the model.

Comparision of pup production estimates ('000s) from Winters (1978) and current population model estimates


Fig. 3. Trajectories of pup production for the period 1955 to 1994 from the model fit to the 6 survey estimates of pup production. The trajectory from the VPA estimates by Winters (1978) is shown for comparison.

Comparison of Winters (1978) VPA estimates of total population ('000s) with those from the current population model


Fig. 4. Trajectories of total population size for the period 1955 to 1994 from the fit of the two formulations of the model to the survey estimates of pup production. The trajectory of total population size from the VPA by Winters (1978) is shown for comparison.

## 50 Realizations of population trajectory



Fig. 5. A random sample of 50 trajectories of total population size from the joint probability distribution of the model parameters for Formulation 1 (pup mortality $=$ mortality on the $1+$ population).

## Total population in 1994



## Total population in 1994



Fig. 6. Frequency and cumulative probability distribution plots for the total population size in 1994 obtained from 500 random samples from the joint probability distribution of the model parameters for Formulation 1.

## Population growth rate 1993-94 Population growth rate 1993-94




Fig. 7. Frequency and cumulative probability distribution plots for the total population percentage growth rate ( 1994 over 1993) obtained from 500 random samples from the joint probability distribution of the model parameters for Formulation 1


Fig. 8. Frequency and cumulative probability distribution plots for the replacement harvest obtained from 500 random samples from the joint probability distribution of the model parameters for Formulation 1.

## Replacement population size



Replacement population size


Fig. 9. Frequency and cumulative probability distribution plots for the population size at replacement from 500 random samples from the joint probability distribution of the model parameters for Formulation 1.

## Replacement exploitation rate Replacement exploitation rate




Fig. 10. Frequency and cumulative probability distribution plots for the replacement harvest exploitation rate (\%) from 500 random samples from the joint probability distribution of the model parameters for Formulation 1.

