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## The Analysis of Comparative Trawl Selectivity Experiments

by<br>Pierre Gagnon<br>Biological Sciences Branch, Department of Fisheries and Oceans<br>Maurice Lamontagne Institute, P.O. Box 1000<br>Mont-Joli (Québec) Canada G5H 3Z4

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#### Abstract

The size selectivity of fishing gear can be measured by comparative selectivity experiments where a non-selective fishing method is compared with a test gear on the same prey density. The results from many such experiments suggest that the efficiency of even closely related gear types is not the same. This possibly confounding effect is modelled in the SELECT analysis method (Millar and Walsh, 1992) as a fixed efficiency difference. We describe an implementation of this method that provides diagnostics for this fixed effect by modelling the ratio of the two efficiencies as a nonlinear function of size. Statistical tests for model selection and graphical diagnostics are also presented with some examples.


## Résumé

La sélectivité des engins de pêche selon la taille peut être mesurée en comparant les résultats d'une méthode de pêche non sélective à ceux que l'on obtient avec un engin expérimental dans une même densité de proies. Il ressort d'une telle comparaison que l'efficacité des engins, même de ceux qui se ressemblent étroitement, diffère. Cette variable confusionnelle possible est modélisé dans la méthode d'analyse SELECT (Millar et Walsh, 1992) sous forme de différence d'efficacité fixe. Nous décrivons ici une application de cette méthode qui permet de déterminer la valeur fixe en question en modélisant le rapport des deux efficacités comme fonction non linéraire de la taille. Nous présentons aussi des test statistiques pour la sélection du modele et des diagnostiques graphiques, ainsi que des exemples.

## Introduction

Selective fishing gear is important for fisheries management because it can improve the survival rate of small fish while maintaining good commercial yield. Trawls are size-selective because smaller fish can escape through the meshes of the codend and also because fish of different sizes will have different probabilities of avoiding the gear (Foster et al. 1981, MacLennan 1992).

The size-selectivity of fishing gear describes the probability of capture as a function of fish size. In general, the size-selectivity of a fishing method can be expressed as the product of the selectivities of all its components. The selectivity of a trawl can be measured by comparing its catch with the catch obtained using a non-selective trawl under the same conditions.

Different fishing methods can have different efficiencies. The efficiency of a fishing method mesures the amount of fish caught per unit of fishing effort. Comparative trawl selectivity experiments are designed to achieve equal efficiency for both trawls (equal tow speeds, swept areas, vertical openings, cable lengths, door configurations, etc.) and exposure to equal fish densities such that the differences in catches can be interpreted directly as differences of selectivity by the codends. This symmetry implies that for sizes fully retained by both codends, the expected number of fish caught should be the same. Nonetheless, despite all the precautions taken to achieve a fair comparison, the situation arises where it is obvious that the two trawls did not have the same efficiency. The assumption of equal efficiency in such cases will lead to erroneous selectivity parameter estimates with possibly adverse consequences for the management of fisheries.

The SELECT method of Millar and Walsh (1992) estimates the conditional odds ratio of entering one trawl or the other by maximum likelihood. This method assumes that the fish caught had an unequal probability of entering both trawls and that this probability is independent of the fish size.

In the implementation of this method for logistic selectivities that we present here, we use profile likelihood to estimate the selectivity parameters and we provide some statistical tests of this independence by expanding the selectivity model to allow size-dependent efficiency differences. We also use an efficient parameterization of the logistic equation and provide some "true" likelihood contours.

## Model

Fish capture is a random process. An experiment involving fish capture must therefore be analysed in probabilistic terms. In a gear comparison experiment, one of the trawls is usually equipped
with a non-selective fine mesh codend and is considered as a reference against which the selectivity of a test trawl is measured.

In probabilistic terms, the question addressed by the experiment is:

## Given that a fish of a certain size is caught in one of the trawls, what is the probability

 that it will be in the test trawl?The catch data from the experiment can be summarized by a model that answers this question. Suppose that a fish having size $x$ has a probability $P_{r}(x)$ of entering and being caught in the reference trawl and $P_{t}(x)$ of entering and being caught in the test trawl. Then if it is caught, it has a probability

$$
\begin{equation*}
\phi_{t}(x)=\frac{P_{t}(x)}{P_{t}(x)+P_{r}(x)} \tag{1}
\end{equation*}
$$

of being caught in the test trawl and $1-\phi_{t}(x)$ of being caught in the reference trawl.
Two types of fish are likely to escape an active fishing gear: smaller fish will escape through the meshes while larger fish may be fast enough to avoid the gear. Thus $P_{t}(x)$ can be expressed as the product $\left(1-P_{t s}(x)\right) P_{t m}(x)$ where $P_{t s}(x)$ is the probability that a fish of size $x$ will avoid the test trawl and $P_{t m}(x)$ is the probability that it will be retained in the test codend. The probability of entering and being caught in the reference trawl can be expressed simply as $P_{r}(x)=1-P_{r g}(x)$ since the reference trawl is non-selective for all sizes of interest. The probability of being caught in the test trawl (1) can thus be written

$$
\begin{equation*}
\phi_{t}(x)=\frac{P_{t m}(x)}{P_{t m}(x)+E_{\mathrm{r}}(x)} \tag{2}
\end{equation*}
$$

where $E_{r}(x)$ is the efficiency ratio defined as

$$
\begin{equation*}
E_{r}(x)=\frac{1-P_{r s}(x)}{1-P_{t s}(x)} \tag{3}
\end{equation*}
$$

The retention odds ratio for the test gear will be assumed to be a logistic function of size with a $50 \%$ retention size $s$ and a range $r$ between the $75 \%$ and the $25 \%$ retention sizes:

$$
\begin{equation*}
\frac{P_{t m}(x)}{1-P_{t m}(x)}=e^{-k\left(\frac{z-2}{r}\right)} \tag{4}
\end{equation*}
$$

where $k=2 \ln (3)$. This parameterization involving "stable" parameters instead of the generic $e^{a x+b}$ will yield parameter estimates which are less correlated and will improve the convergence rate of typical minimization algorithms. It is also easier to find good initial guesses for parameters $s$ and $r$ from the data. These properties of stable parameters are discussed by Ross (1970).

For the efficiency ratio, a more general function of the form

$$
\begin{equation*}
E_{r}(x)=e^{f(x)} \tag{5}
\end{equation*}
$$

will be assumed. The function $f(x)$ will be approximated by its second Taylor polynomial at s

$$
\begin{equation*}
f(x) \approx p+t(x-s)+u(x-s)^{2} \tag{6}
\end{equation*}
$$

The $p$ term in this polynomial represents the component of the efficiency ratio that does not depend on fish size (i.e. this term cannot be associated with a difference in swimming avoidance between trawls because absolute swimming speed increases with size.) The $t$ and $u$ terms will show the main components of size-dependent efficiency. If there is no difference of efficiency between the trawls, $E_{r}(x)$ should be equal to unity for all values of $x$ and thus $p, t$ and $u$ should be zero.

In the SELECT method $E_{r}(x)$ does not depend on the size $x$, it is the conditional odds ratio

$$
\begin{equation*}
E_{r}(x)=\frac{q}{1-q} \tag{7}
\end{equation*}
$$

where $q$ is the probability that when a fish cannot avoid the gear, it enters the reference codend.

## Parameter estimation

The data from a gear comparison experiment can be interpreted as the result of a binomial experiment. It consists of two sets of fish sizes $S_{r}$ and $S_{t}$. The set $S_{r}$ contains the sizes of the $n_{r}$ fish that were caught by the reference gear and $S_{t}$ the $n_{t}$ sizes caught by the test gear. Assuming that fish behave independently, the log-likelihood of obtaining these sets of sizes is

$$
\begin{equation*}
l=\sum_{x_{i} \in S_{t}} \log \left(\phi_{t}\left(x_{i}\right)\right)+\sum_{x_{j} \in S_{r}} \log \left(1-\phi_{t}\left(x_{j}\right)\right) . \tag{8}
\end{equation*}
$$

The maximization of $l$ over the parameters of $\phi_{t}(x)$ will give the value of these parameters that are the most likely for the data.

The function $\phi_{t}(x)$ in the expression of the likelihood describes more than the trawl selectivity. It has five adjustable parameters ( $s, r, p, t, u$ ) of which only $s$ and $r$ describe the trawl selectivity. The other parameters account for a possible difference of efficiency between the trawls, they should be considered as nuisance parameters.

## Size classes

The common practice of rounding the sizes of fish into size classes does not improve the quality of the analysis of selectivity models. In fact, if the width of the size classes is not much smaller than the parameter $r$ of our model (4), its estimation could be quite poor (see Houde and Lepage 1991).

Nevertheless, most selectivity data is kept in the size classes format instead of the size sets format. The log-likelihood (8) for size classes can be expressed as:

$$
l=\sum_{i} n_{i} \log \left(\phi_{l}\left(x_{i}\right)\right)+m_{i} \log \left(1-\phi_{l}\left(x_{i}\right)\right)
$$

plus the constant term ( $\sum_{i} \log \left({ }_{i}{ }_{n_{i}}^{+m_{i}}\right)$ ) that can be ignored, where $n_{i}$ and $m_{i}$ are the number of fish in size class $i$ caught in the test and reference trawls respectively and $x_{i}$ is the centre of size class $i$.

## Sampling ratios

The selectivity data must reflect the true amount of information that was gathered during the experiment. The log-likelihood of the catches of fish in a given size class is directly proportional to the number of fish of that size that were counted. The adjustment of these counts to reflect the total catch when only a fraction of the catch was effectively measured will lead to an artificial deflation of the likelihood and offset all the derived statistics. The proper method for handling large catches is to sample the same fraction from the catches of both codends. When this is not possible or for data that is already collected, the numbers of fish reported should be scaled down to reflect the total number of fish measured.

## Model selection and inferences

Equation (6) can include anywhere from zero to three significant terms. The corresponding nested selectivity ratio models all share the following assumptions: A) the individual fish are behaving independently; $B$ ) the codend of the reference trawl is non-selective; $C$ ) the fishing behaviour of the experimental gear was the same for all sets; D) the selectivity of the test codend is a logistic function of fish size. Assumptions $A, B$ and $C$ can be verified by in situ observation of the catching process while the consistency of assumption $D$ should be checked by graphical and statistical examination of the deviance residuals. Assumption $C$ can also be analysed statistically by looking at individual set results (Fryer, 1991).

Table 1 shows how the models differ in what they assume about the efficiency ratios.

| efficiency ratio model | constraints | hypotheses |
| :--- | :---: | :--- |
| quadratic trend | none | the efficiency ratio appears to depend non-linearly on <br> fish size |
| linear trend | $u=0$ | the efficiency ratio appears to depend linearly on fish <br> size |
| unequal, no trend | $t=u=0$ | the trawls have different efficiencies, but the ratio of <br> efficiencies is constant for all fish sizes |
| equal efficiency | $p=t=u=0$ | both trawls have equal efficiencies for all sixes of fish. |

Table 1: Efficiency ratio models

Because each model in Table 1 is nested in the preceding ones, its relative likelihood can be compared to test the corresponding hypotheses. For example, if the data has maximum log likelihood $l_{1}$ when the quadratic trend model is fitted and of $l_{3}$ when the unequal efficiency model is fitted, then $2\left(l_{1}-l_{3}\right)$ can be compared with $\chi_{2,1-\alpha}^{2}$ to test at level $\alpha$ the presence of an apparent non-linear trend in the efficiency ratio.

In the cases where there is an apparent size-related trend in the efficiency ratio, the applicability of the SELECT model with a logistic codend selectivity function should seriously be called in question. However, the shape of the efficiency ratio relationship should not be interpreted directly in terms of some underlying fish escapement mechanism because escapement and selectivity effects can be confounded if the true selectivity of the codend is not a logistic function of size.

| sire | test | ref. | size | test | ref. | size | test | ref. |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 24 | 0 | 1 | 32 | 91 | 70 | 40 | 17 | 5 |
| 25 | 0 | 1 | 33 | 120 | 108 | 41 | 14 | 6 |
| 26 | 0 | 3 | 34 | 118 | 88 | 42 | 10 | 10 |
| 27 | 1 | 14 | 35 | 107 | 84 | 43 | 4 | 1 |
| 28 | 5 | 30 | 36 | 78 | 68 | 44 | 6 | 6 |
| 29 | 19 | 49 | 37 | 52 | 37 | 45 | 2 | 2 |
| 30 | 29 | 60 | 38 | 40 | 33 | 46 | 5 | 1 |
| 31 | 51 | 50 | 39 | 17 | 12 | 47 | 1 | 0 |

Table 2: Alternate haul selectivity experiment with haddock. Numbers of fish caught by size ( cm ) in 85 mm (test) and 37 mm (reference) mesh codends. Data from Pope et al. (1975) p. 48.

A good measure of function shape adequacy is the residual signs runs test. This non parametric test is based on the number of sequences of size classes with observed selectivities on the same side (over or under) of the predicted selectivity. When the pattern of the observed selectivities does not correspond to the predicted selectivity function shape, the number of runs tends to be lower than would be expected from a random process (see Draper and Smith, 1981).

The lack of fit deviance statistic on the other hand, has a more dubious usefulness for model selection. Its value strongly depends on the size class width that is used. The presence of a single outlier size class will easily inflate the deviance into the poor fit region. The chi-square approximation that is usually assumed for this statistic is valid only if it is independent from the estimated parameters. This independence hypothesis is not even approximately true (McCullagh and Nelder 1989, pp. 118-119).

## Examples

The applicability of these hierarchical selectivity ratio models and some extensions will best be demonstrated by some examples. Some useful graphical diagnostics will also be introduced.

## Textbook case

Let's first consider the data in Table 2, a textbook case by Pope et al. (1975). In this alternate haul experiment, the catches of haddock with a 87 mm mesh codend are compared with the catches obtained with a 35 mm mesh codend. As is often the case, the large mesh codend seems to catch more large fish than the small mesh codend.

The maximum likelihood parameter estimates for this data corresponding to the four models of Table 1 are presented in Table 3. The runs test $p$-values indicate very clearly the difference in the quality of fit between the equal and unequal efficiency models. Given the restricted number of hauls involved in this experiment - two with each mesh size - and the alternate haul design, the constant difference of efficiency can be interpreted simply as a difference in the densities of fish that were encountered by both types of gear. If the difference persisted over more hauls, then a true difference of fishing efficiency should be suspected.

| efficiency ratio | $s$ | $r$ | $p$ | $t$ | $u$ | runs |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| quadratic trend | 30.4 | 2.27 | -0.407 | 0.06 | -0.006 | .5969 |
| linear trend | 29.8 | 2.19 | -0.143 | -0.02 |  | .5827 |
| unequal, no trend | 30.2 | 2.40 | -0.293 |  |  | .5969 |
| equal | 29.4 | 1.78 |  |  |  | .0067 |

Table 3: Selectivity model parameters (italicized values are not significant) and runs test probability on the deviance residuals for alternate haul experiment with haddock (Pope et al. 1975). The models are illustrated in Figure 1.

The data and the models are illustrated in Figure 1. The models assuming a trend in the efficiency ratio differ from the no-trend model only for fish larger than 40 cm . These catch proportions involve only a small number of fish in the experiment and therefore do not give much credibility to the trend in efficiency ratio hypothesis.

The likelihood contours and profile traces for the parameters of the unequal efficiency with no trend model are shown in Figure 2.

This figure shows the most likely values for the parameters and provides some information on their interaction: the curvature and slope of the profile traces indicate how the parameter estimates influence each other (see Bates and Watts, 1988). In a linear regression, the contours would be elliptical and the profile traces would be two strait lines, one vertical and one horizontal.

Data insufficiency or model over-parameterization can be detected in two ways from this type of graph. Strong correlation between the parameter estimates will be indicated by tilted profile traces, the orthogonality of the estimates implies the orthogonality of the profile traces. Pronounced skewness of the $\log$ likelihood function in the neighbourhood of the estimates will result in curved traces crossing at an off-center position in the likelihood confidence region.

## Trouser trawl: comparing mesh types

The second example consists of two trouser trawl cod selectivity experiments extracted from a larger set conducted by C. G. Cooper and R. G. Halliday (Bedford Institute of Oceanography, P.O. Box 1006, Dartmouth, N.S. Canada; unpublished data). In a trouser trawl arrangement, the entire trawl is split in the middle with a fine mesh panel. One side of the divided codend is built with fine mesh netting and serves as a reference while the other side has a construction similar to the tested codend.

In the first experiment of this example, the test side of the codend consists of 155 mm diamond mesh netting while in the second experiment it consists of square mesh netting of the same size. The efficiency ratio models parameters are listed in Table 4 and illustrated in Figure 3. For both experiments the efficiency ratio shows a definite positive linear trend, moreover, the runs test indicates that the no trend model doesn't have the appropriate shape for the diamond mesh data. The experimenter must therefore examine this data with great caution. The presence of an apparent size related efficiency ratio indicates the possible failure of assumption $A, C$ or $D$ for this data.


Figure 1: Proportion of haddock caught in the test trawl as a function of size (cm) during an alternate haul experiment (Pope et al. 1975). The vertical bars are binomial $90 \%$ confidence intervals. The efficiency models (Table 3) are: equal efficiency - short dashes; unequal efficiency, no trend - solid line; linear trend - long dashes; quadratic trend - short-long dashes.


Figure 2: $75 \%$ and $95 \%$ likelihood contours and profile traces for the parameter estimates of the unequal efficiency with no trend model fitted to the data of Table 2. The solid line is the optimum $s$ for fixed $r$ profile and the dashed line is the optimum $r$ for fixed $s$ profile.

| mesh | efficiency ratio | $\boldsymbol{s}$ | $\boldsymbol{r}$ | $\boldsymbol{p}$ | $\boldsymbol{t}$ | $\mathbf{u}$ | runs |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 155D | quadratic trend | 68.1 | 7.8 | -0.870 | 0.072 | -0.00036 | .0580 |
| 155D | linear trend | 68.4 | 8.3 | -0.881 | 0.067 |  | .1447 |
| 155D | unequal, no trend | 64.6 | 10.2 | -0.458 |  |  | .0153 |
| 155D | equal | 60.9 | 9.1 |  |  |  | .0015 |
| 155S | quadratic trend | 69.1 | 6.0 | -0.761 | 0.071 | -0.00103 | .0316 |
| 155S | linear trend | 68.7 | 6.9 | -0.669 | 0.038 |  | .0826 |
| 155S | unequal, no trend | 66.6 | 7.3 | -0.342 |  |  | .2636 |
| 155S | equal | 64.5 | 6.5 |  |  |  | .0046 |

Table 4: Model parameters (italicized values are not significant) and runs test probability on the deviance residuals for a trouser trawl experiment comparing cod catches with diamond and square meshes. The models are illustrated in Figure 3.


Figure 3: Trouser trawl experiment comparing mesh types. Left: proportion of cod caught in the test trawl as a function of size ( cm ). Right: corresponding deviance residuals for the different efficiency ratio models as a function of size ( cm ).

| mesh | efficiency ratio | $\boldsymbol{s}$ | $\boldsymbol{r}$ | $\boldsymbol{p}$ | $\boldsymbol{t}$ | $\boldsymbol{u}$ | runs |
| :---: | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 130 | quadratic trend | 33.5 | 6.8 | -0.130 | -0.01 | -0.00004 | .6162 |
| 130 | linear trend | 33.4 | 6.8 | -0.115 | -0.01 |  | .5249 |
| 130 | unequal, no trend | 34.7 | 7.9 | -0.283 |  |  | .2939 |
| 130 | equal | 32.3 | 4.8 |  |  |  | .2622 |
| 140 | quadratic trend | 45.7 | 13.2 | -0.005 | -0.09 | -0.00086 | .2647 |
| 140 | linear trend | 49.3 | 11.1 | -0.399 | -0.01 |  | .1308 |
| 140 | unequal, no trend | 48.3 | 11.2 | -0.300 |  |  | .1308 |
| 140 | equal | 45.4 | 9.5 |  |  |  | .2993 |

Table 5: Model parameters (italicized values are not significant) and runs test probability on the deviance residuals for trouser trawl experiment comparing cod catches with different diamond mesh sizes. The models are illustrated in Figure 4.

Note that these assumptions will always be more difficult to validate for selectivity experiments that involve a large number of fish. When looked at closely enough, all these simplifying assumptions will be violated to a certain degree.

## Trouser trawl: comparing mesh sizes

The third example is a pair of trouser trawl experiments where a 130 or 140 mm codend is used side by side with a 60 mm codend (D. Gascon, Maurice Lamontagne Institute, P.O. Box 1000, Mont-Joli, Quebec, Canada; unpublished data). The selectivity ratio models summarize the features of the data. Their estimated parameters are presented in Table 5. These experiments also show significant differences between the efficiency of the test and the reference trawls for both mesh sizes but no size-related trend. The data and the model fits are illustrated in Figure 4.

In both cases there is no indication that the efficiency ratio could be a function of fish size, this and the fact that no codend side switches were performed suggest that the differences of efficiency could be simply due to differences in trawl opening sizes.

Experiments involving many mesh sizes paired with the same reference can also be used to compare their selectivities and efficiencies. Under the hypothesis that the reference trawl performed the same way when paired with both mesh sizes, the ratio of the expected retention probabilities gives a measure of the expected catch ratio. For example, the retention ratio between the 140 mm and the 130 mm mesh codends is given by

$$
\frac{P_{t}^{(140)}(x)}{P_{t}^{(130)}(x)}=\frac{\left(1-P_{t=}^{(140)}(x)\right) P_{t m}^{(140)}(x)}{\left(1-P_{t=}^{(130)}(x)\right) P_{t m}^{(130)}(x)}=\frac{E_{r}^{(130)}(x) P_{t m}^{(140)}(x)}{E_{r}^{(140)}(x) P_{t m}^{(130)}(x)}
$$

The shape of this function for this example is shown on Figure 4. The fisheries management preoccupations with mesh changes are addressed directly by this function. For every fish size, it shows what proportion of the catch will be spared by the mesh change. It can also show how much gain in efficiency can be expected for fully selected sizes. In this example, the gain is negligible.


Figure 4: Trouser trawl experiment comparing mesh sizes. Left: proportion of cod caught in the test trawl as a function of size (cm). Right top: deviance residuals from the unequal efficiencies with no trend model. Right bottom: predicted retention probability ratio of 140 mm mesh codend over 130 mm mesh codend as a function of size ( cm ).

## Discussion

The high price of conducting selectivity experiments and the importance of the decisions that depend on the selectivity parameters justify that some time be spent on their proper design and analysis. As for most field experiments, stringent control through standardization, randomization and monitoring is the most important feature of the design of comparative trawl selectivity experiments. A good representation from fish of all sizes including the fully selected sizes must also be ensured. This is in contrast with the study of linear phenomena where the location of the experimental range is less important.

The size selection process of active fishing gear is very complex (Foster et al. 1981). More research is needed on the mechanisms of fish capture before more sophisticated models can be proposed. It is nevertheless important to assess the appropriateness of simple models describing the outcome of this process before making important decisions based on them.

The hypothesis that fishing gear efficiency could depend on fish size cannot be rejected a priori. For comparative fishing experiments such as alternate tows, parallel tows and trouser trawls the presence of such a dependence might invalidate the measurements of selectivity. It is therefore preferable to postulate its existence and show its undetectability than to simply ignore it. However, when efficiency is detected to be size dependent, it cannot safely be compensated for because there is no way to separate the effects of the violation of the assumptions about efficiency and selectivity.

A computer program that calculates the parameter estimates for the four efficiency ratio models from size-class data, some associated statistics and the data needed to make figures similar to Figure 1 and 2 is available from the author.

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