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Estimating partial recruitment from catches and a research survey index: a Monte-Carlo simulation

by

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ABSTRACT

Four methods for estimating partial recruitment coefficients from catch at age and a research index at age are analysed by Monte-Carlo simulations. The author discusses the statistical properties of these four estimates, in particular in terms of their bias and variance.

RESUME

On évalue, au moyen de simulations de Monte-Carlo, quatre méthodes d'estimation des coefficients de recrutement partiel à partir de la prise et d'un indice de recherche, ces derniers étant connus pour chacun des groupes d'âge. L'auteur discute les propriétés statistiques des quatre méthodes d'estimation, en particulier, leur biais et leur variance.

Estimating partial recruitment from catches and a research survey index: a Monte-Carlo simulation.

When an estimate of population abundance at age is available from a research survey, it is common practice to calculate partial recruitment coefficients for a given age from the ratio of the relative frequency at age of catch and of the survey index. This empirical method has the advantage of providing partial recruitment estimates independently of sequential population analysis. In addition, partial recruitment coefficients can be calculated for various fleet components. While this method is generally accepted, its response to various sources of error is still not well understood. We discuss below various estimates of partial recruitment in terms of their statistical properties, such as consistency, bias and variance.

Estimation methods.

The partial recruitment coefficients were estimated from catch-at-age and a research index at age by four different methods.

First, partial recruitment was defined as the ratio of catch-at-age (in percent) and the research index at age (also in percent), normalized so that the maximum value does not exceed unity. In other words,

$$P_a = \frac{C_a/C_t}{R_a/R_t} \Big/ \mathrm{MAX}\left[\frac{C_a/C_t}{R_a/R_t}\right]$$

where C_a = catch at age a;

R_a = research index at age a;

$C_t = \sum_a C_a$;

$R_t = \sum_a R_a$.

This method will be referred to as the RN (Ratio-Normalized) method.

In the second method, the P_a were smoothed by the 3RSR algorithm of McNeil (1977) and the resulting vector was normalized. This method will be referred to as the RSN (Ratio-Smooth-Normalized) method.

The third method requires that a range of fully recruited ages be specified. The range can be as small (say two or three consecutive age-groups) or as large as desired. Large ranges can force, however, a flat-top partial recruitment curve and can thus introduce some bias. For our simulation, we know that ages 6 to 8 were fully recruited for the data generated with both a dome-shaped and a flat-top curve. These ages were thus taken as the range of fully recruited ages. The partial recruitment coefficients were then defined for each age group as

$$P_a = P'_a / (\sum P'_a / \text{nb of values in range})$$

where the summation is over the range of fully recruited ages and where

$$P'_a = (C_a / C_t) / (R_a / R_t)$$

for values outside the range, and as unity (i.e., $P_a = 1$) for values within the range. This method is referred to as the RNF (Ratio-Normalized by Fully recruited ages) method.

The fourth method is an extension of the RNF method. The P_a were smoothed by the 3RSR algorithm of McNeil (1977). This method is referred to as the RNFS (Ratio-Normalized by Fully recruited ages-Smoothed) method.

Monte-Carlo simulations.

Data for the Monte-Carlo simulations were obtained by projecting two stocks over a period of 20 years. Growth and natural mortality values were assumed to be similar to those of a cod stock and two different partial recruitment curves were used. The catch and the population abundance (mid-year) for each age-group in the last year of the projection were used in the Monte-Carlo simulation. These appear in Table 1. From the population abundance at age (say N_a), a survey index was derived in the following manner:

$$R_a = N_a + k N_a e$$

where e was drawn from a normally distributed random variable, with mean=0 and standard deviation=1. Six different data sets were developed, each for k=0.05, 0.10, 0.15, 0.20, 0.25 and 0.30 . For each set, one hundred survey index vectors were drawn, each vector having 13 elements. The catches, say C_a, were assumed to be without error.

In addition, another data set was created, assuming that the variance of the survey index varies with age (see Table 1, last column).

Results.

The results appear in Tables 2-5. With no error in input data (i.e., k=0), the true values of partial recruitment (see Table 1) were obtained from each of the four estimation methods. A dome-shaped curve was found when a dome was present and a flat-top curve was found when all fish were fully recruited above a certain age. The four estimation methods are thus consistent when the research index is representative of the stock abundance at mid-year and when fishing mortality is evenly distributed during the year. These methods may not be consistent, however, when the assumptions regarding the time of the research survey or the temporal distribution of the fishing mortality are violated.

The RN estimates are seriously biased (Table 2; Figure 1) and are quite variable, as illustrated in Fig. 2. The bias increases as the uncertainties of the research index increases. For levels of variation usually encountered in research surveys, bias could be as high as 45 percent for certain age-groups (Table 2, last column).

The RSN estimates have a smaller bias (Table 3; Figure 1) and estimates are closer to the true value, as illustrated in Fig. 2. For levels of variation usually encountered in research surveys, bias is of the order of 10-15 percent for most age-groups (Table 3, last column).

The RNF estimates are not significantly biased (Table 4) but are quite variable for older age groups, as illustrated in Fig. 3. The RNFS estimates (Table 5) are less variable, as illustrated in Fig. 3. Both the RNF and the RNFS methods allow some of the partial recruitment coefficients to be greater than one. Such values (greater than unity) are likely to occur only in older age groups, i.e. those age groups for which the precision of partial recruitment estimates is low. Forcing all the values greater than one to take the value of one would reduce the variance but introduce a bias. The bias introduced is likely to be small in comparison to the gain in precision. Ideally, for each value greater than one adjusted to one, a neighbouring value lower than one should be adjusted to one: the resulting adjustment would be approximately unbiased if the underlying partial recruitment curve is flat-top. For dome-shaped curves, this procedure could introduce a bias but the occurrence of values greater than unity in that case would indicate either that the dome is weak or that the survey index has a low precision. In either case, any adjustment procedure would do equally well (or bad) since data contain no useful information for discriminating between the two shapes.

Among the four methods studied, the RNFS method provides the best compromise in terms of precision and bias. The reader should note, however, that choosing the wrong range of fully recruited ages would introduce a bias for the estimates of some age-groups. The bias should be small if a narrow range is chosen, say two to four age-groups.

Bibliography.

McNeil, D.R. 1977. Interactive data analysis: a practical primer. John Wiley & Sons Inc. New York. 186 pages.

Table 1. Data used for the Monte-Carlo simulation.

Age	Weight	Partial recruitment	Catch numbers	Research index	Partial recruitme	Catch numbers	Research index	Relative error (%)
3	0.25	0.01	226	90371	0.01	226	90371	60
4	0.72	0.10	1828	72979	0.10	1828	72979	30
5	0.98	0.50	6960	55433	0.50	6960	55433	15
6	1.31	1.00	9486	37625	1.00	9486	37625	15
7	1.71	1.00	6048	23991	1.00	6048	23991	15
8	2.21	1.00	3857	15297	1.00	3857	15297	20
9	2.64	1.00	2459	9754	0.90	2239	9877	20
10	3.24	1.00	1568	6220	0.80	1316	6538	25
11	3.72	1.00	1000	3965	0.70	781	4438	25
12	4.41	1.00	638	2529	0.60	466	3088	30
13	6.25	1.00	406	1612	0.55	303	2190	35
14	6.52	1.00	259	1028	0.50	197	1573	40
15	7.23	1.00	165	656	0.50	143	1136	45

Table 2. Results of the Monte-Carlo simulation for the RN method.

		error level (%)							
Age	Real	0	5	10	15	20	25	30	Age related
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
4	0.10	0.10	0.09	0.08	0.08	0.07	0.06	0.06	0.06
5	0.50	0.50	0.46	0.43	0.40	0.37	0.34	0.31	0.27
6	1.00	1.00	0.92	0.84	0.77	0.70	0.63	0.57	0.53
7	1.00	1.00	0.93	0.85	0.79	0.72	0.65	0.59	0.54
8	1.00	1.00	0.93	0.86	0.79	0.72	0.65	0.59	0.55
9	1.00	1.00	0.92	0.84	0.77	0.70	0.63	0.56	0.53
10	1.00	1.00	0.93	0.85	0.78	0.72	0.65	0.58	0.56
11	1.00	1.00	0.93	0.85	0.78	0.72	0.65	0.59	0.56
12	1.00	1.00	0.93	0.85	0.78	0.72	0.65	0.59	0.57
13	1.00	1.00	0.93	0.86	0.79	0.72	0.65	0.59	0.59
14	1.00	1.00	0.93	0.85	0.79	0.72	0.66	0.60	0.61
15	1.00	1.00	0.93	0.86	0.80	0.73	0.67	0.55	0.62

Age	Real	0	5	10	15	20	25	30	Age related
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
4	0.10	0.10	0.09	0.09	0.09	0.08	0.08	0.07	0.08
5	0.50	0.50	0.48	0.46	0.44	0.43	0.41	0.38	0.38
6	1.00	1.00	0.95	0.91	0.86	0.81	0.75	0.68	0.73
7	1.00	1.00	0.96	0.92	0.88	0.83	0.78	0.72	0.75
8	1.00	1.00	0.96	0.92	0.88	0.84	0.78	0.72	0.76
9	0.90	0.90	0.86	0.82	0.78	0.73	0.68	0.63	0.66
10	0.80	0.80	0.77	0.74	0.70	0.67	0.62	0.58	0.62
11	0.70	0.70	0.67	0.64	0.62	0.59	0.55	0.51	0.54
12	0.60	0.60	0.57	0.55	0.53	0.51	0.48	0.45	0.48
13	0.55	0.55	0.53	0.51	0.49	0.46	0.43	0.41	0.46
14	0.50	0.50	0.48	0.46	0.44	0.42	0.40	0.38	0.45
15	0.50	0.50	0.48	0.46	0.45	0.43	0.41	0.35	0.47

N.B.: the values given for each error level are the average of 100 trials.

Table 3. Results of the Monte-Carlo simulation for the RSN method.

		error level (%)							
Age	Real	0	5	10	15	20	25	30	Age related
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
4	0.10	0.10	0.10	0.10	0.09	0.09	0.09	0.09	0.09
5	0.50	0.50	0.49	0.49	0.48	0.48	0.47	0.46	0.44
6	1.00	1.00	0.94	0.89	0.85	0.80	0.76	0.74	0.78
7	1.00	1.00	0.97	0.94	0.91	0.88	0.85	0.83	0.84
8	1.00	1.00	0.98	0.95	0.93	0.91	0.89	0.87	0.86
9	1.00	1.00	0.98	0.96	0.94	0.92	0.90	0.89	0.87
10	1.00	1.00	0.98	0.96	0.94	0.92	0.91	0.89	0.87
11	1.00	1.00	0.98	0.96	0.95	0.93	0.91	0.90	0.88
12	1.00	1.00	0.98	0.96	0.94	0.92	0.91	0.89	0.87
13	1.00	1.00	0.98	0.96	0.94	0.93	0.91	0.90	0.88
14	1.00	1.00	0.98	0.96	0.94	0.93	0.91	0.89	0.88
15	1.00	1.00	0.98	0.97	0.95	0.94	0.92	0.90	0.89

Age	Real	0	5	10	15	20	25	30	Age related
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.00
4	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.11
5	0.50	0.50	0.51	0.53	0.54	0.56	0.58	0.57	0.55
6	1.00	1.00	0.98	0.97	0.94	0.92	0.90	0.87	0.92
7	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.97	0.97
8	1.00	1.00	0.99	0.99	0.99	1.00	0.99	0.99	0.98
9	0.90	0.90	0.91	0.93	0.94	0.96	0.97	0.96	0.94
10	0.80	0.80	0.82	0.84	0.85	0.87	0.89	0.89	0.86
11	0.70	0.70	0.72	0.73	0.75	0.78	0.80	0.79	0.78
12	0.60	0.60	0.61	0.64	0.65	0.67	0.69	0.69	0.68
13	0.55	0.55	0.56	0.58	0.59	0.62	0.64	0.63	0.62
14	0.50	0.50	0.53	0.55	0.56	0.59	0.61	0.60	0.60
15	0.50	0.50	0.51	0.53	0.55	0.57	0.60	0.59	0.60

N.B.: the values given for each error level are the average of 100 trials.

Table 4. Results of the Monte-Carlo simulation for the RNF method.

		error level (%)						
Age	Real	0	5	10	15	20	25	30
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.50	0.50	0.50	0.50	0.51	0.52	0.53	0.55
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	0.99	0.99	1.00	1.01	1.12
10	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01
11	1.00	1.00	1.00	1.01	1.01	1.02	1.03	1.05
12	1.00	1.00	1.00	1.00	1.01	1.02	1.03	1.05
13	1.00	1.00	1.00	1.01	1.01	1.02	1.03	1.04
14	1.00	1.00	1.00	1.01	1.01	1.02	1.04	1.06
15	1.00	1.00	1.00	1.01	1.03	1.05	1.12	0.98

Age	Real	0	5	10	15	20	25	30
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.50	0.50	0.50	0.50	0.51	0.51	0.52	0.55
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	0.90	0.90	0.89	0.89	0.89	0.89	0.91	1.00
10	0.80	0.80	0.79	0.80	0.80	0.80	0.80	0.80
11	0.70	0.70	0.69	0.70	0.70	0.71	0.71	0.73
12	0.60	0.60	0.60	0.60	0.60	0.61	0.61	0.63
13	0.55	0.55	0.55	0.55	0.55	0.56	0.56	0.56
14	0.50	0.50	0.49	0.50	0.50	0.51	0.51	0.53
15	0.50	0.50	0.50	0.50	0.51	0.52	0.56	0.49

N.B.: the values given for each error level are the average of 100 trials.

Table 5. Results of the Monte-Carlo simulation for the RNFS method.

		error level (%)						
Age	Real	0	5	10	15	20	25	30
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.50	0.50	0.50	0.50	0.51	0.51	0.50	0.49
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	0.99	0.98	0.97	0.96	0.93
10	1.00	1.00	1.00	0.99	0.99	0.98	0.96	0.95
11	1.00	1.00	1.00	1.00	0.99	0.99	0.97	0.96
12	1.00	1.00	1.00	1.00	0.99	0.98	0.97	0.95
13	1.00	1.00	1.00	1.00	0.99	0.99	0.98	0.96
14	1.00	1.00	1.00	1.00	0.99	0.99	0.98	0.97
15	1.00	1.00	1.00	1.00	1.00	1.01	1.01	0.99

Age	Real	0	5	10	15	20	25	30
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.50	0.50	0.50	0.50	0.51	0.50	0.50	0.49
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9	0.90	0.90	0.89	0.88	0.88	0.86	0.84	0.83
10	0.80	0.80	0.79	0.79	0.79	0.78	0.77	0.76
11	0.70	0.70	0.69	0.70	0.70	0.69	0.69	0.67
12	0.60	0.60	0.60	0.60	0.60	0.60	0.59	0.59
13	0.55	0.55	0.55	0.55	0.55	0.55	0.54	0.54
14	0.50	0.50	0.51	0.52	0.52	0.52	0.52	0.51
15	0.50	0.50	0.49	0.50	0.51	0.51	0.51	0.50

N.B.: the values given for each error level are the average of 100 trials.

Fig. 1. Bias of partial recruitment estimates (ages 7 and 8, flat–top model) as a function of error level in the survey index.

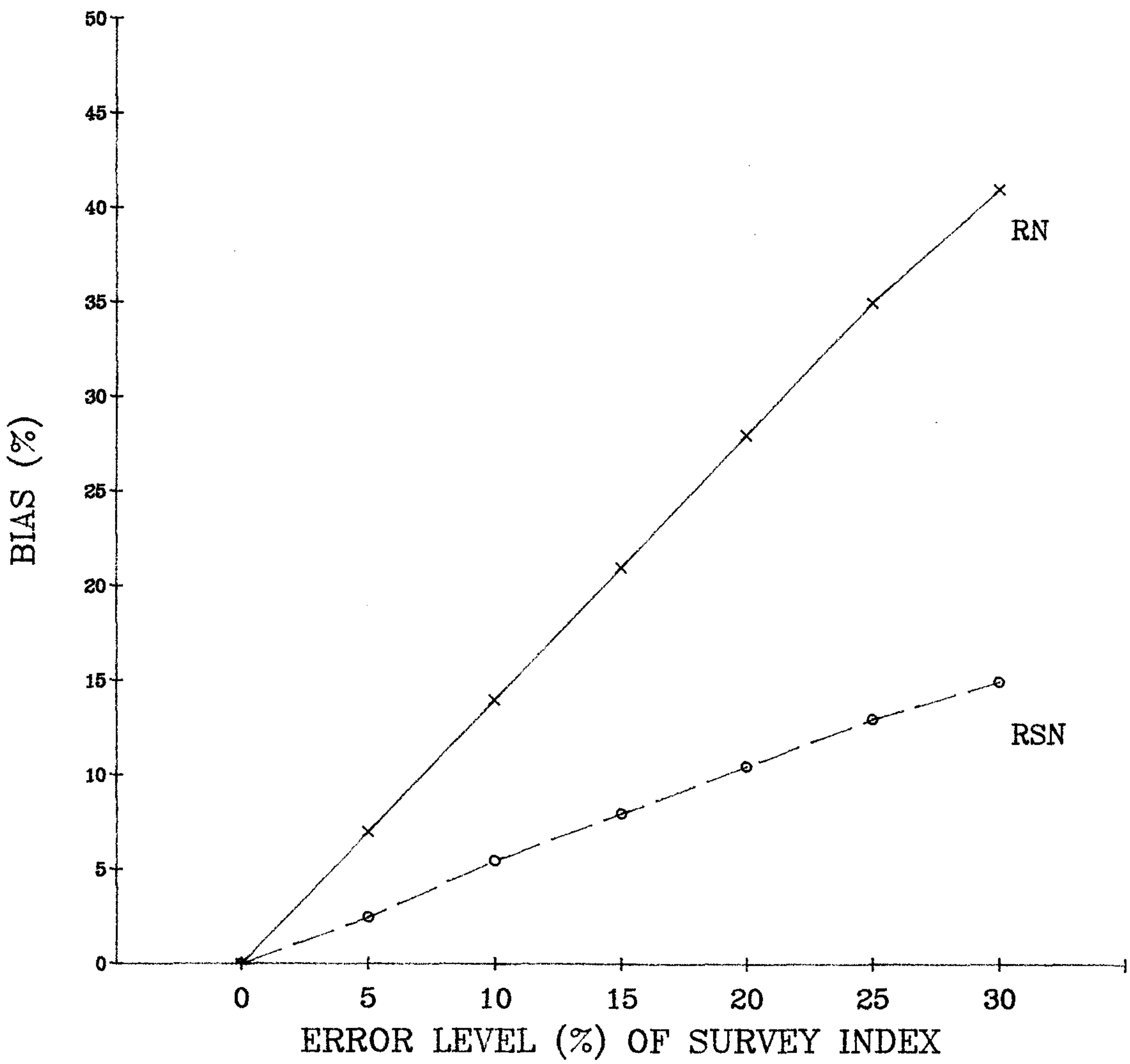

Fig. 2. Frequency distribution of partial recruitment estimates for the RN and the RSN methods. Results are for the estimates obtained at age 6 from the dome-shaped model, when a relative error of 30% is applied.

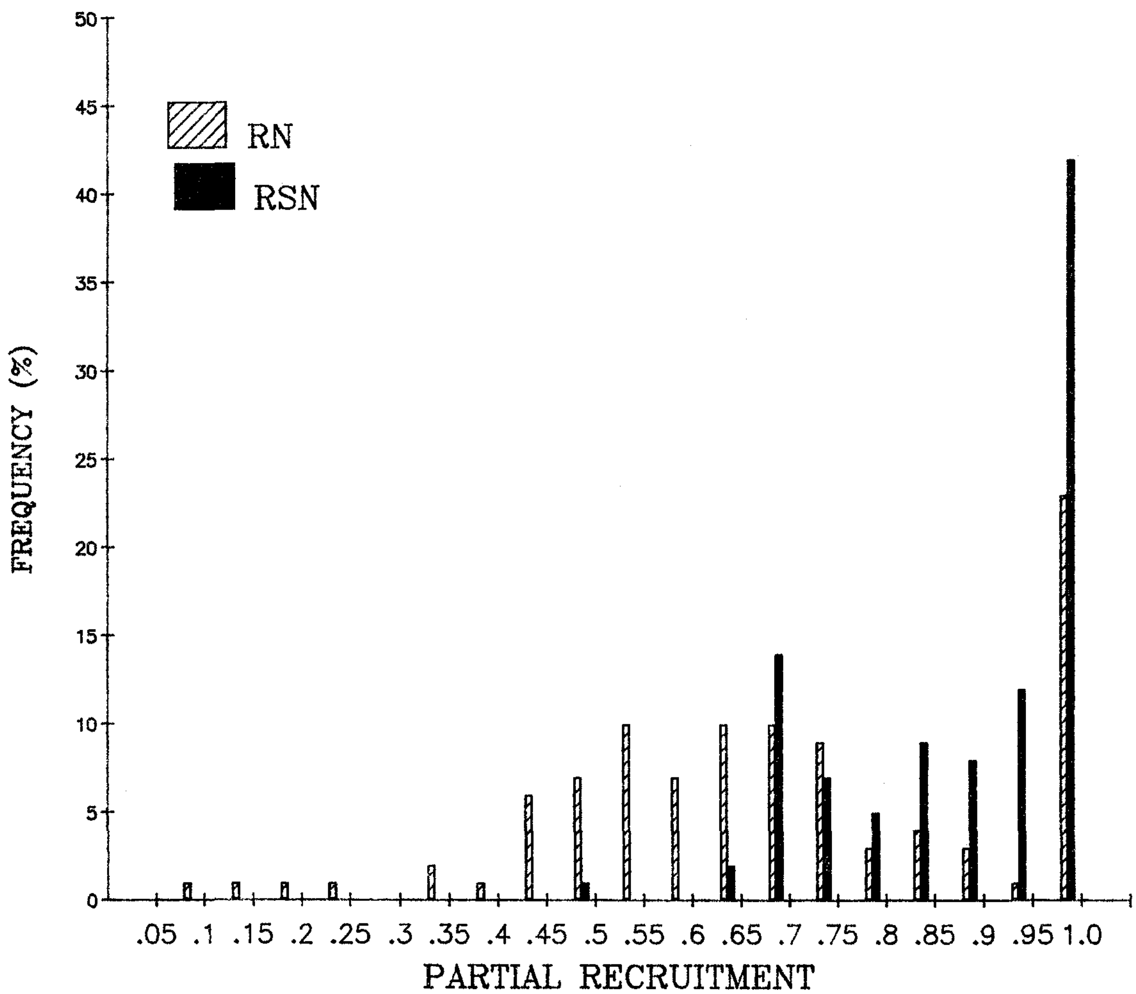

Fig. 3. Frequency distribution of partial recruitment estimates for the RNF and the RNFS methods. Results are for the estimates obtained at age 12 from the dome-shaped model, when a relative error of 30% is applied. The true value is 0.6.