Assessment of haddock in NAFO Division 5Z

by

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ABSTRACT

Canadian landings in 1987 increased to about 4700 t, accounting for nearly 70% of the total, the highest proportion historically. Two year-classes, 1983 and 1985, contributed substantially to the catch with the 1985 year-class, age 2 haddock, being dominant in the Canadian fishery. Results indicate that few old fish remain in the population and the exploitation rate of young haddock is high. Abundance and biomass are near historical low levels. Recruitment has been variable with some of the lowest year-classes occurring in recent years.

RÉSUMÉ

En 1987, les débarquements canadiens ont augmenté, passant à environ 4 700 t. Ils représentaient près de 70 % de tous les débarquements, soit la proportion la plus élevée que l'on ait connue jusqu'ici. Deux classes d'âge, celles de 1983 et de 1985 ont contribué de façon notable aux prises, en particulier la classe de 1985--aiglefin de deux--qui prédominait dans les pêcheries canadiennes. Les résultats révèlent qu'il reste peu de vieux poisson dans la population et que le taux d'exploitation du jeune aiglefin est élevé. L'abondance et la biomasse approchent des plus bas niveaux jamais enregistrés. Le recrutement s'est avéré variable et s'est manifesté dans certaines des plus basses classes d'âge au cours des dernières années.

DESCRIPTION OF THE FISHERY

The haddock stock on Georges Bank has supported an important commercial fishery since the early 1920s (Clark et al. 1982). Canadian catches from this area were not significant until the 1960s, as was the case for catches by USSR and Spain (Table 1). Since 1977, with the extension of jurisdiction by coastal nations, only Canada and USA have had directed fisheries on Georges Bank. Nominal catches of Georges Bank haddock averaged about 46,000 t annually during 1935-60 (Clark et al. 1982). During 1965-66, catches increased to 150,000 t and 121,000 t, respectively, due primarily to the USSR fishery. Since 1967, catches have fluctuated greatly between about 4000 t and 50,000 t. Since the establishment of the international maritime boundary between Canada and USA in 1985, the proportion of the catch taken by Canada has increased to a historical high of almost 70% in 1987. Recent total catches, however, are only about 7000 t, close to the lowest levels.

Canadian fishing activity on Georges Bank in 1987 was concentrated on the northeast peak, as in the past. Since 1970, seasonal closures in the spring have been imposed on the fisheries in this area (Halliday 1988). As a consequence, the catches by Canada have been taken predominantly in June-September (Table 2). The majority of the Canadian catch has been taken with otter trawls, the other significant gear being longlines (Table 3). Between 1978-82, tonnage class 5 trawlers were the dominant component of the fleet but, since then, tonnage classes 2 and 3 trawlers have increased their catches (Table 4).

The USA catch has traditionally been more evenly distributed over the year since those fleets had access to areas which were not subject to seasonal closure (Table 5). Nevertheless, highest catches have tended to occur during the summer months. The catch was taken almost entirely by tonnage classes 3 and 4 vessels using otter trawl gear (W. Overholtz, pers. comm.).

DATA

Catch and Weights at Age

Length frequency and age information were collected for 38 samples (8136 fish measured for length, 1083 fish examined for age) from the Canadian commercial fishery in 1987. Samples were well distributed according to the seasonal and gear composition of the catch (Table 6) and were combined to obtain statistics by age (Table 7, 8) according to the methods described by Gavaris and Gavaris (1983). The length-weight relationship

$$\text{weight (kg)} = 0.0000158 \text{ length (cm)}^{2.91612}$$

derived from Canadian fishery samples (Waiwood and Neilson 1985), was used in these calculations. The USA age statistics (provided by National Marine Fisheries staff) were combined with Canadian results to obtain the total values. The 1983 year-class continued to be the dominant one in the USA catch while the 1985 year-class was dominant in the Canadian catch. The resulting weight at age was similar to values obtained in recent years.

Research Survey

Annual surveys have been conducted on Georges Bank by the Department of Fisheries and Oceans Canada during the spring of 1986-88 (Table 9) and by the National Marine Fisheries Service USA during the spring of 1968-88 (Table 10) and during the fall of 1963-87 (Table 11). A new type of otter trawl door was introduced to both spring and fall USA surveys in 1985 and has been used since then. The impact of this change on the abundance estimates has not been determined. The stratification scheme used for the Canadian survey is shown in Fig. 1.

Both the USA surveys and the Canadian survey show that the 1983 year-class was relatively strong. The fall USA survey and the Canadian survey also indicate that the 1985 year-class, though not as strong, is relatively abundant. Results were not available by age from the spring USA survey conducted in 1988.

ESTIMATION OF STOCK SIZE

Given that the Canadian survey does not provide a long enough time series and the spring USA survey is suspect due to the gear changes, the fall USA survey was used with the annual catch at age data (Table 12) to estimate population size. The adaptive framework described by Gavaris (1988) was employed. It was assumed that the error in the catch at age was negligible relative to the error in the abundance index. Scatterplots of the abundance index versus preliminary population estimates indicated that the relationships for ages greater than 6 were poor and have probably been the cause of the large intercept for the plus group relationship. The fall estimates are related to the beginning of year population estimates for the succeeding year. Based on these observations, the survey was used to compile an annual abundance index for ages 1, 2, 3 and 4-6 aggregated.

Initial attempts to estimate all year-class sizes resulted in high correlations between the parameters being estimated. Therefore, it was assumed that the fishing mortality on age 8 was equal to the total fishing mortality on ages 4-7. The resulting year-class size estimates were not markedly different from those obtained in the initial attempt; however, the correlation matrix of parameters did not contain very high values. The estimates for the 1980-82 year-classes were poorly determined; therefore, it was assumed that ages 5 and older were fully recruited in 1987. These year-classes were not estimated and their size was based on the resulting fishing mortality from the estimate of the size of the 1983 year-class. The intercepts in the relationships between abundance index and the population size were not large and had standard errors of the same order of magnitude. The final formulation arrived at did not include intercepts in the relationships. Ln transformations were used to stabilize the variance of the survey results. The 1986 year-class could not be estimated because the survey did not capture any haddock of this year-class. A value of 350,000 at age 2 for this year-class was used, consistent with the lowest estimate of year-class size. The resulting population matrix is shown in Table 13 and the fishing mortality is in Table 14. The calibration coefficients were 0.117, 0.204, 0.137 and 0.160 for the age groups 1, 2 and 3 and 4-6

aggregated, respectively. Diagnostics are presented in Table 15 and Fig. 2-7. The annual average weight at age (Table 16) was used to derive the estimated average population biomass (Table 17, Fig. 8). The APL functions used for these calculations are shown in Annex 1. These were used in conjunction with the NLLS workspace described by Gavaris (1988).

The population size is close to the lowest values previously observed, although it has increased since 1985. Fishing mortalities since 1985 are at about the $F_{0.1}$ level, having decreased. The population is comprised primarily of the 1983 and 1985 year-classes and recruitment in recent years has been variable (Fig. 9). Exploitation of age 2 haddock is high. Given the low abundance of older haddock, this could result in reduced spawning potential. In conclusion, though the fishing mortality rate on older haddock may not be exceeding $F_{0.1}$, the low abundance of older haddock and the high exploitation rate on young haddock indicate that the stock is not in a healthy state. Management measures to reduce the capture of haddock younger than age 3 would contribute to the conservation and rebuilding of this resource.

FURTHER CONSIDERATIONS

As an aid to developing management strategies, it would be helpful to have an understanding of the distribution of haddock relative to the Canada-USA international maritime boundary. The Canadian surveys are stratified in such a way to facilitate these computations. For the USA surveys, it is necessary to partition strata which are bisected by the boundary. Initially it was intended to use sets on the respective sides of the boundary to estimate density in the partitioned strata. This was not possible due to the absence of sets in stratum partitions for several years. The approach used, therefore, was to apply the partitioned stratum areas to the overall mean catch per tow. This assumes that fish density, within strata which are bisected, is uniform. The results obtained should be considered preliminary until further work is done to examine this assumption. The calculations were done by age group and the results are given in Table 18. There does not appear to be a consistent trend with age; therefore, the annual values for age groups 2-8+ was calculated. The Canadian survey is earlier in spring than the USA survey and closer to the peak spawning period. Indications are that most of the haddock were in the Canadian zone during spawning and dispersed thereafter. During the period when the Canadian commercial fishery occurs, approximated by the USA spring and fall survey distribution patterns, the proportion of haddock in the Canadian zone has ranged between 30 and 45% in recent years. These calculations were repeated using population biomass and yielded similar results to those obtained using numbers.

ACKNOWLEDGMENTS

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Table 1. Nominal catches(t) of haddock from NAFO Division 5Z. Data was obtained from ICNAF/NAFO for 1956-84 and from NMFS and DFO for 1985-87.

Year	USA	Canada	USSR	Spain	Others	Total
1956	51144	0	0	0	0	51144
1957	48561	0	0	0	0	48561
1958	37322	0	0	0	0	37322
1959	36051	0	0	0	0	36051
1960	40800	77	0	0	0	40877
1961	46384	266	0	0	0	46650
1962	49409	3461	1134	0	0	54004
1963	44150	8379	2317	0	0	54846
1964	46512	11625	5483	2	464	64086
1965	52823	14889	81882	10	758	150362
1966	52918	18292	48409	1111	544	121274
1967	34728	13040	2316	1355	30	51469
1968	25469	9223	1397	3014	1318	40421
1969	16456	3990	66	1201	470	22183
1970	8415	1978	103	782	7	11285
1971	7306	1630	374	1310	242	10862
1972	3869	609	137	1098	20	5733
1973	2777	1563	602	386	3	5331
1974	2396	462	109	764	559	4290*
1975	3989	1358	8	61	4	5420
1976	2904	1361	4	46	9	4324
1977	7934	2909	0	0	0	10843*
1978	12160	10179	0	0	0	22339*
1979	14279	5182	0	0	0	19461
1980	17470	10101	0	0	0	27571*
1981	19245	5659	0	0	3	24907
1982	12622	4931	0	0	0	17553
1983	8680	3212	0	0	0	11892
1984	8806	1463	0	0	0	10269
1985	4272	3485	0	0	0	7757
1986	3338	3415	0	0	0	6751
1987	2155	4703	0	0	0	6858

* Values adjusted for discards are 6190, 20531, 26281, and 51084 for 1974, 1977, 1978, and 1980 respectively.

Table 2. Monthly catch(t) of haddock by Canada in NAFO Division 5Z for 1968-1987.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
1968	337	510	990	2337	760	352	693	1240	941	904	120	139	9323
1969	106	73	6	291	587	691	559	607	553	364	119	34	3990
1970	10	105	0	1	576	345	104	456	241	103	25	12	1978
1971	0	9	2	0	400	153	283	279	97	246	140	21	1630
1972	0	118	2	0	4	112	87	116	91	70	9	0	609
1973	4	10	0	0	0	183	198	569	339	233	23	4	1563
1974	19	0	1	0	0	57	64	52	96	60	93	20	462
1975	5	14	0	0	0	167	257	482	104	167	117	45	1358
1976	0	8	61	68	61	595	152	188	186	26	9	7	1361
1977	102	176	6	0	23	519	1098	836	12	58	56	23	2909
1978	103	932	44	21	22	319	407	86	640	5605	2000	0	10179
1979	125	898	398	175	69	1393	905	395	488	261	53	22	5182
1980	39	134	13	33	225	2957	2299	963	1419	1739	103	177	10101
1981	38	482	568	4	254	1354	1242	727	292	82	378	238	5659
1982	131	309	1	12	45	1118	767	684	582	838	400	44	4931
1983	31	67	28	46	60	1288	386	489	527	194	90	6	3212
1984	3	5	80	89	73	433	218	255	212	70	25	0	1463
1985	1	11	33	99	26	354	392	1103	718	594	61	93	3485
1986	11	28	79	99	40	1339	1059	369	233	139	12	8	3415
1987	24	26	138	70	12	1762	1383	665	405	107	97	14	4703

Table 3. Canadian catch(t) of haddock in NAFO Division 5Z by gear for 1968-1987.

Year	OT	LL	MISC.	Total
1968	9170	111	11	9292
1969	3955	22	13	3990
1970	1900	76	2	1978
1971	1475	154	1	1630
1972	411	198	0	609
1973	1461	102	0	1563
1974	374	87	1	462
1975	1247	111	0	1358
1976	1185	154	15	1354
1977	2814	94	1	2909
1978	9716	171	292	10179
1979	4907	274	1	5182
1980	9510	590	1	10101
1981	4644	1015	0	5659
1982	4222	709	0	4931
1983	2396	813	3	3212
1984	624	838	1	1463
1985	2817	626	42	3485
1986	2786	594	35	3415
1987	3569	1046	89	4703

Table 4. Canadian otter trawl catch(t) of haddock in NAFO Division 5Z by tonnage class.

	Side				Stern			
Year	2	3	4	5	2	3	4	5
1968	0	176	3463	0	0	0	580	5041 *
1969	1	8	792	0	0	1	225	2928
1970	0	25	553	0	2	0	134	1186
1971	0	0	494	0	0	0	16	965
1972	0	0	0	0	0	0	148	263
1973	0	25	609	0	0	0	61	766
1974	0	0	26	0	0	6	8	334
1975	0	0	223	0	0	1	60	963
1976	0	1	192	23	0	0	61	908
1977	5	47	358	0	91	243	18	2052
1978	69	17	2485	0	238	822	351	5734
1979	12	116	1573	0	135	855	651	1565
1980	9	16	1426	1	354	365	1016	6323
1981	4	87	389	0	448	484	884	2348
1982	1	25	90	0	190	297	250	3359
1983	16	89	0	0	618	432	107	1134
1984	0	5	0	0	181	269	21	148
1985	0	72	0	0	840	1402	155	348
1986	4	48	0	0	829	1378	96	432
1987	6	41	0	0	782	1448	49	1241

* 1165 t shown as stern tonnage class 2 in the Statistical Bulletin was included with tonnage class 5 pers. com. G. Moulton, NAFO

Table 5. Monthly catch(t) of haddock by USA in NAFO Division 5Z for 1968-1987.

Year	NK	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
1968	0	1680	1756	3023	2381	2499	2750	2696	2261	2372	1971	1376	704	25469
1969	0	990	909	1617	2422	1688	2329	1357	1466	1392	1119	630	537	16456
1970	0	480	419	409	634	1230	1328	1090	822	747	803	245	208	8415
1971	0	305	476	640	719	1148	1075	670	546	556	543	401	227	7306
1972	0	246	308	273	282	632	561	382	371	236	262	158	158	3869
1973	0	180	165	136	179	338	537	329	269	134	253	129	128	2777
1974	0	190	130	138	212	298	327	349	197	125	155	98	177	2396
1975	20	237	201	142	234	569	849	316	351	359	295	236	180	3969
1976	18	187	226	224	277	534	350	132	167	221	166	238	164	2886
1977	50	237	369	361	382	743	902	1017	829	572	773	955	744	7884
1978	53	535	735	675	977	1313	2061	1249	1287	973	1140	914	248	12107
1979	61	757	637	777	1355	1844	1748	1553	1485	1170	1081	1284	527	14218
1980	84	1006	1393	1130	1705	2319	1583	2060	1554	1769	977	1045	845	17386
1981	68	1190	2373	1436	1528	2650	2586	1976	1635	1273	1006	774	750	19177
1982	25	829	1163	752	772	1416	2416	1624	693	1047	707	725	453	12597
1983	30	869	1075	387	759	864	1674	737	590	511	341	484	359	8650
1984	0	832	1157	583	599	1105	1570	735	747	523	412	271	272	8806
1985	0	262	281	367	534	681	710	443	349	226	170	134	115	4272
1986	0	(-	1071	-)	(-	1648	-)	(-	391	-)	(-	229	-)	3338
1987	0	(-	434	-)	(-	1107	-)	(-	394	-)	(-	220	-)	2155

Table 6. Length frequencies sampled from the 1987 Canadian commercial fishery for haddock in NAFO Division 5Z were applied to the weight indicated. The manner in which statistics were pooled is shown by braces. The numbers in brackets are the numbers of age interpretations for age length keys.

Gear	Month	Number measured	Weight(t)	
OT	Jan	-	19.6	
	Feb	-	7.0	
	Mar	-	19.9	
	Apr	-	-	
	May	-	4.8	
	Jun	3017	1721.5	- 1772.8(437)
	Jul	1849	1137.3	
	Aug	867	331.5	
	Sep	400	231.3	- 1700.1(368)
	Oct	-	.1	
	Nov	601	93.6	
	Dec	-	2.0	- 95.7(91)
LL	Jan	-	-	
	Feb	-	8.6	
	Mar	-	112.0	
	Apr	-	65.7	
	May	-	5.8	
	Jun	-	31.2	
	Jul	-	213.3	
	Aug		317.3	
	Sep	1402	172.4	
	Oct	-	104.9	
	Nov	-	2.6	
	Dec	-	11.9	- 1045.7(187)
Misc			88.9	- 4703.2

Table 7. Age composition (000's) for components of the haddock fishery in NAFO Division 5Z during 1987.

Age	a	b	c	d	e	f	g
1	0	0	0	0	0	0	0
2	27	972	910	37	4	18	88
3	17	39	26	3	22	12	8
4	275	443	343	21	105	358	108
5	36	7	19	0	8	30	15
6	14	2	18	0	10	14	8
7	10	7	13	0	13	37	15
8	18	4	3	0	8	34	10
9+	35	14	14	0	16	41	19

a. Can LL Jan-Dec
b. Can OT Jan-Jun
c. Can OT Jul-Sep
d. Can OT Oct-Dec
e. USA Jan-Mar
f. USA Apr-Jun
g. USA Jul-Dec

Table 8. Statistics by age of mean number caught (000's), average length and average weight for the 1987 commercial haddock fishery in NAFO Division 5Z.

Canada

Age	Average Weight	Average Length	Catch Mean	Catch Std Err	Catch CV
1					
2	0.832	41.371	1983	25.379	0.013
3	1.363	48.883	88	14.351	0.163
4	2.066	56.518	1103	26.765	0.024
5	2.221	57.164	64	16.130	0.252
6	2.543	59.748	35	10.485	0.301
7	2.805	62.108	31	8.063	0.261
8	3.510	67.447	26	6.896	0.268
9	3.468	67.328	31	7.065	0.230
10	3.413	67.178	10	4.787	0.491
11	3.905	69.949	7	3.059	0.409
12	4.520	73.996	15	4.473	0.305
13	3.980	70.720	2	1.375	0.619
14					
15	6.571	84.500		0.266	1.323

USA

Age	Average Weight	Average Length	Catch
1			
2	1.144	47.943	110
3	1.453	51.773	41
4	1.804	56.303	570
5	2.073	58.883	54
6	2.783	65.459	32
7	3.181	69.002	64
8	3.808	73.648	52
9	4.342	76.686	74
10	5.878	85.140	2

Table 8. continued

Age	Average Weight	Average Length	Total Catch
1			
2	0.848	41.716	2093
3	1.392	49.810	129
4	1.977	56.445	1673
5	2.153	57.949	118
6	2.657	62.466	66
7	3.059	66.766	95
8	3.710	71.606	78
9	4.086	73.944	105
10	3.815	70.106	12
11	3.905	69.949	7
12	4.520	73.996	15
13	3.980	70.720	2
14			
15	6.571	84.500	

USA data provided by T. Polacheck

Table 9. Stratified mean catch per tow at age (numbers) for haddock in NAFO Division 5Z from the Canadian surveys in spring.

Year	Age Group										
	1	2	3	4	5	6	7	8	9+	1-9+	4-6
1986	4.11	0.22	6.01	1.06	0.18	0.27	0.30	0.33	0.40	12.88	1.51
1987	0.03	3.04	0.69	2.50	0.66	0.08	0.30	0.10	0.86	8.26	3.24
1988	1.47	0.05	8.53	0.17	2.85	0.18	0.17	0.11	0.50	14.03	3.20

Table 10. Stratified mean catch per tow at age(numbers) for haddock on Georges Bank from the spring USA surveys. From 1973-81 a 41 Yankee trawl was used while a 36 Yankee was used in other years.

Year	Age Group										
	1	2	3	4	5	6	7	8	9+	1-9+	4-6
1968	0.27	1.90	0.31	0.47	4.51	1.13	0.17	0.30	0.23	9.29	6.11
1969	0.00	0.05	0.39	0.17	0.28	2.84	0.69	0.19	0.31	4.92	3.29
1970	0.45	0.17	0.00	0.22	0.31	0.31	1.34	0.66	0.57	4.03	0.84
1971	0.00	0.78	0.17	0.00	0.08	0.08	0.06	0.55	0.15	1.87	0.16
1972	2.70	0.06	0.41	0.08	0.02	0.03	0.09	0.02	0.87	4.28	0.13
1973	20.59	3.25	0.00	0.36	0.06	0.00	0.12	0.01	0.86	25.25	0.42
1974	1.43	8.92	1.92	0.00	0.16	0.00	0.01	0.07	0.25	12.76	0.16
1975	0.63	0.65	2.23	0.42	0.00	0.09	0.06	0.01	0.10	4.19	0.51
1976	54.22	0.20	0.40	0.62	0.29	0.00	0.03	0.00	0.07	55.83	0.91
1977	0.41	22.42	0.28	0.82	0.40	0.30	0.00	0.03	0.08	24.74	1.52
1978	0.05	0.65	10.69	0.24	0.63	0.55	0.11	0.04	0.07	13.03	1.42
1979	24.24	1.06	0.76	3.83	0.22	0.11	0.25	0.04	0.03	30.54	4.16
1980	3.49	31.34	0.34	0.70	3.27	0.45	0.25	0.31	0.16	40.31	4.42
1981	2.70	2.69	15.95	1.79	0.62	1.46	0.20	0.09	0.04	25.54	3.87
1982	0.62	1.25	0.77	3.33	0.34	0.23	0.50	0.00	0.00	7.04	3.90
1983	0.29	0.37	0.39	0.15	1.62	0.01	0.03	0.78	0.12	3.76	1.78
1984	1.40	0.79	0.43	0.42	0.39	0.48	0.05	0.03	0.20	4.19	1.29
1985	0.00	4.96	0.76	0.40	0.87	0.34	1.17	0.10	0.25	8.85	1.61
1986	2.49	0.17	2.06	0.24	0.11	0.21	0.12	0.33	0.11	5.84	0.56
1987	0.00	3.62	0.06	0.81	0.08	0.10	0.05	0.22	0.01	4.95	0.99

modified from Overholtz et. al. 1983
data for 1983-86 supplied by W. J. Overholtz
data for 1987 supplied by T. Polacheck

Table 11. Stratified mean catch per tow at age (numbers) for haddock on Georges Bank from the fall USA survey.

Year	Age Group										
	0	1	2	3	4	5	6	7	8+	0-8+	3-5
1963	56.33	17.04	6.19	4.57	5.60	3.99	1.37	1.13	1.10	97.32	14.16
1964	1.59	75.75	42.78	3.91	1.20	2.56	1.05	0.46	0.39	129.69	7.67
1965	0.22	6.82	51.94	6.51	0.72	0.54	0.61	0.54	0.35	68.25	7.77
1966	4.12	0.64	1.94	12.34	2.25	0.35	0.33	0.22	0.13	22.32	14.94
1967	0.02	4.51	0.24	0.67	4.54	1.09	0.33	0.14	0.34	11.88	6.30
1968	0.06	0.04	0.64	0.09	0.22	2.59	0.85	0.18	0.37	5.04	2.90
1969	0.26	0.02	0.00	0.19	0.09	0.11	1.02	0.34	0.24	2.27	0.39
1970	0.03	2.77	0.14	0.01	0.19	0.18	0.34	0.92	0.59	5.17	0.38
1971	1.63	0.00	0.21	0.05	0.01	0.15	0.02	0.06	0.69	2.82	0.21
1972	4.53	1.69	0.00	0.35	0.06	0.00	0.06	0.04	0.89	7.62	0.41
1973	2.17	6.04	1.08	0.00	0.13	0.03	0.00	0.05	0.49	9.99	0.16
1974	0.50	1.19	0.66	0.21	0.00	0.01	0.00	0.00	0.15	2.72	0.22
1975	15.76	0.42	0.48	3.26	0.62	0.00	0.02	0.00	0.21	20.77	3.88
1976	2.90	43.07	0.35	0.36	0.55	0.20	0.00	0.03	0.24	47.70	1.11
1977	0.11	1.75	15.33	0.46	0.47	0.52	0.28	0.03	0.08	19.03	1.45
1978	10.82	0.69	0.85	7.59	0.15	0.21	0.37	0.01	0.01	20.70	7.95
1979	1.08	37.29	0.03	0.74	3.12	0.21	0.23	0.04	0.01	42.75	4.07
1980	9.56	2.22	10.41	0.37	0.25	1.39	0.39	0.38	0.12	25.09	2.01
1981	0.31	5.02	1.70	3.03	0.17	0.34	0.43	0.00	0.01	11.01	3.54
1982	0.89	0.00	0.74	0.32	1.27	0.13	0.07	0.19	0.06	3.67	1.72
1983	3.89	0.16	0.14	0.18	0.20	0.63	0.08	0.00	0.08	5.36	1.01
1984	0.02	2.23	0.59	0.16	0.19	0.04	0.30	0.00	0.08	3.61	0.39
1985	11.35	0.65	1.53	0.22	0.05	0.10	0.01	0.17	0.05	14.13	0.37
1986	0.00	5.11	0.09	1.21	0.06	0.13	0.13	0.02	0.07	6.82	1.40
1987	1.80	0.00	0.79	0.10	0.77	0.06	0.06	0.02	0.02	3.62	0.93

modified from Overholtz et. al. 1983
data for 1983-86 supplied by W. J. Overholtz
data for 1987 supplied by T. Polacheck

Table 12. Catch numbers at age (000's) from the commercial fishery for haddock in NAFO Division 5Z.

Year	Age Group									
	1	2	3	4	5	6	7	8	9+	1-9+
1964	10101	15935	4554	4776	8722	5794	2082	1028	1332	54324
1965	9601	125818	44496	5356	4391	6690	3772	1094	1366	202584
1966	114	6843	100810	19167	2768	2591	2332	1268	867	136760
1967	1150	168	2891	20667	10338	1209	993	917	698	39031
1968	8	2994	709	1921	14519	3499	677	453	842	25622
1969	2	11	1698	448	654	5954	1574	225	570	11136
1970	46	158	16	570	186	214	2308	746	464	4708
1971	0	1375	223	40	289	246	285	1469	928	4855
1972	156	2	450	81	32	120	78	66	1236	2221
1973	2560	2057	3	386	53	30	77	15	447	5628
1974	46	4320	657	2	70	2	2	53	249	5401
1975	192	1034	1864	375	4	42	4	4	88	3607
1976	144	473	550	880	216	1	23	4	112	2403
1977	0	19585	187	680	515	357	4	39	111	21478
1978	0	761	14395	305	567	517	139	14	67	16765
1979	0	26	1726	7169	525	410	315	96	46	10313
1980	8	31000	347	975	6054	594	546	153	81	39758
1981	0	1661	9550	754	699	2278	308	139	80	15469
1982	78	1424	1634	2895	335	645	1387	130	119	8647
1983	0	87	748	709	1923	267	222	875	96	4927
1984	0	94	335	722	367	1532	199	325	456	4030
1985	0	2328	571	225	313	176	656	91	177	4537
1986	6	46	2782	211	168	161	157	272	45	3848
1987	0	2093	129	1673	118	66	95	78	141	4393

modified from Overholtz et. al. 1983
USA data for 1983-86 supplied by W. J. Overholtz
USA data for 1987 supplied by T. Polacheck

Table 13. Estimated population numbers (000's) at the beginning of the year for haddock in NAFO Division 5Z.

Year	Age Group									
	1	2	3	4	5	6	7	8	1-8	4-6
1964	474694	155219	22824	20216	27576	16463	5636	3395	726022	64255
1965	33448	379507	112664	14566	12230	14685	8236	2730	578066	41481
1966	4073	18698	196869	51980	7079	6040	5970	3330	294039	65098
1967	12976	3232	9117	69966	25214	3291	2601	2778	129174	98472
1968	424	9583	2494	4848	38583	11289	1601	1231	70054	54721
1969	988	340	5137	1400	2231	18452	6077	698	35323	22084
1970	4676	807	269	2669	741	1235	9720	3551	23667	4645
1971	397	3786	518	205	1670	439	817	5870	13702	2314
1972	8575	325	1856	222	132	1106	136	411	12764	1459
1973	19565	6880	264	1112	108	79	797	41	28847	1300
1974	10791	13702	3772	214	561	41	38	582	29701	816
1975	8199	8793	7310	2493	173	396	32	29	27425	3063
1976	105190	6539	6264	4298	1702	138	286	22	124439	6138
1977	14370	85992	4926	4630	2723	1198	112	214	114164	8551
1978	6104	11765	52683	3864	3176	1763	658	88	80101	8803
1979	79697	4998	8944	30108	2887	2087	976	413	130109	35082
1980	9899	65250	4068	5761	18163	1889	1338	514	106882	25813
1981	7380	8097	25372	3017	3834	9393	1009	601	58704	16244
1982	2485	6042	5127	12132	1788	2507	5629	547	36257	16426
1983	2757	1964	3659	2719	7313	1161	1469	3354	24395	11193
1984	22462	2257	1529	2319	1584	4248	709	1002	36109	8151
1985	892	18390	1763	949	1245	965	2091	400	26696	3159
1986	29292	731	12950	927	573	736	631	1119	46959	2236
1987	366	23977	557	8085	568	318	457	375	34702	8971
1988	15337	300	17737	339	5106	358	200	288	39665	5803

Table 14. Estimated fishing mortalities for haddock in NAFO Division 5Z. Fishing mortality for age 8 is set equal to "total" fishing mortality for ages 4 to 7.

Year	Age Group							
	1	2	3	4	5	6	7	8
1964	0.02	0.12	0.25	0.30	0.43	0.49	0.52	0.41
1965	0.38	0.46	0.57	0.52	0.51	0.70	0.71	0.58
1966	0.03	0.52	0.83	0.52	0.57	0.64	0.57	0.55
1967	0.10	0.06	0.43	0.40	0.60	0.52	0.55	0.45
1968	0.02	0.42	0.38	0.58	0.54	0.42	0.63	0.52
1969	0.00	0.04	0.45	0.44	0.39	0.44	0.34	0.44
1970	0.01	0.24	0.07	0.27	0.32	0.21	0.30	0.26
1971	0.00	0.51	0.65	0.24	0.21	0.97	0.49	0.32
1972	0.02	0.01	0.31	0.52	0.31	0.13	1.00	0.20
1973	0.16	0.40	0.01	0.48	0.78	0.54	0.11	0.52
1974	0.00	0.43	0.21	0.01	0.15	0.06	0.06	0.11
1975	0.03	0.14	0.33	0.18	0.03	0.12	0.15	0.17
1976	0.00	0.08	0.10	0.26	0.15	0.01	0.09	0.22
1977	0.00	0.29	0.04	0.18	0.23	0.40	0.04	0.23
1978	0.00	0.07	0.36	0.09	0.22	0.39	0.27	0.19
1979	0.00	0.01	0.24	0.31	0.22	0.24	0.44	0.30
1980	0.00	0.74	0.10	0.21	0.46	0.43	0.60	0.40
1981	0.00	0.26	0.54	0.32	0.22	0.31	0.41	0.30
1982	0.04	0.30	0.43	0.31	0.23	0.33	0.32	0.30
1983	0.00	0.05	0.26	0.34	0.34	0.29	0.18	0.34
1984	0.00	0.05	0.28	0.42	0.30	0.51	0.37	0.44
1985	0.00	0.15	0.44	0.30	0.33	0.23	0.43	0.29
1986	0.00	0.07	0.27	0.29	0.39	0.28	0.32	0.31
1987	0.00	0.10	0.30	0.26	0.26	0.26	0.26	0.26

Table 15. Observed ln abundance index and residuals of the ln abundance index for haddock in NAFO Division 5Z.

Year	Index Age Group				Residual Age Group				
	1	2	3	4-6	1	2	3	4-6	Total
1964	10.94	9.74	8.73	9.56	0.01	-0.62	0.69	0.32	0.40
1965	7.37	11.24	10.66	8.95	-0.90	-0.02	1.02	0.15	0.24
1966	5.39	8.83	10.86	8.96	-0.78	0.58	0.66	-0.29	0.17
1967	8.32	6.46	7.57	9.61	1.00	-0.03	0.44	-0.05	1.36
1968	3.00	8.41	5.48	8.75	-0.91	0.84	-0.35	-0.33	-0.75
1969	4.09	3.69	6.46	7.97	-0.66	-0.55	-0.09	-0.20	-1.50
1970	5.56	3.00	3.00	5.97	-0.75	-2.11	-0.61	-0.64	-4.10
1971	3.40	7.93	4.94	5.94	-0.44	1.28	0.68	0.03	1.55
1972	7.40	3.22	5.35	5.35	0.48	-0.97	-0.19	-0.10	-0.79
1973	8.42	7.43	3.00	6.02	0.68	0.19	-0.59	0.68	0.96
1974	7.68	8.71	6.98	5.08	0.54	0.77	0.74	0.20	2.26
1975	6.21	7.08	6.49	5.39	-0.65	-0.41	-0.41	-0.80	-2.28
1976	9.67	6.04	6.17	8.26	0.24	-1.15	-0.58	1.38	-0.11
1977	7.97	10.67	5.86	7.01	0.54	0.90	-0.65	-0.21	0.58
1978	4.70	7.47	9.64	7.28	-1.87	-0.31	0.76	0.03	-1.40
1979	9.29	6.54	6.75	8.98	0.15	-0.39	-0.36	0.35	-0.26
1980	6.98	10.53	3.40	8.31	-0.07	1.03	-2.92	-0.01	-1.97
1981	9.17	7.71	9.25	7.61	2.40	0.30	1.10	-0.26	3.54
1982	5.74	8.52	7.44	8.17	0.06	1.41	0.89	0.30	2.65
1983	6.79	5.30	6.61	7.45	1.01	-0.69	0.39	-0.04	0.67
1984	8.27	5.08	4.94	6.92	0.39	-1.05	-0.40	-0.25	-1.32
1985	3.00	7.71	6.38	5.97	-1.66	-0.52	0.90	-0.26	-1.54
1986	9.34	6.48	7.33	5.91	1.19	1.47	-0.15	0.03	2.56
1987		8.54	4.50	7.24		0.05	0.17	-0.02	0.19
1988	7.50		6.67	6.84	0.00		-1.12	0.00	-1.12
Total					0.00	0.00	0.00	0.00	

Table 16. Average weight(kg) at age of haddock caught by the commercial fishery in NAFO Division 5Z.

Year	Age Group 1	2	3	4	5	6	7	8	9+	Avg
1964	0.50	0.83	1.12	1.43	1.64	2.01	2.40	2.64	2.97	1.25
1965	0.58	0.69	1.03	1.35	1.67	1.99	2.26	2.66	3.11	0.90
1966	0.58	0.73	0.89	1.26	1.70	2.07	2.28	2.87	3.18	1.03
1967	0.66	0.70	0.95	1.18	1.42	2.05	2.31	2.66	3.10	1.33
1968	0.59	0.81	1.05	1.32	1.57	2.10	2.32	2.62	2.86	1.60
1969	0.52	0.78	1.10	1.69	1.75	1.99	2.52	2.99	3.63	2.01
1970	0.71	1.27	1.22	1.93	2.19	2.39	2.58	3.23	3.75	2.63
1971	0.60	1.03	1.31	1.74	2.39	2.81	2.92	3.10	3.72	2.47
1972	0.62	1.03	1.74	2.04	2.42	2.92	3.06	3.44	3.66	2.91
1973	0.60	1.03	1.58	2.13	2.41	3.29	3.42	3.86	3.94	1.21
1974	0.72	1.06	1.82	2.32	2.83	3.76	4.05	3.92	4.26	1.35
1975	0.62	0.98	1.63	2.21	2.20	2.94	4.00	4.05	4.33	1.54
1976	0.60	0.99	1.39	1.99	2.66	2.63	3.69	4.67	4.94	1.79
1977	0.60	1.07	1.44	2.17	2.73	3.21	4.15	4.00	4.99	1.21
1978	0.60	0.94	1.50	2.04	2.79	3.19	3.37	3.61	5.11	1.61
1979	0.60	1.00	1.28	2.02	2.51	3.14	3.78	3.79	4.87	2.05
1980	0.60	0.72	1.20	1.93	2.30	2.94	3.86	4.13	4.83	1.09
1981	0.60	0.91	1.24	1.80	2.40	2.80	3.73	4.44	4.04	1.61
1982	0.60	0.92	1.41	1.94	2.44	2.83	3.35	4.00	3.73	2.03
1983	0.60	1.00	1.43	1.94	2.31	2.80	3.38	3.59	3.89	2.43
1984	0.60	0.86	1.31	1.74	2.17	2.67	2.98	3.39	3.62	2.48
1985	0.60	0.98	1.26	1.91	2.39	2.86	3.03	3.53	3.92	1.69
1986	0.60	0.95	1.38	1.84	2.42	2.86	3.04	3.54	4.08	1.76
1987	0.60	0.85	1.39	1.98	2.15	2.66	3.06	3.71	4.10	1.56

modified from Clark et. al. 1982
USA data for 1980-86 supplied by W. J. Overholtz
USA data for 1987 supplied by T. Polacheck

Table 17. Estimated average annual population biomass(t) for haddock in NAFO Division 5Z.

Year	Age Group								
	1	2	3	4	5	6	7	8	1-8
1964	212663	110230	20593	22722	33551	23876	9621	6718	439976
1965	14710	192006	80802	14008	14652	19268	12245	5032	352722
1966	2109	9737	109174	46616	8408	8449	9513	6735	200741
1967	7388	1993	6421	62218	24608	4808	4229	5424	117089
1968	225	5776	1990	4451	42847	17673	2523	2296	77781
1969	465	236	4146	1751	2947	27109	11848	1541	50044
1970	2993	827	287	4112	1263	2418	19694	9176	40772
1971	216	2789	457	289	3270	727	1727	14164	23639
1972	4772	303	2527	323	250	2752	243	1169	12340
1973	9876	5326	376	1716	167	184	2339	113	20098
1974	7026	10784	5621	447	1342	135	134	1967	27456
1975	4549	7308	9244	4580	341	995	107	98	27223
1976	57161	5637	7514	6867	3818	328	916	85	82327
1977	7814	72741	6297	8371	6029	2893	414	696	105256
1978	3320	9673	60522	6838	7236	4245	1772	264	93869
1979	43340	4517	9262	47742	5906	5290	2723	1233	120013
1980	5381	30397	4220	9134	30590	4126	3555	1597	88997
1981	4013	5914	22252	4228	7497	20584	2816	2106	69409
1982	1329	4371	5353	18469	3542	5495	14719	1720	54998
1983	1499	1738	4201	4076	13034	2565	4125	9303	40540
1984	12215	1720	1593	3004	2711	8125	1608	2504	33481
1985	485	15186	1637	1421	2319	2249	4712	1116	29125
1986	15927	611	14211	1348	1046	1676	1495	3098	39412
1987	199	17554	611	12815	979	676	1120	1113	35067

Table 18. Proportion of Georges Bank haddock, in numbers, which were captured on the Canadian side of the Canada-USA internatinal maritime boundary during research vessel surveys.

Year	Age Group									
USA fall										
	0	1	2	3	4	5	6	7	8+	2-8+
1963	0.29	0.31	0.19	0.12	0.12	0.12	0.10	0.13	0.15	0.14
1964	0.12	0.18	0.11	0.09	0.09	0.09	0.08	0.08	0.05	0.11
1965	0.27	0.14	0.09	0.10	0.15	0.17	0.18	0.18	0.15	0.10
1966	0.21	0.06	0.21	0.18	0.13	0.14	0.16	0.22	0.20	0.17
1967	0.00	0.14	0.06	0.09	0.07	0.07	0.12	0.13	0.14	0.08
1968	0.26	0.53	0.24	0.22	0.16	0.20	0.20	0.17	0.17	0.20
1969	0.40			0.17	0.07	0.04	0.07	0.06	0.07	0.08
1970	0.14	0.07	0.12	0.17	0.13	0.14	0.11	0.11	0.09	0.11
1971	0.31	0.33	0.15	0.14	0.25	0.14	0.27	0.15	0.10	0.12
1972	0.19	0.27		0.23	0.22		0.29	0.02	0.12	0.15
1973	0.10	0.52	0.28		0.32	0.09		0.10	0.06	0.21
1974	0.06	0.04	0.29	0.14		0.24			0.08	0.23
1975	0.33	0.21	0.52	0.74	0.78		1.00		0.77	0.73
1976	0.13	0.50	0.42	0.35	0.19	0.19		0.22	0.20	0.27
1977	0.08	0.11	0.37	0.24	0.13	0.11	0.10	0.30	0.16	0.35
1978	0.21	0.21	0.34	0.33	0.27	0.22	0.27	0.03	0.14	0.33
1979	0.32	0.29	0.14	0.20	0.20	0.28	0.30	0.33	0.19	0.21
1980	0.24	0.31	0.15	0.13	0.26	0.30	0.12	0.15	0.07	0.16
1982	0.01		0.27	0.44	0.53	0.54	0.32	0.48	0.20	0.44
1983	0.16	0.61	0.64	0.56	0.23	0.17	0.02	0.12	0.08	0.29
1984	0.21	0.29	0.25	0.26	0.28	0.26	0.17		0.18	0.23
1985	0.28	0.40	0.30	0.35	0.55	0.50	0.35	0.29	0.41	0.32
1986		0.40	0.35	0.34	0.30	0.17	0.33	0.43	0.32	0.33

Table 18. continued

USA spring

Year	Age Group									
	1	2	3	4	5	6	7	8	9+	2-9+
1970	0.16	0.25		0.46	0.44	0.44	0.44	0.44	0.42	0.43
1971		0.24	0.25		0.17	0.16	0.22	0.18	0.22	0.21
1972	0.23	0.10	0.36	0.30	0.27	0.26	0.24	0.36	0.24	0.27
1973	0.02	0.29		0.44	0.43		0.32	0.00	0.22	0.29
1974	0.17	0.34	0.35		0.32		0.27	0.11	0.17	0.34
1975	0.11	0.24	0.42	0.41		0.46	0.44	0.52	0.37	0.38
1976	0.03	0.35	0.24	0.22	0.19		0.03		0.27	0.24
1977	0.08	0.25	0.23	0.31	0.29	0.25		0.17	0.10	0.25
1978	0.08	0.22	0.23	0.17	0.20	0.21	0.19	0.21	0.22	0.22
1979	0.08	0.08	0.39	0.39	0.30	0.20	0.20	0.15	0.23	0.32
1980	0.23	0.18	0.08	0.30	0.29	0.26	0.25	0.21	0.28	0.19
1982	0.59	0.48	0.44	0.41	0.29	0.26	0.13	0.03	0.35	0.39
1983	0.26	0.52	0.43	0.33	0.27	0.33	0.08	0.13	0.05	0.28
1984	0.20	0.32	0.45	0.39	0.36	0.28	0.23	0.20	0.30	0.34
1985		0.48	0.47	0.47	0.47	0.44	0.43	0.41	0.40	0.46
1986	0.35	0.45	0.44	0.41	0.51	0.46	0.49	0.43	0.42	0.44

Canada spring

	1	2	3	4	5	6	7	8	9+
1987	1.00	0.98	0.95	0.96	0.91	0.83	0.82	0.81	0.82
1988	0.26	0.71	0.78	0.67	0.70	0.72	0.68	0.69	0.62

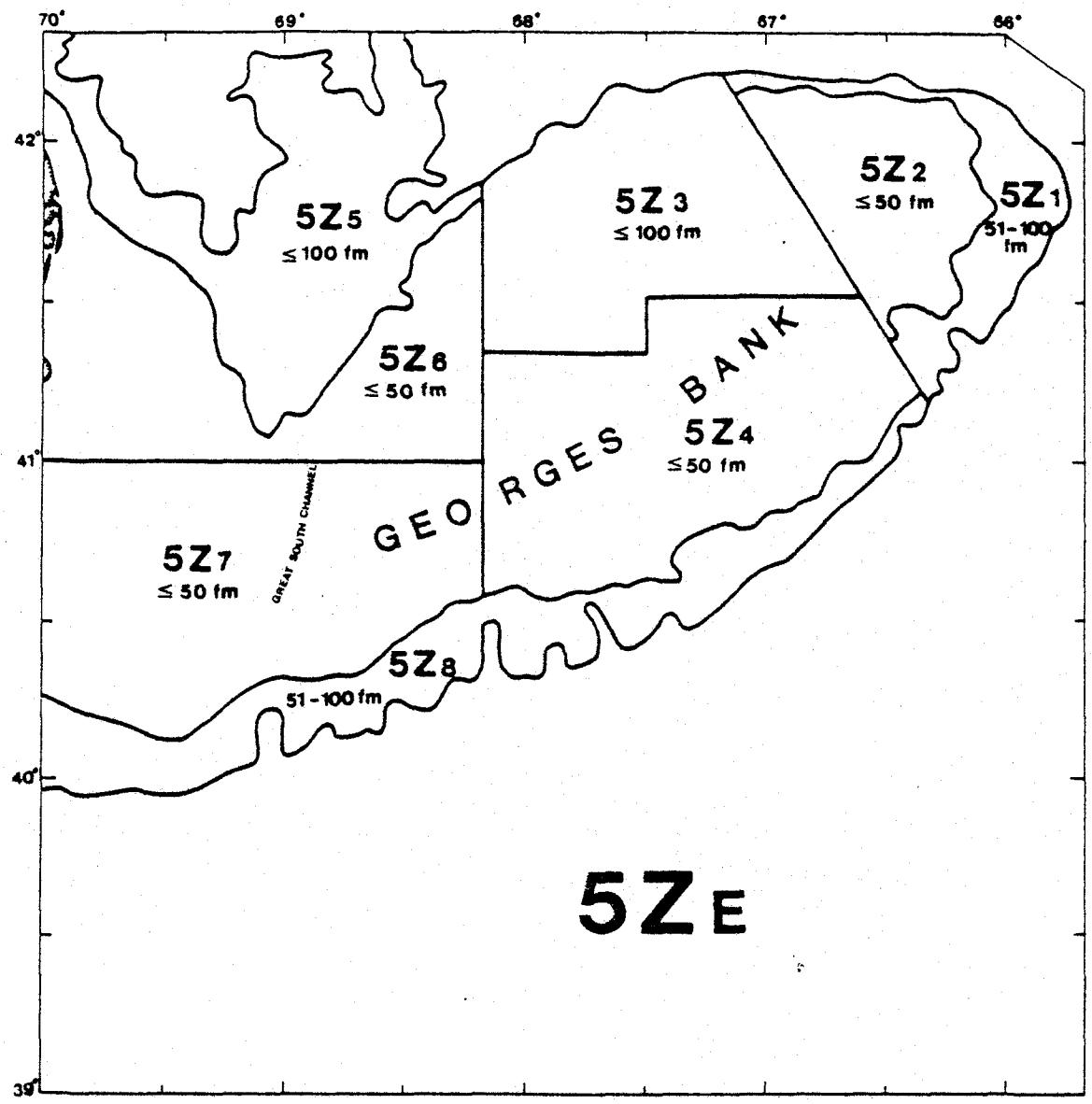

Fig. 1 Stratification scheme used for the Canadian survey of Georges Bank since 1987.

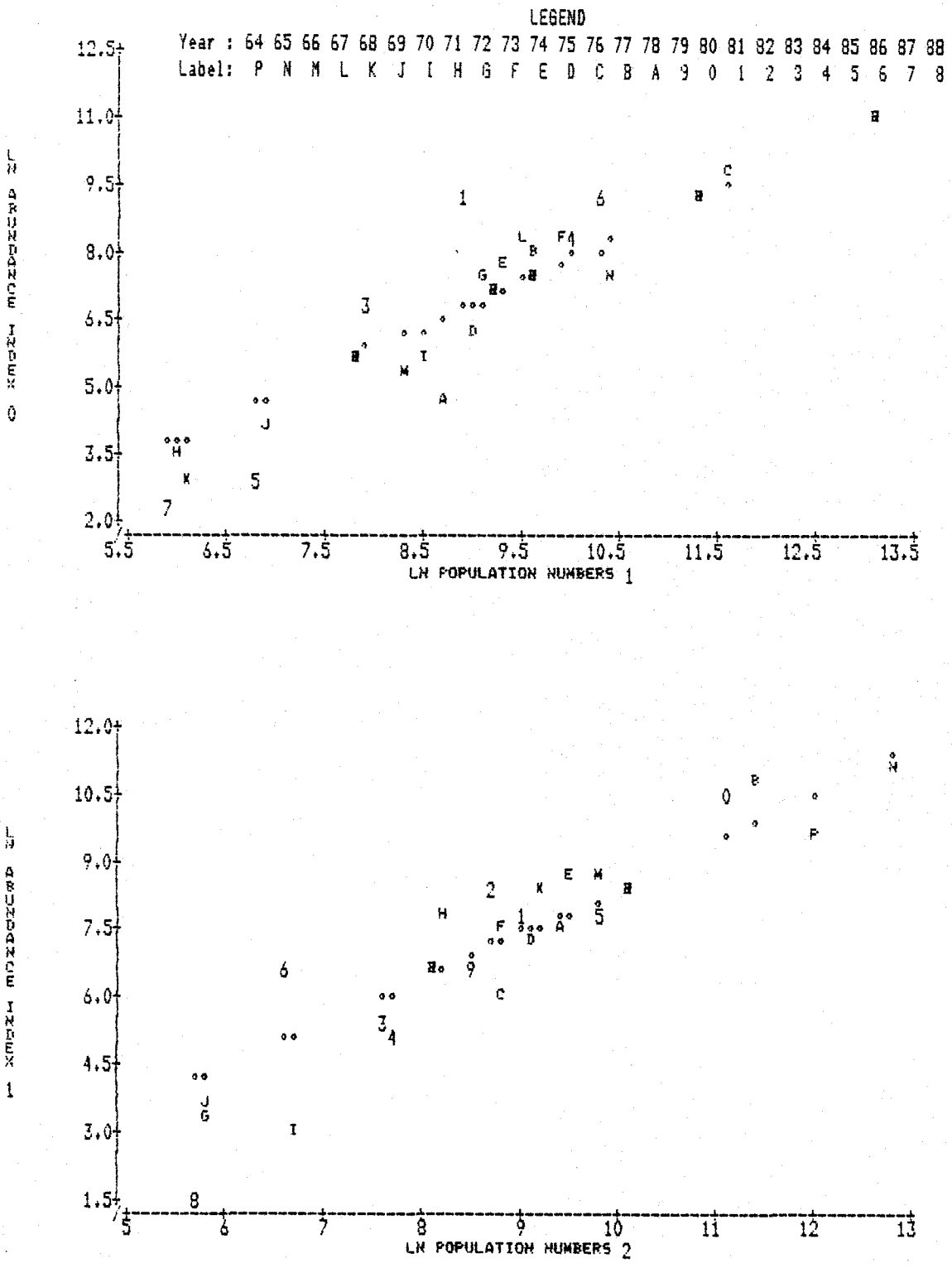

Fig. 2 Age by age plots of observed (characters) and predicted (o) ln abundance index from research surveys versus ln population numbers for haddock in NAFO Division 5Z.

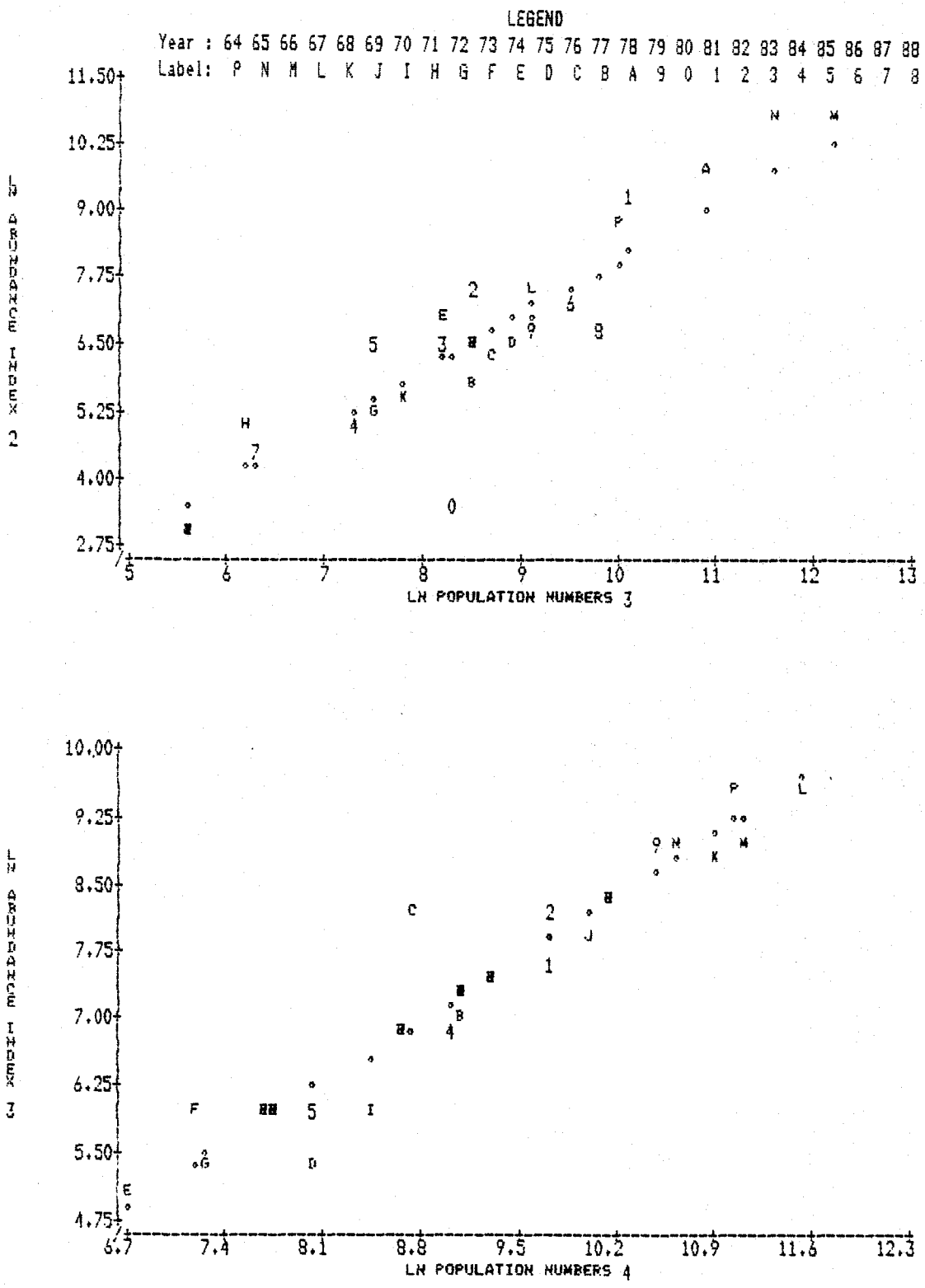

Fig. 2 continued

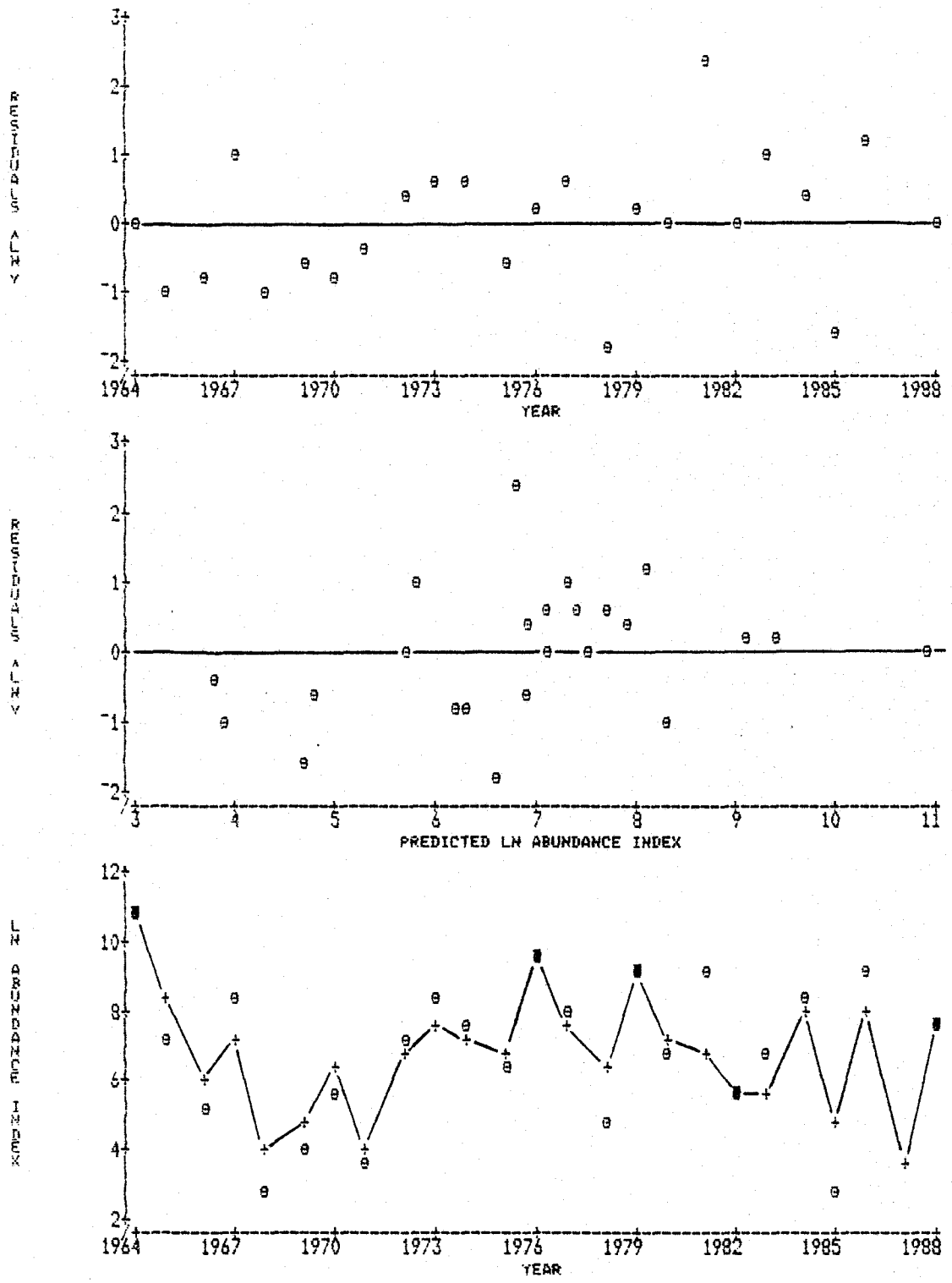

Fig. 3 Residual plots and time series plot of the observed (o) and predicted (+) ln abundance index for age 1 haddock in NAFO Division 5Z.

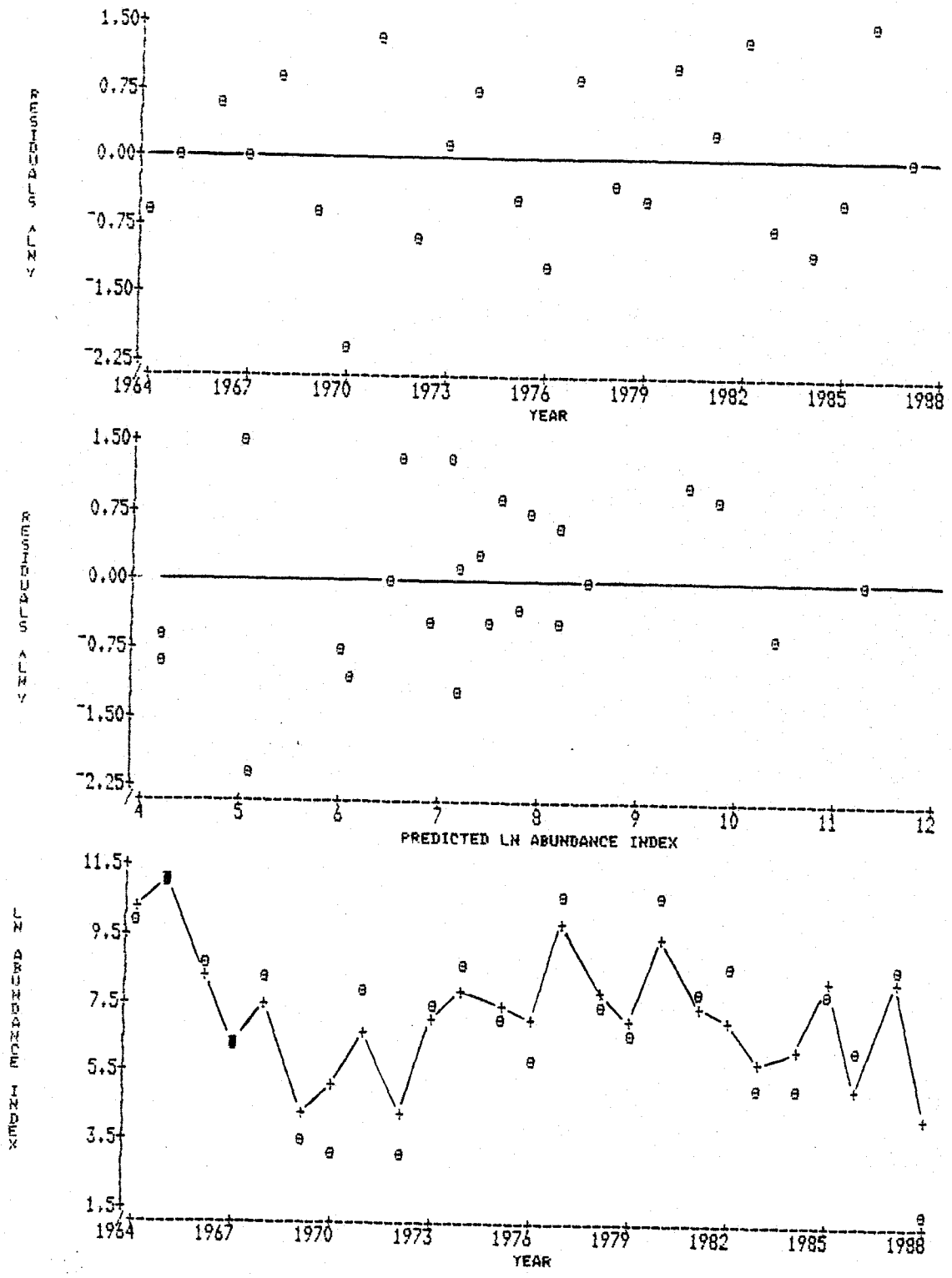

Fig. 4 Residual plots and time series plot of the observed (o) and predicted (+) ln abundance index for age 2 haddock in NAFO Division 5Z.

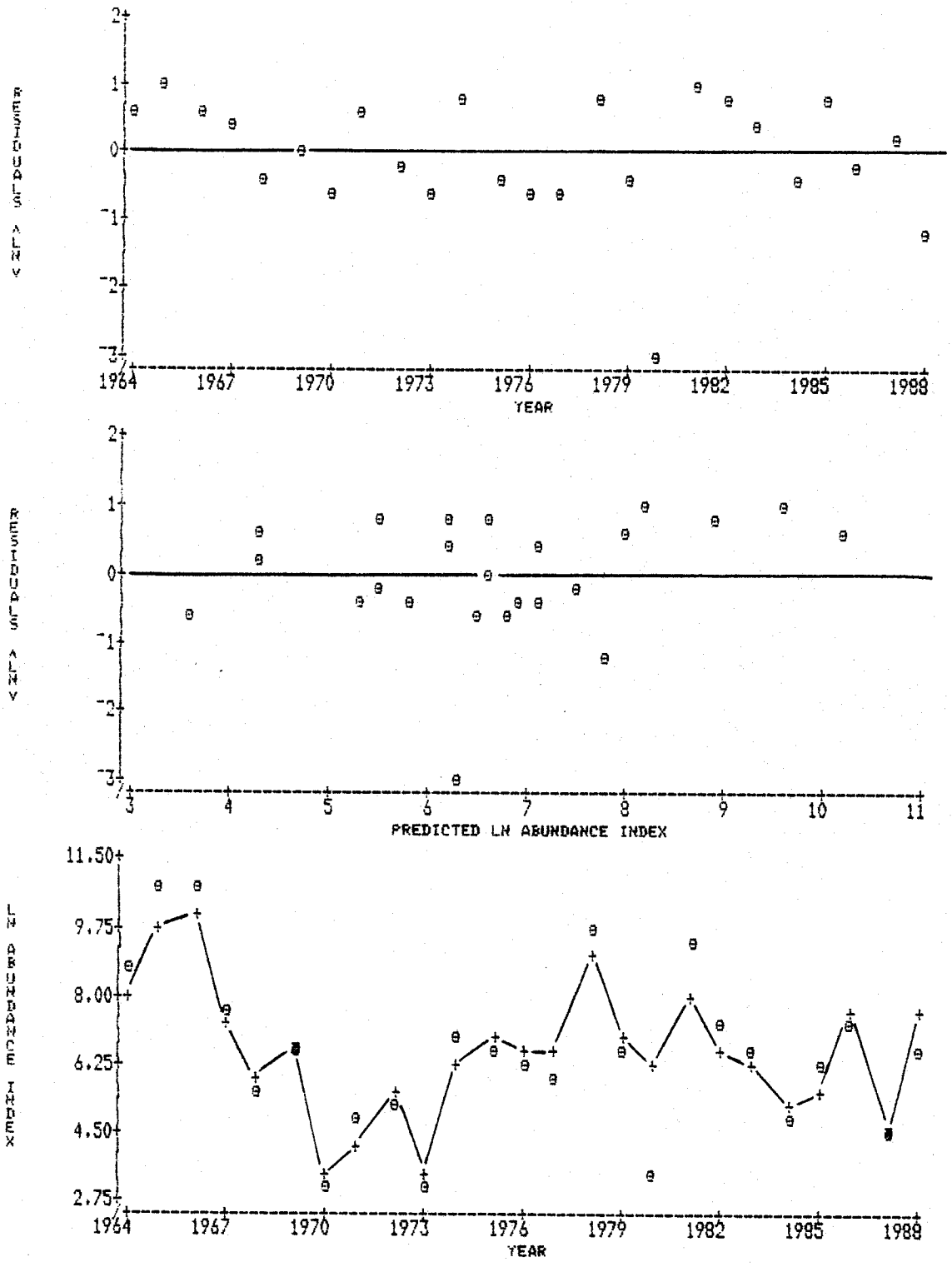

Fig. 5 Residual plots and time series plot of the observed (o) and predicted (+) ln abundance index for age 3 haddock in NAFO Division 5Z.

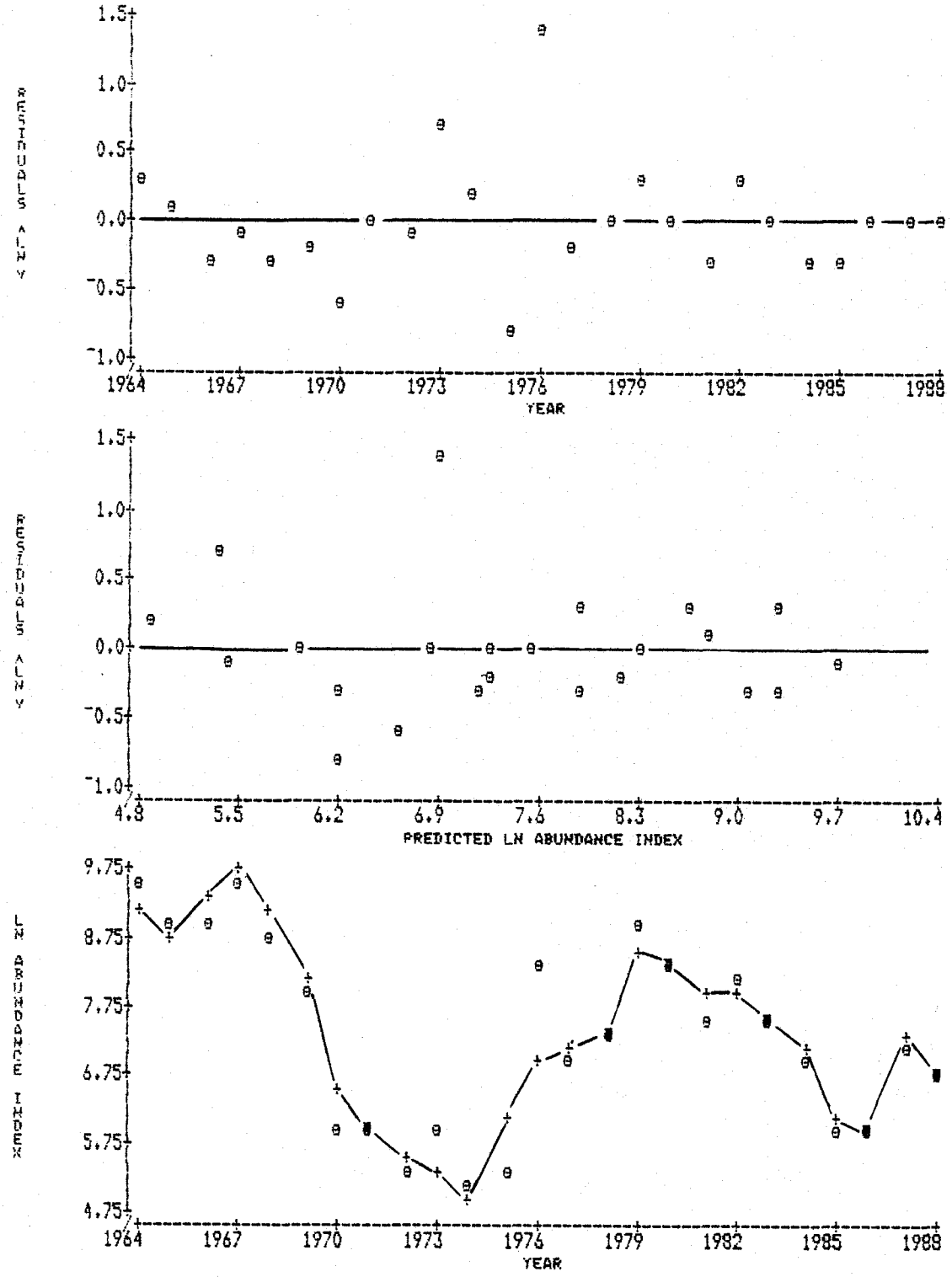

Fig. 6 Residual plots and time series plot of the observed (o) and predicted (+) ln abundance index for ages 4-6 haddock in NAFO Division 5Z.

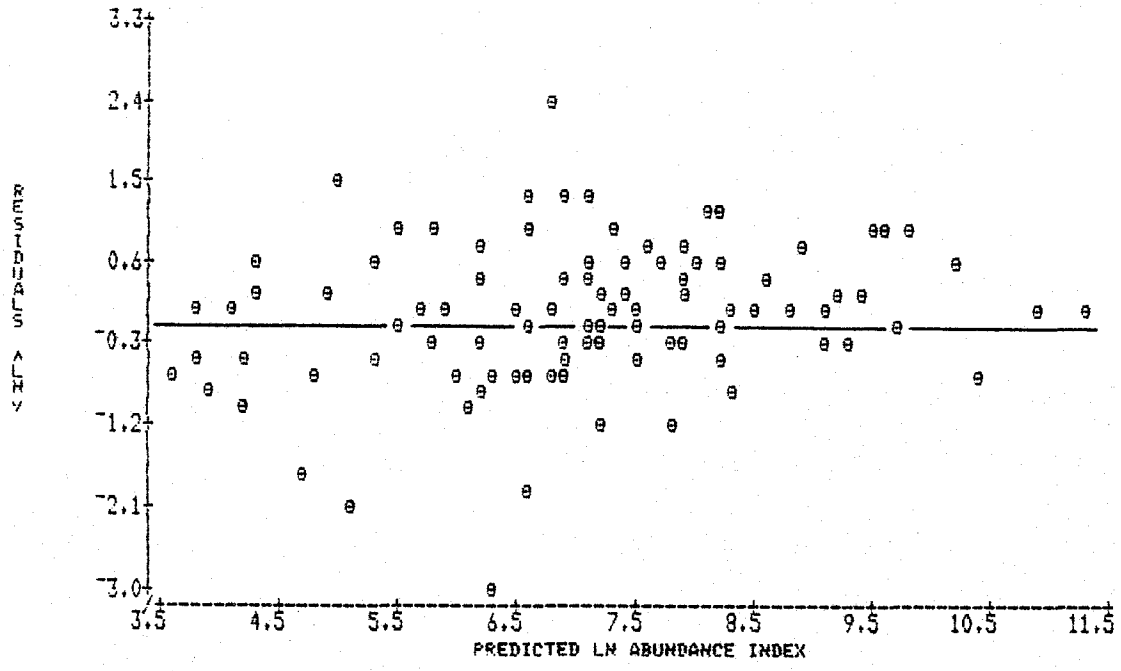

Fig. 7 Plot of residuals versus predicted ln abundance index for all age groups of haddock in NAFO Division 5Z.

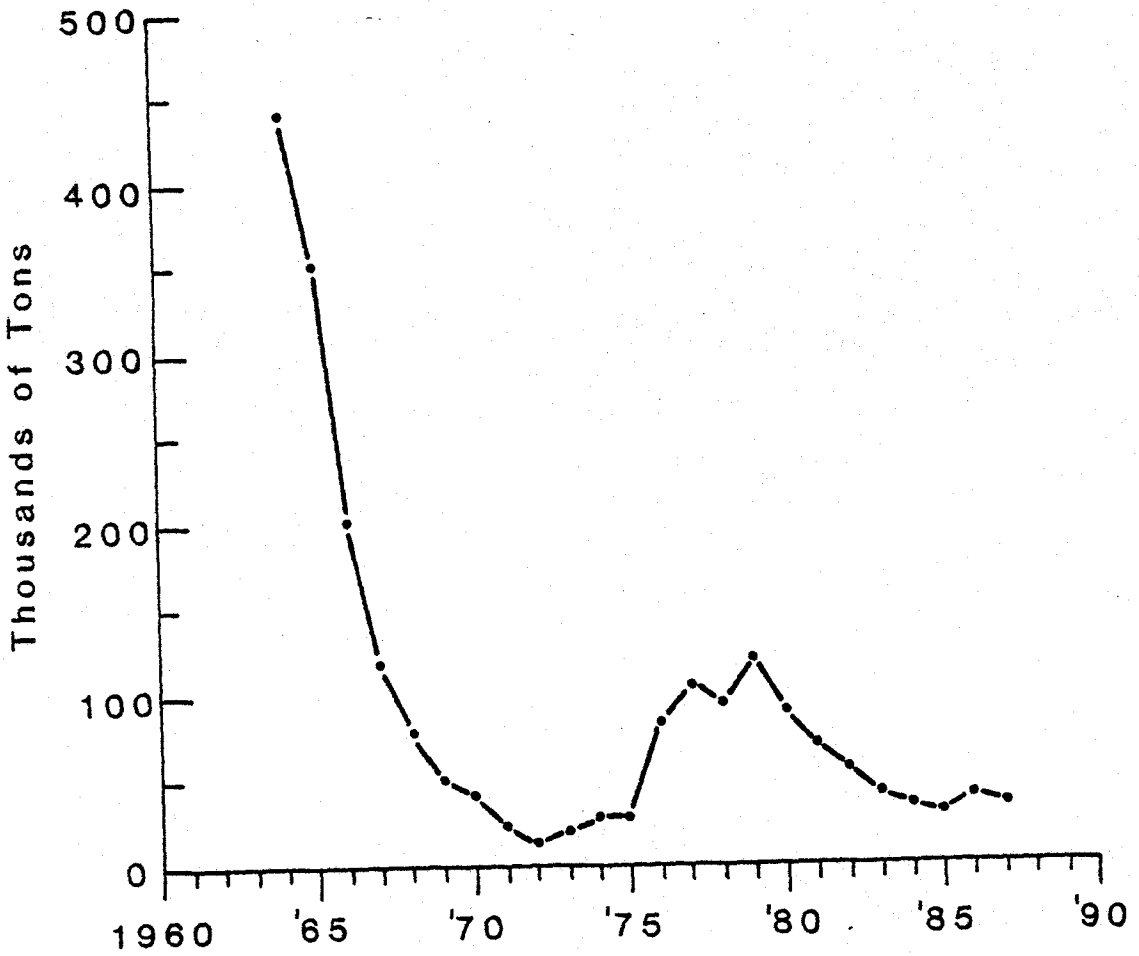

Fig. 8 Average biomass(t) for ages 1-8 haddock in NAFO Division 5Z.

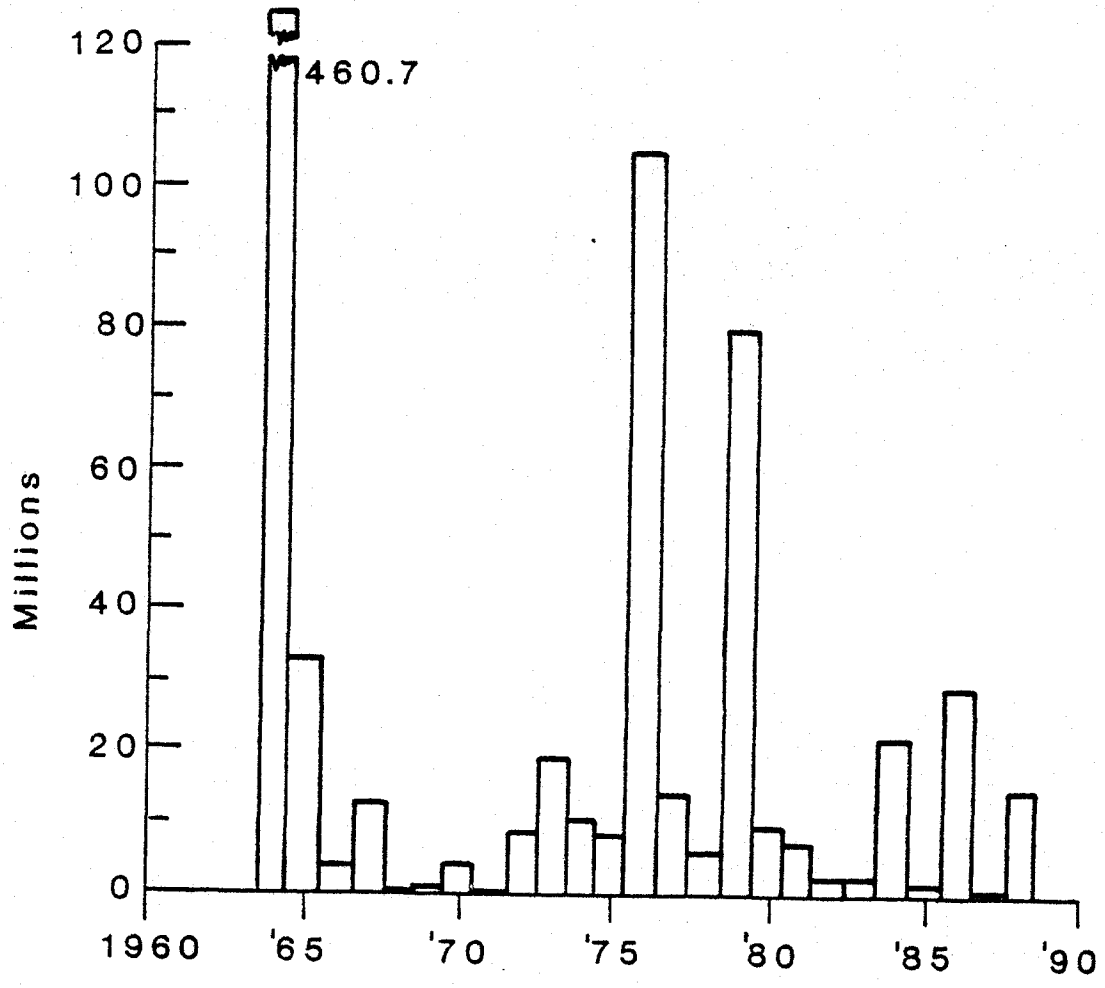

Fig. 9 Age 1 recruitment of haddock in NAFO Division 5Z.

ANNEX 1 SOFTWARE

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      ∇ R←FFILL V;AG;C;AGES;F
[1]     AG←1↑⍴C ⍝ NUMBER OF AGES
[2]     C←,(AG,¯1)↑C ⍝ CATCH IN LAST YEAR
[3]     AGES←5 ⍝ ROW INDEX FOR FULLY RECRUITED AGES, V
[4]     F←(⍟((V×⋆M)÷C[AGES-1]×⋆M÷2)÷V)-M
[5]     R←(AGES-1)↓¯1↓((C×F+M)÷F×(1-⋆-F+M))×⋆-F+M
      ∇
      ∇ R←FRGN∆FN A
[1]     ⍝ THIS FUNCTION SHOULD RETURN A 1 IF THE PARAMETERS
[2]     ⍝ ARE IN THE FEASIBLE REGION AND 0 OTHERWISE
[3]     R←∧/(A>LBND),A<UBND
      ∇
      ∇ R←OBJ∆FN A
[1]     S←4↑A ⍝ SURVIVORS AT DESIGNATED AGE
[2]     S←BOOL\S
[3]     S[(~BOOL)/⍳⍴BOOL]←300
[4]     S←S,FFILL ¯1↑S
[5]     K←4↓A ⍝ CALIBRATION COEFFICIENTS
[6]     POPN←4 5 6 7 SPAS S ⍝ CALCULATE POPULATION NUMBERS
[7]     E←(⍟(0 ¯1 ↓POPN)÷((0 ¯1 ↓POPN)×⋆-M)-C×⋆-M÷2)-M
[8]     POPN∆MID←(0 ¯1 ↓POPN)×(1-⋆-E+M)÷E+M
[9]     EI←POPN RESI K ⍝ CALCULATE INDEX RESIDUALS
[10]    ⍝EC←POPN∆MID RESC E  CALCULATE CATCH RESIDUALS
[11]    R←((,DEL)/,EI) ⍝ ,EC  REMOVE COMMENT FOR CATCH ERROR
      ∇
      ∇ R←ALPHA PNLTY∆FN A
[1]     ⍝ THE PENALTY FUNCTION FOR THE CONSTRAINTS
[2]     ⍝ R IS THE RESULTANT MAGNITUDE
[3]     ⍝ A IS THE VECTOR OF PARAMETERS
[4]     ⍝ ALPHA IS THE VECTOR OF CONSTANTS FOR THE CONSTRAINTS
[5]     R←+/ALPHA÷(A-LCNSTRNT),UCNSTRNT-A
      ∇
      ∇ R←POPN RESI K
[1]     R←3 6 SUMM POPN
[2]     IHAT←⍟↑K×⍤¯1↓R ⍝ PREDICTED MATRIX FOR INDEX
[3]     R←I-IHAT ⍝ WEIGHTED RESIDUALS
      ∇
      ∇ R←ROWS SPAS S;J;AG;YR;F
[1]     ⍝ ASSUMES LAST ROW IS NOT A PLUS GROUP
[2]     ⍝ ASSUMES F ON LAST AGE GROUP EQUAL TO AVERAGE FOR ROWS
[3]     AG←1↑⍴C ⍝ NUMBER OF AGE GROUPS
[4]     YR←1+¯1↑⍴C ⍝ NUMBER OF YEARS
[5]     R←(AG,YR)⍴0 ⍝ INITIALIZE MATRIX
[6]     R[;YR]←S ⍝ LAST YEAR SURVIVORS
[7]     J←YR
[8]       L10:→(0≥J←J-1)/L99 ⍝ CALCULATE
[9]       R[¯1↓⍳AG;J]←(C[¯1↓⍳AG;J]×⋆M÷2)+R[1↓⍳AG;J+1]×⋆M ⍝ POP NUMBS
[10]      F←(⍟(+⌿R[¯1↓ROWS;J])÷+⌿R[1↓ROWS;J+1])-M ⍝ AVERAGE F ON ROWS
[11]      R[AG;J]←(C[AG;J]×F+M)÷F×1-⋆-F+M
[12]      →L10
[13]    L99:
      ∇
      ∇ R←I SUMM M
[1]     ⍝ UTILITY TO COMPRESS MATRIX FOR PLUS GROUP BETWEEN ROW I[1] AND I[2]
[2]     R←M[⍳I[1];]
[3]     R←R⍪+⌿M[I[1]↓⍳I[2];]
      ∇
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