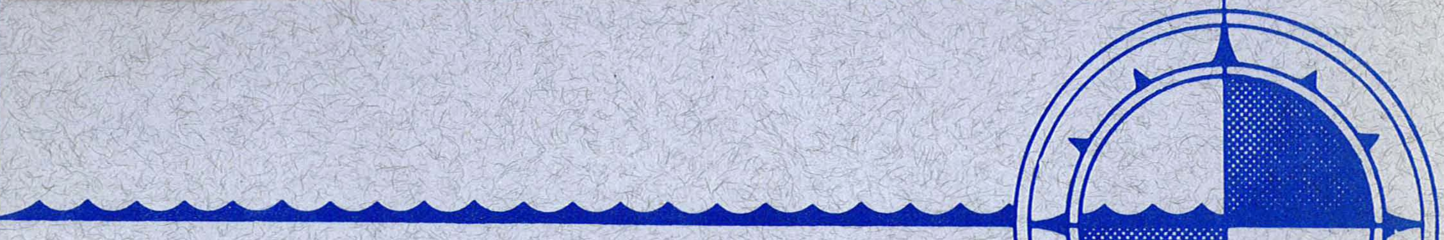
Pacific Marine Science Report 77-10

Manual for Tidal Heights Analysis and Prediction

M.G.G. Foreman



INSTITUTE OF OCEAN SCIENCES, PATRICIA BAY
Victoria, B.C.

Pacific Marine Science Report 77-10

MANUAL FOR TIDAL HEIGHTS ANALYSIS AND PREDICTION

by

M.G.G. Foreman

Institute of Ocean Sciences, Patricia Bay
Victoria, B.C.

July 1977

This is a manuscript which has received only limited circulation. On citing this report in a bibliography, the title should be followed by the words "UNPUBLISHED MANUSCRIPT" which is in accordance with accepted bibliographic custom.

Table of Contents

3. Use of the Tidal Heights Prediction Computer Program

4. Tidal Heights Prediction Program Details

6. Appendices

Preface

This report is intended to serve as a user's manual to G. Godin's tidal heights analysis and prediction programs, revised along lines suggested by Godin. In addition to describing input and output of these programs, the report gives an outline of the methods used; a full presentation of which can be found in Godin [1972] and Godin and Taylor [1973].

Users who wish to receive updates of these programs and manual should send their names, addresses and type of computer used for implementation to the author.

ACKNOWLEDGEMENTS

The writer wishes to thank Dr. G. Godin for his guidance during the computer program revisions, Mr. J. Taylor and Dr. R.F. Henry for their helpful suggestions, Mrs. C. Wallace for assisting in the preparation of the diagrams, and Ms. E. Peirson and Mrs. S. McKenzie for typing the report.

1. USE OF THE TIDAL HEIGHTS ANALYSIS COMPUTER PROGRAM

1.1 General Description

This program analyzes the hourly height tide gauge data for a given period of time. Amplitudes and Greenwich phase lags are calculated via a least squares fit method coupled with nodal modulation for only those constituents that can be resolved over the length of record. Unless specified otherwise, a standard data package of 69 constituents will be considered for inclusion in the analysis, however, up to 77 additional shallow water constituents can be requested. If the record length is such that certain important constituents are not included directly in the analysis, provision is made for the inference of the amplitude and phase of these constituents from others. Gaps within the tidal record are permitted.

1.2 Routines Required

1) MAIN......... reads in some of data, controls most of the output and calls other routines.

2) INPUT........ reads in the hourly height data for the desired time period and checks for errors.

3) UCON......... chooses the constituents to be included in the analysis via the Rayleigh criterion.

4) SCFIT2....... finds the least squares fit to an equally spaced time series using sines and cosines of specified frequencies as fitting functions.

5) VUF.......... reads required information and calculates the nodal corrections for all constituents.

6) INFER........ reads required information and calculates the amplitude and phase of inferred constituents, as well as adjusting the amplitude and phase of the constituent used for the inference.

7) CHLSKY....... solves the symmetric positive definite matrix equation resulting from a linear least squares fit.

8) CDAY......... returns the consecutive day number from a specific origin for any given date in the range 1901 to 1999, and vice versa.

9) SETTAB....... sorts results in preparation for 'J' card output.

10) DGM388....... produces 'J' card output and lists the constituents in species by increasing frequency.

1.3 Data Input

For a computer run of the tidal heights analysis program, two logical units are used for data input. Logical unit number 8 contains the tidal constituent information while logical unit 4 contains the hourly heights and information relating to the type of analysis and output required. A listing of the standard constituent information for logical unit 8 and a sample set of input for logical unit 4 are given in Appendices 1 and 2 respectively.

Logical unit 8 expects four types of data:

i) One card each for all the possible constituents, KONTAB, to be included in the analysis along with their frequencies, FREQ, in cycles/hour and the constituent with which they should be compared under the Rayleigh criterion, KMPR. The format used is (4X,A5,3X,F13.10,4X,A5). Unless

KONTAB is specifically designated on logical unit 4 for inclusion, a blank data field for KMPR results in the constituent not being included in the analysis. A blank card terminates this data type.

ii) Two cards specifying values for the astronomical arguments SO,HO,PO,ENPO, PPO,DS,DH,DP,DNP,DPP in the format (5F13.10).

SO = mean longitude of the moon (cycles) at the reference time origin.

HO = mean longitude of the sun (cycles) at the reference time origin.

PO = mean longitude of the lunar perigee (cycles) at the reference time origin.

ENPO = negative of the mean longitude of the ascending node (cycles) at the reference time origin.

PPO = mean longitude of the solar perigee (perihelion) at the reference time origin.

DS,DH,DP,DNP,DPP are their respective rates of change over a 365 day period at the reference time origin.

iii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shift along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format (6X,A5,1X,6I3,F5.2,I4) and contains the following information:

KON = constituent name,

II,JJ,KK,LL,MM,NN = the six Doodson numbers for KON,

SEMI = the phase correction for KON,

NJ = the number of satellite constituents.

A blank card terminates this data type.

If NJ > 0, information on the satellite constituents follows, three satellites per card, in the format (11X,3(3I3,F4.2,F7.4,1X,I1,1X)). For each satellite the values read are:

LDEL,MDEL,NDEL = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;

PH = the phase correction of the satellite constituent relative to the phase of the main constituent;

EE = the amplitude ratio of the satellite tidal potential to that of the main constituent;

IR = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,

= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semidiurnal constituents,

= otherwise if no correction is required to the amplitude ratio.

iv) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is (6X,A5,I1,2X,4(F5.2,A5,5X)) and the respective values read are:

KON = the name of the shallow water constituent,

NJ = the number of main constituents from which it is derived,

COEF,KONCO = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

Logical unit 4 contains six types of data.

i) One card for the variables IOUT1,RAYOPT,ZOFF,ICHK in the format (I2,2X,F4.2,2X,F10.0,I2).

IOUT1 = 6 if the only output desired is a line printer listing of results,
= 2 if both analysis card output and listing are desired.
RAYOPT = Rayleigh criterion constant value if different from 1.0.
ZOFF = constant to be subtracted from all the hourly heights.
ICHK = 0 if the hourly height input data is to be checked for format errors,
= otherwise if this checking to be waived.

ii) One card for each possible inference pair. The format is (2(4X,A5,E16.10),2F10.3) and the respective values read are:

KONAN & SIGAN = the name and frequency of the analyzed constituent to be used for the inference,
KONIN & SIGIN = the name and frequency of the inferred constituent,
R = amplitude ratio of KONIN to KONAN,
ZETA = the Greenwich phase lag of the inferred constituent subtracted from the Greenwich phase lag of the analyzed constituent.

These are terminated by one blank card.

iii) One card for each shallow water constituent, other than those in the standard 69 constituent data package, to be considered for inclusion in the analysis. The Rayleigh comparison constituent is also required and the additional shallow water constituent must be found in data type i) of logical unit 8 but have a blank data field where the Rayleigh comparison constituent is expected. The format is (6X,A5,4X,A5) and a blank card is required at the end.

iv) One card in the format (I1,1X,4I2,2X,4I2) specifying the following information on the period of the analysis:

INDY = 8 indicates an analysis is desired for the upcoming period;
= 0 indicates no further analyses are required;
1HH1,1DD1,1MM1,1YY1= hour, day, month and year of the beginning of the analysis (measured in time ITZONE of input data v));
1HHL,1DDL,1MML,1YYL= hour, day, month and year of the end of the analysis.

v) One card in the format (I1,4X,A5,3A6,A4,A3,1X,2I2,I3,I2,5X,A5) containing the following information on the tidal station:

INDIC = 1 if J card output is desired (see 1.4),
= otherwise if not;
KSTN = tidal station number;
(NA(J),J=1,4) = tidal station name (22 characters maximum length);
ITZONE = time zone of the hourly observations;
LAD,LAM = station latitude in degrees and minutes;
LOD,LOM = station longitude in degrees and minutes;
IREF = reference station number.

vi) The hourly height data cards contain the following information in the format (I1,1X,I5,7X,3I2,12A4).

KOLI = 1 or 2 indicates whether this specific card is the first or second one for that day,
= otherwise indicates a non-data card which is ignored.

JSTN = tidal station number.

ID,IM,IY = day, month and year of the heights on this card.

(KARD(J),J=1,12) = the hourly heights in integer form. The final constituent amplitudes are in units 1/100 of those for the hourly height. Missing values should be specified as a blank field or 9999.

When KOLI=1 the first hourly height on the data card is assumed to be at 0100 hr and when KOLI=2 it is assumed to be at 1300 hr. Although it is not necessary, for reasons outlined in 2.3.1, it is recommended that these observations be recorded in Greenwich mean time.

After the initial analysis of a computer run is completed, control returns to input iv). Successive cards are read then until either a 0 or 8 value is found for INDY.

The hourly height data cards need not begin and end so as to include exactly the analysis period. The program ignores data outside this range. However if more than one analysis is desired from a single job submission and hourly height data cards do extend beyond the first analysis period, care should be taken to ensure that one of these cards does not have KOLI=0 or blank, otherwise the job will be terminated. This is because all successive cards after the one containing the last hour of the desired analysis period are read in input iv) format.

1.4 Output

Four logical units are used for the output of results from the tidal heights analysis program. Device number 6 is the line printer; 2 and 7 are files used for analysis punched card, and alphatext or flexowriter J punched card outputs respectively; and 9 is a temporary disc file used for storing preliminary punch output and program termination. 6 and 9 are required for all program runs whereas the use of 2 and 7 is controlled by the input variables IOUT1 and INDIC which are read from device 4.

When IOUT1 is 6, INDIC is other than 1, and there are no inferred constituents, the only output is two pages on the line printer. The first of these lists the constituents included in the least squares fit, their frequencies in cycles per hour (although eight decimal places are given, depending on computer accuracy, less than this number may be significant), the C and S coefficient values (see 2.2.1) measured in units 1/100 of those for the hourly heights, and their respective standard deviation estimates. It also specifies the number of hourly height observations (excluding gaps) within the analysis period, the average and standard deviation of the original observations, the root mean square residual error, and the matrix condition number. In the columns titled AL, GL, A, and G, the second page respectively lists the amplitudes and phases (degrees) obtained for each constituent from the C and S coefficient values, and the same amplitudes and phases after nodal modulation and astronomical argument adjustments. The initial and final hour of the analysis are also specified along with the Rayleigh criterion constant ('separation'), the midpoint of the analysis period, the total number of possible hourly observations in the analysis period, and the total number of possible observations used in the analysis. This last value includes gaps in the record and is the largest odd number less than or equal to the total number of possible hourly observations (if the total number of possible hourly observations is an even number, the last hour is ignored). If there is at least one inferred constituent, page 2 results are repeated with the inclusion of inferred constituents and appropriate adjustments to the constituents from which the inferences were made. Appendix 3 lists the final page of results obtained from the input values of Appendix 2.

The only effect of changing the value of IOUT1 to 2 (regardless of INDIC's value) is to store on file 2, the same information as the second (and third) page(s) of the line printer. The list of constituent names, amplitudes and Greenwich phase lags begins on line 5 of this file and is in the correct format for input to the tidal heights prediction program, namely (5X,A5,28X,F8.4,F7.2).

When INDIC is 1, J card output for the alphatext or flexowriter is produced on logical units 6 and 7. This should not be required by the general user as it is simply a reorganization of the results in a manner so that they can be used for further processing, such as lunitidal computations and the production of tide tables. On the line printer these results are given on two pages following the standard output. The first of these lists the phase and amplitude results in a specific constituent order while the second page rearranges them according to increasing frequency and prints them in a format similar to that of device 7. Blank lines separate different constituent groups, and on the card punch, these groups are padded if necessary by a blank J card to give them an even number of elements.

1.5 Program Conversion, Modifications, Storage, and Dimension Guidelines

The source program and constituent data package described in this manual were tested on the IBM 370-168 computer installation at the University of British Columbia and the UNIVAC 1106 at Institute of Ocean Sciences, Patricia Bay. Although as much of the program as possible was written in basic ASCII FORTRAN, some changes will have to be made before the program and data package can be used on other installations. These may include:

i) converting the character code from EBCDIC to BCD,

ii) switching the numeric-character to integer conversion technique in subroutine INPUT from DECODE to the local equivalent.

iii) deleting some or all REAL*8 declaration statements. These are used either to permit alphanumeric strings longer than 4 characters (which is the ASCII FORTRAN single precision word length) to be read (e.g. constituent names are read in A5 format), or to gain computational precision in the average, standard deviation, and sine/cosine iteration formula calculations of subroutine SCFIT2. CDC installations should not require double precision variables for either of these cases because their single precision is about 1.5 that of IBM and UNIVAC, and because there are no alphanumeric strings longer than 6 characters (which is the CDC word length) in the program.

iv) altering the variable list structure for ENTRY statements and reference to them.

v) changing some or all of the logical unit (file reference) numbers from their present values (namely 2,4,6,7,8,9) in order to conform with local machine restrictions.

vi) deleting, or changing the form of, the END FILE statement in subroutine DGM388 which writes an end of file mark before rewinding device 9.

The program in its present form requires approximately 5000, and 52000 UNIVAC words, for the storage of its instructions, and arrays respectively. A large part of this is due to AS, the 170 by 170 matrix resulting from the least squares fit for constituent amplitudes and phases (see 2.2.1). If memory requirements are restrictive on a particular installation, array storage can be cut to approximately 38000 words by storing only the upper triangle of this matrix in a one dimensional array. A program version that does this is available upon request, however it will have somewhat slower processing times than the one described in this manual (see 2.2.2 for details).

Changing the number or type of constituents in the standard data package may require some alterations to the analysis program. The arrays NAME and ARRAY in subroutines SETTAB and DGM388 respectively, are used for producing J card output, and their contents to have been designed specifically for the present data package. If J card output is desired, appropriate changes to these arrays should accompany any to the data package. If constituents are added to the standard data package, the dimensions of several arrays may have to be altered. Restrictions on the minimum dimension of such arrays are now given.

Let

	MTOT	be the total number of possible constituents contained in the data package (presently 146),
	M	be the number of constituents considered for inclusion in the analysis (presently 69 + the number of shallow water constituents specifically designated for inclusion),
	MCON	be the number of main constituents in the standard data package (presently 45),
	MSAT	be the sum of the total number of satellites for these main constituents and the number of main constituents with no satellites (presently 162+8),
and	MSHAL	be the sum for all shallow water constituents, of the number of main constituents from which each is derived (presently 251).

Then in the main program, arrays KONTAB, FREQ, and KMPR should have minimum dimension MTOT; arrays KON,C,S,SIG,ERC,ERS,A,EPS,KO,AA and GD should have minimum dimension M; array AS should have minimum dimension 2M-1 by 2M-1; array NKON should have dimension at least as large as the number of extra shallow water constituents specifically designated for analysis inclusion (its present maximum is 15); and array Z should be large enough to contain the hourly heights (and gaps) in the analysis period (its present maximum is 375 days).

In subroutine INPUT, because arrays Z,AS,A, and EPS are in a common block, they should be dimensioned the same as in the main program.

In subroutine VUF, arrays KON,VU,F and NJ should have minimum dimension MTOT; arrays II,JJ,KK,LL,MM,NN, and SEMI should have minimum dimension MCON; arrays EE,LDEL,MDEL, NDEL, IR, and PH should have minimum dimension MSAT; and KONCO, COEFF should have minimum dimension MSHAL.

In subroutine INFER, arrays KONAN,KONIN,SIGAN,SIGIN,R, and ZETA can presently accommodate a maximum of 10 inferred constituents.

In subroutine SETTAB, arrays KO,AA,GD and NKON should be dimensioned as in the main program, while NAME should have minimum dimension M.

In subroutine DGM388,ARRAY should have minimum dimension M; and arrays IZONE,CT,DATAN,NUM,JJ,STATNO,G, and H should have minimum dimensions M+1.

In subroutine SCFIT2, arrays X,A,AM, and EPM are in COMMON with Z,AS,A, and EPS of the main program and hence should have the same dimensions. Arrays CW,SW,RHSC, and RHSS should have minimum dimension M, and array RHS should have minimum dimension 2M-1. AC and AS should have the same first dimension as A and care should be taken that through their equivalence relationships, neither AC and AS, nor RHSC and RHSS overlap.

Finally, in subroutine CHLSKY, arrays A and F should have minimum dimensions 2M-1 by 2M-1,and 2M-1 respectively.

2. TIDAL HEIGHTS ANALYSIS PROGRAM DETAILS

2.1 The Constituent Data Package

2.1.1 The astronomical variables

The astronomical variables required by the tidal analysis program were used by Doodson [1921] in his development of the tidal potential. From them one can calculate the position of the sun or moon, and hence the tide generating forces, at any time. These variables are:

$S(t)$ = mean longitude of the moon,
$H(t)$ = mean longitude of the sun,
$P(t)$ = mean longitude of the lunar perigee,
$N'(t)$ = negative of the longitude of the mean ascending node,
and $P'(t)$ = mean longitude of the solar perigee (perihelion).

For H, N' and P' these longitudes are measured along the ecliptic eastward from the mean vernal equinox position at time t; while for S and P they are measured in the ecliptic eastward from the mean vernal equinox position at time t to the mean ascending mode of the lunar orbit, and then along this orbit. Together with the rates of change of these variables, τ the local mean lunar time, and the Doodson numbers for each tidal constituent, one can calculate the constituent frequencies, their astronomical argument phase angles V, and their nodal modulation phase, u, and amplitude, f, corrections.

The values of the astronomical variables in the constituent data package were calculated from power series expansion formulae in the Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac [1961], and the Astronomical Ephemeris [1974]. These formulae were derived from Newcomb's Tables of the Sun and a revision of Brown's lunar theory (used in the development of his Tables of Motion of the Moon) so that it is in accord with Newcomb's. Calculations were done in double precision on an IBM computer and have approximately 14 significant digits accuracy.

In the program, the astronomical variables used in the calculation of V(t), u(t) and f(t), are evaluated to the first term in their Taylor expansion; e.g. $S(t) = S(t_0) + (t-t_0)\ \frac{dS(t_0)}{dt}$. Provided they are consistent with other time dependent calculations, t and t_0 can be chosen arbitrarily. However, in order to facilitate computations (see 2.2.1), t is chosen to be the central hour of the desired analysis, and in order to gain precision t_0, the reference time origin, is taken to be 000 ET[1] January 1, 1976. This latter date, it was felt, would be closer to the analysis period of most records than the previous choice of 000 ET January 1, 1901, and hence would yield more accurate results via the linear approximation.

In keeping with the choice of reference time origin and astronomical variable specifications, t should be measured in Ephemeris time. However, the correction from Universal time is irregular and in most cases small, so it has been assumed for computational purposes that all observations are recorded in E.T.

The frequencies specified for all constituents have been calculated from the rates of change of the astronomical variables and the respective Doodson numbers. This means that they too are calculated at the time origin January 1, 1976; although there would be very little change were they taken at January 1, 1901 instead.

---

[1] Ephemeris Time (ET) is the uniform measure of time defined by the laws of dynamics and determined in principle from the orbital motion of the Earth as represented by Newcomb's Tables of the Sun. Universal or Greenwich Mean Time is defined by the rotational motion of the Earth and is not rigorously uniform.

2.1.2 Choice of constituents and Rayleigh comparison pairs

There is a maximum of 146 possible tidal constituents that can be included in the tidal analysis, 45 of these are astronomical in origin (main constituents) while the remaining 101 are shallow water constituents.(1) Because computation time (and cost) of the computer program increases approximately as the square of the number of constituents included in the analysis, and because for many tidal stations, most of the shallow water constituents are insignificant, a smaller standard package was seen as adequate for general use. Based on the suggestions of G. Godin, it was decided that this package contain all the main constituents and 24 of the shallow water. However, provision was made so that other shallow water constituents among the 77 remaining could be included if desired.

The Rayleigh comparison constituent is used for the purpose of deciding whether or not a specific constituent should be included in the analysis. If F_0 is the frequency of such a constituent, F_1 is the frequency of its Rayleigh comparison constituent and T is the time span of the proposed record to be analyzed, then the constituent will be included in the analysis only if $|F_0 - F_1|\ T \geq \mathrm{RAY}$. RAY is commonly given the value 1 although it can be specified differently in the program.

---

(1) The criterion for selecting these main constituents was to include all the diurnal and semidiurnal constituents with Cartwright [1973] tidal potential amplitudes greater than 0.00250, along with M3 and the most important low frequency constituents. Section 2.1.3 gives the analogous shallow water constituent criterion.

In order to determine the set of Rayleigh comparison pairs, it is important to consider, within a given constituent group (e.g. diurnal or semi-diurnal), the order of constituent inclusion in the analysis as T (the time span of the record to be analyzed) increases. Assuming this point of view, the specific objectives used when constructing the set listed in Appendix 1 were:

i) within each constituent group, when possible, have the order of constituent selection correspond with decreasing magnitude of tidal potential amplitude (as calculated by Cartwright [1973]),

ii) when possible, compare a candidate constituent with whichever of the neighbouring, already selected constituents, that is nearest in frequency,

iii) when there are two neighbouring constituents of relatively equal tidal potential amplitude, rather than waiting until the record length is sufficient to permit the selection of both at the same time (i.e. by comparing them to each other), choose a representative of the pair whose inclusion will be as early as possible. This will give information sooner about that frequency range, and via inference, still enable some information to be obtained on both constituents.

The Rayleigh comparison pairs chosen for the low frequency, diurnal, semidiurnal and terdiurnal constituent groups are given in Tables 1, 2, 3 and 4 respectively. Figures given for the length of record required for constituent inclusion assume a Rayleigh criterion constant value (input variable RAYOPT) of 1.0.

2Q1 and SIG1 provide an example of objective iii). Because 2Q1 has a greater frequency separation from Q1 and hence would appear in an analysis of shorter record length than SIG1, it was chosen as the representative.

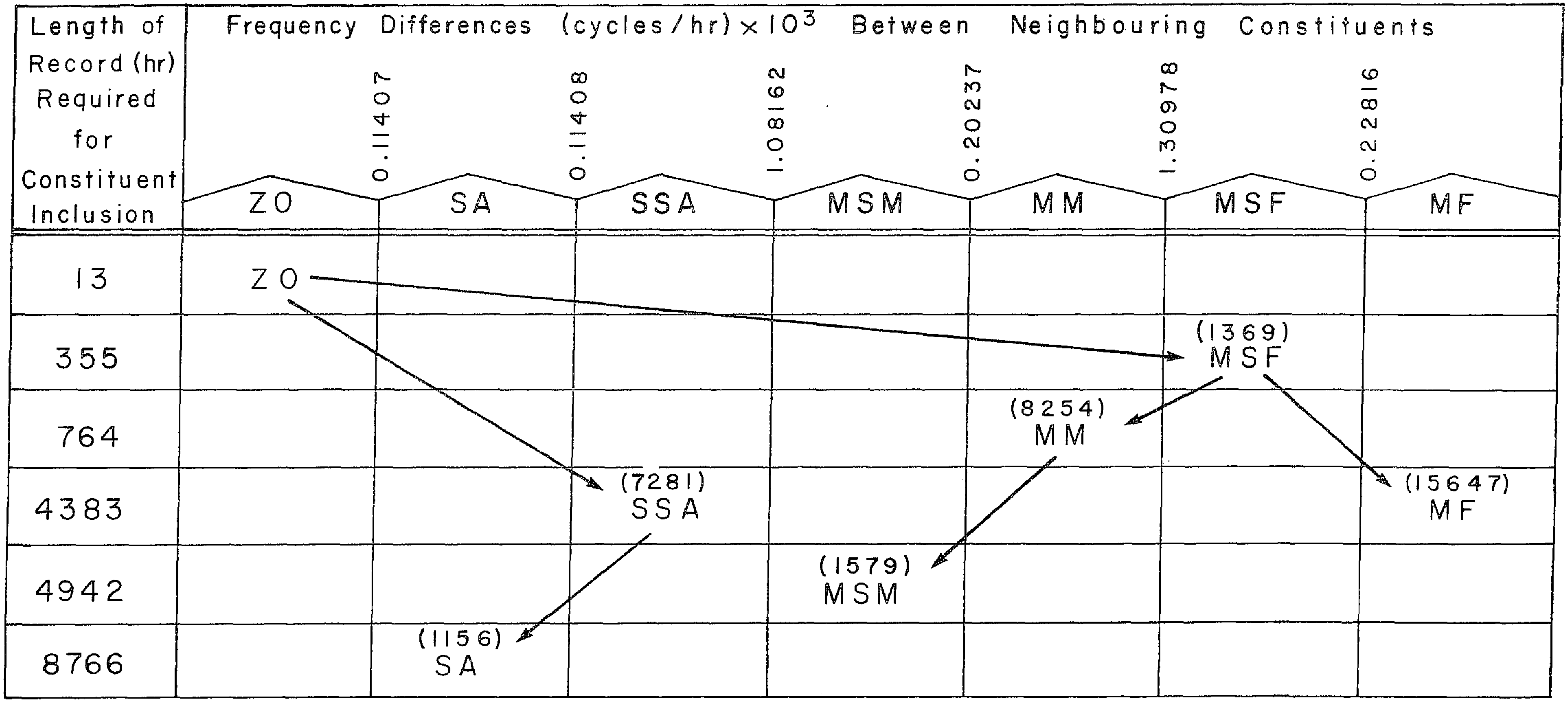
However, it can be seen in several cases, that it was not possible or feasible to adhere to all the objectives just outlined. Choosing a Rayleigh comparison constituent from the list of those constituents already included in the analysis proved to be difficult near the frequency edges of constituent groups. Upward arrows indicate failure to uphold this objective. OO1 is such a case. For it, the potential comparison pairs were SO1, K1 and J1. The first of these would result in both SO1 and OO1 appearing at the same later time than had J1 or K1 been chosen. Hence, information about OO1 would be unnecessarily delayed. Although, due to the tidal potential amplitude of J1, objective i) is violated with both the second and third choices, it was felt that the third was a better compromise. With it, OO1 only appears 11 hours sooner than J1.

Table I

Order of Slower than Diurnal Constituent Selection in Accordance with the Rayleigh Criterion

←— Link Rayleigh Comparison Pairs

( ) Tidal Potential Amplitudes for Main Constituents



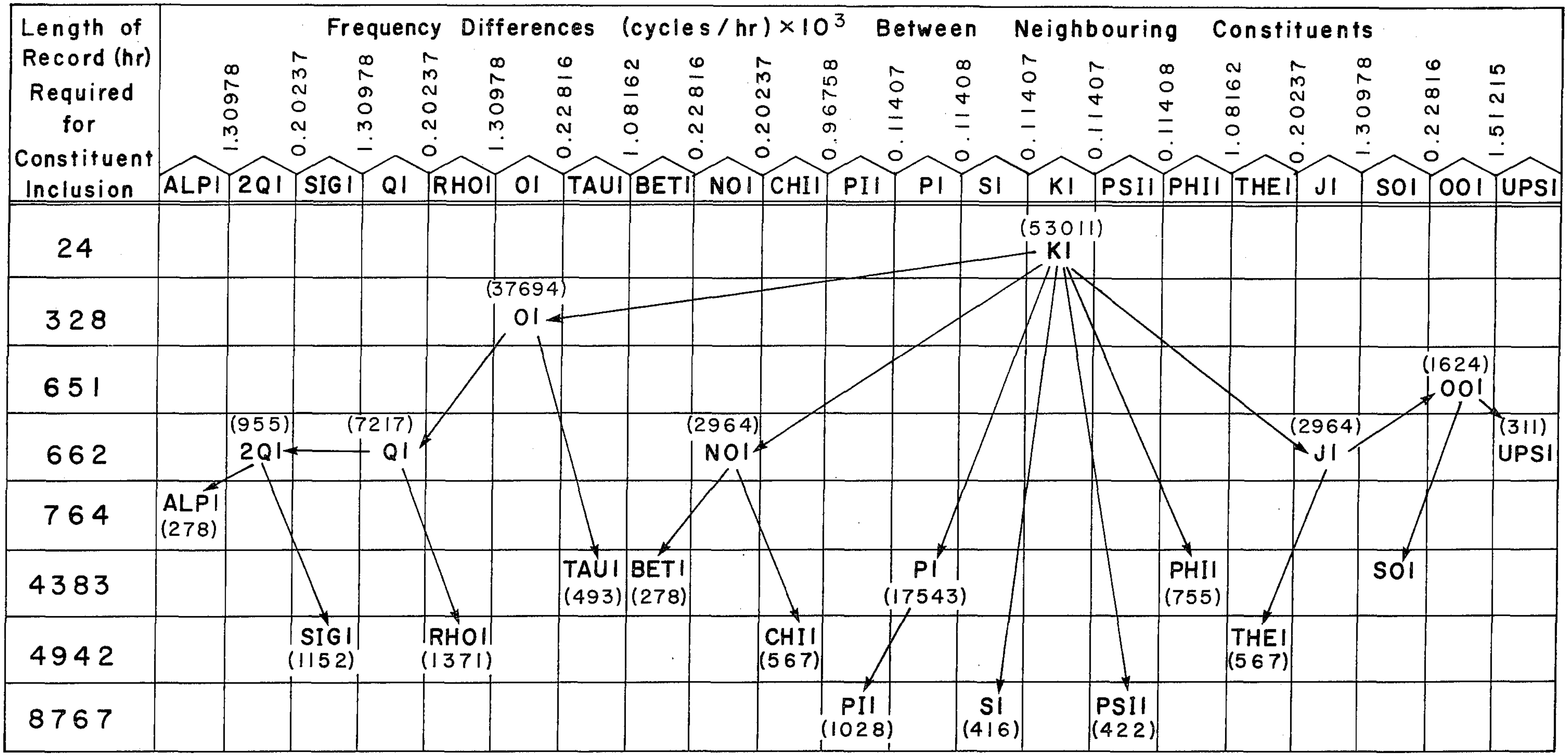
Length of Record (hr) Required for Constituent Inclusion	Frequency Differences (cycles/hr) × 10^3 Between Neighbouring Constituents					
		0.11407	0.11408	1.08162	0.20237	1.30978	0.22816
	ZO	SA	SSA	MSM	MM	MSF	MF
13	ZO						
355						(1369) MSF	
764					(8254) MM		
4383			(7281) SSA				(15647) MF
4942				(1579) MSM			
8766		(1156) SA					

Table 2

Order of Diurnal Constituent Selection in Accordance with the Rayleigh Criterion

←— Link Rayleigh Comparison Pairs

( ) Tidal Potential Amplitudes for Main Constituents



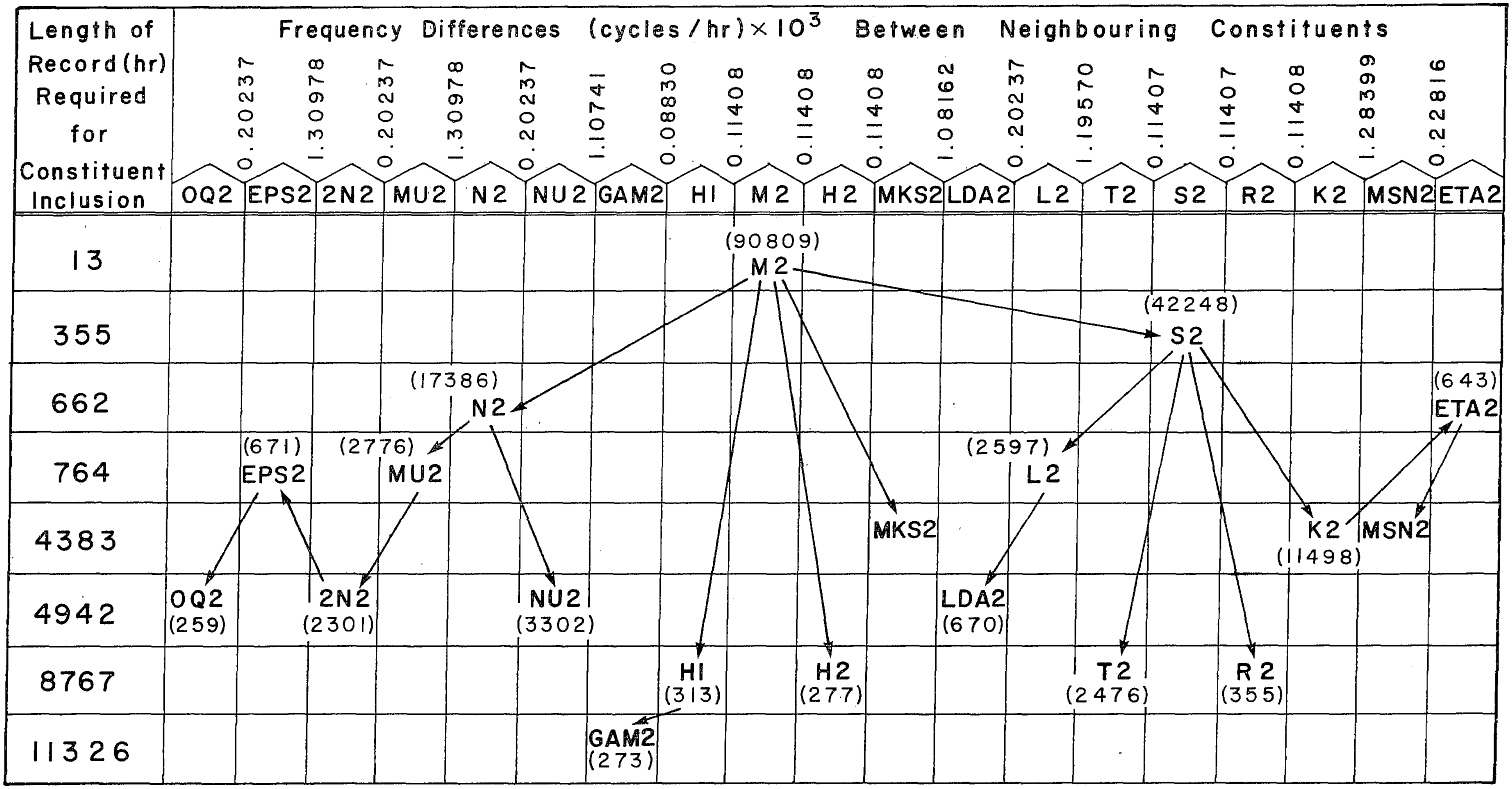
Length of Record (hr) Required for Constituent Inclusion	Frequency Differences (cycles/hr) ×10^3 Between Neighbouring Constituents																																								
	ALP1	1.30978	2Q1	0.20237	SIG1	1.30978	Q1	0.20237	RHO1	1.30978	O1	0.22816	TAU1	1.08162	BET1	0.22816	NO1	0.20237	CHI1	0.96758	PI1	0.11407	P1	0.11408	S1	0.11407	K1	0.11407	PSI1	0.11408	PHI1	1.08162	THE1	0.20237	J1	1.30978	SO1	0.22816	OO1	1.51215	UPS1
24																											K1 (53011)														
328											O1 (37694)																														
651																																							OO1 (1624)		
662			2Q1 (955)				Q1 (7217)										NO1 (2964)																		J1 (2964)						UPS1 (311)
764	ALP1 (278)																																								
4383													TAU1 (493)		BET1 (278)								P1 (17543)								PHI1 (755)						SO1				
4942					SIG1 (1152)				RHO1 (1371)										CHI1 (567)														THE1 (567)								
8767																					PI1 (1028)				S1 (416)				PSI1 (422)												

Table 3

Order of Semi Diurnal Constituent Selection in Accordance with the Rayleigh Criterion

←— Link Rayleigh Comparison Pairs

( ) Tidal Potential Amplitudes for Main Constituents



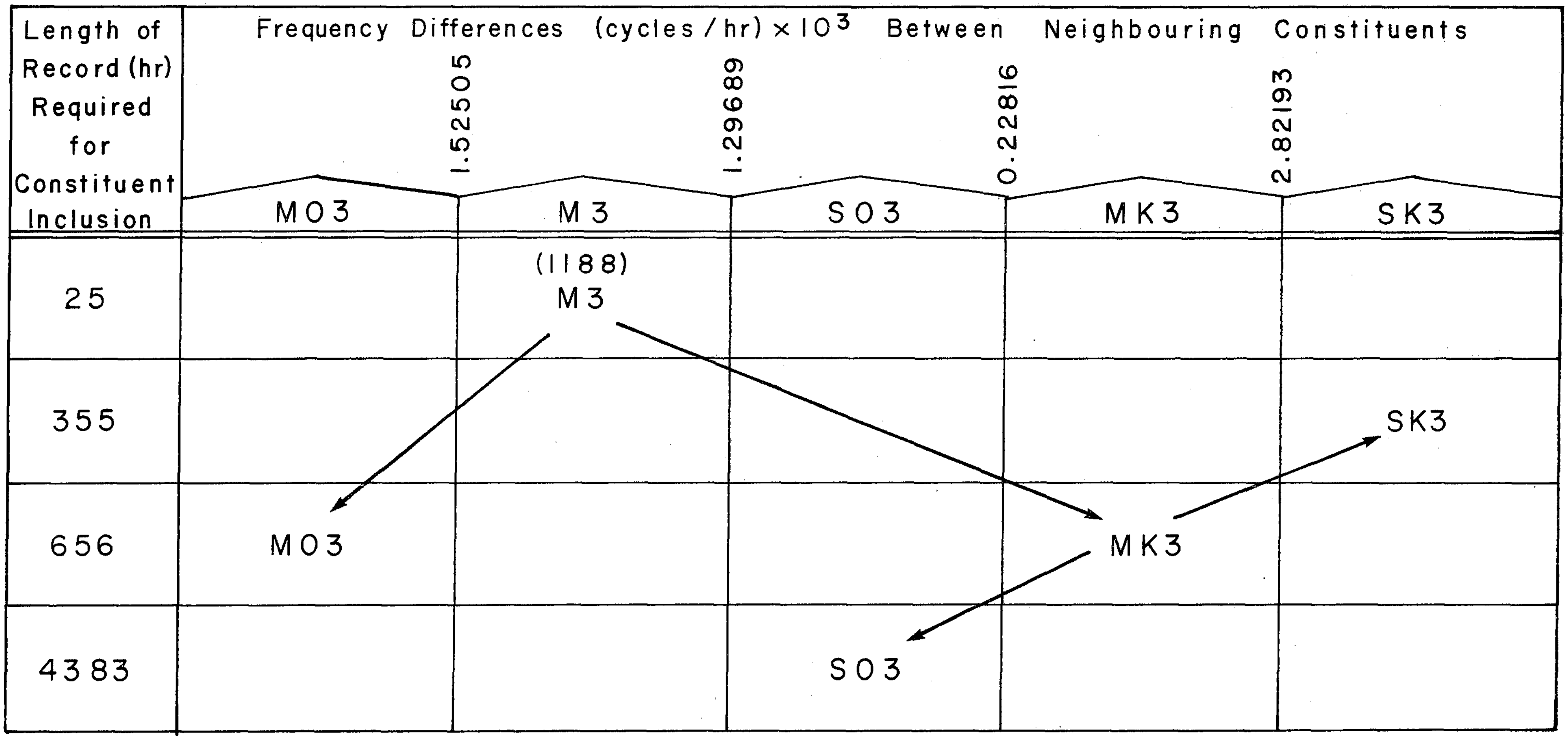
Length of Record (hr) Required for Constituent Inclusion	Frequency Differences $(cycles/hr)\times10^3$ Between Neighbouring Constituents																		
	0.20237 1.30978 0.20237 1.30978 0.20237 1.10741 0.08830 0.11408 0.11408 0.11408 1.08162 0.20237 1.19570 0.11407 0.11407 0.11408 1.28399 0.22816																		
	OQ2	EPS2	2N2	MU2	N2	NU2	GAM2	H1	M2	H2	MKS2	LDA2	L2	T2	S2	R2	K2	MSN2	ETA2
13									(90809) M2										
355															(42248) S2				
662					(17386) N2														(643) ETA2
764		(671) EPS2		(2776) MU2									(2597) L2						
4383											MKS2						K2 (11498)	MSN2	
4942	OQ2 (259)		2N2 (2301)			NU2 (3302)						LDA2 (670)							
8767								H1 (313)		H2 (277)				T2 (2476)		R2 (355)			
11326							GAM2 (273)												

Table 4

Order of Terdiurnal Constituent Selection in Accordance with the Rayleigh Criterion

← Link Rayleigh Comparison Pairs

( ) Tidal Potential Amplitude for Main Constituent



Frequency Differences (cycles / hr) × 10^3 Between Neighbouring Constituents

1.52505 1.29689 0.22816 2.82193

Length of Record (hr) Required for Constituent Inclusion	MO3	M3	SO3	MK3	SK3
25		(1188) M3			
355					SK3
656	MO3			MK3	
4383			SO3		

Table 5

Shallow Water Constituents in the Standard Data Package

Shallow Water Constituent	Length of Record (hr) Required for Constituent Inclusion	Component Main Constituents and Lengths (hr) of Record Required for Their Inclusion in the Analysis					
SO1	4383	S2	355	O1	328		
MKS2	4383	M2	13	K2	4383	S2	356
MSN2	4383	M2	13	S2	355	N2	662
MO3	656	M2	13	O1	328		
SO3	4383	S2	355	O1	328		
MK3	656	M2	13	K1	24		
SK3	355	S2	355	K1	24		
MN4	662	M2	13	N2	662		
M4	25	M2	13				
SN4	764	S2	355	N2	662		
MS4	355	M2	13	S2	355		
MK4	4383	M2	13	K2	4383		
S4	355	S2	355				
SK4	4383	S2	355	K2	4383		
2MK5	24	M2	13	K1	24		
2SK5	178	S2	355	K1	24		
2MN6	662	M2	13	N2	662		
M6	26	M2	13				
2MS6	355	M2	13	S2	355		
2MK6	4383	M2	13	K2	4383		
2SM6	355	S2	355	M2	13		
2SK6	4383	M2	13	S2	355	K2	4383
3MK7	24	M2	13	K1	24		
M8	26	M2	13				

K2 is an example of an unavoidable violation of objective i). Because it is so close in frequency to S2, its importance as a major semidiurnal constituent does not insure it an early inclusion in the analysis package.

Because shallow water constituents do not have a tidal potential amplitude, objective i) does not apply to them. However, based on his experience, Godin was able to suggest a hierarchy of their relative importance. A further criteria used when selecting comparison pairs for them was that no shallow water constituent should appear in an analysis before all the main constituents, from which it is derived, have also been selected. Table 5 shows that this has been upheld for all the shallow water constituents in the standard 69 constituent data package.

I suggest that the objectives outlined here be employed when choosing the Rayleigh comparison constituent for any additions to the list of possible constituents to be included in the analysis.

2.1.3 Satellite constituents and nodal modulation

Doodson's [1921] development of the tidal potential contains a very large number of constituents. Due to the great length of record required for their separation, several of these can be considered, for all intents and purposes, unanalysable. The standard approach to this problem is to form clusters consisting of all constituents with the same first three Doodson numbers. The major contributor in terms of tidal potential amplitude lends its name to the cluster and the lesser constituents are called satellites.

The method of analysis uses this main and satellite constituent approach in the following manner. The Rayleigh criteria is applied to the main constituent frequencies to determine whether or not they are to be included in the analysis.

For each of those so chosen, we analyse at its frequency and obtain an apparent amplitude and phase. However, because these results are actually due to the cumulative effect of all the constituents in that cluster, an adjustment is made so that only the contribution due to the main constituent is found. This adjustment is called the nodal modulation.

In order to make the nodal modulation correction to the amplitude and phase of a main constituent, it is necessary to know the relative amplitudes and phases of the satellites. As is commonly done, it is assumed in this program that the same relationship as is found with the equilibrium tide (tidal potential), holds with the actual tide. That is, the tidal potential amplitude ratio of a satellite to its main constituent is assumed to be equal to the corresponding tidal heights amplitude ratio, and the difference in tidal potential phase equals the difference in tidal height phase.

The source of the tidal potential amplitude ratio, as found in the constituent data package of Appendix 1, is Cartwright [1971,1973]. Using new computation methods and the latest values for the astronomical constants, he obtains more accurate results than those from the previously used Doodson computations. It should be noted that in several cases (whenever the satellite arises via a third order term), this version of the constituent data package requires that the amplitude ratio be multiplied by a latitude correction factor. This represents a refinement of the previous version where a latitude of 50°N was assumed and the multiplication factor was incorporated directly into the ratio.

Phase differences between satellites and main constituents arise when the tidal potential development yields different trigonometric terms for these constituents. The common convention is to express all terms in cosine form and so an extra -¼ cycle phase shift is introduced if the term was originally a sine. Satellites requiring such a shift are called third order. A further ½ cycle change is also introduced when all negative amplitudes are made positive.

Because several test analyses indicate less consistent results when third order satellites are included in the N2 and L2 nodal modulation, Godin has decided to delete these from the present standard constituent data package. Instead he suggests that the results of analyses with this package should be compared with those of previous analyses in order to find the most suitable adjustment for these constituents.

The only other main constituents that do not have all their satellites included for nodal modulation are the slow frequency constituents. For them, no satellites are specified. Because low frequency noise may be as much as an order of magnitude greater than the satellite contributions, and Mm, MSf and Mf when they are detectable are often of shallow water origin, the effect of making corrections for the expected satellites would be to obscure further, rather than clarify the actual low frequency periodic signal.

Section 2.3.2 gives further details on the nodal modulation correction.

2.1.4 Shallow water constituents

Shallow water tidal constituents arise from the distortion of main constituent tidal oscillations in shallow water. Because the speed of propagation of a progressive wave is approximately proportional to the square root of the depth of water in which it is travelling, shallow water has the effect of retarding the trough of a wave more than the crest. This distorts the original sinusoidal wave shape and introduces harmonic signals that are not predicted in tidal potential development. The frequencies of these derived harmonics can be found by calculating the effect of non-linear terms in the hydrodynamic equations of motion on a signal due to one or more main constituents (see Godin, [1972] pages 154-164 for further details).

The shallow water constituents chosen for inclusion in the standard 69 constituent data package were suggested by G. Godin. They are listed in Table 5 and are derived only from the largest main constituents, namely M2,S2,N2,K2,K1 and O1, using the lowest types of possible interaction. The 77 additional shallow water constituents that can be included in the analysis if so desired are derived from lesser main constituents and higher types of interaction. In the constituent data package listing of Appendix 1, they can be spotted by their lack of a Rayleigh comparison constituent.

When shallow water effects are noticeable, main constituents, if they are close in frequency, may coexist or be masked by constituents of non-linear origin. The resultant nodal modulation will be due to the pair and thus will not coincide to the calculated modulation of the main constituent. In suspected cases, the effectiveness of nodal corrections in a series of successive analyses will indicate the presence of pairs or emphasize the predominance of one constituent over the other. The following table (taken from unpublished notes of Godin) lists compound constituents which may coexist with or mask constituents of direct astronomical origin. In all cases except SO1 and MO3, the main rather than the compound constituent is included in the standard constituent data package.

TABLE 6

Shallow Water Constituents that may Mask Main Constituents

Main Constituent	Compound Constituent which may coexist at or near its frequency
Q1	NK1
O1	MK1 *
TAU1	MP1 *
NO1 (with M1 as a satellite)	NO1 *
P1	SK1 *
K1	MO1
J1	MQ1
SO1	SO1
OQ2	OQ2 *
EPS2	MNS2
2N2	O2 *
MU2	2MS2
N2	KQ2 *
GAM2	OP2 *
M2	KO2 *
L2	2MN2 *
S2	KP2
K2	K2
MO3	MO3 *
M3	NK3 *

* The modulation or frequency of the compound constituent is sufficiently different that the pair could be separated if a long enough record of high precision were available.

2.2 The Least Squares Method of Analysis

2.2.1 Formulation of the problem

The first stage in the actual analysis of tidal records is the least squares fit for constituent amplitude and phase. If the tidal record is of minimum length 13 hours, the present program and data package insure that the constant constituent Z0 is always included in the analysis. If σ_j for $j = 1, M$ are the frequencies (cycles/hr) of the other tidal constituents chosen for inclusion in the analysis by the Rayleigh criterion, then the problem is to find the amplitudes A_j and phases ϕ_j of the function $C_o + \sum_{j=1}^{M} A_j \cos(2\pi(\sigma_j t_i - \phi_j))$ that best fit the series of observations $y(t_i)$, $i = 1,N$[(1)]. Assuming $N > 2M + 1$ we see that it is impossible to solve the system $y(t_i)=C_o + \sum_{j=1}^{M} A_j \cos(2\pi(\sigma_j t_i - \phi_j))$ exactly because it is overdetermined. Hence, it is necessary to adopt a criterion which will enable unique optimum values for the parameters A_j and ϕ_j to be found. The most common optimization criterion used, and the one chosen here, is the least squares technique.

Re-expressing $\sum_{j=1}^{M} A_j \cos(2\pi(\sigma_j t_i - \phi_j))$ as

$$\sum_{j=1}^{M} (C_j \cos(2\pi\sigma_j t_i) + S_j \sin(2\pi\sigma_j t_i)),$$

where $A_j = \sqrt{C_j^2 + S_j^2}$ and $\phi_j = \arctan S_j/C_j$, so that the fitting function is linear in the parameters S_j and C_j and hence more easily solved; and rewriting $y(t_i)$ as y_i; the objective of the least squares technique is to minimize

---

(1) In order to minimize the loss of accuracy due to round off, the average of the hourly heights observations is subtracted from all original values. The $y(t_i)$ values mentioned in all computations henceforth are actually the resultant deviations. At the end of all calculations, C_o is adjusted by this mean value.

$$T = \sum_{i=1}^{N} (y_i - C_o - \sum_{j=1}^{M} (C_j \cos 2\pi\sigma_j t_i + S_j \sin 2\pi\sigma_j t_i))^2 ,$$

for C_o and all C_j, S_j $j=1,M$.

This is done by solving the following 2M + 1 simultaneous equations for J=1,M:

$$0 = \frac{\partial T}{\partial C_o} = 2 \sum_{i=1}^{N} (y_i - C_o - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i) (-1) ;$$

$$0 = \frac{\partial T}{\partial C_j} = 2 \sum_{i=1}^{N} (y_i - C_o - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i) (- \cos 2\pi\sigma_j t_i) ;$$

$$0 = \frac{\partial T}{\partial S_j} = 2 \sum_{i=1}^{N} (y_i - C_o - \sum_{j=1}^{M} C_j \cos 2\pi\sigma_j t_i - \sum_{j=1}^{M} S_j \sin 2\pi\sigma_j t_i) (- \sin 2\pi\sigma_j t_i).$$

This results in the matrix equation $B\underline{x} = \underline{y}$ of Figure 1 (page 33).

Gaps in the data record (i.e. missing hourly observations) are easily handled by the least squares method because it is not necessary that the observation times t_i $i = 1,N$ be evenly spaced. For example, if the analysis covers the total time period of 100 hr but hours 50 to 74 inclusive are missing, then t_{50} will correspond to the 75th hour. However, because the following identities which simplify the summations require that the observation times be evenly spaced, it is necessary that each of the matrix terms be calculated as the sum of contributions over the data periods that contain no gaps. Assuming that $[n_0,n_1]$ is the hour range of a section of record containing no gaps, we can substitute $t_k = k$ in the matrix coefficients expressions since the times are at successive hours.

Using the relationships

$$\cos a \cos b = \tfrac{1}{2}(\cos(a+b) + \cos(a-b))$$

$$\sin a \sin b = \tfrac{1}{2}(\cos(a-b) - \cos(a+b))$$

$$\sin a \cos b = \tfrac{1}{2}(\sin(a+b) + \sin(a-b)),$$

the formula for the sum of a geometric series, namely

$$a + ar + \ldots + ar^n = a(r^{n+1} - 1)/(r - 1),$$

and expressing cos x and sin x as the real and imaginary parts of e^{ix}, we obtain the identities:

$$\sum_{k=n_0}^{n_1} \cos kx = \sin\left[\left(\frac{n_1-n_0+1}{2}\right)x\right] \cos\left[\left(\frac{n_1+n_0}{2}\right)x\right] / \sin\,(x/2),$$

and

$$\sum_{k=n_0}^{n_1} \sin kx = \sin\left[\left(\frac{n_1-n_0+1}{2}\right)x\right] \sin\left[\left(\frac{n_1+n_0}{2}\right)x\right] / \sin\,(x/2)$$

Hence, the summation expressions in the least squares matrix can be simplified (with regard to computer execution time) as follows.

$$\sum_{k=n_0}^{n_1} \cos\,(2\pi\sigma_1 k)\,\cos\,(2\pi\sigma_2 k) = 1/2 \sum_{k=n_0}^{n_1} \left[\cos\,(2\pi k(\sigma_1+\sigma_2)) + \cos\,(2\pi k(\sigma_1-\sigma_2))\right]$$

$$= 1/2\,\big(\sin\,((n_1-n_0+1)\pi\,(\sigma_1+\sigma_2))\,\cos\,((n_1+n_0)\pi\,(\sigma_1+\sigma_2))\,/\,\sin\pi\,(\sigma_1+\sigma_2)$$

$$+ \sin\,((n_1-n_0+1)\pi\,(\sigma_1-\sigma_2))\,\cos\,((n_1+n_0)\pi\,(\sigma_1-\sigma_2))\,/\,\sin\pi\,(\sigma_1-\sigma_2)\big)$$

$$\sum_{k=n_0}^{n_1} \sin\,(2\pi\sigma_1 k)\,\sin\,(2\pi\sigma_2 k) = 1/2 \sum_{k=n_0}^{n_1} \left[\cos\,(2\pi k(\sigma_1-\sigma_2)) - \cos\,(2\pi k(\sigma_2+\sigma_2))\right]$$

$$= 1/2\,\big(\sin\,((n_1-n_0+1)\pi\,(\sigma_1-\sigma_2))\,\cos\,((n_1+n_0)\pi\,(\sigma_1-\sigma_2))\,/\,\sin\pi\,(\sigma_1-\sigma_2)$$

$$- \sin\,((n_1-n_0+1)\pi\,(\sigma_1+\sigma_2))\,\sin\,((n_1+n_0)\pi\,(\sigma_1+\sigma_2))\,/\,\sin\pi\,(\sigma_1+\sigma_2)\big).$$

$$\sum_{k=n_0}^{n_1} \sin(2\pi\sigma_1 k)\,\cos\,(2\pi\sigma_2 k) = 1/2 \sum_{k=n_0}^{n_1} \left[\sin\,(2\pi k(\sigma_1+\sigma_2)) + \sin\,(2\pi k(\sigma_1-\sigma_2))\right]$$

$$= 1/2\,\big((\sin\,((n_1-n_0+1)\pi\,(\sigma_1+\sigma_2))\,\sin\,((n_1+n_0)\pi\,(\sigma_1+\sigma_2))\,/\,\sin\pi\,(\sigma_1+\sigma_2)$$

$$+ \sin\,((n_1-n_0+1)\pi\,(\sigma_1-\sigma_2))\,\sin\,((n_1+n_0)\pi\,(\sigma_1-\sigma_2))\,/\,\sin\pi\,(\sigma_1-\sigma_2)).$$

With these substitutions made in Fig. 1 we have the least squares matrix equation $B\underline{x} = \underline{y}$ generated in subrouting SCFIT2.

Because B is symmetric it is sufficient to store only its upper triangle consisting of $2M^2 + 3M + 1$ elements instead of the entire matrix of $(2M + 1)^2$ elements. However this saving does not come without its price as the complicated subscripting scheme required to keep track of these elements when stored in a one dimensional array significantly affects the solution computation time (see 2.2.2 for details). Assuming that this program will not have storage problems on most computer installations, it was decided to waive the storage saving in favour of improved computational time.

Partitioning the matrix equation $B\underline{x} = \underline{y}$ into the form

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} \underline{c} \\ \underline{s} \end{pmatrix} = \begin{pmatrix} \underline{y}_c \\ \underline{y}_s \end{pmatrix} ,$$

where B_{11}, B_{12}, B_{21}, B_{22}, $\underline{c}$, $\underline{s}$, $\underline{y}_c$, $\underline{y}_s$ have dimensions $(M+1) \times (M+1)$, $(M+1) \times M$, $M \times (M+1)$, $M \times M$, $(M+1) \times 1$, $M \times 1$, $(M+1) \times 1$, $M \times 1$ respectively, it is easily seen when $n_0=-n_1$, that B_{12} and B_{21} become zero matrices and two smaller matrix equations, $B_{11}\underline{c} = \underline{y}_c$ and $B_{22}\underline{s} = \underline{y}_s$ result. The combined computation time to solve these equations is less than that of the original (see 2.2.2) so it is desirable to attain this condition when possible. Since the time origin of the hourly observations is arbitrary provided it is consistent with that of the astronomical argument V, we can attain the desired condition for a record with no gaps by choosing the central hour of the record as the origin. (This requires that the total number of observations be odd and is satisfied by ignoring the last observation, if the total is even). Although there is generally no corresponding matrix simplification in the case of a record with gaps, for consistency with the foregoing choice, it is convenient to choose the central hour of the record universally as the time origin.

FIGURE 1

The Matrix Equation $B\underline{x} = \underline{y}$ Resulting from the Least Squares Fit for Constituent Amplitudes and Phases

$$C_k = \sum_{i=1}^{N} \cos 2\pi\sigma_k t_i$$

$$S_k = \sum_{i=1}^{N} \sin 2\pi\sigma_k t_i$$

$$CC_{kj} = \sum_{i=1}^{N} (\cos 2\pi\sigma_k t_i)\ (\cos 2\pi\sigma_j t_i) = CC_{jk}$$

$$SS_{kj} = \sum_{i=1}^{N} (\sin 2\pi\sigma_k t_i)\ (\sin 2\pi\sigma_j t_i) = SS_{jk}$$

$$CS_{kj} = \sum_{i=1}^{N} (\cos 2\pi\sigma_k t_i)\ (\sin 2\pi\sigma_j t_i) = SC_{jk}$$

$$\begin{bmatrix}
N & C_1 & C_2 & \cdots\cdots & C_M & S_1 & S_2 & \cdots\cdots & S_M \\
C_1 & CC_{11} & CC_{12} & \cdots\cdots & CC_{1M} & CS_{11} & CS_{12} & \cdots\cdots & CS_{1M} \\
C_2 & CC_{21} & CC_{22} & \cdots\cdots & CC_{2M} & CS_{21} & CS_{22} & \cdots\cdots & CS_{2M} \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
C_M & CC_{M1} & CC_{M2} & \cdots\cdots & CC_{MM} & CS_{M1} & CS_{M2} & \cdots\cdots & CS_{MM} \\
S_1 & SC_{11} & SC_{12} & \cdots\cdots & SC_{1M} & SS_{11} & SS_{12} & \cdots\cdots & SS_{1M} \\
\vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
S_M & SC_{M1} & SC_{M2} & \cdots\cdots & SC_{MM} & SS_{M1} & SS_{M2} & \cdots\cdots & SS_{MM}
\end{bmatrix}
\begin{bmatrix}
C_0 \\ C_1 \\ C_2 \\ \vdots \\ C_M \\ S_1 \\ \vdots \\ S_M
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N} y_i \\
\sum_{i=1}^{N} y_i \cos 2\pi\sigma_1 t_i \\
\sum_{i=1}^{N} y_i \cos 2\pi\sigma_2 t_i \\
\vdots \\
\sum_{i=1}^{N} y_i \cos 2\pi\sigma_M t_i \\
\sum_{i=1}^{N} y_i \sin 2\pi\sigma_1 t_i \\
\vdots \\
\sum_{i=1}^{N} y_i \sin 2\pi\sigma_M t_i
\end{bmatrix}$$

2.2.2 Solution of the matrix equation, the condition number, and statistical properties

Most of the discussion and development of the Cholesky factorization algorithm introduced in this section is taken directly from Forsythe and Moler [1967]. Although all results and discussion are now stated only for the matrix B and the equation Bx = y, they apply as well for the partitioned systems B_{11}, $B_{11}C = Y_C$ and B_{22}, $B_{22}S = Y_S$.

In addition to symmetry, a useful property of the matrix B in Fig. 1, is that it qualifies the matrix equation for a simpler solution than were it a general matrix, is its positive definiteness. This property requires that for all $(2M+1) \times 1$ dimensional vectors $\underline{x} \neq 0$, $\underline{x}^T B \underline{x} > 0$.

The positive definiteness of B can be demonstrated by considering the overdetermined matrix equation $\underline{y} = A\underline{x} + \underline{e}$ resulting from the system of equations $y(t_i) = C_0 + \sum_{j=1}^{M} (C_j \cos 2\pi\sigma_j t_i + S_j \sin 2\pi\sigma_j t_i) + e_i$ for i = 1,N where the vector $\underline{x}^T = (C_0, C_1, C_2, \ldots C_M, S_1, S_2, \ldots S_M)$, $\underline{y}^T = (y(t_1), \ldots y(t_N))$ and $\underline{e}$ is the vector of residuals. It is easily seen that $A^T A = B$, and so for any $\underline{x} \neq 0$,

$$\underline{x}^T B \underline{x} = \underline{x}^T A^T A \underline{x} = \underline{z}^T \underline{z} = \sum_{i=1}^{N} z_i^2, \text{ where } \underline{x}^T A^T = \underline{z}^T = (z_1, \ldots z_N).$$

It is worth mentioning that the overdetermined system $\underline{y} = A\underline{x} + \underline{e}$ can be solved in many ways, depending on the criterion chosen for minimizing $\underline{e}$. For our purposes, those methods which solve the system without changing the form of the matrix are impractical from a storage, processing time and rounding error point of view because the first dimension of A(= the number of hourly observations) is commonly 9000. However, minimizing $\underline{e}^T\underline{e}$ is equivalent to the least squares criterion adopted here.

An important result for any positive definite symmetric matrix B is that it can be uniquely decomposed in the form $B = GG^T$, where G is a lower triangular matrix with positive diagonal elements (1). Expanding this relationship leads to the matrix element equalities:

$$b_{jj} = \sum_{k=1}^{j} g^2_{jk},$$

$$b_{ij} = \sum_{k=1}^{j} g_{ik}g_{jk} \quad \text{for all } i > j.$$

The algorithm resulting from using these equations in the proper order to find the elements of G is known as Cholesky's square root method for factoring a positive definite matrix (also attributed to Banachiewicz; see Faddeev and Faddeeva [1963]). Unlike other matrix decomposition methods such as Gaussian elimination, it does not have to search for and divide by pivots. Such techniques must insure that the reduced matrix elements are not too large so that rounding errors and loss of accuracy do not occur. In Cholesky's method however, we can see that $|g_{ij}| \leq \sqrt{b_{ii}}$ for all i,j and so upper bounds for the elements of G always exist.

Prior to revision of SCFIT2 the constituent Z0 was treated like all others in the sense that both C_0 and S_0 were found. In order to insure that Z0's phase was zero, S_0 was forced to zero by setting the corresponding diagonal element of B to a very large number. Although this did not affect the symmetry or positive definiteness of the matrix, there was a loss of accuracy (which is more evident on computers, such as IBM, with a smaller number of single precision significant digits) due to the corresponding

---

(1) If B is symmetric but not positive definite a similar decomposition exists. However some elements of G may be complex or, in the degenerate case, zero along the diagonal.

large element in the reduced matrix. For this reason and the slightly reduced array storage required, in the revised subroutine, only C_0 is now sought in the least squares fit.

Once B has been decomposed into the upper and lower triangular matrices, it is a relatively easy matter to solve the matrix solution. This is done by breaking down the equation $GG^T\underline{x} = \underline{y}$ into $G\underline{b} = \underline{y}$ and $G^T\underline{x} = \underline{b}$. Because of the triangular nature of G, these equations can be solved by forward and backward substitution for $\underline{b}$ and $\underline{x}$ respectively.

The amount of arithmetic in a matrix algorithm is usually measured by the number of multiplicative operations (i.e. multiplications and divisions) used, since there are normally approximately the same number of additive operations. For a matrix of dimension n x n, the Cholesky factorization algorithm requires n square roots and approximately 1/6 n^3 multiplications. This compares favourably with the 1/3 n^3 multiplications required by Gaussian elimination (Wilkinson [1967]) to produce a triangular matrix.

Wilkinson [1967] suggests a factorization of B into LDL^T, where L is a lower triangular matrix and D is a positive diagonal matrix, that involves no more multiplications than Cholesky and avoids the square roots. However, assuming that the time ratio of a square root operation to a multiplication is 15:1 (approximate ratio for IBM 370-168) and that all 69 constituents in the data package are included in the analysis (i.e. n = 137) the time saved by eliminating the square roots is only 0.5%. Furthermore some of this gain would be replaced by time required for storing and retrieving information from the additional matrix D, and for the n additional division operations each time a solution is calculated by forward and backward substitution. Hence, the factorization was not adopted in the present program.

Because the time required for the factorization of B varies as the cube of the number of unknowns, an approximate four-fold time reduction should result when the tidal record has no gaps and the partitioned rather than the original matrix equations are solved. However as the following table of execution times for sections of subroutine SCFIT2 demonstrates, significant improvements can also be expected in the time required for matrix generation, and error calculation. The values shown were obtained on an IBM 370-168 computer with a 34 constituent analysis of a 38 day tidal record.

TABLE 7

Comparison of Processing Times between the Partitioned and Non-partitioned Matrix Equation Solutions

	Partitioned matrix system times (sec.)	Non-partitioned matrix system (sec.)
Parameter initializations & right hand generation	.347	.346
Matrix generation	.059	.178
Matrix factorization	.049	.146
Solution	.010	.018
Error calculations	.128	.403

The advantage of storing the lower triangle of B in a two dimensional array and thus avoiding the complicated subscripting scheme required for one dimensional storage is illustrated in TABLE 3. The values shown were obtained from solving a matrix equation of order 110 resulting from a yearly tidal analysis. For comparison, the times required when using the Forsythe and Moler technique (Gaussian elimination with partial pivoting, forward and backward substitution) with two dimensional matrix storage are also included.

TABLE 8

Comparison of Processing Times for the Cholesky and Forsythe & Moler Methods of Solving the Least Squares Matrix Equation

	PROCESSING TIMES (SEC.)		
	Cholesky method with two dimensional array storage	Cholesky method with one dimensional array storage	Forsythe and Moler method with two dimensional array storage
Matrix factorization	.753	1.299	2.313
Solution	.053	.059	.047

A rough indication of the roundoff difficulties associated with solving the equation Bx = y is given by the matrix condition number. Although several different definitions for a condition number exist, an appropriate one for our purposes, in the sense that it pertains to least squares matrices and is easily calculated, is specified by Davis and Rabinowitz [1961]. Its development is as follows.

If $\{\underline{b}_1, \ldots \underline{b}_n\}$ are n-dimensional vectors such that the matrix

$$B = \begin{pmatrix} \underline{b}_1 \\ \vdots \\ \underline{b}_n \end{pmatrix} (\underline{b}_1 \ldots\ldots \underline{b}_n) = \begin{pmatrix} \underline{b}_1\cdot\underline{b}_1 & \ldots\ldots & \underline{b}_1\cdot\underline{b}_n \\ \vdots & & \vdots \\ \underline{b}_n\cdot\underline{b}_1 & \ldots\ldots & \underline{b}_n\cdot\underline{b}_n \end{pmatrix},$$

then it can be shown that $0 \leq \det(B) \leq \|\underline{b}_1\| \, \|\underline{b}_2\| \ldots\ldots \|\underline{b}_n\|$ where if $\underline{b}_j = (b_{j1}, \ldots\ldots b_{jn})$, the norm $\|\underline{b}_j\| = \sqrt{\sum_{i=1}^{n} b^2_{ji}}$. Furthermore det(B) = 0 if and only if the vectors are linearly dependent, and $\det(B) = \|b_1\| \ldots\ldots \|\underline{b}_n\|$ if and only if they are orthogonal (i.e. $\underline{b}_i\cdot\underline{b}_j = 0$ for $i \neq j$). This determinant is known as the Gram determinant of the system $\{\underline{b}_1, \ldots \underline{b}_n\}$ and is the square of the n-dimensional volume of the parallelepiped whose edges are these vectors.

Since it can be shown that all least squares matrices can be expressed in this manner, this result can be applied to our situation. In particular when the vectors are normalized so that $||\underline{b}_i|| = 1$, the actual value of det(B) will always be bounded and provide a measure of the linear independence of the system, and hence roundoff difficulties encountered in solving the equation. A value close to 1 will mean near orthogonality, a virtually diagonal matrix for B, and thus an easy solution. On the other hand, a value close to 0 will mean that at least two rows are near scalar multiples of one another, and thus greater accuracy problems will occur when their difference is calculated during the equation solution.

For our particular case observe that $\det(B) = \det(GG^T) = (\det G)^2 = \prod_{i=1}^{n} g_{ii}^2$, and that B can be written as

$$GG^T = \begin{bmatrix} \underline{g}_1 \cdot \underline{g}_1 & \cdots & \underline{g}_1 \cdot \underline{g}_n \\ \vdots & & \\ \underline{g}_n \cdot \underline{g}_n & & \underline{g}_n \cdot \underline{g}_n \end{bmatrix},$$

$$\text{where } G^T = \begin{vmatrix} g_{11} & g_{21} & \cdots & g_{n1} \\ 0 & g_{22} & \cdots & g_{n2} \\ & & \ddots & \\ 0 & \ldots\ldots & 0 & g_{nn} \end{vmatrix} = (\underline{g}_1, \underline{g}_2 \cdots \underline{g}_n).$$

Since $b_{jj} = \sum_{k=1}^{j} g_{jk}^2$, $||\underline{g}_j|| = \sqrt{b_{jj}}$ and the determinant of the matrix resulting from normalizing the $\underline{g}_j$ vectors is $\prod_{i=1}^{n} (g_{ii}^2/b_{ii})$. The square root of this value is the volume of the n-dimensional parallelepiped whose edges are these normalized vectors and is the quantity calculated as the condition number of the matrix B.

The statistical properties of the least squares fit solution can be found in any analysis of variance or regression model text. They are outlined briefly as follows.

Reverting to the overdetermined problem statement, the least squares objective can be stated as finding the vector $\underline{x}$ in $\underline{y} = A\underline{x} + \underline{e}$ such that $\underline{e}^T\underline{e}$ is minimized. This yields the solution $\hat{\underline{x}} = (A^TA)^{-1} A^T\underline{y}$.

The total sum of squares is $\underline{y}^T\underline{y}$ and the sum of squares due to regression is $\hat{\underline{x}}^TA^Ty$. Their difference is the residual error sum of squares and this difference divided by the degrees of freedom in the fit is the residual mean square error (MSE). 'Degrees of freedom' is the difference between the number of hourly observations (excluding gaps) and A the number of parameters fit in the analysis. If there were M constituents including Z0 chosen for the analysis,the degrees of freedom would be N - 2M + 1.

If it is assumed, as is commonly done, that the vector $\underline{e}$ is distributed normally with 0 standard deviation and $\sigma^2 I$ variance, where I is the unit diagonal matrix, then the variance of $\hat{\underline{x}}$ is $(A^TA)^{-1} \sigma^2$. Since the mean square residual error is an unbiased estimator for σ^2, an estimate of the standard deviation of $\hat{x}_i$, the ith element of $\hat{\underline{x}}$, is

$$\left((\underline{\mu}_i^T (A^TA)^{-1} \underline{\mu}_i) \text{ MSE}\right)^{1/2}$$

where $\underline{\mu}_i$ is the vector with 1 in the ith position and 0's elsewhere.

2.3 Modifications to the Least Squares Analysis Results

2.3.1 Astronomical argument and Greenwich Phase Lag

Instead of regarding each tidal constituent as the result of some particular component of the tidal potential, an artificial causal agent can be attributed to each constituent in the form of a fictitious star which travels around the equator with an angular speed equal to that of its corresponding constituent. Making use of this conceptual aid, the astronomical argument, V(L,t), of a tidal constituent can then be viewed as the angular position of this fictitious star relative to longitude L, and at time t. Although the longitudinal dependence is easily calculated, for historical reasons L is generally assumed to be the Greenwich meridian, and V is reduced to a function of one variable.

The Greenwich phase lag, g, is the difference between this astronomical argument for Greenwich and the phase of the observed constituent signal. Its value is dependent upon the time zone in which the hourly heights of the record were taken. This means that when phases at various stations, not necessarily in the same time zone, are compared, they must be reduced to a common zone in order to avoid spurious differences due to difference relative times. Specifically, if σ is the constituent frequency and $g(j+\Delta_j)$ and $g(j)$ are the Greenwich phase lags evaluated for time zones $j+\Delta_j$ and j respectively (e.g. Pacific Standard Time is +8), then

$$g(j+\Delta_j) = g(j) + (\Delta_j)\sigma.$$

Although these adjustments are easily calculated, they can be tedious because each constituent must be handled individually. Therefore, to avoid possible misinterpretation of phases from nearby stations or subsequent phase alterations, it is suggested that all observations be recorded in, or converted to, GMT.

The calculation of g (see 2.3.3) requires that the astronomical argument need only be evaluated at one time, the central hour of the analysis period. For a particular main constituent, it is calculated as

$$V = i_0 \tau + j_0 S + k_0 H + \ell_0 P + m_0 N' + n_0 P'$$

where $i_0, j_0, k_0, \ell_0, m_0, n_0$ are the Doodson numbers of the constituent and S, H, P, N', P' are the astronomical variables defined in 2.1.1. τ, the number of mean lunar days from the absolute time origin of 000 ET January 1, 1976, is calculated as sum of the local mean solar time from this origin and (H-S), and so need not be read from data cards.

For shallow water constituents, the astronomical argument is calculated as the linear combination of the coefficient number and the astronomical argument of the main constituents from which it is derived. For example, $V_{MSN2} = V_{M2} + V_{S2} - V_{N2}$ and $V_{2MK5} = 2\ V_{M2} + V_{K1}$.

2.3.2 Nodal corrections

Most of this section has been taken from the unpublished notes of G. Godin which were written subsequent to Cartwright's [1971,1973] recalculation of the tide generating potential. The material presented here is intended to give greater detail than that of section 2.1.3.

Due to the presence of satellites to the major contributor in a given cluster, it is known from tidal potential theory that the analyzed signal found at the frequency σ_j of the main constituent is actually the result of

$$a_j \sin (V_j - g_j) + \sum_k A_{jk}\, a_{jk} \sin (V_{jk} - g_{jk}) + \sum_\ell A_{j\ell}\, a_{j\ell} \cos (V_{j\ell} - g_{j\ell})$$

for the diurnal and terdiurnal constituents of direct gravitational origin, and

$$a_j \cos (V_j - g_j) + \sum_k A_{jk}\, a_{jk} \cos (V_{jk} - g_{jk}) + \sum_\ell A_{j\ell}\, a_{j\ell} \sin (V_{j\ell} - g_{j\ell})$$

for the slow and semidiurnal constituents. a, g and V are the true amplitude, Greenwich phase, and astronomical argument at the central time of the record for all the constituents. Single j subscripts refer to the major contributor while jk and $j\ell$ subscripts refer to satellites originating from tidal potential terms of the second and third order respectively. A is the element of the interaction matrix resulting from the interference of a satellite with the main constituent.

It is the convention in tides and an assumption for our least squares fit that all constituents arise through a cosine term and positive amplitude. That is, the contribution for a constituent whose astronomical argument is V_j and whose Greenwich phase is g_j, is expected to be in the form $a_j \cos (V_j - g_j)$ for $a_j > 0$. However, the diurnal and terdiurnal constituents, assuming that they are due to second order terms in the tidal potential, actually arise through a $b_j \sin (V_j - g_j)$ term where b_j may be negative. Hence, a phase correction (variable SEMI read in data input iii) from logical unit 8) of either -1/4 or -3/4 cycles is necessary.

i.e. $$\begin{aligned} b_j \sin (V_j - g_j) &= |b_j| \cos (V_j - g_j - 1/4) \text{ if } b_j \geq 0 \\ &= |b_j| \cos (V_j - g_j - 3/4) \text{ if } b_j < 0. \end{aligned}$$

Similarly, an adjustment of 1/2 cycle will only be necessary for slow and semidiurnal main constituents if the tidal potential amplitude is negative.

Making these changes, the combined result of a constituent cluster in the diurnal and terdiurnal cases is

$$|a_j| \cos (V'_j - g_j) + \sum_k A_{jk}\, a_{jk} \cos (V'_{jk} + \alpha_{jk} - g_k) + \sum_\ell A_{j\ell}\, a_{j\ell} \cos (V'_{j\ell} + \alpha_{j\ell} - g_{j\ell})$$

where if $a_j < 0$, $V' = V - 3/4$, $\alpha_{jk} = 1/2$ and $\alpha_{j\ell} = 3/4$,

and if $a_j > 0$, $V' = V - 1/4$, $\alpha_{jk} = 0$ and $\alpha_{j\ell} = 1/4$.

A further phase adjustment to satellite constituents can be made if we wish to ensure that their amplitudes are positive. This convention was adopted for the data package of Appendix 1 (variable PH read in data input iv) from logical unit 8). Replacing a_{jk} and $a_{j\ell}$ by their absolute values we now see that

$$\begin{aligned} \alpha_{jk} &= 0 \quad \text{if both } a_{jk} \text{ and } a_j \text{ have the same sign,} \\ &= 1/2 \quad \text{otherwise;} \end{aligned}$$

$\alpha_{j\ell}$ = 1/4 if both $a_{j\ell}$ and a_j have the same sign,

= 3/4 otherwise.

Similarly, for the slow and semidiurnal constituents the cluster contribution can be rewritten as

$$|a_j| \cos (V'_j - g_j) + \sum_k A_{jk} |a_{jk}| \cos (V'_{jk} + \alpha_{jk} - g_{jk}) + \sum_\ell A_{j\ell} |a_{j\ell}| \cos (V'_{j\ell} + \alpha_{j\ell} - g_{j\ell})$$

where $V' = V + 1/2$ if $a_j < 0$,

V otherwise;

$\alpha_{jk} = 0$ if a_{jk} and a_j have the same sign,

1/2 otherwise;

$\alpha_{j\ell} = -1/4$ if $a_{j\ell}$ and a_j have the same sign

1/4 otherwise.

Special note should be made of the terdiurnal M3 because both it and its only satellite are due to third order terms in the tidal potential. Hence, both contribute directly through a cosine term and so behave as if they were second order semidiurnals.

In order to determine the amplitude and phase of the major contributor, we assume that the result actually found in the analysis was $f_j a_j \cos (V'_j - g_j + u_j)$, where f_j and u_j are called the nodal modulation corrections in amplitude and phase respectively. To avoid a possible misunderstanding, it is worth mentioning here that the term nodal modulation is actually a misnomer. It and the symbols f and u were first used before the advent of modern computers to designate corrections for the moons nodal progression that were not incorporated into the calculations of the astronomical argument for the main constituent. However, now the term satellite modulation is more appropriate because our correction is due to the presence of satellite constituents differing not only in the contribution of the lunar node to their astronomical argument, but also in the lunar and solar perigee effect.

For the purpose of calculating f_j and u_j it is assumed that the admittance is very nearly a constant over the frequency range within a constituent cluster, and so $g_j = g_{jk} = g_{j\ell}$; and $r_{jk} = |a_{jk}|/|a_j|$, $r_{j\ell} = |a_{j\ell}|/|a_j|$ are equal to the ratio of the tidal equilibrium amplitudes of the satellite to the major contributor. These ratios are latitude dependent when satellites of the third order are involved, necessitating the correction factors mentioned in 2.1.3. However, the ratios are usually small and the correction is slight.

Dropping the 'prime' notation and grouping the second and third order terms in one summation, the relationship between the analysed results for a main constituent and the actual cluster contribution is

$$f_j|a_j| \cos (V_j+u_j-g_j) = |a_j| (\cos (V_j-g_j) + \sum_k A_{jk} r_{jk} \cos (V_j-g_j+\Delta_{jk}+\alpha_{jk}))$$

where $\Delta_{jk} = V_{jk} - V_j$.

Expanding this result and observing that it must be true for all $V_j(t)$, the following explicit formulae are found for f and u:

$$f_j = \sqrt{(1 + \sum_k A_{jk} r_{jk} \cos (\Delta_{jk} + \alpha_{jk}))^2 + (\sum_k A_{jk} r_{jk} \sin (\Delta_{jk} + \alpha_{jk}))^2},$$

$$u_j = \arctan \left[\frac{\sum_k A_{jk} r_{jk} \sin (\Delta_{jk} + \alpha_{jk})}{1 + \sum_k A_{jk} r_{jk} \cos (\Delta_{jk} + \alpha_{jk})} \right] .$$

For an analysis carried out over 2N + 1 consecutive observations, Δt time units apart, A_{jk} is given by $A_{jk} = \dfrac{\sin [(2N+1)\Delta t\ (\sigma_{jk}-\sigma_j)/2]}{(2N+1) \sin [\Delta t\ (\sigma_{jk}-\sigma_j)/2]}$

where σ_j is the frequency of the main contributor and σ_{jk} is that of its satellite. However, A_{jk} is very nearly one even for a one year analysis and in the program is approximated by this value.

For a shallow water constituent whose frequency is calculated as $\sum_{j=1}^{N_0} c_j\sigma_j$, where σ_j is the frequency of the j^{th} main constituent from which it is derived and c_j is the linear coefficient, the nodal modulation corrections for amplitude and phase are computed as $f = \prod_{j=1}^{N_0} f_j^{|c_j|}$ and $u = \sum_{j=1}^{N_0} c_j u_j$.

2.3.3 Final amplitude and phase results

The result of the least squares analysis was to find for a constituent with frequency σ_j, the optimal amplitude A_j and phase ϕ_j value for the tidal signal $A_j \cos 2\pi (\sigma_j t - \phi_j)$. However, due to nodal corrections, when the astronomical argument is calculated at the central time origin t = 0 of the record, we know that the actual contribution of the constituent cluster is $f_j a_j \cos 2\pi (V_j + u_j - g_j)$. Hence, the amplitude and Greenwich phase lag of the constituent corresponding to frequency σ_j can be calculated as $a_j = A_j/f_j$ and $g_j = V_j + u_j + \phi_j$.

2.3.4 Inferred constituents

In accordance with previous notation, tidal signals in this section are assumed to be real in nature. However an alternative presentation using complex numbers, and the basis for the following development is given by Godin [1972].

If the length of a specific tidal record is such that certain important constituents will not be included directly in the analysis, provision is made via the data input on logical unit 4 to include these constituents indirectly by inferring their amplitudes and phases from neighbouring constituents that are included. If accurate amplitude ratios and phase differences are specified, inference has the effect of significantly reducing any periodic behaviour in the amplitudes and phases of the constituent used for the inference. This is due to the removal of interaction from the neighbouring inferred constituent. If it so happens that a constituent specified for inference is included directly in the analysis, the program will ignore the inference calculations.

The actual adjustments are as follows. Assume that the constituent with frequency σ_2 is to be inferred from the constituent with frequency σ_1, and that the least squares fit analysis found the latters contribution to be $A_1^0 \cos 2\pi(\sigma_1 t - \phi_1^0)$, where A_1^0 and ϕ_1^0 are the amplitude and phase respectively (σ_1 and ϕ_1^0 are measured in cycles/hour and cycles respectively). Letting

VU_1 be the astronomical argument + nodal modulation phase correction,
g_1 be the Greenwich phase lag,
f_1 be the nodal modulation amplitude correction factor,
and a_1 be the corrected amplitude,

then from 2.3.3 we know that

$$- \phi_1 = VU_1 - g_1$$

$$\text{and} \quad a_1 = A_1/f_1.$$

Assuming that A_1 and ϕ_1 are the post-inference amplitude and phase for the constituent with frequency σ_1,

$$r_{12} = a_2/a_1 = (A_2/f_2)/(A_1/f_1),$$

$$\text{and} \ \zeta = g_1 - g_2 = VU_1 + \phi_1 - VU_2 - \phi_2$$

(the latter two being data input variables R and ZETA respectively), then the presence of the inferred constituent in the analyzed signal yields the relationship:

$$\begin{aligned}A_1^0 \cos 2\pi(\sigma_1 t-\phi_1^0) &= A_1 \cos 2\pi(\sigma_1 t-\phi_1) + A_2 \cos 2\pi(\sigma_2 t-\phi_2)\\ &= A_1 \cos 2\pi(\sigma_1 t-\phi_1)\left[1+r_{12}\left(\frac{f_2}{f_1}\right)\cos 2\pi\{(\sigma_2-\sigma_1)t+VU_2 - VU_1 + \zeta\}\right]\\ &\quad - A_1 \sin 2\pi(\sigma_1 t-\phi_1)\left[r_{12}\left(\frac{f_2}{f_1}\right)\sin 2\pi\{(\sigma_2-\sigma_1)t+VU_2 - VU_1 + \zeta\}\right].\end{aligned}$$

Since the constituent with frequency σ_2 was not chosen for inclusion in the least squares analysis, $|\sigma_2-\sigma_1|$ $N<RAY$, where N is the record length in hours and RAY is the Rayleigh criterion constant (usually 1.0). Assuming in general that $|\sigma_2-\sigma_1|N$ is small, good approximations to $\cos 2\pi\{(\sigma_2-\sigma_1)t + VU_2-VU_1+\zeta\}$ and $\sin 2\pi\{(\sigma_2-\sigma_1)t + VU_2-VU_1+\zeta\}$ are their average values over the interval [-N/2, N/2], namely

$\sin [\pi N(\sigma_2-\sigma_1)] \cos [2\pi(VU_2 - VU_1 +\zeta)]/\pi N(\sigma_2-\sigma_1)$ and

$\sin [\pi N(\sigma_2-\sigma_1)] \sin [2\pi(VU_2 - VU_1 +\zeta)]/\pi N(\sigma_2-\sigma_1)$ respectively.

Making these substitutions and setting

$$S = r_{12}\left(\frac{f_2}{f_1}\right)\sin [\pi N(\sigma_2-\alpha_1)] \sin [2\pi(VU_2 - VU_1 + \zeta)]/\pi N(\sigma_2-\sigma_1),$$

and $$C = 1 + r_{12}\left(\frac{f_2}{f_1}\right)\sin [\pi N(\sigma_2-\sigma_1)] \cos[2\pi(VU_2 - VU_1 +\zeta)]/\pi N(\sigma_2-\sigma_1),$$

we obtain

$$\frac{A_1^0}{A_1} \cos 2\pi(\sigma_1 t - \phi_1^0) = C \cos 2\pi(\sigma_1 t - \phi_1) - S \sin 2\pi(\sigma_1 t - \phi_1).$$

Expanding and regrouping this result yields

$$\cos 2\pi\sigma_1 t\left[\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 -C \cos 2\pi\phi_1- S \sin 2\pi\phi_1\right]=$$

$$\sin 2\pi\sigma_1 t\left[-\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 + C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1\right].$$

Now since this relationship must hold for all t, both terms in brackets are equal to zero. Hence

$$\frac{A_1^0}{A_1} \cos 2\pi\phi_1^0 = C \cos 2\pi\phi_1 + S \sin 2\pi\phi_1,$$

$$\frac{A_1^0}{A_1} \sin 2\pi\phi_1^0 = C \sin 2\pi\phi_1 - S \cos 2\pi\phi_1,$$

and so

$$A_1 = A_1^0 / \sqrt{C^2 + S^2},$$

$$\phi_1 = \phi_1^0 + [\arctan (S/C)]/2\pi.$$

The relative phase and amplitude of the inferred constituent are then calculated as

$$\phi_2 = VU_1 - VU_2 + \phi_1 - \zeta,$$

and $A_2 = r_{12} A_1 (f_2/f_1)$.

3 USE OF THE TIDAL HEIGHTS PREDICTION COMPUTER PROGRAM

3.1 General Description

This program produces tidal height values at a given location for a specified period of time. Amplitudes and Greenwich phase lags of the tidal constituents to be used in the prediction are required as input and either equally spaced heights or all the high and low values can be produced.

3.2 Routines Required

1) MAIN..................reads in tidal station and time period information, amplitudes and Greenwich phases of constituents to be used in the prediction, and calculates the desired tidal heights.
2) ASTRO.................reads the standard constituent data package and calculates the frequencies, astronomical arguments, and nodal corrections for all constituents.
3) PUT...................controls the output for high-low predictions.
4) HPUT..................controls the output for equally spaced predictions.
5) CDAY..................returns the consecutive day number from a specific origin for any given date in the range 1901 to 1999, and vice versa.

3.3 Data Input

All input data required by the tidal heights prediction program is from logical unit 8. A sample set is given in Appendix 4. Although data types i), ii) and iii) are identical to types ii), iii) and iv) expected on logical unit 8 by the analysis program, for completeness they are repeated here.

i) Two cards specifying values for the astronomical arguments SO, HO, PO, ENPO, PPO, DS, DH, DP, DNP, DPP in the format (5F13.10).

SO = mean longitude of the moon (cycles) at the reference time origin.

HO = mean longitude of the sun (cycles) at the reference time origin.

PO = mean longitude of the lunar perigee (cycles) at the reference time origin.

ENPO = negative of the mean longitude of the ascending node (cycles) at the reference time origin.

PPO = mean longitude of the solar perigee (perihelion) at the reference time origin.

DS, DH, DP, DNP, DPP are their respective rates of change over a 365 day period at the reference time origin.

ii) At least one card for all the main tidal constituents specifying their Doodson numbers and phase shift along with as many cards as are necessary for the satellite constituents. The first card for each such constituent is in the format (6X,A5,1X,6I3,F5.2,I4) and contains the following information:

KON = constituent name,
II,JJ,KK,LL,MM,NN = the six Doodson numbers for KON,
SEMI = the phase correction for KON,
NJ = the number of satellite constituents.

A blank card terminates this data type.

If NJ > 0, information on the satellite constituents follows, three satellites per card, in the format (11X,3(3I3,F4.2,F7.4,1X,I1,1X)). For each satellite the values read are:

LDEL, MDEL, NDEL = the last three Doodson numbers of the main constituent subtracted from the last three Doodson numbers of the satellite constituent;

PH = the phase correction of the satellite constituent relative to the phase of the main constituent;

EE = the amplitude ratio of the satellite tidal potential to that of the main constituent;

IR = 1 if the amplitude ratio has to be multiplied by the latitude correction factor for diurnal constituents,

= 2 if the amplitude ratio has to be multiplied by the latitude correction factor for semi-diurnal constituents,

= otherwise if no correction is required to the amplitude ratio.

iii) One card specifying each of the shallow water constituents and the main constituents from which they are derived. The format is (6X,A5,I1,2X,4(F5.2,A5,5X)) and the respective values read are:

KON = the name of the shallow water constituent,

NJ = the number of main constituents from which it is derived,

COEF, KONCO = combination number and name of these main constituents.

The end of these shallow water constituents is denoted by a blank card.

iv) One card with the tidal station information ISTN, (NA(J),J=1,4), ITZONE, LAD, LAM, LOD, LOM in the format:

(5X,I4,1X,3A6,A4,A3,1X,I2,1X,I2,2X,I3,1X,I2).

ISTN = the station number.

(NA(J),J=1,4) = the station name.

ITZONE = the time zone reference for the 'Greenwich' phases.

LAD, LAM = the station latitude in degrees and minutes.

LOD, LOM = the station longitude in degrees and minutes.

v) One card for each constituent to be included in the prediction, with the constituent name (KON), amplitude (AMP) and phase lag (G) in the format (5X,A5,28X,F8.4,F7.2). (This format is compatible with the analysis program results produced on output device 2). The phase lag units should be degrees (measured in time zone ITZONE) while the units of the predicted tidal heights will be the same as those of the input amplitudes. The last constituent is followed by a blank card.

vi) One card containing the following information on the period and type of prediction desired. The format is (3I3,1X,3I3,1X,A4,F9.5).

IDYO, IMOO, IYRO = the first day, month, and year of the prediction period.

IDYE, IMOE, IYRE = the last day, month and year of the prediction period.

ITYPE = 'EQUI' if equally spaced predictions are desired,
= 'EXTR' if all the high and low tide times and heights are desired.

DT = the time spacing of the predicted values if ITYPE = 'EQUI",
= the time step increment used to initially bracket a high or low value if ITYPE = 'EXTR'.

Equally spaced predictions begin at DT hours on the first day and extend to 2400 hours (assuming 24 is a multiple of DT) of the last day. When ITYPE = 'EXTR', Godin and Taylor [1973] recommend using the following values for DT: 3 hours for a semi-diurnal tide, 6 hours for a diurnal tide, and 0.5 hours for a mixed tide.

Type vi) data may be repeated any number of times. One blank card following a type vi) record will return the program to type iv) input, while two blank cards will end the program execution.

3.4 Output

Two logical units are used for the output of results in the tidal heights prediction program. Device number 6 is the line printer and 10 is a data file. Both equally spaced and high-low predictions are put onto both devices with the same format. However the line printer also records the station name and location along with the amplitudes and phase lags of the constituents used in the prediction. Appendix 5 lists device 10 output resulting from the input of Appendix 4.

When daily high-low values are desired, the date, station number and a series of up to six heights and occurrence times are listed per record. Each record begins with the variable HL whose value is 0 if the first height for that day is a high (i.e. larger than the second height) and 1 if the first height is a low. If there are less than six high-low values for a day, they are padded up to six with the values 9999 and 99.9 for the times and heights respectively. On device 10, the format used for the variables HL, the station number, the day, month, year, and six times and heights is

```
(1X,I1,I5,2I3,I2,6(I5,F5.1)).
```

When equally spaced heights are requested, 8 values are listed on each record preceded by the station number, the time, day, month and year of the first value, and followed by the time increment between heights. On device number 10, the format for these variables is;

```
(1X,I4,F8.4,I3,2I2,8F6.2,F12.4).
```

3.5 Program Conversion, Modifications, Storage and Dimension Guidelines

The source program and constituent data package described in this manual were tested on the UNIVAC 1106 at Patricia Bay Institute of Ocean Sciences. Although the program was written in basic ASCII FORTRAN, there may be some changes needed before the program and data package can be used on other installations. These may include:

i) converting the character code from EBCDIC to BCD.

ii) deleting some or all the REAL*8 declaration statements. Presently these are used only to permit alphanumeric strings longer than 4 characters (which is the ASCII FORTRAN single precision word length) to be read (eg. constituent names are read in A5 format). CDC installations should not require these double precision variables because there are no alphanumeric strings longer than 6 characters (which is the CDC word length) in the program.

iii) switching the Chebyshev iteration formula to double precision in order to maintain accuracy. Tests performed on the UNIVAC demonstrate that this change is not necessary, however on less accurate installations, it may be (UNIVAC real variable single precision has 8.1 significant digits). As a test for machine accuracy, I suggest using the input of Appendix 4 and comparing the results with those listed in Appendix 5. In the event that more precision is needed, the following variables in the main program should be declared REAL*8: CH,CHA, CHB, CHM, CHP, SUM TWOC, BTWOC, HGT, Z, ZA, ZB, ZP, ZM, DTPY;
and all related assignments and library function calls should be altered accordingly (e.g. '=0.0' should become '0.DO' and SQRT should be DSQRT).

iv altering the variable list structure for ENTRY statements and references to them.

v) changing some or all of the logical unit (file reference) numbers from their present values in order to conform with local machine restrictions.

The program in its present form requires approximately 3000 and 15000 UNIVAC words for the storage of its instructions and arrays respectively. As with the analysis program, changing the number or type of constituents in the standard data package may require alteration to the dimensions of some arrays. Restrictions on the minimum dimension of all arrays are now given.

Let

	MTAB	be the total number of possible constituents contained in the data package (presently 146),
	M	be the number of constituents to be included in the prediction,
	MCON	be the number of main constituents in the standard data package (presently 45),
	MSAT	be the sum of the number of satellites for these main constituents and the number of main constituents with no satellites (presently 162 + 8)
	MSHAL	be the sum for all shallow water constituents of the number of main constituents from which each is derived (presently 251),
and	NITER	be the number of iterations required to reduce the time interval within which it is known that a high or low tide exists, to a desired length (with the largest initial interval size of 6 hours and a 6 minute final interval, NITER is 6).

Then in the main program, arrays KONTAB, SIGTAB, V, U, and F should have minimum dimension MTAB; KON, SIG, AMP, G, INDX, TWOC, CH, CHP, CHA, CHB, CHM, ANGO and AMPNC should have minimum dimension M; and the two dimensional array BTWDC should have a minimum dimension of M by NITER. Array COSINE which stores pre-calculated cosine function values over the range of 0° to 360° and is used as a look-up table, presently has 2002 elements.

In subroutine ASTRO, the arrays KON, FREQ, V, U, F and NJ should have minimum dimension MTAB; arrays II, JJ, KK, LL, MM, NN and SEMI should have minimum dimension MCON; arrays EE, LDEL, MDEL, NDEL, IR and PH should have minimum dimension MSAT; and arrays KONCO and COEFF should have minimum dimension MSHAL.

In subroutine PUT, the dimensions of arrays HGTK and ITIME should be at least as large as the maximum number of high and low values per day (this is presently assumed to be 9).

In subroutine HPUT, the dimension of array H should be at least equal to the number of equally spaced tidal height values per output record of logical unit 10 or 6 (presently, this is 8).

In subroutine CDAY, both arrays NDM and NDP should have dimension 12.

4. Tidal Heights Prediction Program Details

4.1 Problem Formulation and the Equally Spaced Predictions Method

The tidal height, $h(t)$, at a particular station may be represented by the harmonic summation (see section 2.3.3)

$$h(t) = \sum_{j=1}^{m} f_j(t)A_j \cos(2\pi(V_j(t) + u_j(t) - g_j)) \quad , \tag{1}$$

where A_j, g_j = the amplitude and phase lag of constituent j,

$f_j(t)$, $u_j(t)$ = the nodal modulation amplitude and phase correction factors for constituent j,

and $V_j(t)$ = the astronomical argument for constituent j.

Expanding $V(t)$ as in section 2.3.1 and using the first order Taylor approximations for the astronomical arguments as in section 2.1.1, $V(t)$ can be re-expressed as

$$\begin{aligned} V(t) &= i\tau(t) + jS(t) + kH(t) + \ell P(t) + mN'(t) + nP'(t) \\ &= i\tau(t_\circ) + jS(t_\circ) + kH(t_\circ) + \ell P(t_\circ) + mN'(t_\circ) + nP'(t_\circ) \\ &\quad + (t-t_\circ)\frac{\partial}{\partial t}[i\tau(t)+jS(t)+kH(t)+\ell P(t)+mN'(t)+nP'(t)]_{t=t_\circ} \\ &= V(t_\circ) + (t - t_\circ)\sigma, \end{aligned}$$

where $t_\circ$ is the reference time origin of 000 ET January 1 1976, and σ is the constituent frequency at this time origin. It follows from this result that $V(t_2) = V(t_1) + (t_2 - t_1)\sigma$ for arbitrary times t_1, t_2, and so $V_j(t)$ can be replaced in (1) by $V_j(t_1) + (t - t_1)\sigma_j$ for some convenient time t_1.

From section 2.3.2 it is seen that f(t) and u(t) are time dependent only through the $\Delta_{jk}(t)$ variable. Since satellites differ from main constituents in only the last three Doodson numbers (see section 2.1.3),

$$\Delta_{jk}(t) = V_{jk}(t) - V_j(t)$$

$$= \Delta \ell P(t) + \Delta m N'(t) + \Delta n P'(t) \ .$$

Using the first order Taylor approximations for P,N', and P', it follows that over a time period $[t_1,t_2]$ the change in $\Delta_{jk}(t)$ is

$$\Delta_{jk}(t_2) - \Delta_{jk}(t_1) = \Delta\ell(P(t_2)-P(t_1))+\Delta m(N'(t_2)-N'(t_1))+\Delta n(P'(t_2)-P'(t_1))$$

$$= (t_2-t_1)\ \frac{d}{dt}[\Delta\ell P(t) + \Delta m N'(t) + \Delta n P'(t)]_{t=t_o}$$

$$= (t_2-t_1)(\sigma_{jk}-\sigma_j).$$

Since $\frac{d}{dt}[P(t)+N'(t)+P'(t)]_{t=t_o}$ is .16668884 cycles/365 days and $|\Delta\ell|,|\Delta m|,|\Delta n|$ are always less than or equal to 4, if $|t_2-t_1| \leq 16$ days, $|\Delta_{jk}(t_2)-\Delta_{jk}(t_1)| \leq .03$ cycles. This small variation in $\Delta_{jk}(t)$ leads to a similar behaviour in $\cos(\Delta_{jk}(t))$ and $\sin(\Delta_{jk}(t))$, and hence f(t) and u(t). Thus only a small loss in accuracy but a considerable calculation time saving will result if f(t) and u(t) are approximated by a constant value throughout the period of a month. Consequently f(t) and u(t) are assumed to equal their value at 000 hr of the 16th day of the month for the entire monthly period; and for convenience, V(t) is set to $V(t_{16}) + (t-t_{16})\sigma$, where t_{16} is this same time.

The procedure for calculating a series of tidal heights is then as follows. Since the tidal prediction data package does not contain constituent frequencies, they must be calculated via the astronomical variable derivatives and the constituent Doodson numbers. f, u and V values are then calculated for the 16th day of the first month of the desired prediction period and, as required, for subsequent months. Tidal heights for the desired values of t can then be calculated as

$$h(t) = \sum_{j=1}^{m} f_j(t_{16})A_j\cos(2\pi(V_j(t_{16})+(t-t_{16})\sigma_j+u_j(t_{16})-g_j)). \qquad (2)$$

In order to avoid calling a trigonometric library function for each new value of t, when a sequence of equally spaced heights are required, the following Chebyshev iteration formula is used for each constituent contribution,

$$f(n+1) = 2\cos(\sigma\Delta t)\, f(n) - f(n-1)\ , \qquad (3)$$

where $f(n) = \cos(n\sigma\Delta t)$ or $\sin(n\sigma\Delta t)$.

4.2 The High and Low Tide Prediction Method

The material presented here is taken from Godin and Taylor [1973].

In 4.1 we saw that the tidal height at a given location can be represented by the harmonic sum

$$h(t) = \sum_{j=1}^{m} f_j(t_\circ)A_j\cos(2\pi(V_j(t_\circ) + (t-t_\circ)\sigma_j + u(t_\circ) - g_j)) \qquad (1)$$

where:

A_j, g_j, σ_j = the amplitude, phase lag and frequency of constituent j,

$f_j(t_\circ), u_j(t_\circ)$ = the nodal modulation amplitude and phase correction factors for constituent j at the time origin $t_\circ$,

$V_j(t_\circ)$ = the astronomical argument for constituent j at the time origin $t_\circ$.

Letting D(t) be the derivative of h(t), i.e.

$$D(t) = -\sum_{j=1}^{m} f_j(t_\circ)\, A_j 2\pi\sigma_j \sin(2\pi(V_j(t_\circ) + (t-t_\circ)\sigma_j + u(t_\circ) - g_j)) \quad , \quad (2)$$

the high-low tide prediction method uses the following calculus results. If D(t) is a continuous function on the interval $[t_1, t_2]$ and t_k is a point in this interval, then:

i) $D(t_k) = 0$ if and only if t_k is an extreme point, or saddle point,[1] or h(t) is constant in the neighbourhood of t_k;

ii) if $D(t_1)$ and $D(t_2)$ have opposite signs then there exists a t_k in (t_1, t_2) with $D(t_k) = 0$.

---

[1] An example of a saddle point is $x = 0$ for the function $f(x) = x^3$.

Now for computational purposes we can assume that saddle points do not exist. That is to say, due to accuracy limitations of the computer, a zero derivative will be approximated by a number with a very small absolute value, and thus perturb a saddle point so that it becomes either a maximum and minimum, or a near saddle point (in the neighbourhood of a 'near saddle point', the derivative is of constant sign and almost assumes the value zero). And since, from its definition, we can reasonably assume that h(t) is not constant over any arbitrarily small interval, the continuity of D(t) everywhere implies that an interval $[t_1,t_2]$ with $D(t_1)$ and $D(t_2)$ having opposite signs contains an extremum.

However this result alone is not sufficient to guarantee the location of all extrema because, it does not eliminate the possibility of having more than one extremum in an interval whose endpoints have different signs, nor does it imply that if the endpoints have the same derivative sign there is no extremum in the interval. In order to ensure these conditions and thus be assured of bracketing all extreme values, it is necessary that a minumum interval size be specified in which we can assume that there exists at most one high or low tide.

Clearly the interval size Δt will be dependent upon the nature of the tide at a particular station. The time between successive high and low waters for predominantly semi-diurnal and diurnal tides is approximately 6 and 12 hours respectively. However if the tide is mixed, the extremal pattern is more complicated. FIGURE 2 shows the water level at Victoria, British Columbia between July 24 and 31, 1976. It is a mixed tide where the shorter period fluctuations override the major diurnal oscillations with a continuous shift in their position and amplitude.

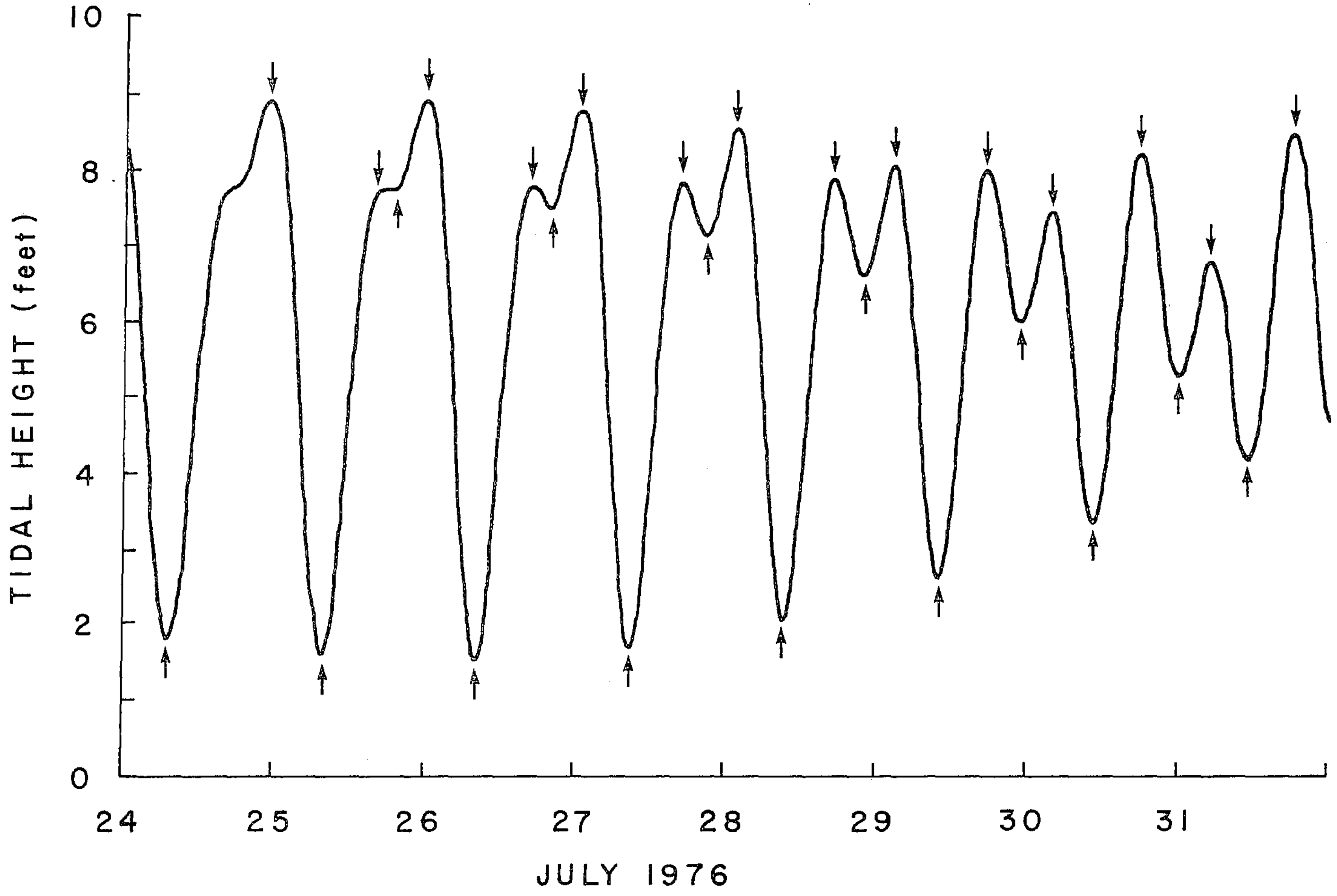


FIGURE 2

Synthesized water level at Victoria British Columbia over the period of July 24-31 1976. The tide is of a mixed character with F = 2.1. The arrows indicate the time and height of the extrema predicted using the method described in 4.2.

One characterization of the tide may be obtained by calculating the ratio of the amplitudes of the major harmonic constituents M2, S2, K1 and O1. This value is called the form number (Dietrich [1963]) and is defined precisely as

$$F = \frac{K1 + O1}{M2 + S2} \quad .$$

The tide is then said to be

i) semi-diurnal if $0 \leq F \leq .25$,
ii) mixed if $.25 < F \leq 3.00$,
iii) diurnal if $F > 3.00$.

For Victoria, $F = 2.1$.

In accordance with this determination Godin suggests the following maximum time interval values in which it can be assumed that there exists at most one extremum:

i) $\Delta t = 3$ hours for semi-diurnal tide,
ii) $\Delta t = .5$ hours for mixed tide,
iii) $\Delta t = 6$ hours for diurnal tide.

Although in fact, a mixed tide may have extrema closer than .5 hours, he feels that for practical purposes it is sufficient to note just one of them.

With these values of Δt we can then bracket all extrema by moving forward in time with steps of size Δt, and comparing signs of the interval endpoints. Once such upper and lower bounds have been found, the extreme point can be located exactly by any one of a number of search techniques. Because it requires a minimal amount of time, the one chosen is Bolzano's method of bisection coupled with linear interpolation. Although the bisection method does not take the minimal number of iterations when compared to more sophisticated search techniques, it is able to make significant time savings by computing new sine function values as a linear combination of old ones, and thus, unlike the other methods, avoid calls to the fortran library function SIN.

In more detail, the search algorithm for an extremum is then as follows:

i) move forward in time from the origin, or the last extremum, in steps of Δt until either a change in sign exists between the derivative values at the end points of the interval (t_a, t_b), or t_b extends beyond the desired prediction period. Each constituent contribution in the summation $D(t)$ is evaluated by the Chebyshev iteration formula (3) of 4.1. When an interval containing an extremum is located, set $k = 1$ and proceed to ii).

ii) calculate $t_k = t_a + \frac{1}{2^k} \Delta t$, and for each constituent in the sum, evaluate $D(t_k)$ by using the formula

$$\sin(t_k) = (\sin(t_a) + \sin(t_b))/(2 \cos(\frac{1}{2^k} \Delta t)).$$

If $|D(t_k)| \leq 10^{-16}$, set $D(t_k) = 10^{-16}$.

iii) re-assign whichever of t_a or t_b has the same derivative sign as $D(t_k)$, by t_k. If the new interval length $t_b - t_a$ is less than .1 hours, proceed to iv). Otherwise set $k = k+1$ and return to ii).

iv) use the following linear interpolation formula to find the extremum t_E,

$$t_E = t_a + D(t_a)(t_b - t_a)/(D(t_a) - D(t_b)) \quad ,$$

and evaluate $h(t_E)$ via (1). For each constituent term in this sum, obtain the function value by using a pre-calculated stored table of 2002 cosine values with arguments in the range of 0° to 360°. Return to i).

FIGURE 3 illustrates an example of the sequence of steps involved in the search for an extreme value. It is easily calculated that the number of iterations required to reduce the bracketing interval from Δt to .1 hour is 6 for diurnal tides, 3 for mixed tides, and 5 for semi-diurnal tides.

Arrows in FIGURE 2 indicate the extrema predicted for Victoria using the technique just described; the shaft of the arrow locates the time abscissa while the tip ends at the predicted height. The predicted hourly heights and the times and heights of all extrema are listed in Appendix 5.

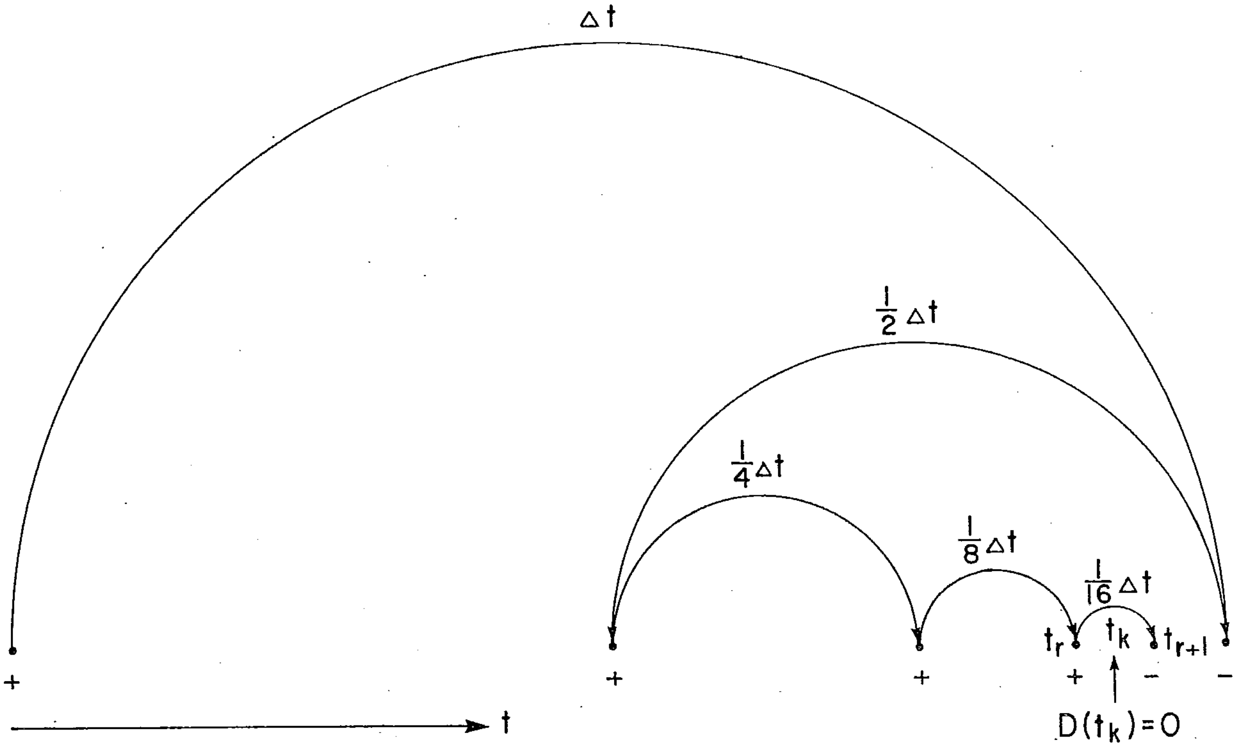


FIGURE 3

An example of the sequence of steps involved in locating a zero t_k of the derivative $D(t)$. The sign of $D(t)$ at the various points tested is denoted by + or –. After a step Δt, the sign has changed; by a retrogression of $\frac{1}{2}\Delta t$, the sign has reverted to plus, forcing a forward step of $\frac{1}{4}\Delta t$ where the sign is still unchanged. Two further forward steps of $\frac{1}{8}\Delta t$ and $\frac{1}{16}\Delta t$ locate the minimum width interval (t_r, t_{r+1}) over which the position of t_k is determined by linear interpolation from the values of $D(t)$ at t_r and t_{r+1}.

5. Consistency of the Analysis and Prediction Programs

Although consistency between the tidal heights analysis program and the tidal heights prediction program was a major objective in their revision, they do have one difference. In particular, if a pseudo-tidal record were generated by the prediction program and analysed using the same constituents, the amplitude and phase results given by the analysis program would not be identical to those used as input for the prediction program.

In a small part, this discrepancy is due to round-off accumulated during the calculations. However a test performed on the UNIVAC 1106 at Patricia Bay with a two month period of synthesized hourly heights indicates that such errors occur no sooner than the fourth digit. The remainder of the difference (which is, at worst, in the third digit) can be attributed to different approximating assumptions for the calculation of f and u, the nodal modulation amplitude and phase correction factors. Whereas the prediction program calculates these values at the 16th day of each month in the desired time period and keeps them constant throughout the entire month, the analyses program assumes them to be constant over the entire analyses period and equal to their true values at the central hour of that period.

It is important to note, though, that significantly different results can be expected in a similar test run if there is at least one more constituent used in the synthesis than analysis. This is because the least squares fit technique will adjust the amplitudes and phases of constituents included in the analysis to partially account for contributions due to constituents included in the synthesis but not the analysis. In fact this will occur even if the extra constituents are inferred (e.g. P1 is included in the synthesis and in the analysis via inference from K1) because of small inaccuracies in the approximating inference assumptions. However, except for round-off errors and the slightly different f and u values, having more constituents in the analysis than the synthesis will not affect the results.

Appendix 1

Standard Constituent Input Data for the Tidal Heights Analysis Computer Program

This data is read by the program from logical unit 8

```
Z0          0.0             M2
SA          0.0001140741    SSA
SSA         0.0002281591    Z0
MSM         0.0013097808    MM
MM          0.0015121518    MSF
MSF         0.0028219327    Z0
MF          0.0030500918    MSF
ALP1        0.0343965699    2Q1
2Q1         0.0357063507    Q1
SIG1        0.0359087218    2Q1
Q1          0.0372185026    O1
RHO1        0.0374208736    Q1
O1          0.0387306544    K1
TAU1        0.0389588136    O1
BET1        0.0400404353    NO1
NO1         0.0402685944    K1
CHI1        0.0404709654    NO1
PI1         0.0414385130    P1
P1          0.0415525871    K1
S1          0.0416666721    K1
K1          0.0417807462    Z0
PSI1        0.0418948203    K1
PHI1        0.0420089053    K1
THE1        0.0430905270    J1
J1          0.0432928981    K1
2PO1        0.0443745198
SO1         0.0446026789    OO1
OO1         0.0448308380    J1
UPS1        0.0463429898    OO1
ST36        0.0733553835
2NS2        0.0746651643
ST37        0.0748675353
ST1         0.0748933234
OQ2         0.0759749451    EPS2
EPS2        0.0761773161    2N2
```

ST2	0.0764054753	
ST3	0.0772331498	
O2	0.0774613089	
2N2	0.0774870970	MU2
MU2	0.0776894680	N2
SNK2	0.0787710897	
N2	0.0789992488	M2
NU2	0.0792016198	N2
ST4	0.0794555670	
OP2	0.0802832416	
GAM2	0.0803090296	H1
H1	0.0803973266	M2
M2	0.0805114007	Z0
H2	0.0806254748	M2
MKS2	0.0807395598	M2
ST5	0.0809677189	
ST6	0.0815930224	
LDA2	0.0818211815	L2
L2	0.0820235525	S2
2SK2	0.0831051742	
T2	0.0832192592	S2
S2	0.0833333333	M2
R2	0.0834474074	S2
K2	0.0835614924	S2
MSN2	0.0848454852	ETA2
ETA2	0.0850736443	K2
ST7	0.0853018034	
2SM2	0.0861552660	
ST38	0.0863576370	
SKM2	0.0863834251	
2SN2	0.0876674179	
NO3	0.1177299033	
MO3	0.1192420551	M3
M3	0.1207671010	M2
NK3	0.1207799950	
SO3	0.1220639878	MK3
MK3	0.1222921469	M3
SP3	0.1248859204	
SK3	0.1251140796	MK3
ST8	0.1566887168	

N4	0.1579984976	
3MS4	0.1582008687	
ST39	0.1592824904	
MN4	0.1595106495	M4
ST9	0.1597388086	
ST40	0.1607946422	
M4	0.1610228013	M3
ST10	0.1612509604	
SN4	0.1623325821	M4
KN4	0.1625607413	
MS4	0.1638447340	M4
MK4	0.1640728931	MS4
SL4	0.1653568858	
S4	0.1666666667	MS4
SK4	0.1668948258	S4
MNO5	0.1982413039	
2MO5	0.1997534558	
3MP5	0.1999816149	
MNK5	0.2012913957	
2MP5	0.2025753884	
2MK5	0.2028035475	M4
MSK5	0.2056254802	
3KM5	0.2058536393	
2SK5	0.2084474129	2MK5
ST11	0.2372259056	
2NM6	0.2385098983	
ST12	0.2387380574	
2MN6	0.2400220501	M6
ST13	0.2402502093	
ST41	0.2413060429	
M6	0.2415342020	2MK5
MSN6	0.2428439828	
MKN6	0.2430721419	
ST42	0.2441279756	
2MS6	0.2443561347	M6
2MK6	0.2445842938	2MS6
NSK6	0.2458940746	
2SM6	0.2471780673	2MS6
MSK6	0.2474062264	2SM6
S6	0.2500000000	

```
    ST14         0.2787527046
    ST15         0.2802906445
    M7           0.2817899023
    ST16         0.2830867891
    3MK7         0.2833149482     M6
    ST17         0.2861368809
    ST18         0.3190212990
    3MN8         0.3205334508
    ST19         0.3207616099
    M8           0.3220456027     3MK7
    ST20         0.3233553835
    ST21         0.3235835426
    3MS8         0.3248675353
    3MK8         0.3250956944
    ST22         0.3264054753
    ST23         0.3276894680
    ST24         0.3279176271
    ST25         0.3608020452
    ST26         0.3623141970
    4MK9         0.3638263489
    ST27         0.3666482815
    ST28         0.4010448515
    M10          0.4025570033
    ST29         0.4038667841
    ST30         0.4053789360
    ST31         0.4069168759
    ST32         0.4082008687
    ST33         0.4471596822
    M12          0.4830684040
    ST34         0.4858903367
    ST35         0.4874282766

  .7428797055   .7771900329  .5187051308  .3631582592  .7847990160  000GMT 1/1/76
13.3594019864   .9993368945  .1129517942  .0536893056  .0000477414  INCR./365DAYS
      Z0       0  0  0  0  0  0 0.0    0
      SA       0  0  1  0  0 -1 0.0    0
      SSA      0  0  2  0  0  0 0.0    0
      MSM      0  1 -2  1  0  0  .00   0
      MM       0  1  0 -1  0  0 0.0    0
      MSF      0  2 -2  0  0  0 0.0    0
```

```
MF      0  2  0  0  0  0 0.0    0
ALP1    1 -4  2  1  0  0 -.25   2
ALP1  -1  0  0 .75 0.0360R1   0 -1  0 .00 0.1906
2Q1     1 -3  0  2  0  0-0.25   5
2Q1   -2 -2  0 .50 0.0063    -1 -1  0 .75 0.0241R1  -1  0  0 .75 0.0607R1
2Q1    0 -2  0 .50 0.0063     0 -1  0 .0  0.1885
SIG1    1 -3  2  0  0  0-0.25   4
SIG1  -1  0  0 .75 0.0095R1   0 -2  0 .50 0.0061     0 -1  0 .0  0.1884
SIG1   2  0  0 .50 0.0087
Q1      1 -2  0  1  0  0-0.25  10
Q1    -2 -3  0 .50 0.0007    -2 -2  0 .50 0.0039    -1 -2  0 .75 0.0010R1
Q1    -1 -1  0 .75 0.0115R1  -1  0  0 .75 0.0292R1   0 -2  0 .50 0.0057
Q1    -1  0  1 .0  0.0008     0 -1  0 .0  0.1884     1  0  0 .75 0.0018R1
Q1     2  0  0 .50 0.0028
RHO1    1 -2  2 -1  0  0-0.25   5
RHO1   0 -2  0 .50 0.0058     0 -1  0 .0  0.1882     1  0  0 .75 0.0131R1
RHO1   2  0  0 .50 0.0576     2  1  0 .0  0.0175
O1      1 -1  0  0  0  0-0.25   8
O1    -1  0  0 .25 0.0003R1   0 -2  0 .50 0.0058     0 -1  0 .0  0.1885
O1     1 -1  0 .25 0.0004R1   1  0  0 .75 0.0029R1   1  1  0 .25 0.0004R1
O1     2  0  0 .50 0.0064     2  1  0 .50 0.0010
TAU1    1 -1  2  0  0  0-0.75   5
TAU1  -2  0  0 .0  0.0446    -1  0  0 .25 0.0426R1   0 -1  0 .50 0.0284
TAU1   0  1  0 .50 0.2170     0  2  0 .50 0.0142
BET1    1  0 -2  1  0  0 -.75   1
BET1   0 -1  0 .00 0.2266
NO1     1  0  0  1  0  0-0.75   9
NO1   -2 -2  0 .50 0.0057    -2 -1  0 .0  0.0665    -2  0  0 .0  0.3596
NO1   -1 -1  0 .75 0.0331R1  -1  0  0 .25 0.2227R1  -1  1  0 .75 0.0290R1
NO1    0 -1  0 .50 0.0290     0  1  0 .0  0.2004     0  2  0 .50 0.0054
CHI1    1  0  2 -1  0  0-0.75   2
CHI1   0 -1  0 .50 0.0282     0  1  0 .0  0.2187
PI1     1  1 -3  0  0  1-0.25   1
PI1    0 -1  0 .50 0.0078
P1      1  1 -2  0  0  0-0.25   6
P1     0 -2  0 .0  0.0008     0 -1  0 .50 0.0112     0  0  2 .50 0.0004
P1     1  0  0 .75 0.0004R1   2  0  0 .50 0.0015     2  1  0 .50 0.0003
S1      1  1 -1  0  0  1-0.75   2
S1     0  0 -2 .0  0.3534     0  1  0 .50 0.0264
K1      1  1  0  0  0  0-0.75  10
```

```
K1      -2 -1  0 .0  0.0002    -1 -1  0 .75 0.0001R1  -1  0  0 .25 0.0007R1
K1      -1  1  0 .75 0.0001R1   0 -2  0 .0  0.0001     0 -1  0 .50 0.0198
K1       0  1  0 .0  0.1356     0  2  0 .50 0.0029     1  0  0 .25 0.0002R1
K1       1  1  0 .25 0.0001R1
PSI1      1  1  1  0  0 -1-0.75   1
PSI1     0  1  0 .0  0.0190
PHI1      1  1  2  0  0  0-0.75   5
PHI1    -2  0  0 .0  0.0344    -2  1  0 .0  0.0106     0  0 -2 .0  0.0132
PHI1     0  1  0 .50 0.0384     0  2  0 .50 0.0185
THE1      1  2 -2  1  0  0 -.75   4
THE1    -2 -1  0 .00  .0300    -1  0  0 .25 0.0141R1   0 -1  0 .50  .0317
THE1     0  1  0 .00  .1993
J1        1  2  0 -1  0  0-0.75  10
J1       0 -1  0 .50 0.0294     0  1  0 .0  0.1980     0  2  0 .50 0.0047
J1       1 -1  0 .75 0.0027R1   1  0  0 .25 0.0816R1   1  1  0 .25 0.0331R1
J1       1  2  0 .25 0.0027R1   2  0  0 .50 0.0152     2  1  0 .50 0.0098
J1       2  2  0 .50 0.0057
001       1  3  0  0  0  0-0.75   8
001     -2 -1  0 .50 0.0037    -2  0  0 .0  0.1496    -2  1  0 .0  0.0296
001     -1  0  0 .25 0.0240R1  -1  1  0 .25 0.0099R1   0  1  0 .0  0.6398
001      0  2  0 .0  0.1342     0  3  0 .0  0.0086
UPS1      1  4  0 -1  0  0 -.75   5
UPS1    -2  0  0 .00 0.0611     0  1  0 .00 0.6399     0  2  0 .00 0.1318
UPS1     1  0  0 .25 0.0289R1   1  1  0 .25 0.0257R1
OQ2       2 -3  0  3  0  0 0.0    2
OQ2     -1  0  0 .25 0.1042R2   0 -1  0 .50 0.0386
EPS2      2 -3  2  1  0  0 0.0    3
EPS2    -1 -1  0 .25 0.0075R2  -1  0  0 .25 0.0402R2   0 -1  0 .50 0.0373
2N2       2 -2  0  2  0  0 0.0    4
2N2     -2 -2  0 .50 0.0061    -1 -1  0 .25 0.0117R2  -1  0  0 .25 0.0678R2
2N2      0 -1  0 .50 0.0374
MU2       2 -2  2  0  0  0 0.0    3
MU2     -1 -1  0 .25 0.0018R2  -1  0  0 .25 0.0104R2   0 -1  0 .50 0.0375
N2        2 -1  0  1  0  0 0.0    4
N2      -2 -2  0 .50 0.0039    -1  0  1 .00 0.0008     0 -2  0 .00 0.0005
N2       0 -1  0 .50 0.0373
NU2       2 -1  2 -1  0  0 0.0    4
NU2      0 -1  0 .50 0.0373     1  0  0 .75 0.0042R2   2  0  0 .0  0.0042
NU2      2  1  0 .50 0.0036
GAM2      2  0 -2  2  0  0 -.50   3
```

```
GAM2  -2 -2  0 .00 0.1429    -1  0  0 .25 0.0293R2   0 -1  0 .50 0.0330
H1      2  0 -1  0  0  1-0.50   2
H1     0 -1  0 .50 0.0224     1  0 -1 .50 0.0447
M2      2  0  0  0  0  0 0.0    9
M2    -1 -1  0 .75 0.0001R2  -1  0  0 .75 0.0004R2   0 -2  0 .0  0.0005
M2     0 -1  0 .50 0.0373     1 -1  0 .25 0.0001R2   1  0  0 .75 0.0009R2
M2     1  1  0 .75 0.0002R2   2  0  0 .0  0.0006     2  1  0 .0  0.0002
H2      2  0  1  0  0 -1 0.0    1
H2     0 -1  0 .50 0.0217
LDA2    2  1 -2  1  0  0-0.50   1
LDA2   0 -1  0 .50 0.0448
L2      2  1  0 -1  0  0-0.50   5
L2     0 -1  0 .50 0.0366     2 -1  0 .00 0.0047     2  0  0 .50 0.2505
L2     2  1  0 .50 0.1102     2  2  0 .50 0.0156
T2      2  2 -3  0  0  1 0.0    0
S2      2  2 -2  0  0  0 0.0    3
S2     0 -1  0 .0  0.0022     1  0  0 .75 0.0001R2   2  0  0 .0  0.0001
R2      2  2 -1  0  0 -1-0.50   2
R2     0  0  2 .50 0.2535     0  1  2 .0  0.0141
K2      2  2  0  0  0  0 0.0    5
K2    -1  0  0 .75 0.0024R2  -1  1  0 .75 0.0004R2   0 -1  0 .50 0.0128
K2     0  1  0 .0  0.2980     0  2  0 .0  0.0324
ETA2    2  3  0 -1  0  0 0.0    7
ETA2   0 -1  0 .50 0.0187     0  1  0 .0  0.4355     0  2  0 .0  0.0467
ETA2   1  0  0 .75 0.0747R2   1  1  0 .75 0.0482R2   1  2  0 .75 0.0093R2
ETA2   2  0  0 .50 0.0078
M3      3  0  0  0  0  0 -.50   1
M3     0 -1  0 .50  .0564

2P01 2   2.0 P1        -1.0 01
S01  2   1.0 S2        -1.0 01
ST36 3   2.0 M2         1.0 N2        -2.0 S2
2NS2 2   2.0 N2        -1.0 S2
ST37 2   3.0 M2        -2.0 S2
ST1  3   2.0 N2         1.0 K2        -2.0 S2
ST2  4   1.0 M2         1.0 N2         1.0 K2        -2.0 S2
ST3  3   2.0 M2         1.0 S2        -2.0 K2
02   1   2.0 01
ST4  3   2.0 K2         1.0 N2        -2.0 S2
SNK2 3   1.0 S2         1.0 N2        -1.0 K2
```

```
OP2   2    1.0 O1          1.0 P1
MKS2  3    1.0 M2          1.0 K2          -1.0 S2
ST5   3    1.0 M2          2.0 K2          -2.0 S2
ST6   4    2.0 S2          1.0 N2          -1.0 M2          -1.0 K2
2SK2  2    2.0 S2         -1.0 K2
MSN2  3    1.0 M2          1.0 S2          -1.0 N2
ST7   4    2.0 K2          1.0 M2          -1.0 S2          -1.0 N2
2SM2  2    2.0 S2         -1.0 M2
ST38  3    2.0 M2          1.0 S2          -2.0 N2
SKM2  3    1.0 S2          1.0 K2          -1.0 M2
2SN2  2    2.0 S2         -1.0 N2
NO3   2    1.0 N2          1.0 O1
MO3   2    1.0 M2          1.0 O1
NK3   2    1.0 N2          1.0 K1
SO3   2    1.0 S2          1.0 O1
MK3   2    1.0 M2          1.0 K1
SP3   2    1.0 S2          1.0 P1
SK3   2    1.0 S2          1.0 K1
ST8   3    2.0 M2          1.0 N2          -1.0 S2
N4    1    2.0 N2
3MS4  2    3.0 M2         -1.0 S2
ST39  4    1.0 M2          1.0 S2           1.0 N2          -1.0 K2
MN4   2    1.0 M2          1.0 N2
ST40  3    2.0 M2          1.0 S2          -1.0 K2
ST9   4    1.0 M2          1.0 N2           1.0 K2          -1.0 S2
M4    1    2.0 M2
ST10  3    2.0 M2          1.0 K2          -1.0 S2
SN4   2    1.0 S2          1.0 N2
KN4   2    1.0 K2          1.0 N2
MS4   2    1.0 M2          1.0 S2
MK4   2    1.0 M2          1.0 K2
SL4   2    1.0 S2          1.0 L2
S4    1    2.0 S2
SK4   2    1.0 S2          1.0 K2
MNO5  3    1.0 M2          1.0 N2           1.0 O1
2MO5  2    2.0 M2          1.0 O1
3MP5  2    3.0 M2         -1.0 P1
MNK5  3    1.0 M2          1.0 N2           1.0 K1
2MP5  2    2.0 M2          1.0 P1
2MK5  2    2.0 M2          1.0 K1
```

```
MSK5 3   1.0 M2        1.0 S2        1.0 K1
3KM5 3   1.0 K2        1.0 K1        1.0 M2
2SK5 2   2.0 S2        1.0 K1
ST11 3   3.0 N2        1.0 K2       -1.0 S2
2NM6 2   2.0 N2        1.0 M2
ST12 4   2.0 N2        1.0 M2        1.0 K2       -1.0 S2
ST41 3   3.0 M2        1.0 S2       -1.0 K2
2MN6 2   2.0 M2        1.0 N2
ST13 4   2.0 M2        1.0 N2        1.0 K2       -1.0 S2
M6   1   3.0 M2
MSN6 3   1.0 M2        1.0 S2        1.0 N2
MKN6 3   1.0 M2        1.0 K2        1.0 N2
2MS6 2   2.0 M2        1.0 S2
2MK6 2   2.0 M2        1.0 K2
NSK6 3   1.0 N2        1.0 S2        1.0 K2
2SM6 2   2.0 S2        1.0 M2
MSK6 3   1.0 M2        1.0 S2        1.0 K2
ST42 3   2.0 M2        2.0 S2       -1.0 K2
S6   1   3.0 S2
ST14 3   2.0 M2        1.0 N2        1.0 O1
ST15 3   2.0 N2        1.0 M2        1.0 K1
M7   1   3.5 M2
ST16 3   2.0 M2        1.0 S2        1.0 O1
3MK7 2   3.0 M2        1.0 K1
ST17 4   1.0 M2        1.0 S2        1.0 K2        1.0 O1
ST18 2   2.0 M2        2.0 N2
3MN8 2   3.0 M2        1.0 N2
ST19 4   3.0 M2        1.0 N2        1.0 K2       -1.0 S2
M8   1   4.0 M2
ST20 3   2.0 M2        1.0 S2        1.0 N2
ST21 3   2.0 M2        1.0 N2        1.0 K2
3MS8 2   3.0 M2        1.0 S2
3MK8 2   3.0 M2        1.0 K2
ST22 4   1.0 M2        1.0 S2        1.0 N2        1.0 K2
ST23 2   2.0 M2        2.0 S2
ST24 3   2.0 M2        1.0 S2        1.0 K2
ST25 3   2.0 M2        2.0 N2        1.0 K1
ST26 3   3.0 M2        1.0 N2        1.0 K1
4MK9 2   4.0 M2        1.0 K1
ST27 3   3.0 M2        1.0 S2        1.0 K1
```

```
ST28 2   4.0 M2        1.0 N2
M10  1   5.0 M2
ST29 3   3.0 M2        1.0 N2        1.0 S2
ST30 2   4.0 M2        1.0 S2
ST31 4   2.0 M2        1.0 N2        1.0 S2        1.0 K2
ST32 2   3.0 M2        2.0 S2
ST33 3   4.0 M2        1.0 S2        1.0 K1
M12  1   6.0 M2
ST34 2   5.0 M2        1.0 S2
ST35 4   3.0 M2        1.0 N2        1.0 K2        1.0 S2
```

Appendix 2

Sample Tidal Station Input Data for the Analysis Program

The following sample input for logical unit 4 will produce an analysis of Tuktoyaktuk data for the period of 1600 MST July 6 1975 to 1400 MST September 9 1975 inclusive, with constituents P1 and K2 inferred, shallow water constituent M10 specifically designated for analysis inclusion, and only line printer output of the results. The final analysis results are listed in Appendix 3.

```
 6   1.0          0.0
    K1         0.0417807462    P1         0.0415525871   0.33093    -7.07
    S2         0.0833333333    K2         0.0835614924   0.27215   -22.40

      M10       M8

8 16060775  14090975
     6485  TUKTOYUKTUK NWT        MST 6927 3302
1  6485          6 775
2  6485          6 775             215 224 215 202 215 227 234 242 238
1  6485          7 775 229 218 206 200 193 187 179 176 183 199 215 231
2  6485          7 775 252 263 260 244 210 176 154 145 153 162 182 203
1  6485          8 775 221 232 230 195 153 119 105 115 132 159 192 218
2  6485          8 775 246 262 264 252 228 197 166 154 159 178 201 229
1  6485          9 775 251 267 291 257 225 204 183 176 188 183 204 232
2  6485          9 775 255 272 285 298 296 253 199 152 112 111 132 167
1  6485         10 775 201 221 227 223 201 166 131  99  70  82 121 161
2  6485         10 775 209 264 302 321 329 303 254 205 168 148 163 180
1  6485         11 775 212 244 271 282 278 258 221 169 135 126 134 156
2  6485         11 775 182 219 249 257 262 243 205 168 135 110 105 118
1  6485         12 775 142 178 213 242 247 233 203 159 119  85  72  89
2  6485         12 775 116 148 180 205 223 222 186 147 105  66  43  55
1  6485         13 775  78 104 136 167 194 199 182 148 107  72  54  61
2  6485         13 775  87 108 139 165 182 190 185 158 125  89  59  49
1  6485         14 775  55  84 113 138 164 182 194 184 154 118  83  66
2  6485         14 775  66  87 117 146 165 180 181 164 134 100  62  40
1  6485         15 775  36  48  72 103 129 157 167 167 156 131 105  87
2  6485         15 775  71  72  80  95 114 128 137 144 132 110  83  63
1  6485         16 775  42  24  29  57  94 125 147 156 155 134 105  82
2  6485         16 775  70  64  63  74  95 116 131 135 130 115  93  75
```

```
1   6485        17 775  60  50  50  61  84 111 135 149 154 154 143 122
2   6485        17 775  99  81  72  73  85  99 115 127 127 115 108  94
1   6485        18 775  78  59  50  50  64  81 100 132 164 184 188 179
2   6485        18 775 159 137 133 135 137 143 147 148 153 161 170 161
1   6485        19 775 147 141 142 128 130 143 160 177 193 211 230 236
2   6485        19 775 226 204 181 165 157 161 178 178 178 186 196 198
1   6485        20 775 201 190 172 155 138 130 136 156 174 199 227 245
2   6485        20 775 254 256 245 199 162 141 129 134 157 183 206 220
1   6485        21 775 219 222 210 194 182 169 171 183 206 240 255 265
2   6485        21 775 288 296 292 282 262 238 212 190 186 198 220 240
1   6485        22 775 259 268 271 264 249 228 203 184 187 206 230 257
2   6485        22 775 283 293 295 282 261 232 204 182 165 171 192 218
1   6485        23 775 232 247 255 249 230 205 181 158 148 152 180 209
2   6485        23 775 234 260 272 261 231 196 160 130 111 109 125 157
1   6485        24 775 187 209 224 231 209 181 155 125 110 111 130 159
2   6485        24 775 195 227 249 250 233 200 161 123  94  87  97 123
1   6485        25 775 153 183 196 202 195 174 138 101  71  58  60  87
2   6485        25 775 122 159 185 202 199 179 144 103  66  40  35  48
1   6485        26 775  75 104 132 151 160 155 129  98  66  39  34  47
2   6485        26 775  79 113 144 163 172 167 151 117  85  50  20  19
1   6485        27 775  39  74 107 136 148 158 141 118  89  54  29  16
2   6485        27 775  41  76 105 143 189 202 196 185 185 162 160 163
1   6485        28 775 168 187 222 254 260 275 281 268 256 241 221 198
2   6485        28 775 208 230 258 264 285 301 291 270 247 212 188 176
1   6485        29 775 183 200 224 245 256 269 280 270 243 216 194 164
2   6485        29 775 163 177 201 232 263 282 281 290 259 238 202 179
1   6485        30 775 179 184 205 226 242 272 281 279 263 233 205 279
2   6485        30 775 168 184 210 235 247 253 263 259 244 221 193 183
1   6485        31 775 180 176 194 208 215 224 235 243 241 225 207 188
2   6485        31 775 176
1   6485         1 875
2   6485         1 875
1   6485         2 875
2   6485         2 875         104  95  93  95 103 112 118 118 116 108
1   6485         3 875  97  83  68  56  51  54  75  95 117 130 138 139
2   6485         3 875 133 120 103  87  71  56  52  66  81  98 109 107
1   6485         4 875  98  77  49  28  14   4   7  17  44  70  94 110
2   6485         4 875 117 116 107  88  71  55  46  44  60  84 108 125
1   6485         5 875 133 136 114  86  70  62  62  79 113 143 175 208
2   6485         5 875 238 256 266 240 203 179 143 117 118 146 167 186
```

```
1   6485        6 875 224 243 227 204 180 158 154 170 201 222 234 243
2   6485        6 875 254 260 247 231 211 188 160 143 137 145 167 195
1   6485        7 875 221 239 249 249 227 184 144 111 102 129 170 201
2   6485        7 875 233 255 260 252 227 195 156 123 107 118 149 180
1   6485        8 875 211 232 245 257 229 200 171 138 102  95 122 163
2   6485        8 875 207 253 295 338 369 353 318 285 221 184 165 175
1   6485        9 875 212 240 260 283 282 259 229 196 174 176 187 204
2   6485        9 875 244 288 329 356 369 370 324 281 289 294 293 287
1   6485       10 875 329 380 426 441 447 453 418 387 353 337 322 314
2   6485       10 875 342 365 404 438 470 482 487 456 441 423 438 448
1   6485       11 875 464 478 491 505 538 528 493 488 472 425 398 390
2   6485       11 875 393 408 421 438 444 433 412 379 337 300 262 247
1   6485       12 875 245 252 277 304 327 339 339 308 257 208 182 182
2   6485       12 875 203 235 260 281 319 315 297 273 237 198 168 158
1   6485       13 875 157 171 195 217 239 252 258 253 242 225 202 179
2   6485       13 875 167 172 190 217 242 257 266 263 244 217 187 155
1   6485       14 875 132 134 163 195 228 246 259 256 236 209 180 150
2   6485       14 875 129 122 136 161 184 200 207 205 195 177 158 136
1   6485       15 875 116 105 104 115 140 164 193 203 216 208 196 187
2   6485       15 875 159 142 147 164 175 183 197 202 202 202 192 176
1   6485       16 875 160 147 137 136 152 172 195 211 224 228 222 210
2   6485       16 875 199 186 171 165 163 169 180 190 201 203 200 193
1   6485       17 875 185 175 162 152 156 169 201 227 249 272 284 285
2   6485       17 875 295 280 259 241 225 211 211 226 247 268 286 297
1   6485       18 875 296 272 245 214 196 194 209 226 239 244 245 248
2   6485       18 875 246 239 229 218 201 183 165 158 160 183 207 221
1   6485       19 875 227 224 209 187 159 138 131 139 162 185 209 228
2   6485       19 875 239 242 233 212 183 152 129 119 132 167 193 218
1   6485       20 875 237 241 230 205 178 151 130 114 122 145 172 203
2   6485       20 875 226 237 237 223 197 165 131 108 103 118 144 173
1   6485       21 875 203 225 229 223 200 175 150 129 131 146 173 202
2   6485       21 875 236 258 263 256 233 198 165 137 127 133 159 190
1   6485       22 875 221 241 252 252 231 200 167 137 119 114 134 166
2   6485       22 875 201 234 256 264 249 212 176 140 111 103 115 140
1   6485       23 875 171 203 232 244 242 214 180 146 121 115 126 157
2   6485       23 875 187 211 235 249 247 229 194 151 114  88  87 110
1   6485       24 875 143 177 206 230 237 223 186 140  93  64  66  94
2   6485       24 875 129 165 197 215 220 205 177 144 112  84  80 100
1   6485       25 875 136 173 208 238 252 244 217 181 139 105  89  93
2   6485       25 875 121 159 188 199 200 185 155 121  84  64  45  28
```

```
1  6485          26 875  32  72 121 174 215 237 211 197 234 243 176 196
2  6485          26 875 250 219 272 361 391 376 355 389 370 321 300 285
1  6485          27 875 288 323 350 380 422 415 405 389 412 430 453 509
2  6485          27 875 557 559 548 560 576 557 513 489 462 422 388 383
1  6485          28 875 371 393 413 419 443 472 444 423 384 340 304 284
2  6485          28 875 280 286 300 309 312 324 319 299 270 227 203 181
1  6485          29 875 193 240 281 317 352 351 361 350 354 348 358 350
2  6485          29 875 326 317 316 324 327 313 298 283 264 244 230 215
1  6485          30 875 194 194 217 241 256 262 261 259 247 229 213 195
2  6485          30 875 175 163 168 177 187 198 203 204 191 171 144 119
1  6485          31 875  97  92 102 125 150 168 176 188 197 197 206 202
2  6485          31 875 191 186 192 200 197 199 206 205 207 208 205 198
1  6485           1 975 185 187 194 209 234 255 275 285 305 327 332 320
2  6485           1 975 301 295 291 275 277 294 312 328 344 335 321 328
1  6485           2 975 323 315 324 316 318 329 321 314 317 329 336 336
2  6485           2 975 327 316 301 284 263 245 236 231 233 240 250 262
1  6485           3 975 261 250 227 202 172 153 153 162 171 172 190 214
2  6485           3 975 226 228 214 186 160 143 142 155 173 201 236 255
1  6485           4 975 274 284 282 255 216 183 165 179 203 231 258 294
2  6485           4 975 327 364 353 332 299 262 227 207 219 240 256 275
1  6485           5 975 302 309 298 274 240 196 159 142 158 192 222 249
2  6485           5 975 270 280 282 269 239 197 154 110  99 125 159 187
1  6485           6 975 214 235 236 221 189 153 118  83  63  52  65 106
2  6485           6 975 132 151 165 175 169 148 115  74  42  18   5  30
1  6485           7 975  68 123 189 218 198 167 126  81  52  40  56  81
2  6485           7 975 121 173 203 211 199 173 137 100  72  58  57  92
1  6485           8 975 135 179 218 238 240 224 190 136  91  58  48  65
2  6485           8 975  99 140 173 194 195 175 141  96  57  33  30  50
1  6485           9 975  86 129 173 204 217 202 171 125  79  47  40  59
2  6485           9 975  88 121
```

Appendix 3

Final analysis results arising from the input data of Appendix 2
and the standard constituent data package of Appendix 1.

```
ANALYSIS OF HOURLY TIDAL HEIGHTS  STN  6485    16H  6/ 7/75 TO  14H  9/ 9/75
NO.OBS.=  1559   NO.PTS.ANAL.=  1559    MIDPT= 3H  8/ 8/75  SEPARATION =1.00

 NO NAME     FREQUENCY   STN   M-Y/ M-Y        A       G                       AL       GL
  1 Z0       .00000000  6485   775/ 975   1.9806     .00                  1.9806      .00
  2 MM       .00151215  6485   775/ 975    .2121  263.34                   .2121   288.50
  3 MSF      .00282193  6485   775/ 975    .1561  133.80                   .1561   115.15
  4 ALP1     .03439657  6485   775/ 975    .0152  334.95                   .0141   180.96
  5 2Q1      .03570635  6485   775/ 975    .0246   82.69                   .0226   246.82
  6 Q1       .03721850  6485   775/ 975    .0158   65.74                   .0144   252.75
  7 O1       .03873065  6485   775/ 975    .0764   74.23                   .0694   284.43
  8 NO1      .04026859  6485   775/ 975    .0290  238.14                   .0380   275.85
  9 P1       .04155259  6485   775/ 975    .0465   71.76  INF FR K1        .0468   252.20
 10 K1       .04178075  6485   775/ 975    .1406   64.69                   .1332   145.54
 11 J1       .04329290  6485   775/ 975    .0253    7.32                   .0234   103.62
 12 OO1      .04483084  6485   775/ 975    .0531  235.74                   .0463   358.47
 13 UPS1     .04634299  6485   775/ 975    .0298   91.73                   .0233   239.11
 14 EPS2     .07617732  6485   775/ 975    .0211  184.59                   .0216   109.98
 15 MU2      .07768947  6485   775/ 975    .0419   83.23                   .0428    30.06
 16 N2       .07899925  6485   775/ 975    .0838   44.52                   .0857   306.35
 17 M2       .08051140  6485   775/ 975    .4904   77.70                   .5007     4.40
 18 L2       .08202355  6485   775/ 975    .0213   35.21                   .0174   168.03
 19 S2       .08333333  6485   775/ 975    .2195  126.65                   .2193    36.74
 20 K2       .08356149  6485   775/ 975    .0597  149.05  INF FR S2        .0515   131.15
 21 ETA2     .08507364  6485   775/ 975    .0071  246.05                   .0059   235.38
 22 MO3      .11924206  6485   775/ 975    .0148  234.97                   .0138    11.86
 23 M3       .12076710  6485   775/ 975    .0123  261.57                   .0126   331.91
 24 MK3      .12229215  6485   775/ 975    .0049  331.60                   .0048   339.15
 25 SK3      .12511408  6485   775/ 975    .0023  237.69                   .0022   228.64
 26 MN4      .15951065  6485   775/ 975    .0092  256.47                   .0096    85.00
 27 M4       .16102280  6485   775/ 975    .0126  291.78                   .0131   145.17
 28 SN4      .16233258  6485   775/ 975    .0083  270.85                   .0085    82.78
 29 MS4      .16384473  6485   775/ 975    .0010  339.35                   .0011   176.14
 30 S4       .16666667  6485   775/ 975    .0047  299.56                   .0047   119.75
 31 2MK5     .20280355  6485   775/ 975    .0013  310.10                   .0013   244.35
```

```
32 2SK5      .20844741  6485   775/ 975     .0045 104.00                          .0043   5.04
33 2MN6      .24002205  6485   775/ 975     .0035 271.23                          .0038  26.46
34 M6        .24153420  6485   775/ 975     .0017 158.88                          .0018 298.97
35 2MS6      .24435613  6485   775/ 975     .0056 306.10                          .0059  69.59
36 2SM6      .24717807  6485   775/ 975     .0023 298.92                          .0023  45.80
37 3MK7      .28331495  6485   775/ 975     .0086 212.25                          .0086  73.19
38 M8        .32204560  6485   775/ 975     .0030  42.43                          .0033 109.22
39 M10       .40255700  6485   775/ 975     .0009 198.23                          .0010 191.71
```

Appendix 4

The following sample input for logical unit 8 will synthesize hourly heights, and the times and heights of all extrema at Victoria for the period of 0100 PST July 1 1976 to 2400 PST July 31 1976 inclusive. The output results are listed in Appendix 5.

```
  .7428797055  .7771900329  .5187051308  .3631582592  .7847990160  000GMT 1/1/76
13.3594019864  .9993368945  .1129517942  .0536893056  .0000477414  INCR./365DAYS
      Z0      0  0  0  0  0  0 0.0    0
      SA      0  0  1  0  0 -1 0.0    0
      SSA     0  0  2  0  0  0 0.0    0
      MSM     0  1 -2  1  0  0  .00   0
      MM      0  1  0 -1  0  0 0.0    0
      MSF     0  2 -2  0  0  0 0.0    0
      MF      0  2  0  0  0  0 0.0    0
      ALP1    1 -4  2  1  0  0 -.25   2
      ALP1  -1  0  0 .75 0.0360R1   0 -1  0 .00 0.1906
      2Q1     1 -3  0  2  0  0-0.25   5
      2Q1   -2 -2  0 .50 0.0063    -1 -1  0 .75 0.0241R1  -1  0  0 .75 0.0607R1
      2Q1    0 -2  0 .50 0.0063     0 -1  0 .0  0.1885
      SIG1    1 -3  2  0  0  0-0.25   4
      SIG1  -1  0  0 .75 0.0095R1   0 -2  0 .50 0.0061     0 -1  0 .0  0.1884
      SIG1   2  0  0 .50 0.0087
      Q1      1 -2  0  1  0  0-0.25  10
      Q1    -2 -3  0 .50 0.0007    -2 -2  0 .50 0.0039    -1 -2  0 .75 0.0010R1
      Q1    -1 -1  0 .75 0.0115R1  -1  0  0 .75 0.0292R1   0 -2  0 .50 0.0057
      Q1    -1  0  1 .0  0.0008     0 -1  0 .0  0.1884     1  0  0 .75 0.0018R1
      Q1     2  0  0 .50 0.0028
      RHO1    1 -2  2 -1  0  0-0.25   5
      RHO1   0 -2  0 .50 0.0058     0 -1  0 .0  0.1882     1  0  0 .75 0.0131R1
      RHO1   2  0  0 .50 0.0576     2  1  0 .0  0.0175
      O1      1 -1  0  0  0  0-0.25   8
      O1    -1  0  0 .25 0.0003R1   0 -2  0 .50 0.0058     0 -1  0 .0  0.1885
      O1     1 -1  0 .25 0.0004R1   1  0  0 .75 0.0029R1   1  1  0 .25 0.0004R1
      O1     2  0  0 .50 0.0064     2  1  0 .50 0.0010
```

```
TAU1     1 -1  2  0  0  0-0.75   5
TAU1  -2  0  0 .0  0.0446    -1  0  0 .25 0.0426R1   0 -1  0 .50 0.0284
TAU1   0  1  0 .50 0.2170     0  2  0 .50 0.0142
BET1     1  0 -2  1  0  0 -.75   1
BET1   0 -1  0 .00 0.2266
NO1      1  0  0  1  0  0-0.75   9
NO1   -2 -2  0 .50 0.0057    -2 -1  0 .0  0.0665    -2  0  0 .0  0.3596
NO1   -1 -1  0 .75 0.0331R1  -1  0  0 .25 0.2227R1  -1  1  0 .75 0.0290R1
NO1    0 -1  0 .50 0.0290     0  1  0 .0  0.2004     0  2  0 .50 0.0054
CHI1     1  0  2 -1  0  0-0.75   2
CHI1   0 -1  0 .50 0.0282     0  1  0 .0  0.2187
PI1      1  1 -3  0  0  1-0.25   1
PI1    0 -1  0 .50 0.0078
P1       1  1 -2  0  0  0-0.25   6
P1     0 -2  0 .0  0.0008     0 -1  0 .50 0.0112     0  0  2 .50 0.0004
P1     1  0  0 .75 0.0004R1   2  0  0 .50 0.0015     2  1  0 .50 0.0003
S1       1  1 -1  0  0  1-0.75   2
S1     0  0 -2 .0  0.3534     0  1  0 .50 0.0264
K1       1  1  0  0  0  0-0.75  10
K1    -2 -1  0 .0  0.0002    -1 -1  0 .75 0.0001R1  -1  0  0 .25 0.0007R1
K1    -1  1  0 .75 0.0001R1   0 -2  0 .0  0.0001     0 -1  0 .50 0.0198
K1     0  1  0 .0  0.1356     0  2  0 .50 0.0029     1  0  0 .25 0.0002R1
K1     1  1  0 .25 0.0001R1
PSI1     1  1  1  0  0 -1-0.75   1
PSI1   0  1  0 .0  0.0190
PHI1     1  1  2  0  0  0-0.75   5
PHI1  -2  0  0 .0  0.0344    -2  1  0 .0  0.0106     0  0 -2 .0  0.0132
PHI1   0  1  0 .50 0.0384     0  2  0 .50 0.0185
THE1     1  2 -2  1  0  0 -.75   4
THE1  -2 -1  0 .00  .0300    -1  0  0 .25 0.0141R1   0 -1  0 .50  .0317
THE1   0  1  0 .00  .1993
J1       1  2  0 -1  0  0-0.75  10
J1     0 -1  0 .50 0.0294     0  1  0 .0  0.1980     0  2  0 .50 0.0047
J1     1 -1  0 .75 0.0027R1   1  0  0 .25 0.0816R1   1  1  0 .25 0.0331R1
J1     1  2  0 .25 0.0027R1   2  0  0 .50 0.0152     2  1  0 .50 0.0098
J1     2  2  0 .50 0.0057
OO1      1  3  0  0  0  0-0.75   8
OO1   -2 -1  0 .50 0.0037    -2  0  0 .0  0.1496    -2  1  0 .0  0.0296
OO1   -1  0  0 .25 0.0240R1  -1  1  0 .25 0.0099R1   0  1  0 .0  0.6398
OO1    0  2  0 .0  0.1342     0  3  0 .0  0.0086
```

```
UPS1     1  4  0 -1  0  0 -.75    5
UPS1    -2  0  0 .00 0.0611      0  1  0 .00 0.6399      0  2  0 .00 0.1318
UPS1     1  0  0 .25 0.0289R1    1  1  0 .25 0.0257R1
OQ2      2 -3  0  3  0  0 0.0     2
OQ2     -1  0  0 .25 0.1042R2    0 -1  0 .50 0.0386
EPS2     2 -3  2  1  0  0 0.0     3
EPS2    -1 -1  0 .25 0.0075R2   -1  0  0 .25 0.0402R2    0 -1  0 .50 0.0373
2N2      2 -2  0  2  0  0 0.0     4
2N2     -2 -2  0 .50 0.0061     -1 -1  0 .25 0.0117R2   -1  0  0 .25 0.0678R2
2N2      0 -1  0 .50 0.0374
MU2      2 -2  2  0  0  0 0.0     3
MU2     -1 -1  0 .25 0.0018R2   -1  0  0 .25 0.0104R2    0 -1  0 .50 0.0375
N2       2 -1  0  1  0  0 0.0     4
N2      -2 -2  0 .50 0.0039     -1  0  1 .00 0.0008      0 -2  0 .00 0.0005
N2       0 -1  0 .50 0.0373
NU2      2 -1  2 -1  0  0 0.0     4
NU2      0 -1  0 .50 0.0373      1  0  0 .75 0.0042R2    2  0  0 .0  0.0042
NU2      2  1  0 .50 0.0036
GAM2     2  0 -2  2  0  0 -.50    3
GAM2    -2 -2  0 .00 0.1429     -1  0  0 .25 0.0293R2    0 -1  0 .50 0.0330
H1       2  0 -1  0  0  1-0.50    2
H1       0 -1  0 .50 0.0224      1  0 -1 .50 0.0447
M2       2  0  0  0  0  0 0.0     9
M2      -1 -1  0 .75 0.0001R2   -1  0  0 .75 0.0004R2    0 -2  0 .0  0.0005
M2       0 -1  0 .50 0.0373      1 -1  0 .25 0.0001R2    1  0  0 .75 0.0009R2
M2       1  1  0 .75 0.0002R2    2  0  0 .0  0.0006      2  1  0 .0  0.0002
H2       2  0  1  0  0 -1 0.0     1
H2       0 -1  0 .50 0.0217
LDA2     2  1 -2  1  0  0-0.50    1
LDA2     0 -1  0 .50 0.0448
L2       2  1  0 -1  0  0-0.50    5
L2       0 -1  0 .50 0.0366      2 -1  0 .00 0.0047      2  0  0 .50 0.2505
L2       2  1  0 .50 0.1102      2  2  0 .50 0.0156
T2       2  2 -3  0  0  1 0.0     0
S2       2  2 -2  0  0  0 0.0     3
S2       0 -1  0 .0  0.0022      1  0  0 .75 0.0001R2    2  0  0 .0  0.0001
R2       2  2 -1  0  0 -1-0.50    2
R2       0  0  2 .50 0.2535      0  1  2 .0  0.0141
K2       2  2  0  0  0  0 0.0     5
K2      -1  0  0 .75 0.0024R2   -1  1  0 .75 0.0004R2    0 -1  0 .50 0.0128
```

```
K2     0  1  0 .0  0.2980     0  2  0 .0  0.0324
ETA2    2  3  0 -1  0  0 0.0      7
ETA2   0 -1  0 .50 0.0187     0  1  0 .0  0.4355     0  2  0 .0  0.0467
ETA2   1  0  0 .75 0.0747R2   1  1  0 .75 0.0482R2   1  2  0 .75 0.0093R2
ETA2   2  0  0 .50 0.0078
M3      3  0  0  0  0  0 -.50    1
M3     0 -1  0 .50  .0564

2P01 2   2.0 P1        -1.0 01
S01  2   1.0 S2        -1.0 01
ST36 3   2.0 M2         1.0 N2        -2.0 S2
2NS2 2   2.0 N2        -1.0 S2
ST37 2   3.0 M2        -2.0 S2
ST1  3   2.0 N2         1.0 K2        -2.0 S2
ST2  4   1.0 M2         1.0 N2         1.0 K2        -2.0 S2
ST3  3   2.0 M2         1.0 S2        -2.0 K2
02   1   2.0 01
ST4  3   2.0 K2         1.0 N2        -2.0 S2
SNK2 3   1.0 S2         1.0 N2        -1.0 K2
OP2  2   1.0 01         1.0 P1
MKS2 3   1.0 M2         1.0 K2        -1.0 S2
ST5  3   1.0 M2         2.0 K2        -2.0 S2
ST6  4   2.0 S2         1.0 N2        -1.0 M2        -1.0 K2
2SK2 2   2.0 S2        -1.0 K2
MSN2 3   1.0 M2         1.0 S2        -1.0 N2
ST7  4   2.0 K2         1.0 M2        -1.0 S2        -1.0 N2
2SM2 2   2.0 S2        -1.0 M2
ST38 3   2.0 M2         1.0 S2        -2.0 N2
SKM2 3   1.0 S2         1.0 K2        -1.0 M2
2SN2 2   2.0 S2        -1.0 N2
N03  2   1.0 N2         1.0 01
M03  2   1.0 M2         1.0 01
NK3  2   1.0 N2         1.0 K1
S03  2   1.0 S2         1.0 01
MK3  2   1.0 M2         1.0 K1
SP3  2   1.0 S2         1.0 P1
SK3  2   1.0 S2         1.0 K1
ST8  3   2.0 M2         1.0 N2        -1.0 S2
N4   1   2.0 N2
3MS4 2   3.0 M2        -1.0 S2
```

```
ST39 4    1.0 M2         1.0 S2         1.0 N2        -1.0 K2
MN4  2    1.0 M2         1.0 N2
ST40 3    2.0 M2         1.0 S2        -1.0 K2
ST9  4    1.0 M2         1.0 N2         1.0 K2        -1.0 S2
M4   1    2.0 M2
ST10 3    2.0 M2         1.0 K2        -1.0 S2
SN4  2    1.0 S2         1.0 N2
KN4  2    1.0 K2         1.0 N2
MS4  2    1.0 M2         1.0 S2
MK4  2    1.0 M2         1.0 K2
SL4  2    1.0 S2         1.0 L2
S4   1    2.0 S2
SK4  2    1.0 S2         1.0 K2
MNO5 3    1.0 M2         1.0 N2         1.0 O1
2MO5 2    2.0 M2         1.0 O1
3MP5 2    3.0 M2        -1.0 P1
MNK5 3    1.0 M2         1.0 N2         1.0 K1
2MP5 2    2.0 M2         1.0 P1
2MK5 2    2.0 M2         1.0 K1
MSK5 3    1.0 M2         1.0 S2         1.0 K1
3KM5 3    1.0 K2         1.0 K1         1.0 M2
2SK5 2    2.0 S2         1.0 K1
ST11 3    3.0 N2         1.0 K2        -1.0 S2
2NM6 2    2.0 N2         1.0 M2
ST12 4    2.0 N2         1.0 M2         1.0 K2        -1.0 S2
ST41 3    3.0 M2         1.0 S2        -1.0 K2
2MN6 2    2.0 M2         1.0 N2
ST13 4    2.0 M2         1.0 N2         1.0 K2        -1.0 S2
M6   1    3.0 M2
MSN6 3    1.0 M2         1.0 S2         1.0 N2
MKN6 3    1.0 M2         1.0 K2         1.0 N2
2MS6 2    2.0 M2         1.0 S2
2MK6 2    2.0 M2         1.0 K2
NSK6 3    1.0 N2         1.0 S2         1.0 K2
2SM6 2    2.0 S2         1.0 M2
MSK6 3    1.0 M2         1.0 S2         1.0 K2
ST42 3    2.0 M2         2.0 S2        -1.0 K2
S6   1    3.0 S2
ST14 3    2.0 M2         1.0 N2         1.0 O1
ST15 3    2.0 N2         1.0 M2         1.0 K1
```

```
 M7    1    3.5 M2
 ST16 3    2.0 M2              1.0 S2              1.0 O1
 3MK7 2    3.0 M2              1.0 K1
 ST17 4    1.0 M2              1.0 S2              1.0 K2              1.0 O1
 ST18 2    2.0 M2              2.0 N2
 3MN8 2    3.0 M2              1.0 N2
 ST19 4    3.0 M2              1.0 N2              1.0 K2             -1.0 S2
 M8    1    4.0 M2
 ST20 3    2.0 M2              1.0 S2              1.0 N2
 ST21 3    2.0 M2              1.0 N2              1.0 K2
 3MS8 2    3.0 M2              1.0 S2
 3MK8 2    3.0 M2              1.0 K2
 ST22 4    1.0 M2              1.0 S2              1.0 N2              1.0 K2
 ST23 2    2.0 M2              2.0 S2
 ST24 3    2.0 M2              1.0 S2              1.0 K2
 ST25 3    2.0 M2              2.0 N2              1.0 K1
 ST26 3    3.0 M2              1.0 N2              1.0 K1
 4MK9 2    4.0 M2              1.0 K1
 ST27 3    3.0 M2              1.0 S2              1.0 K1
 ST28 2    4.0 M2              1.0 N2
 M10   1    5.0 M2
 ST29 3    3.0 M2              1.0 N2              1.0 S2
 ST30 2    4.0 M2              1.0 S2
 ST31 4    2.0 M2              1.0 N2              1.0 S2              1.0 K2
 ST32 2    3.0 M2              2.0 S2
 ST33 3    4.0 M2              1.0 S2              1.0 K1
 M12   1    6.0 M2
 ST34 2    5.0 M2              1.0 S2
 ST35 4    3.0 M2              1.0 N2              1.0 K2              1.0 S2

7120 VICTORIA HARBOUR BC   PST 48 23  123 22
Z0                                      6.067  000.0
Q1                                      0.197  130.3
O1                                      1.211  137.0
NO1                                     0.112  120.8
P1                                      0.674  148.5
S1                                      0.098  154.1
K1                                      2.070  149.4
J1                                      0.117  166.4
N2                                      0.294   63.4
```

```
     M2                                                    1.213   87.0
     S2                                                    0.332   93.9

001007076 031007076 EQUI  1.0
001007076 031007076 EXTR  0.5
```

Appendix 5

Tidal heights prediction results arising from the input data of Appendix 4. FIGURE 2 is a plot of these hourly heights over the period of 0100 PST July 24 1976 to 2400 PST July 31 1976.

```
 STN   1ST HR   DATE     1     2     3     4     5     6     7     8        DT HRS

7120   1.0000   1 776  7.46  7.74  7.93  7.89  7.52  6.80  5.80  4.66      1.0000
7120   9.0000   1 776  3.58  2.76  2.36  2.47  3.07  4.05  5.23  6.39      1.0000
7120  17.0000   1 776  7.36  7.98  8.21  8.09  7.73  7.28  6.90  6.68      1.0000
7120   1.0000   2 776  6.66  6.82  7.05  7.22  7.19  6.89  6.30  5.48      1.0000
7120   9.0000   2 776  4.58  3.77  3.21  3.04  3.32  4.01  5.00  6.10      1.0000
7120  17.0000   2 776  7.12  7.90  8.31  8.34  8.02  7.48  6.88  6.35      1.0000
7120   1.0000   3 776  6.01  5.90  5.99  6.19  6.38  6.44  6.29  5.91      1.0000
7120   9.0000   3 776  5.36  4.75  4.21  3.90  3.91  4.29  5.00  5.93      1.0000
7120  17.0000   3 776  6.92  7.78  8.37  8.59  8.41  7.89  7.16  6.38      1.0000
7120   1.0000   4 776  5.68  5.20  4.98  5.01  5.22  5.50  5.72  5.81      1.0000
7120   9.0000   4 776  5.71  5.46  5.14  4.85  4.73  4.86  5.28  5.96      1.0000
7120  17.0000   4 776  6.80  7.65  8.36  8.77  8.80  8.42  7.71  6.77      1.0000
7120   1.0000   5 776  5.78  4.90  4.27  3.96  3.98  4.26  4.69  5.14      1.0000
7120   9.0000   5 776  5.51  5.72  5.76  5.69  5.60  5.60  5.78  6.18      1.0000
7120  17.0000   5 776  6.80  7.54  8.27  8.84  9.09  8.95  8.38  7.45      1.0000
7120   1.0000   6 776  6.30  5.10  4.05  3.29  2.94  3.00  3.41  4.05      1.0000
7120   9.0000   6 776  4.77  5.43  5.92  6.21  6.32  6.35  6.40  6.57      1.0000
7120  17.0000   6 776  6.93  7.48  8.13  8.76  9.21  9.33  9.02  8.26      1.0000
7120   1.0000   7 776  7.13  5.76  4.38  3.18  2.36  2.02  2.19  2.79      1.0000
7120   9.0000   7 776  3.67  4.66  5.58  6.29  6.74  6.95  7.01  7.05      1.0000
7120  17.0000   7 776  7.19  7.49  7.98  8.56  9.11  9.46  9.46  8.99      1.0000
7120   1.0000   8 776  8.05  6.72  5.18  3.65  2.38  1.56  1.31  1.65      1.0000
7120   9.0000   8 776  2.47  3.59  4.80  5.90  6.74  7.26  7.48  7.52      1.0000
7120  17.0000   8 776  7.51  7.59  7.85  8.29  8.83  9.32  9.58  9.46      1.0000
7120   1.0000   9 776  8.84  7.73  6.24  4.57  2.98  1.72   .99   .91      1.0000
7120   9.0000   9 776  1.44  2.48  3.80  5.16  6.35  7.21  7.70  7.86      1.0000
7120  17.0000   9 776  7.82  7.74  7.78  8.00  8.42  8.93  9.37  9.55      1.0000
7120   1.0000  10 776  9.30  8.55  7.31  5.72  4.00  2.44  1.28   .73      1.0000
7120   9.0000  10 776   .85  1.60  2.81  4.26  5.67  6.84  7.63  8.01      1.0000
7120  17.0000  10 776  8.06  7.92  7.78  7.78  8.00  8.40  8.88  9.26      1.0000
7120   1.0000  11 776  9.34  8.99  8.12  6.80  5.19  3.52  2.09  1.12      1.0000
```

```
7120   9.0000 11 776    .79   1.14   2.07   3.41   4.89   6.25   7.31   7.94        1.0000
7120  17.0000 11 776   8.16   8.08   7.85   7.66   7.64   7.85   8.24   8.68        1.0000
7120   1.0000 12 776   8.98   8.97   8.52   7.59   6.25   4.71   3.19   1.97        1.0000
7120   9.0000 12 776   1.26   1.18   1.74   2.81   4.18   5.59   6.82   7.68        1.0000
7120  17.0000 12 776   8.12   8.16   7.95   7.65   7.42   7.40   7.60   7.96        1.0000
7120   1.0000 13 776   8.33   8.55   8.45   7.93   6.99   5.73   4.34   3.05        1.0000
7120   9.0000 13 776   2.11   1.69   1.85   2.57   3.70   5.02   6.28   7.30        1.0000
7120  17.0000 13 776   7.94   8.16   8.04   7.71   7.36   7.11   7.08   7.25        1.0000
7120   1.0000 14 776   7.56   7.86   7.99   7.82   7.28   6.40   5.29   4.13        1.0000
7120   9.0000 14 776   3.14   2.50   2.35   2.72   3.54   4.64   5.83   6.89        1.0000
7120  17.0000 14 776   7.67   8.06   8.09   7.82   7.42   7.02   6.76   6.70        1.0000
7120   1.0000 15 776   6.83   7.06   7.28   7.34   7.14   6.64   5.89   4.99        1.0000
7120   9.0000 15 776   4.11   3.43   3.10   3.18   3.69   4.53   5.54   6.55        1.0000
7120  17.0000 15 776   7.38   7.91   8.08   7.93   7.55   7.08   6.65   6.36        1.0000
7120   1.0000 16 776   6.26   6.33   6.49   6.64   6.66   6.48   6.08   5.51        1.0000
7120   9.0000 16 776   4.87   4.30   3.91   3.83   4.09   4.67   5.46   6.33        1.0000
7120  17.0000 16 776   7.14   7.73   8.03   8.01   7.72   7.25   6.72   6.24        1.0000
7120   1.0000 17 776   5.91   5.76   5.77   5.87   5.98   6.02   5.92   5.68        1.0000
7120   9.0000 17 776   5.33   4.96   4.66   4.53   4.63   4.99   5.57   6.27        1.0000
7120  17.0000 17 776   6.98   7.58   7.96   8.06   7.88   7.47   6.91   6.32        1.0000
7120   1.0000 18 776   5.79   5.41   5.20   5.17   5.25   5.38   5.49   5.52        1.0000
7120   9.0000 18 776   5.47   5.35   5.23   5.15   5.21   5.43   5.82   6.34        1.0000
7120  17.0000 18 776   6.93   7.48   7.90   8.10   8.03   7.71   7.18   6.53        1.0000
7120   1.0000 19 776   5.85   5.27   4.84   4.60   4.56   4.68   4.89   5.13        1.0000
7120   9.0000 19 776   5.33   5.48   5.57   5.63   5.72   5.88   6.15   6.52        1.0000
7120  17.0000 19 776   6.98   7.46   7.87   8.13   8.17   7.95   7.49   6.83        1.0000
7120   1.0000 20 776   6.07   5.31   4.66   4.21   3.99   4.01   4.23   4.58        1.0000
7120   9.0000 20 776   4.98   5.37   5.69   5.94   6.12   6.29   6.49   6.75        1.0000
7120  17.0000 20 776   7.09   7.48   7.86   8.16   8.29   8.19   7.83   7.21        1.0000
7120   1.0000 21 776   6.41   5.53   4.69   4.00   3.56   3.41   3.55   3.94        1.0000
7120   9.0000 21 776   4.47   5.05   5.60   6.05   6.38   6.62   6.80   6.99        1.0000
7120  17.0000 21 776   7.22   7.52   7.86   8.19   8.40   8.42   8.18   7.65        1.0000
7120   1.0000 22 776   6.87   5.91   4.90   3.99   3.31   2.94   2.93   3.26        1.0000
7120   9.0000 22 776   3.85   4.58   5.32   5.98   6.49   6.84   7.05   7.20        1.0000
7120  17.0000 22 776   7.35   7.56   7.84   8.16   8.45   8.59   8.50   8.11        1.0000
7120   1.0000 23 776   7.41   6.44   5.33   4.22   3.28   2.66   2.43   2.63        1.0000
7120   9.0000 23 776   3.19   3.99   4.89   5.75   6.45   6.94   7.22   7.36        1.0000
7120  17.0000 23 776   7.45   7.57   7.77   8.06   8.39   8.65   8.74   8.53        1.0000
7120   1.0000 24 776   7.97   7.08   5.94   4.69   3.52   2.62   2.13   2.11        1.0000
7120   9.0000 24 776   2.54   3.34   4.34   5.36   6.25   6.92   7.32   7.49        1.0000
```

```
7120 17.0000 24 776  7.53   7.55   7.65   7.87   8.19   8.54   8.79   8.80        1.0000
7120  1.0000 25 776  8.47   7.75   6.68   5.39   4.06   2.90   2.11   1.81        1.0000
7120  9.0000 25 776  2.03   2.72   3.72   4.86   5.93   6.77   7.32   7.57        1.0000
7120 17.0000 25 776  7.60   7.53   7.50   7.59   7.85   8.23   8.61   8.83        1.0000
7120  1.0000 26 776  8.77   8.30   7.43   6.22   4.84   3.51   2.44   1.82        1.0000
7120  9.0000 26 776  1.76   2.23   3.14   4.30   5.49   6.52   7.24   7.61        1.0000
7120 17.0000 26 776  7.67   7.54   7.37   7.30   7.42   7.73   8.15   8.55        1.0000
7120  1.0000 27 776  8.74   8.60   8.02   7.04   5.76   4.37   3.11   2.21        1.0000
7120  9.0000 27 776  1.82   2.01   2.71   3.79   5.02   6.18   7.08   7.61        1.0000
7120 17.0000 27 776  7.76   7.62   7.33   7.07   6.98   7.12   7.48   7.93        1.0000
7120  1.0000 28 776  8.33   8.50   8.28   7.64   6.62   5.34   4.04   2.95        1.0000
7120  9.0000 28 776  2.27   2.13   2.55   3.44   4.60   5.81   6.86   7.56        1.0000
7120 17.0000 28 776  7.85   7.77   7.42   7.00   6.66   6.55   6.71   7.08        1.0000
7120  1.0000 29 776  7.56   7.95   8.08   7.85   7.20   6.22   5.05   3.92        1.0000
7120  9.0000 29 776  3.06   2.63   2.73   3.34   4.33   5.49   6.61   7.46        1.0000
7120 17.0000 29 776  7.93   7.97   7.66   7.13   6.57   6.17   6.02   6.17        1.0000
7120  1.0000 30 776  6.54   7.01   7.39   7.54   7.33   6.76   5.91   4.93        1.0000
7120  9.0000 30 776  4.05   3.44   3.26   3.56   4.28   5.29   6.38   7.34        1.0000
7120 17.0000 30 776  7.97   8.19   7.99   7.46   6.77   6.09   5.60   5.40        1.0000
7120  1.0000 31 776  5.52   5.87   6.32   6.72   6.90   6.79   6.38   5.74        1.0000
7120  9.0000 31 776  5.02   4.40   4.05   4.08   4.51   5.28   6.24   7.20        1.0000
7120 17.0000 31 776  7.96   8.37   8.36   7.94   7.22   6.37   5.58   4.99        1.0000


HL STN   DATE    TIME   HGT TIME   HGT TIME   HGT TIME   HGT TIME   HGT TIME   HGT

0 7120   1   776   322   7.9 1117   2.3 1907   8.2 9999 99.9 9999 99.9 9999 99.9
1 7120   2   776    33   6.6  424   7.2 1152   3.0 1933   8.4 9999 99.9 9999 99.9
1 7120   3   776   200   5.9  550   6.4 1228   3.9 2002   8.6 9999 99.9 9999 99.9
1 7120   4   776   321   5.0  759   5.8 1302   4.7 2034   8.8 9999 99.9 9999 99.9
1 7120   5   776   426   3.9 1047   5.8 1332   5.6 2109   9.1 9999 99.9 9999 99.9
1 7120   6   776   521   2.9 2148   9.3 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120   7   776   609   2.0 2230   9.5 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120   8   776   654   1.3 1547   7.5 1648   7.5 2313   9.6 9999 99.9 9999 99.9
1 7120   9   776   737    .9 1611   7.9 1819   7.7 2358   9.5 9999 99.9 9999 99.9
1 7120  10   776   819    .7 1639   8.1 1931   7.8 9999 99.9 9999 99.9 9999 99.9
0 7120  11   776    44   9.4  859    .8 1708   8.2 2035   7.6 9999 99.9 9999 99.9
0 7120  12   776   129   9.0  937   1.1 1737   8.2 2137   7.4 9999 99.9 9999 99.9
0 7120  13   776   214   8.6 1013   1.7 1806   8.2 2239   7.1 9999 99.9 9999 99.9
0 7120  14   776   300   8.0 1047   2.3 1833   8.1 2347   6.7 9999 99.9 9999 99.9
```

```
0 7120 15  776  346  7.3 1118  3.1 1900  8.1 9999 99.9 9999 99.9 9999 99.9
1 7120 16  776  102  6.3  438  6.7 1145  3.8 1926  8.1 9999 99.9 9999 99.9
1 7120 17  776  226  5.8  548  6.0 1205  4.5 1951  8.1 9999 99.9 9999 99.9
1 7120 18  776  345  5.2  754  5.5 1208  5.2 2015  8.1 9999 99.9 9999 99.9
1 7120 19  776  442  4.6 2040  8.2 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 20  776  524  4.0 2106  8.3 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 21  776  559  3.4 2136  8.4 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 22  776  630  2.9 2210  8.6 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 23  776  701  2.4 2250  8.7 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 24  776  732  2.1 2333  8.8 9999 99.9 9999 99.9 9999 99.9 9999 99.9
1 7120 25  776  804  1.8 1638  7.6 1850  7.5 9999 99.9 9999 99.9 9999 99.9
0 7120 26  776   19  8.8  837  1.7 1643  7.7 1955  7.3 9999 99.9 9999 99.9
0 7120 27  776  107  8.7  910  1.8 1656  7.8 2055  7.0 9999 99.9 9999 99.9
0 7120 28  776  159  8.5  945  2.1 1714  7.9 2155  6.6 9999 99.9 9999 99.9
0 7120 29  776  254  8.1 1019  2.6 1736  8.0 2259  6.0 9999 99.9 9999 99.9
0 7120 30  776  356  7.5 1052  3.3 1800  8.2 9999 99.9 9999 99.9 9999 99.9
1 7120 31  776    7  5.4  509  6.9 1126  4.0 1828  8.4 9999 99.9 9999 99.9
```

REFERENCES

Cartwright, D.E. and R.J. Taylor, 1971. New Computations of the Tide-Generating Potential. Geophys. J.R. Astr. Soc. 23:45-74.

Cartwright, D.E. and A.C. Edden, 1973. Corrected Tables of Tidal Harmonics. Geophys. J.R. Astr. Soc. 33:253-264.

Davis, P.J. and P. Rabinowitz, 1961. Advances in Orthonormalizing Computation. Advances in Computing. 2:56-57.

Dietrich, G., 1963. General Oceanography. Interscience Publishers. New York, London.

Doodson, A.T., 1921. The Harmonic Development of the Tide-Generating Potential. Proc. Roy. Soc. Series A. 100:306-323. Re-issued in the International Hydrographic Review, May 1954.

Faddeev, D.K. and V.N. Faddeeva, 1963. Computational Methods of Linear Algebra. W.H. Freeman and Company, San Francisco.

Forsythe, G.E. and C.B. Moler, 1967. Computer Solution of Linear Algebraic Systems. Prentice-Hall, Englewood Cliffs, N.J.

Godin, G., 1972. The Analysis of Tides, University of Toronto Press, Toronto.

Godin, G. and J. Taylor, 1973. A Simple Method for the Prediction of the Time and Height of High and Low Water. Reprint from the International Hydrographic Review. Vol. L, N° 2, July 1973.

Godin, G., 1974. Nodal Corrections (unpublished notes).

Her Majesty's Nautical Almanac Office. The Astronomical Ephemeris, 1974. Her Majesty's Stationery Office, London.

Her Majesty's Nautical Almanac Office, 1961. Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac. Her Majesty's Stationery Office, London.

Searle, S.R., 1971. Linear Models. John Wiley and Sons, New York.

Wilkinson, J.H., 1967. The Solution of Ill-Conditioned Linear Equations in Mathematical Methods for Digital Computers, Vol. 2, edited by Ralston and Wilf.