Pacific Marine Science Report 76-5

THE TIDES IN THE FRASER ESTUARY

by

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INSTITUTE OF OCEAN SCIENCES, PATRICIA BAY
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ABSTRACT

The interaction between the tides in the Strait of Georgia and the discharge of the Fraser River is examined by a one-dimensional numerical model of the Fraser estuary. The model computes water surface elevations between the head of the tide water at Chilliwack and the mouth of the Fraser, including all four delta arms and Pitt Lake.

After a brief description of the conventional computation method, the report covers some practical aspects in the development of a tidal model, such as the field work preceding the model's calibration, the schematization, the effect of inaccuracies in the boundary conditions upon the computed water levels, and other sources of error. A relationship between the high and low waters in the navigable part of the river and those outside the mouth is examined for a variety of discharges.

Other uses of the model are discussed, such as an estimate of energy dissipation, and the effect of proposed hydraulic structures upon the tides in the estuary.

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ACKNOWLEDGEMENTS

The field work to collect the necessary tidal data for the model's calibration was carried out jointly by Water Survey of Canada in Vancouver, and the Tides and Currents Section of the Canadian Hydrographic Service in Victoria. Several staff members of both organizations participated in the data acquisition, and in particular, we acknowledge the generous help of Jack Wallace, Dick Brawn and Margaret Key of Water Survey, and of Syd Wigen, Willie Rapatz and Bob Brown of Tides and Currents.

We would like to thank Sus Tabata, Rick Thomson and John Garrett of Offshore Oceanography, and Pat Crean and Falconer Henry of Numerical Modelling for their helpful discussions and constructive criticism. We are also indebted to Dave Prandle of the Institute of Coastal Oceanography at Liverpool, and Norm Crookshank of the National Research Council at Ottawa for their expert advice in the initial development of the model.

Finally, we appreciate the assistance of Anne Harrison who drew the sketches, Lynne Egan who typed the report, and Sharon Thomson who did the proof-reading.

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INTRODUCTION

This report discusses a method to compute tidal heights in the Fraser estuary from the upstream discharges and the tides in the Strait of Georgia outside the delta. The method is based on a conventional one-dimensional numerical model, which was developed to improve tidal height predictions in the navigable portion of the Fraser River. Water surface elevations and velocities are computed every 2½ minutes at two-mile intervals. A graphical relationship is subsequently established between maximum and minimum tidal heights at selected points along the river, and corresponding extrema outside the mouth.

The model was calibrated with 15 tide gauges. Although the calculated and observed water levels show good agreement in height and phase at all stations, the accuracy of the model's flow computations cannot be established until more velocity measurements have been made.

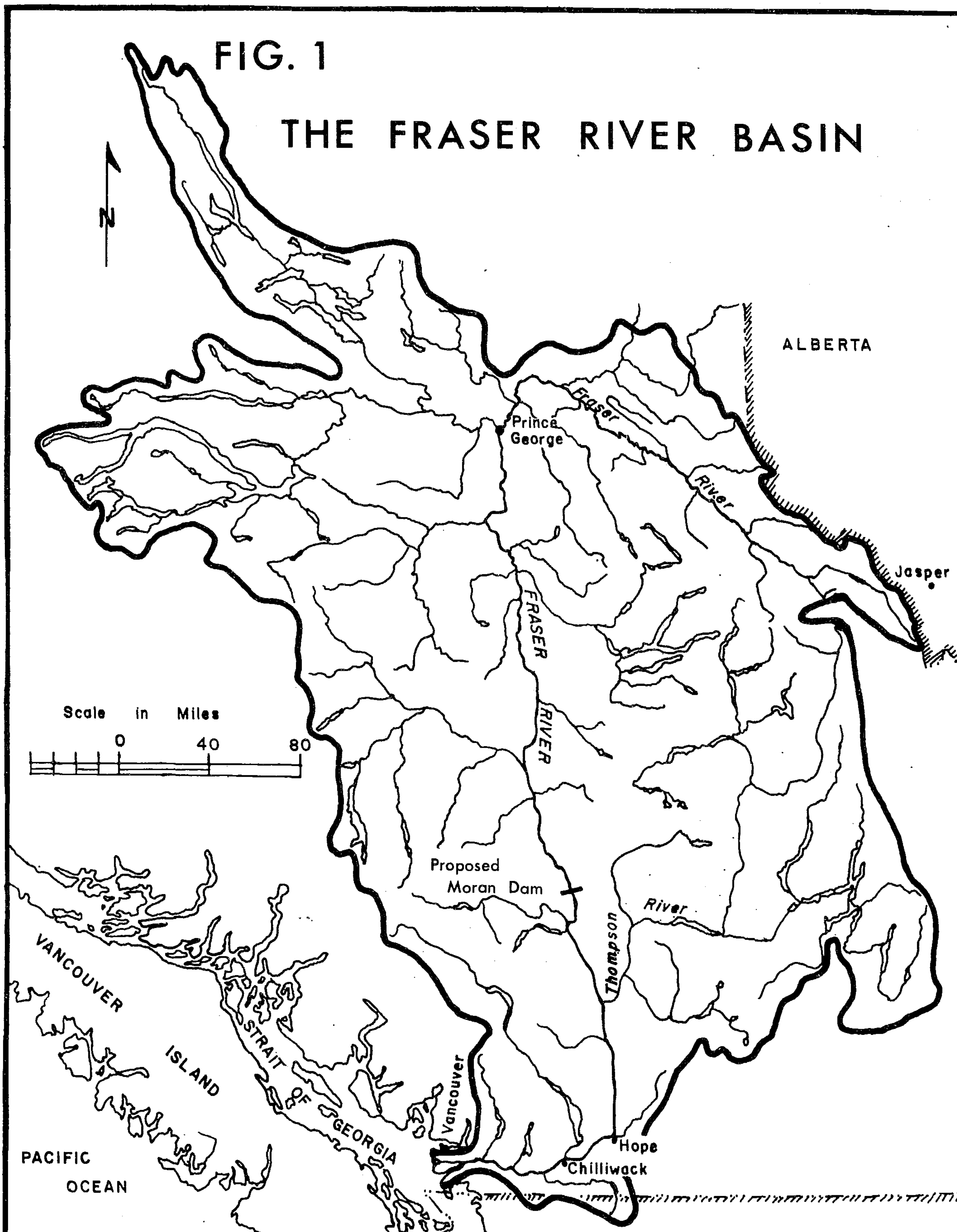
Rather than restricting itself to a discussion of the numerical technique followed, the report covers several phases in the development of a hydrodynamic model, i.e. the field measurements, the schematization, the calibration, and an error analysis. The model's accuracy is tested by comparing the computed heights and times of maximum and minimum tides at New Westminster with the observed values over a four-year period.

Some other practical applications of the model are mentioned, such as an assessment of the energy dissipated by friction in the tidal portion of the river.

NOTE: Because of its engineering applications and the nature of the data input, this report generally uses British units. Where necessary, the MKS values are given in parentheses.

FIG. 1

THE FRASER RIVER BASIN



THE FRASER

The Fraser River, one of the major rivers in North America, drains about 90,000 square miles of British Columbia (one quarter of the province) before entering the Strait of Georgia (Figure 1).

The source of the river is near Jasper at the Alberta border, 850 miles from the mouth, with an elevation of 6,000 feet above mean sea level. The river descends rapidly to an elevation of 2,400 feet in the first 80 miles, then flows at moderate slopes through plateau country and canyons. At Hope, about 100 miles east of the mouth, the Fraser reaches an alluvial valley, widens and flattens out to a mature stream, and, at low discharges, begins to "feel" the tidal influence near Chilliwack, 60 river miles from the mouth. Below New Westminster, the river divides into four distributaries - Main Arm, North Arm, Middle Arm, and Canoe Pass.

The profile in Figure 2 shows the Fraser water surface and mean discharges (1).

Fig.2 PROFILE OF THE FRASER RIVER

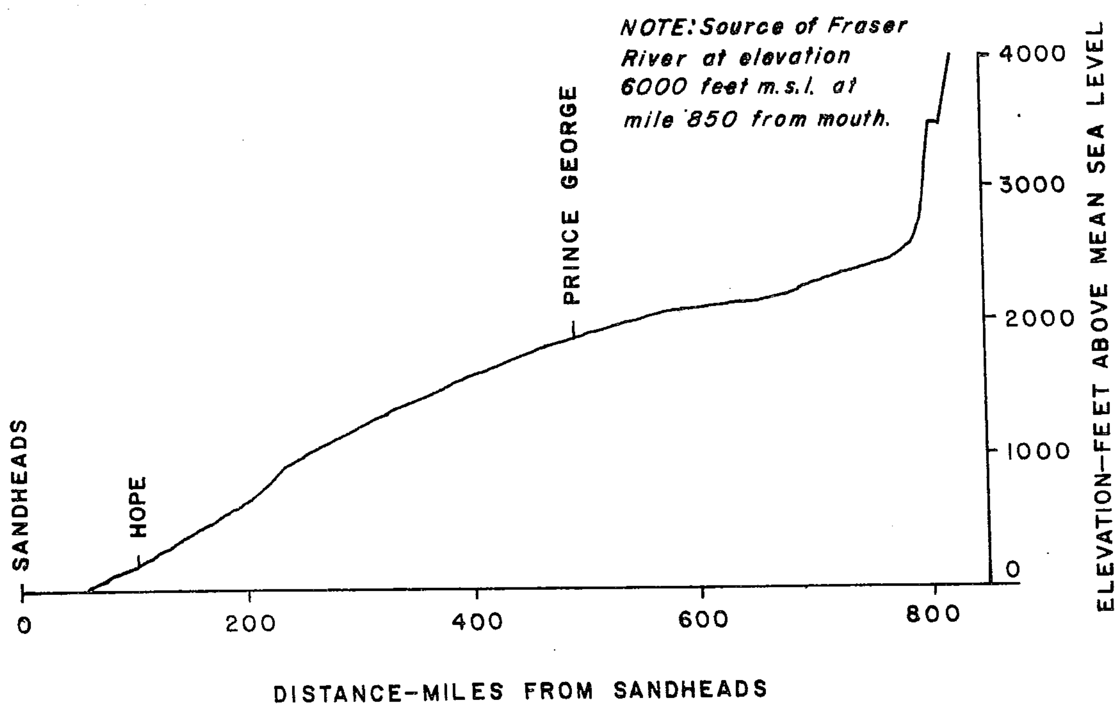


Fig.3 HYDROGRAPH FOR FRASER RIVER AT HOPE 1972

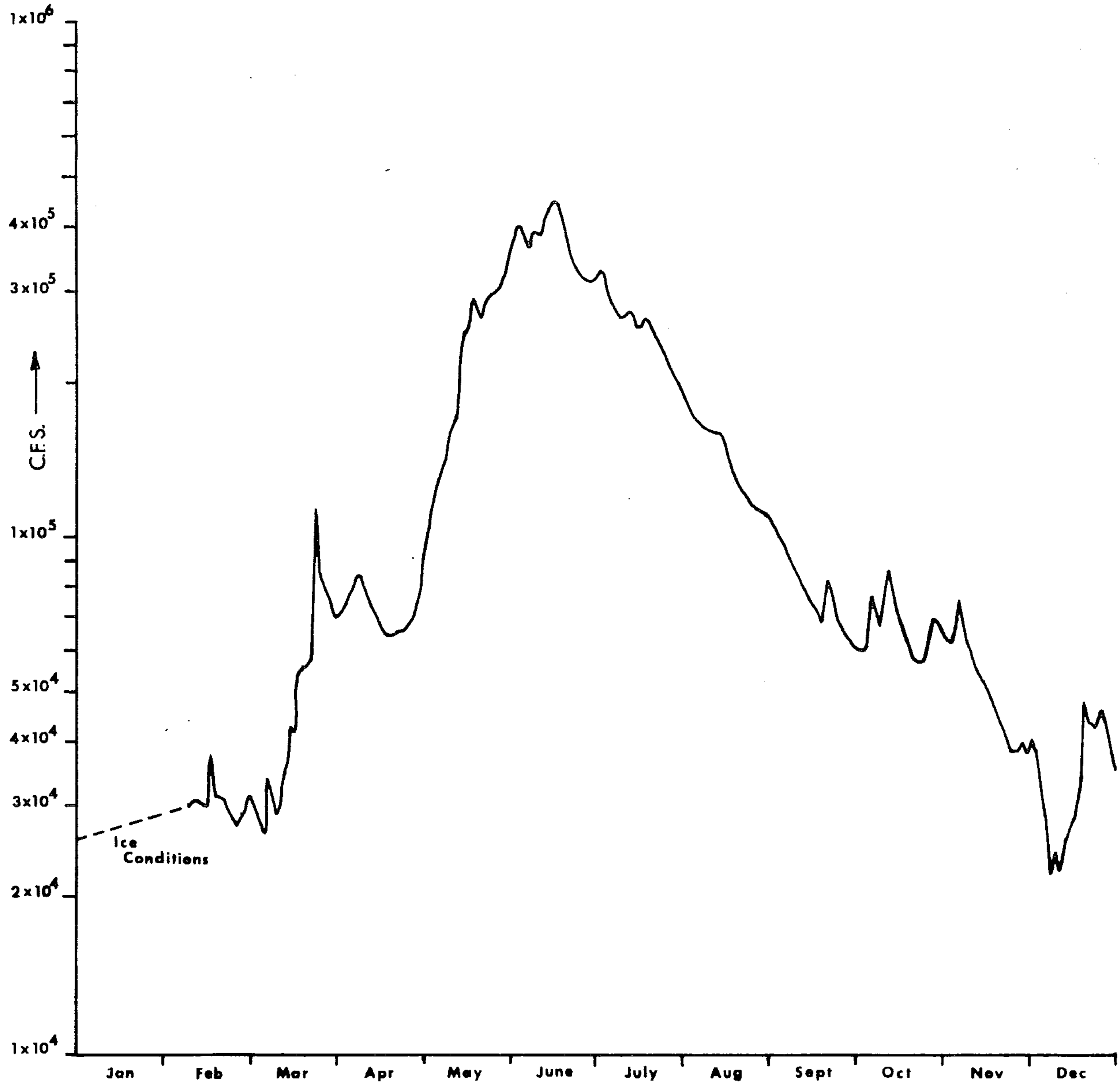
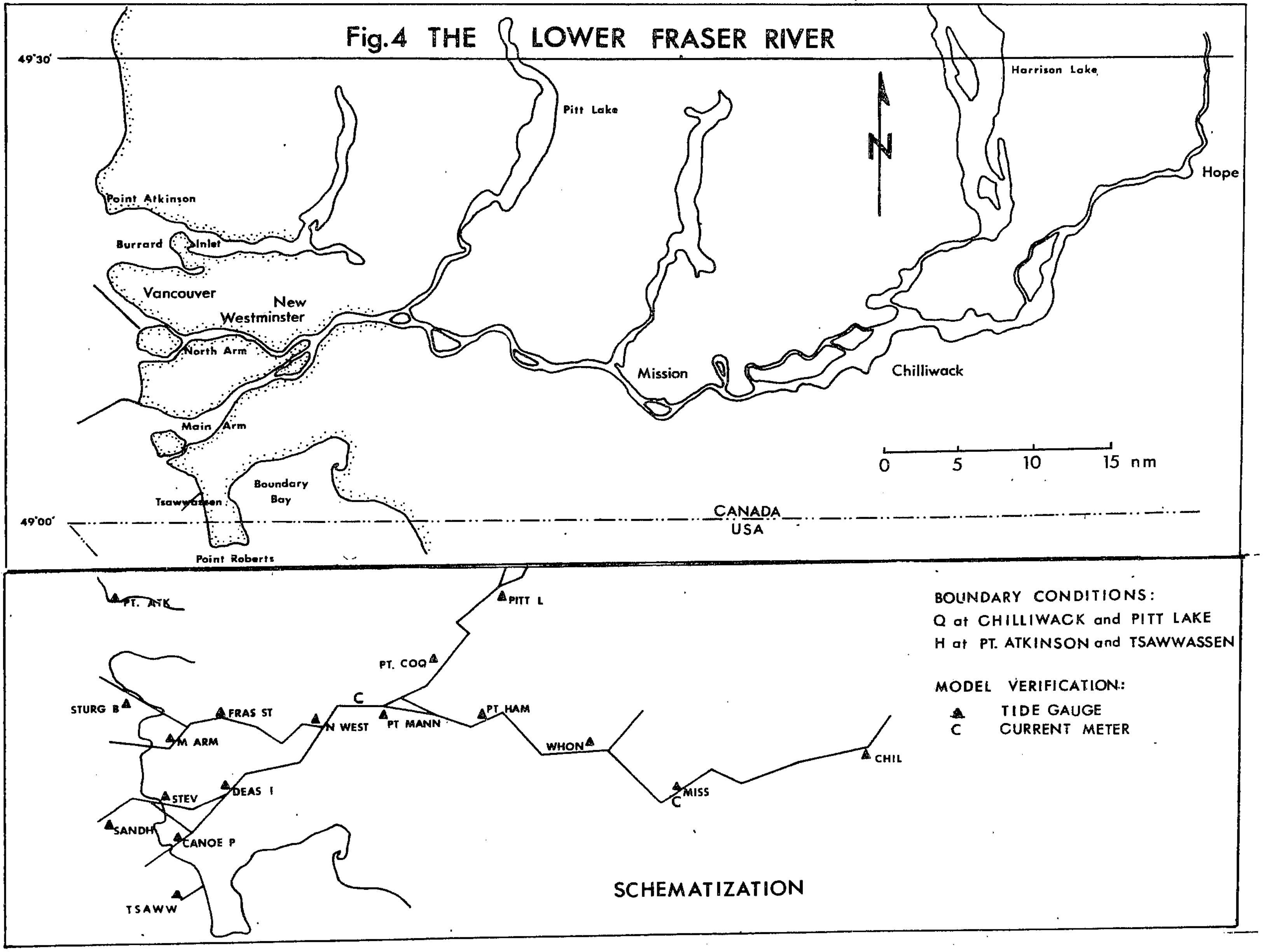


Fig.4 THE LOWER FRASER RIVER



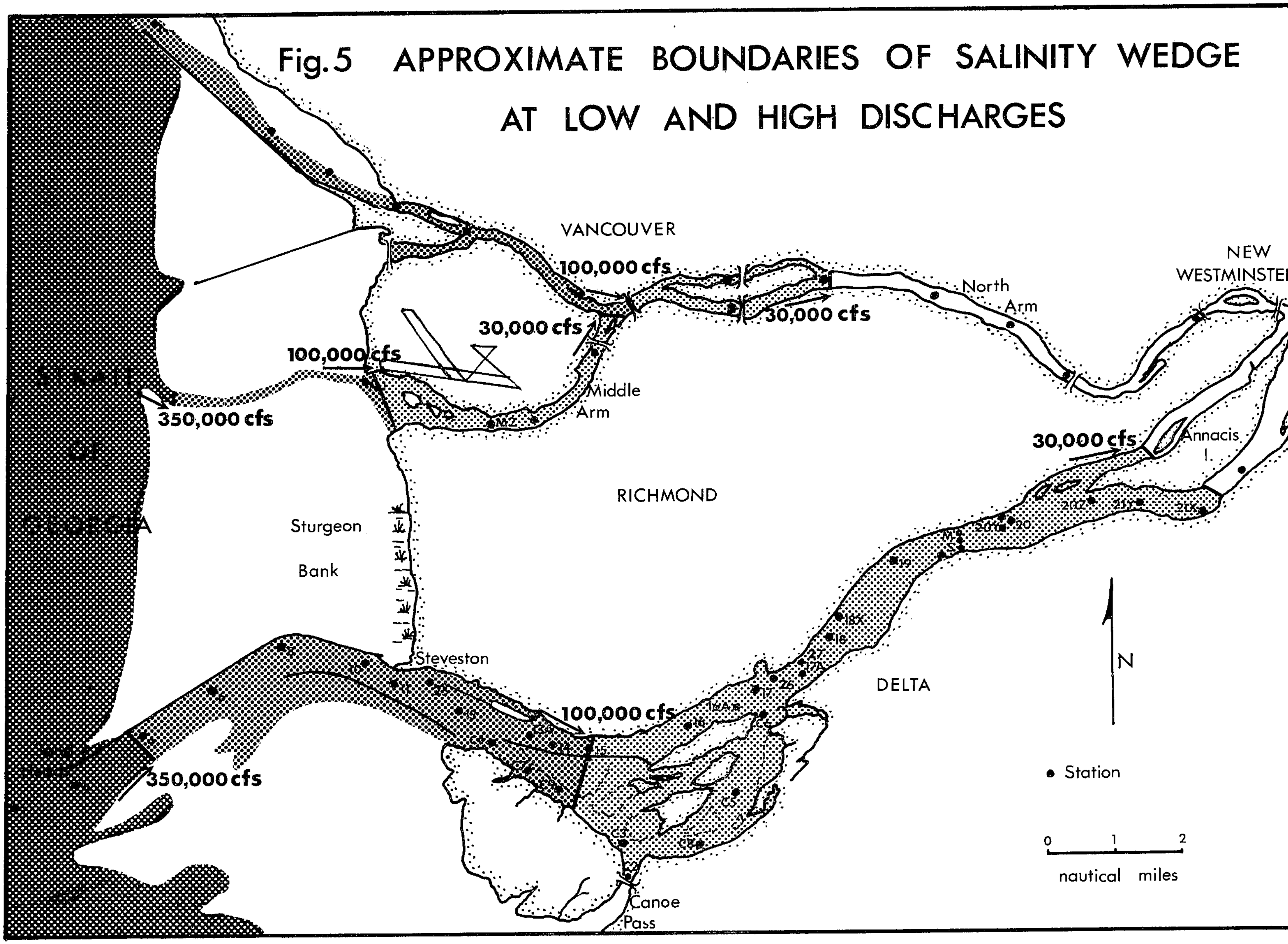


Fig. 5 APPROXIMATE BOUNDARIES OF SALINITY WEDGE AT LOW AND HIGH DISCHARGES

Snow forms about two-thirds of the precipitation in the drainage basin. The snow starts to melt in April, increasing the run-off to a maximum in late May or early June. This run-off has been measured since 1912 at Hope, a gauging station at the head of the Lower Fraser Valley, well upstream of the tidal influence.

The mean daily discharge measured at Hope varies between 20,000 cfs (600 m^3/sec) in the winter and 310,000 cfs (8800 m^3/sec) in the summer, averaged over 44 years (1912-1956). A peak flow of 536,000 cfs (15,200 m^3/sec) was recorded on May 31, 1948, when 55,000 acres in the Lower Fraser Valley were flooded. Figure 3 shows a typical hydrograph (1972).

The tide in the Strait of Georgia propagates to Chilliwack in the winter, during periods of low discharges (non-freshet), and to Mission in the summer during the freshet. The tide also propagates into Pitt Lake, with ranges of more than three feet. Water levels are recorded continuously by several gauges throughout the Valley (see Figure 4), and currents are measured with non-directional meters at Mission and Port Mann. Although the tidal records show a periodic rise and fall of the water level as far upstream as Chilliwack, actual flow reversals due to the flooding tide do not occur above Mission. During a strong freshet, the flow is outward all the way to the mouth, at all stages of the tide.

The upper limit of the salinity wedge has been observed as far upstream as Annacis Island at low discharges of 30,000 cfs (2) (see Figure 5). At average discharges (100,000 cfs or 2800 m^3/sec), the salinity wedge reaches the proximity of the Deas Island tunnel in the Main Arm, and the Oak Street Bridge in the shallower North Arm.

The mean annual total sediment load of the Fraser at Hope has been estimated at 25,000,000 tons (3), which is the sum of suspended load (including wash load and saltation load) and bed load. At New Westminster, the laboratory of the Sediment Survey of Water Survey of Canada has been analyzing sediment samples collected at Hope, Agassiz, Mission, and Port Mann since approximately 1965. Of the sediment load transported by the Fraser, only a small percentage has been found to be bed load; for instance, the total sediment load measured at Port Mann in 1966 was about 22,000,000 tons, of which 20,000,000 tons was suspended load and 2,000,000 tons bed load. However, it is the movement of the bed load which may alter the configuration of the river bed in the delta and affect the accuracy of model-predictions. During freshets, large bed waves migrate slowly downstream and continually change the cross-sectional areas. These bed waves can be measured by echo sounders. During the 1950 freshet, a bed wave with a height of 15 feet from trough to crest was followed downstream from the Deas Island tunnel. The wave was 500 feet long, and moved downstream at a rate of 250 feet per day (4).

Along the western delta front, there appears to be a sediment movement in a net northerly direction, with the principal deposition taking place off the main channel. Recent measurements (5) indicate an advancement of the delta front of up to 60 feet per year just south of Sandheads.

During periods of high discharge, the harbour of New Westminster (the only freshwater port in Western Canada) is subject to heavy siltation. At New Westminster, the river trifurcates into the North Arm, Annacis Channel and

Annieville Channel, with a consequent decrease in flow and increase in sedimentation in the Main Arm immediately below New Westminster. The Department of Public Works carries out an annual dredging program between the port of New Westminster and the mouth of the Fraser River, 21 miles downstream. Four million tons of bed load (the equivalent of three million cubic yards) are dredged annually from the entire estuary between New Westminster and the Strait of Georgia, 80-90% being from the Main Arm (6). To alleviate shoaling in the navigable part of the Fraser, a series of training walls was recently constructed to increase the river's sediment-carrying capacity in certain critical areas. This project, generally known as the Trifurcation Scheme (7), was designed and tested during the 1950's in a hydraulic movable-bed model at the University of British Columbia (horizontal scale 1:600). The Trifurcation Scheme, completed in 1973, was one of the major contributing factors in the increase in the Fraser River's annual gross shipping tonnage (8) from 4,868,248 tons in 1973 to 5,631,937 tons in 1974 (9).

At present, the maximum permissible draught for deep sea vessels entering the port of New Westminster is 32 feet on a 12 foot tide (10). To accommodate large container ships and bulk carriers, dredging operations are being considered to increase the permissible draught to 40 feet.

The North Arm is navigable by ships with a 12 foot draught.

While the primary purpose of the hydraulic model at UBC was to investigate methods to improve the regulation of the navigable channels in the Fraser, a second hydraulic model (fixed-bed, horizontal scale 1:1440), built shortly afterwards by the National Research Council at Ottawa, studied the flood danger in the Lower Fraser Valley. An intriguing feature of this NRC model was a proposal to construct a diversion canal from Annacis Island to Boundary Bay, which would reduce the flood water levels noticeably (11). This scheme and other related proposals were verified at NRC by a numerical model (12). Both hydraulic models have since been discontinued. In recent years, Western Canada Hydraulic Laboratories Ltd. at Port Coquitlam constructed a third model similar to the UBC model but on a larger horizontal scale of 1:480. All three physical models extended from the Strait of Georgia to a point beyond Chilliwack, the upper limit of tidal influence.

The possibility of regulating the Fraser River by dams, both to control floods and to generate power, has been studied in detail during the past two decades, but has met with stiff opposition from fisheries interests and environmental groups. The Fraser is the largest salmon-producing river in North America, and has created a multi-million dollar protein food industry. At present, no device exists which can pass migrating salmon safely either way over a high dam such as the proposed Moran Dam (Figure 1).

Although the demand for hydro-electric power in the Lower Mainland rises with increasing population and industry, the construction of a major dam in the Fraser would have to be a compromise among several interests. This project would require a joint study which has yet to be undertaken.

A dam would, of course, have a significant effect upon the tidal characteristics of the delta; it would smooth out seasonal fluctuations in the tides and currents and related phenomena such as sedimentation and the boundaries of the salinity wedge.

The reduction in maximum river heights and currents due to the decrease in discharge from freshet to regulated flow would, however, be partly compensated for by the tides: the tidal wave would penetrate further upstream, raising the high water levels and also adding to peak flows at ebb, due to storage of the preceding flood tides.

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THE TIDES IN THE STRAIT OF GEORGIA

The tides in the Strait of Georgia at the mouth of the Fraser River are mixed, mainly semi-diurnal, as demonstrated by the ratio of the sum of the diurnal constituents ($K_1 + O_1$) to that of the semi-diurnal constituents ($M_2 + S_2$) (13): this ratio is 1.16 for Point Atkinson, a principal reference port in the strait located on the north side of Burrard Inlet, about 12 nautical miles north of the entrance to the main channel of the Fraser Delta (see Figure 13). The Point Atkinson gauge has been in operation since 1914, and its recorded or predicted tides form one of the boundary conditions of the numerical model discussed later. A second gauge, operating since 1967, and also used for a boundary condition, is located at Tsawwassen, nine nautical miles south of the entrance to the main channel (see Figure 13). Its records show a $(K_1 + O_1)/(M_2 + S_2)$ ratio of 1.27.

The tidal range for large tides is 16.2 feet (4.9 m) at Point Atkinson, and 15.4 feet (4.7 m) at Tsawwassen.

At the mouth of the Main Arm of the Fraser, the sea water level is slightly raised by the river outflow. A tide gauge operating at Sandheads from March to September 1969 recorded water levels with monthly averages 0.6 feet higher than those at Tsawwassen during the freshet, and 0.3 feet higher at very low discharges. It is interesting to note that oceanographic records (14) of a station two nautical miles west of Sandheads, at high discharges show geopotential anomalies about five dynamic centimetres (0.17 feet) higher than those observed outside the main river plume, all dynamic heights being referred to a depth of no motion of 328 feet (100 m). There was much less consistency at very low discharges.

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TIDAL COMPUTATIONS

The tidal computations extend from the head of the tide water at Chilliwack to the mouth of the Fraser (Figure 4), including all four delta arms and Pitt Lake. The one-dimensional model used for these computations is based on the shallow-water wave equations (15) and has been described in detail in a previous publication (16). It will be discussed only briefly here.

The one-dimensional partial differential equations of continuity and motion can be written in the form

(CONTINUITY) $$\frac{\partial Q}{\partial x} + W \frac{\partial h}{\partial t} = 0,$$

where Q is the discharge in the x-direction (downstream) in feet^3/sec, W is the width of the water surface in feet, h is the elevation of the water surface in feet above geodetic datum, and x and t are the variables for distance and time, respectively;

(MOTION) $$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - g \frac{\partial h}{\partial x} - g \frac{u|u|}{C^2 d},$$

where u is the water velocity in the x-direction in feet/sec, g is the acceleration of gravity in feet/sec^2, d is the elevation of the water surface above the river bottom in feet, and C is the friction coefficient in feet$^{\frac{1}{2}}$/sec. Considering u to be the average velocity over the river cross-section A (in feet^2), and expressing the equation of motion in terms of the discharge Q (in feet^3/sec), we may write

(MOTION) $$\frac{1}{A}\frac{\partial Q}{\partial t} - \frac{Q}{A^2}\frac{\partial A}{\partial t} + \frac{Q}{A}\left\{\frac{1}{A}\frac{\partial Q}{\partial x} - \frac{Q}{A^2}\frac{\partial A}{\partial x}\right\} = - g \frac{\partial h}{\partial x} - g \frac{Q|Q|}{A^2 C^2 d} .$$

After some additional minor modifications, both the Equation of Continuity and the Equation of Motion can be written in finite difference form ($\Delta x, \Delta t$), and solved at the intersections of a space-time grid, with the river discharge at one end of the model and the tides at the other (seaward) end as boundary conditions.

To express the equations in finite-difference form, the first derivatives are approximated by central differences (17), e.g.

$$\frac{\partial h}{\partial x} = \frac{H^k_{m+1} - H^k_{m-1}}{2\Delta x}$$

where k and m indicate time and distance steps, respectively.

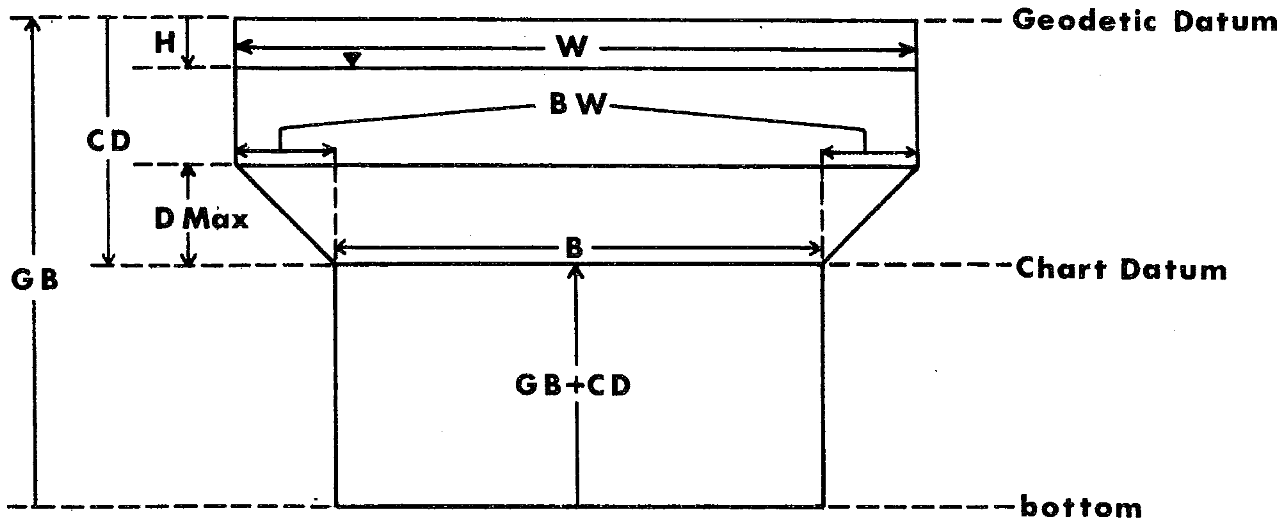
The term $\partial Q/\partial x$ may be inaccurate in finite-difference form because of the relatively long length (Δx) of the sections of the Fraser model, and is therefore replaced by $- W\, \partial h/\partial t$ (from continuity).

Putting $\partial A/\partial t = \partial A/\partial h \,.\, \partial h/\partial t$, we can rewrite the equation of motion as

$$\frac{1}{gA}\frac{\partial Q}{\partial t} - \left(\frac{\partial A}{\partial h} + W\right)\frac{Q}{gA^2}\frac{\partial h}{\partial t} - \frac{Q^2}{gA^3}\frac{\partial A}{\partial x} = - \frac{\partial h}{\partial x} - \frac{Q|Q|}{C^2 A^2 d} .$$

The term $\partial A/\partial h$ is approximated by the width of the conveyance channel, B in Figure 6, while W also includes shoals.

Fig. 6 SCHEMATIZATION OF SECTIONS



B – Mean width of a section at Chart Datum (fixed)

W – Mean width at time t (variable)

BW – Bank width (fixed)

GB – Geodetic Datum wrt bottom (fixed)

CD – Chart Datum wrt Geodetic Datum (fixed)

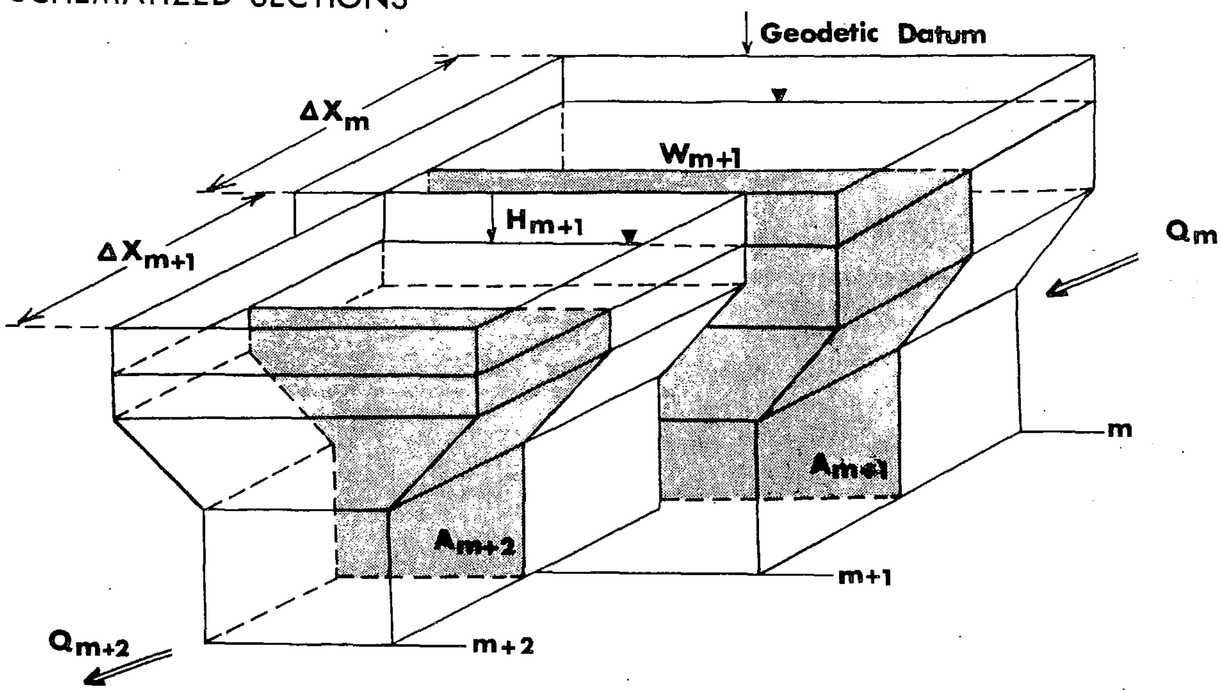
D Max – Maximum bank height (fixed)

H – Water level wrt Geodetic Datum (variable; negative in figure)

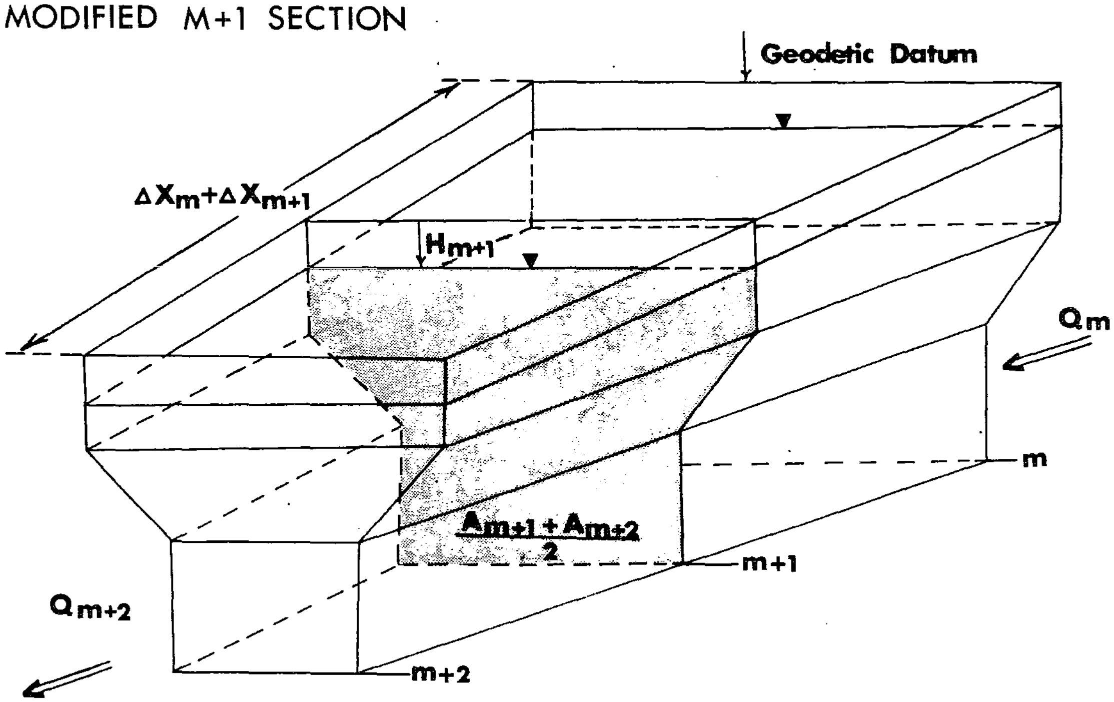
GB + CD – Chart Datum wrt bottom (fixed), obtained from hydrographic charts

Fig.7 SMOOTHING OF CROSS-SECTIONAL AREAS

SCHEMATIZED SECTIONS



MODIFIED M+1 SECTION



With these modifications, the equations of continuity and motion are respectively expressed as follows:

$$\frac{Q_{m+2}^{k-1} - Q_m^{k-1}}{\Delta x_m + \Delta x_{m+1}} + W_{m+1}^{k-1} \cdot \frac{H_{m+1}^{k} - H_{m+1}^{k-2}}{2\Delta t} = 0.$$

$$\frac{1}{gA_m^k} \cdot \frac{Q_m^{k+1} - Q_m^{k-1}}{2\Delta t} - (B+W)_m^k \frac{Q_m^{k+1}}{g(A_m^k)^2} \cdot \frac{(H_{m+1}^k + H_{m-1}^k) - (H_{m+1}^{k-2} + H_{m-1}^{k-2})}{4\Delta t}$$

$$- \frac{Q_m^{k-1}\,Q_m^{k+1}}{g(A_m^k)^3} \cdot \left(\frac{A_{m+1}^k - A_{m-1}^k}{\Delta x_m + \Delta x_{m-1}}\right) = - \frac{H_{m+1}^k - H_{m-1}^k}{\Delta x_m + \Delta x_{m-1}} - \frac{Q_m^{k+1}|Q_m^{k-1}|}{(C_m)^2 (A_m^k)^2 D_m^k}.$$

The subscript m for the friction coefficient C enables the program to vary C with each section. The product $Q_m^{k+1}\,Q_m^{k-1}$ linearizes Q^2, because Q_m^{k-1} is obtained from the previous time step (k-1).

In their final form, when the model's matrix has been compressed and the rows relabelled, the equations are as follows:

(CONTINUITY)

$$H_{m+1}^{n+1} = H_{m+1}^{n} - \left\{4\Delta t \cdot (Q_{m+2}^{n} - Q_m^{n})\right\} \cdot \left\{(W_{m+1}^{n} + W_{m+2}^{n}) \cdot (\Delta x_m + \Delta x_{m+1})\right\}^{-1}.$$

The term 1/2 $(W_{m+1} + W_{m+2})$ represents the width at section line m+1 in Figure 7.

(MOTION)

$$Q_m^{n+1} = \left\{\frac{\Delta x_m + \Delta x_{m-1}}{g.\Delta t.(A_m^{n+1} + A_{m+1}^{n+1})} Q_m^n - (H_{m+1}^{n+1} - H_{m-1}^{n+1})\right\}$$

$$\cdot \left\{\frac{\Delta x_m + \Delta x_{m-1}}{g.\Delta t.(A_m^{n+1} + A_{m+1}^{n+1})} + \frac{(\Delta x_m + \Delta x_{m-1}) \cdot |Q_m^n|}{C_m^2 \cdot \left(\frac{A_m^{n+1} + A_{m+1}^{n+1}}{2}\right)^2 \cdot \left(GB_m + \frac{H_{m+1}^{n+1} + H_{m-1}^{n+1}}{2}\right)}\right.$$

$$- \frac{Q_m^n \cdot \left\{(A_{m+1}^{n+1} + A_{m+2}^{n+1}) - (A_m^{n+1} + A_{m-1}^{n+1})\right\}}{g\,\frac{(A_m^{n+1} + A_{m+1}^{n+1})^3}{4}}$$

$$- \frac{(\Delta x_m + \Delta x_{m-1}) \cdot (B_m^{n+1} + W_m^{n+1} + B_{m+1}^{n+1} + W_{m+1}^{n+1})}{8g.\Delta t.\left(\frac{A_m^{n+1} + A_{m+1}^{n+1}}{2}\right)^2}$$

$$\left. \cdot \left[(H_{m+1}^{n+1} + H_{m-1}^{n+1}) - (H_{m+1}^{n} + H_{m-1}^{n})\right]\right\}^{-1}.$$

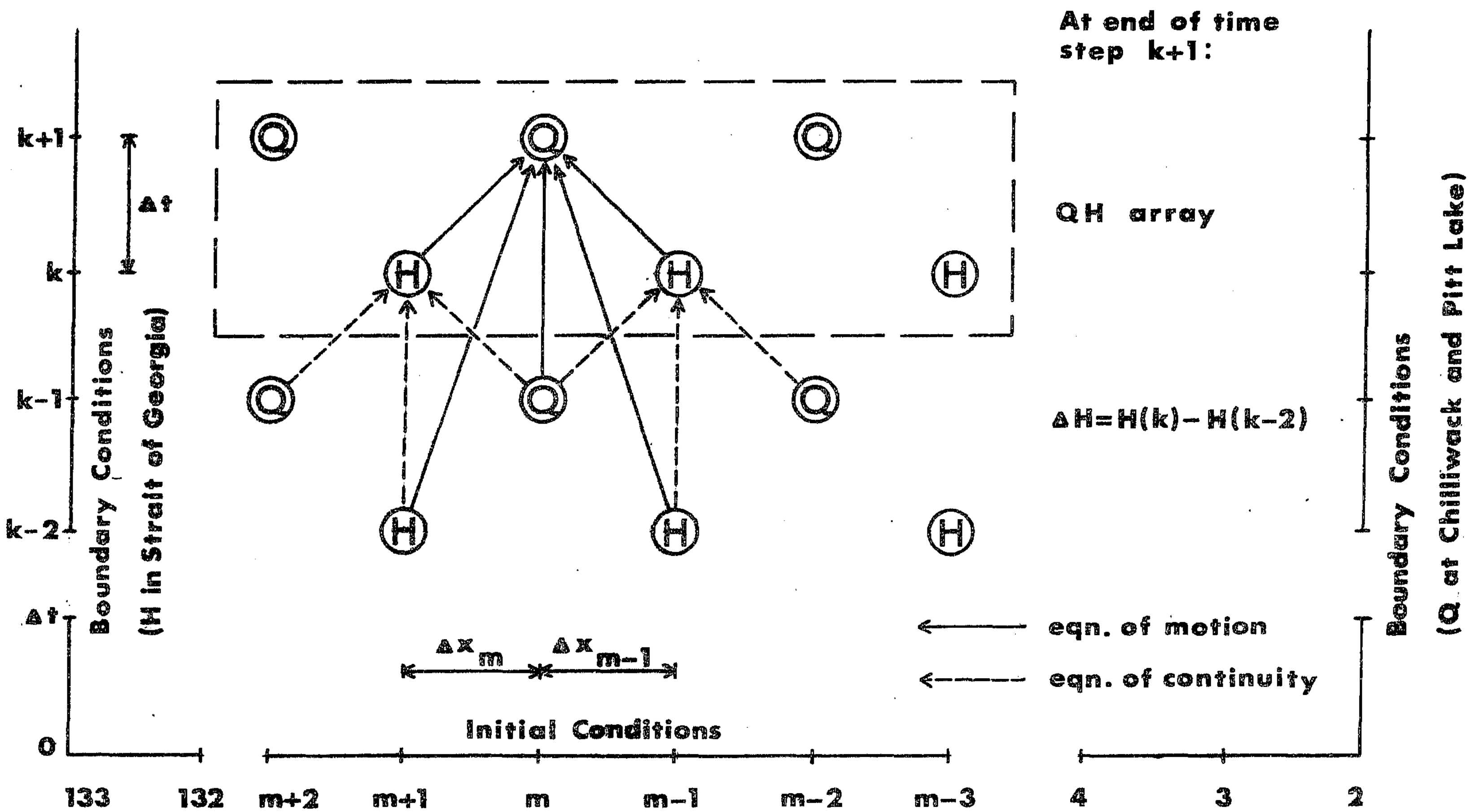


Fig.8 COMPUTER SCHEME

As the computation scheme in Figure 8 illustrates, the tidal heights H are computed at the odd-numbered sections in the x-direction (starting at the river mouth with the observed tidal heights as the downstream boundary conditions) and the discharges Q at the even-numbered sections (starting at Chilliwack and Pitt Lake with the observed discharges as the upstream boundary conditions).

ASSUMPTIONS

The tidal computations assume one-dimensional, vertically integrated flow throughout the delta.

The downstream boundary conditions at the mouth of the Fraser are assumed to be truly represented by the vertical tides at Point Atkinson and Tsawwassen (with minor adjustments for river discharge).

Except for the tidal interaction with Pitt Lake, tributary inflow between Chilliwack and the mouth of the Fraser has been omitted.

The effects of wind, barometric pressure and centrifugal forces in river bends are ignored.

Changes in cross-sectional areas due to sedimentation (bed waves, etc.) have been neglected.

Salinity intrusion has been neglected.

Fig. 9 SCHEMATIZATION AT DEAS ISLAND

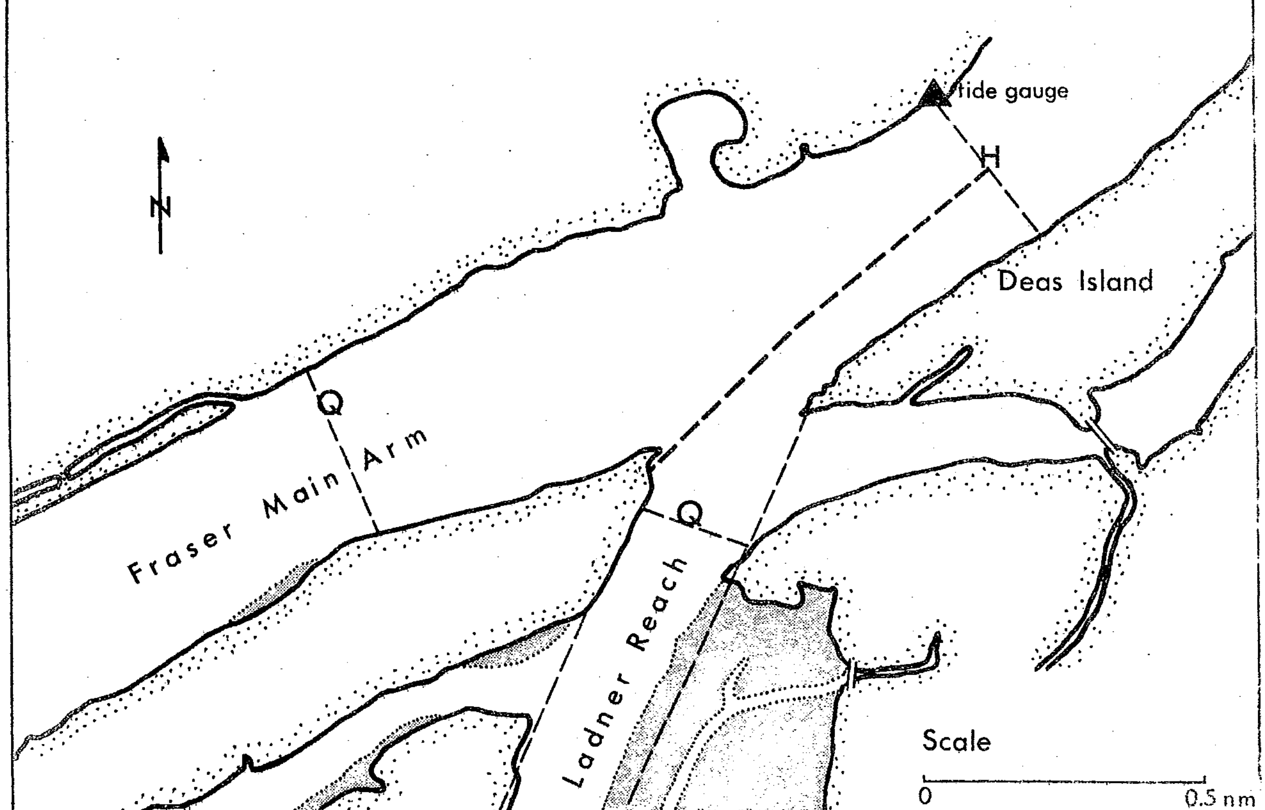
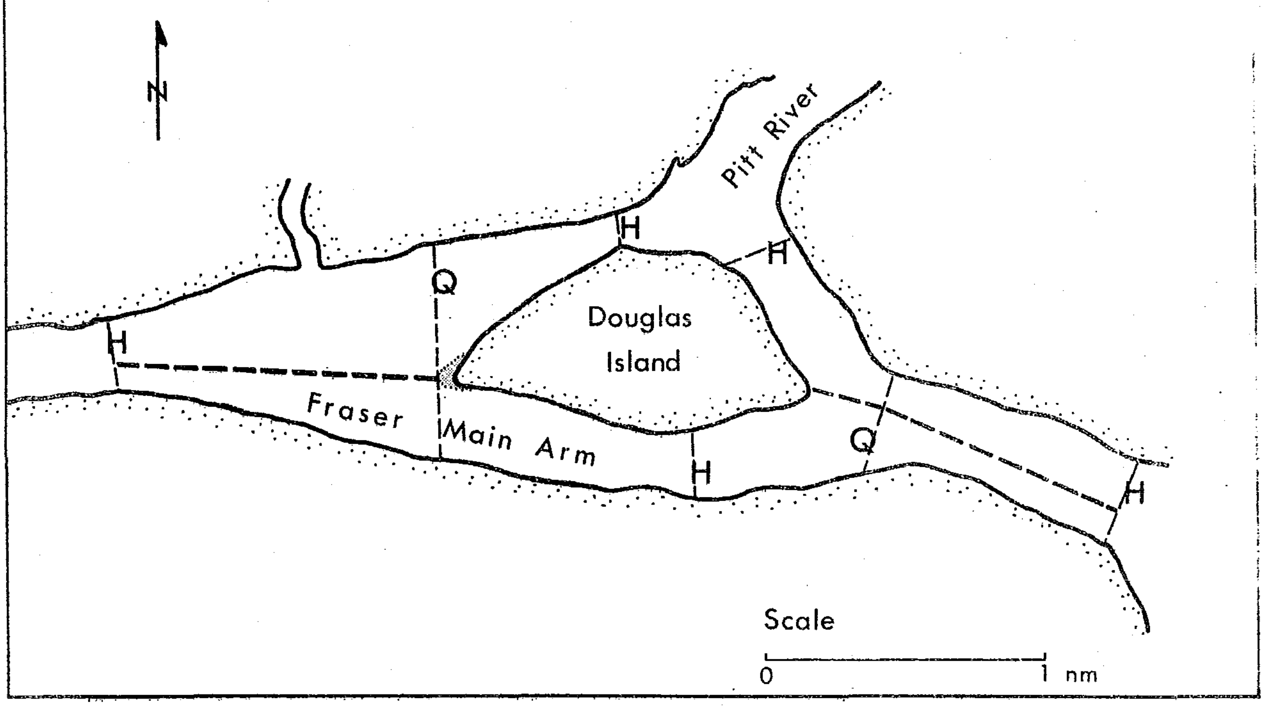


Fig. 10 SCHEMATIZATION AT DOUGLAS ISLAND



SCHEMATIZATION

The river and its tributaries were divided into segments, about 6000 feet long. The depth of each segment with respect to the chart datum was obtained from a fieldsheet by overlaying the soundings with a transparent grid, and tabulating the average sounding per square. The sum of these average soundings was divided by the total number of squares to find the representative depth. The width was determined by division of the surface area (the total number of squares multiplied by the area per square) by the length of the segment. Since geodetic datum was used as a reference level for tidal heights, the depths were adjusted accordingly. Geodetic datum was selected as a reference level because it remains the same throughout the model, while chart datum is raised at regular intervals in an upstream direction.

To avoid abrupt changes in cross-sectional areas, the dimensions were smoothed out as in Figure 7.

The values B, BW, DMAX, CD and GB (see Figure 6) were taken from the charts, and are part of the data input.

To facilitate calibration, the segments were arranged so that the H-sections as sketched in Figure 8 would coincide with the locations of the tide gauges. Another criterion for the schematization was that a common H-section should be assigned to each river arm at a bifurcation or confluence (rather than a Q section and an uncertain flow distribution). If this arrangement was not feasible, the division was extended upstream (bifurcation) or downstream (confluence) to the nearest H-section by a hypothetical training wall (see Figures 9 and 10).

At Douglas Island, where Pitt River enters the Fraser, both confluences and bifurcations occur (see Figure 10). In addition to the usual modifications in the calculations arising from these conditions, the configuration of the schematized flow made it necessary to perform the calculations in one reach in a direction opposite to those in an adjoining arm.

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STABILITY AND THE TIME STEP

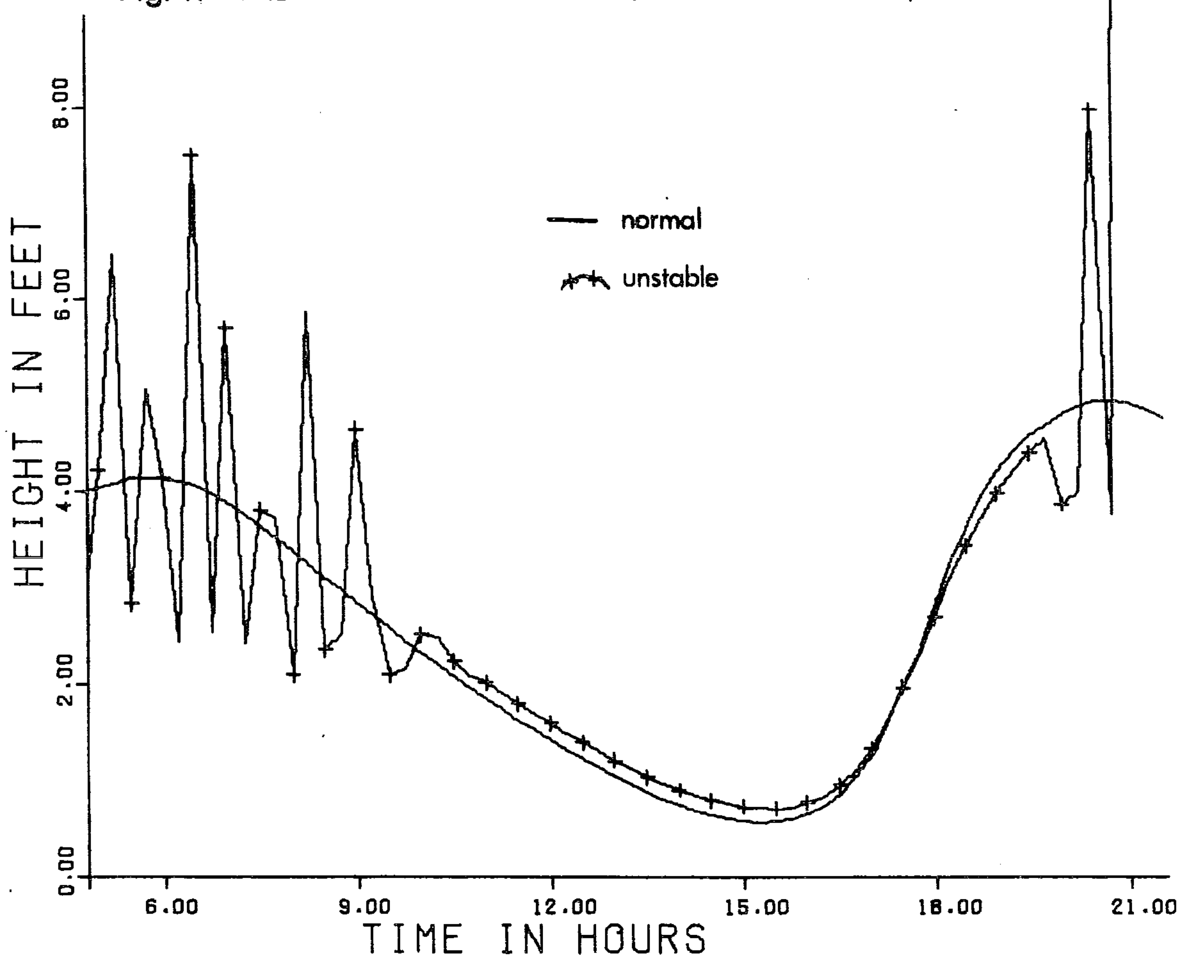
Stability is essential in an explicit scheme to prevent the progressive amplification of numerical errors introduced by finite-difference approximations to differential equations. The accepted criterion for the unconditional stability of a one-dimensional explicit scheme is

$$\frac{\Delta x}{\Delta t} > c \;;$$

where c is the velocity of propagation of a tidal wave. ($c = \sqrt{gh}$, where h is the maximum water depth in the system.)

In the Fraser model, the (minimum) section length Δx had been determined by the schematization (as 3700 feet). Therefore, the time step Δt was adjusted in order to attain stability in the calculations. Although the model was unstable for a time step of 112.5 seconds, it appeared to be stable with $\Delta t = 75$ seconds. A subsequent run of the model with $\Delta t = 37.5$ seconds produced the same values for the predicted heights, confirming that stability had been reached. Therefore, a time step of 75 seconds was selected for the model.

Fig. 11 INDUCED INSTABILITY (New Westminster)



To investigate the importance of the section lengths in establishing stability, the lengths of several sections near New Westminster were reduced to 1600 feet from about 6000 feet. Figure 11 shows the predicted heights at New Westminster for both the normal and modified schemes. Although the calculations for the modified scheme are unstable and fluctuate rapidly at 5 hrs, the predicted heights return to normal when the water depth decreases to within stability bounds. However, when the critical depth is exceeded (i.e. when the stability criterion is no longer satisfied) at 20 hrs, the calculated heights oscillate wildly outside the range of the normal computations.

BOUNDARY CONDITIONS

The model's *upstream* boundary conditions are the steady-state river discharges Q at points outside the tidal influence in the Fraser and Pitt Rivers:

For the Fraser River, the limit of the tidal influence was considered to be at Chilliwack, the location of the first upstream river gauge without daily fluctuations in its records of water surface elevations. River discharges are not measured at Chilliwack, and the model's eastern input is based on the discharge of the Fraser at Hope, 30 miles upstream from Chilliwack, and of the Harrison River at Harrison Hot Springs. Between Hope and Chilliwack, the Harrison River is the only tributary with a significant discharge. The records of the gauges at Hope and at Harrison Hot Springs were compiled from the 1969-72 surface water records of Water Survey of Canada (18), and their sums plotted against the corresponding discharges at Hope. Although the data for all four years showed the same linear relationship, the 1972 data were the most useful because of their large range, and were used to determine the model input at Chilliwack (Figure 12). The sum of the discharges at Hope and at Harrison Hot Springs was assumed to be a reasonable estimate for the discharge at Chilliwack, and a linear approximation to the data points gave the relationship $Q_{CHILL} = 1.12\ Q_{HOPE} + 5000$ cfs. The x-intercept of 5000 cfs might be considered to be the outflow of the Harrison River in the hypothetical case that the discharge of the Fraser at Hope becomes zero.

The predicted discharges at Chilliwack (i.e. values obtained using the relationship in Figure 12) were compared with discharges measured at Mission (15 miles downstream from Chilliwack) for four periods during the freshet, when the tidal effect upon the flow at Mission would be minimal. The observed discharges were an average of 4.5% higher than predicted which might be accounted for by local run-off and tributary inflow (e.g. Chilliwack River) between the two stations during the freshet.

A considerable part of the tide propagates through Pitt River into Pitt Lake and this system therefore was included in the model, with the discharge Q at the head of Pitt Lake as a boundary condition. Records of discharges into the head of Pitt Lake were not available; however, one estimate of 4000 cfs was obtained from Water Survey of Canada for a discharge at Hope of 150,000 cfs. To arrive at an approximate relationship between Q_{PITT} and Q_{HOPE}, the available discharge records of other rivers with their sources in the same area as the Pitt River (i.e. Mamquam, Cheakamus and Lillooet) were compared with those of the Fraser at Hope. These comparisons suggested the existence of a linear relationship. A similar relationship was assumed to exist between Q_{HOPE} and Q_{PITT}; thus for a discharge at Hope of 400,000 cfs (11,300 m^3/sec), the model input at the head of Pitt Lake was 10,000 cfs (300 m^3/sec).

The *downstream* boundary conditions are the tides in the Strait of Georgia. Initially, they were derived from records of tide gauges established in the four distributaries, as illustrated in Figure 13. These gauges were operated by the Tides and Currents Section of the Canadian Hydrographic Service for several months during the pre-freshet and freshet periods of 1969. However, the gauges were temporary, their purpose being to provide boundary conditions for the strictly one-dimensional portion of the estuary during the

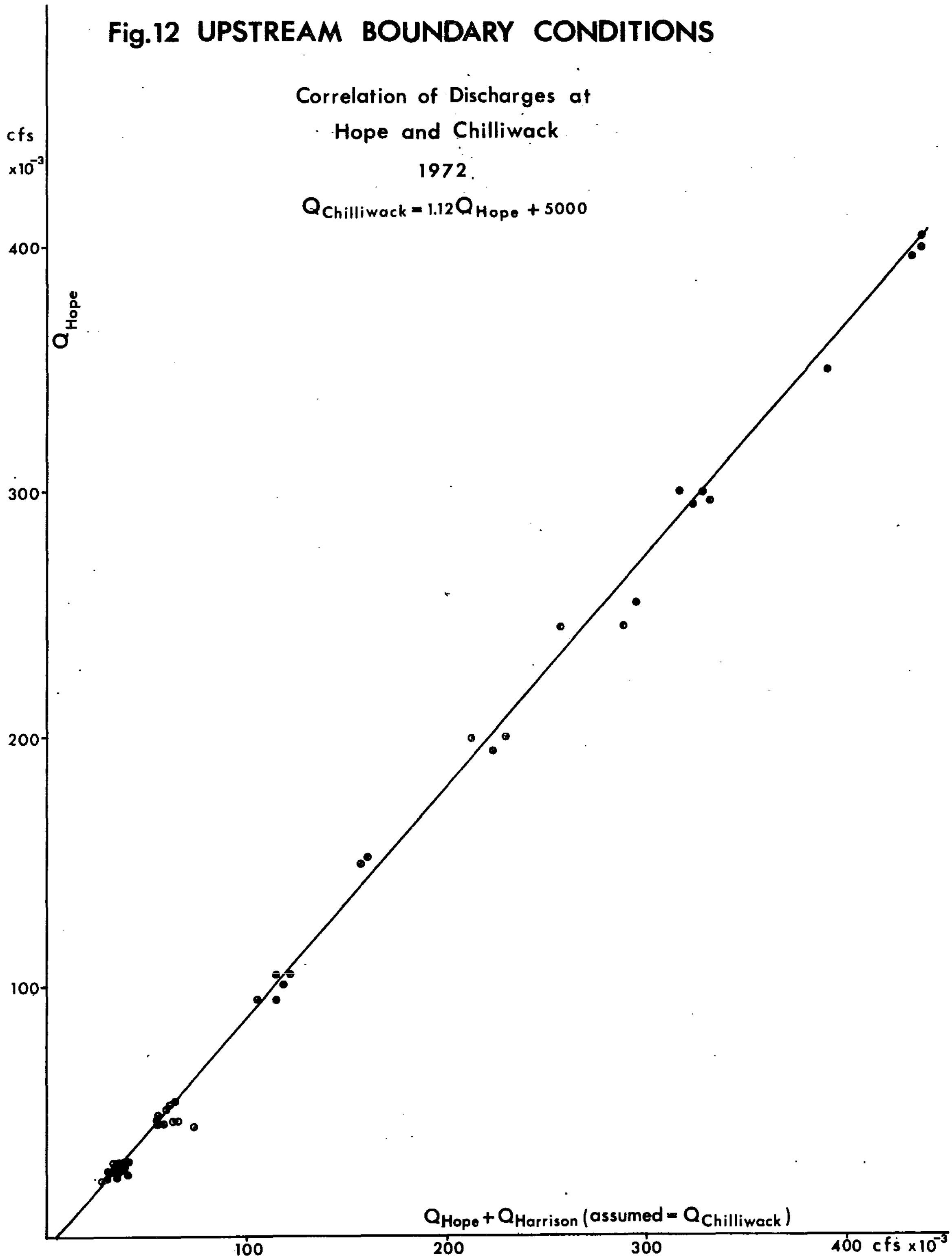


Fig. 12 UPSTREAM BOUNDARY CONDITIONS
Correlation of Discharges at
Hope and Chilliwack
1972
$Q_{Chilliwack} = 1.12 Q_{Hope} + 5000$
cfs
$\times 10^{-3}$
400
300
200
100
Q_{Hope}
$Q_{Hope} + Q_{Harrison}$ (assumed = $Q_{Chilliwack}$)
100
200
300
400 cfs $\times 10^{-3}$

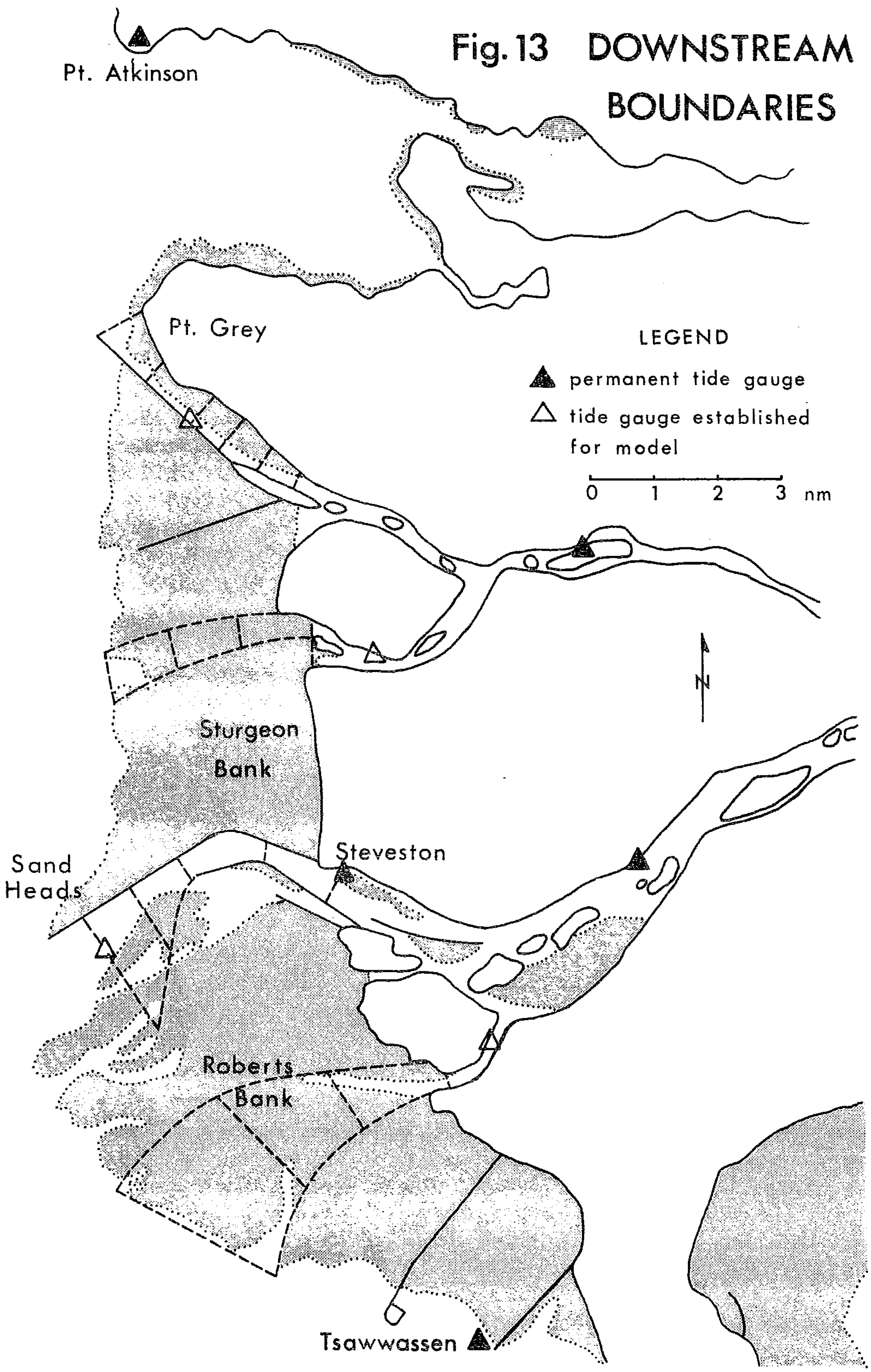


Fig. 13 DOWNSTREAM BOUNDARIES
Pt. Atkinson
Pt. Grey
LEGEND
permanent tide gauge
tide gauge established for model
0 1 2 3 nm
N
Sturgeon Bank
Sand Heads
Steveston
Roberts Bank
Tsawwassen

preliminary calibration. When the friction coefficients had been established, the model was extended to the Strait of Georgia and the boundary conditions transferred to the permanent tide gauges at Point Atkinson and Tsawwassen. The mean sea levels obtained from the Sandheads gauge during the freshet and non-freshet were compared with those of Point Atkinson and Tsawwassen for the same periods. The resulting small corrections (between 0.3 and 0.6 feet) were then applied to the records of Point Atkinson and Tsawwassen to obtain respectively the boundary conditions at North Arm and Middle Arm; and Main Arm and Canoe Pass. In essence, these height corrections accounted for the slight rise in water level along the outer edge of the delta, due to the fresh water outflow. The outermost sections of the river arms in the model were subsequently gradually widened to allow the main channel to expand laterally into the Strait of Georgia. This simplification, although less realistic than a two-dimensional scheme of the approaches to the Fraser, proved to be quite satisfactory.

The model's final version is run with the slightly modified observed or predicted tides at Point Atkinson and Tsawwassen downstream; and the measured or anticipated discharges at Hope (adjusted for Chilliwack) and Pitt Lake upstream.

INITIAL CONDITIONS

For calibration of the model, the initial conditions used at the odd-numbered (H) sections were the water surface elevations obtained from tide gauges along the river, interpolated linearly for sections without gauges. The discharge used at the even-numbered (Q) sections was the measured discharge at Hope (adjusted for Chilliwack) assumed to be uniform initially, and distributed among the four arms in proportion to the cross-sectional areas.

When no actual records are available, the initial conditions estimated for an average discharge are used. To avoid errors in the predictions due to inaccurate initial conditions, in other words, to allow the model to "settle down", the program is normally run for one complete tidal cycle prior to its required output.

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FIELD OBSERVATIONS

Between Chilliwack and Steveston, eleven float gauges are operated by Water Survey of Canada. Their records were used to calibrate the model. To provide the calibration with accurate height and phase data, the heights and times of the gauge records were checked at intervals of a few days whenever feasible. Water Survey also modified some of the chart driving mechanisms when there appeared to be a need for higher resolution in the recorded tide curves. In addition to the river gauges, four pressure gauges were installed by the Tides and Currents Section in the four distributaries (Figure 4). They were levelled in to the nearest geodetic bench marks.

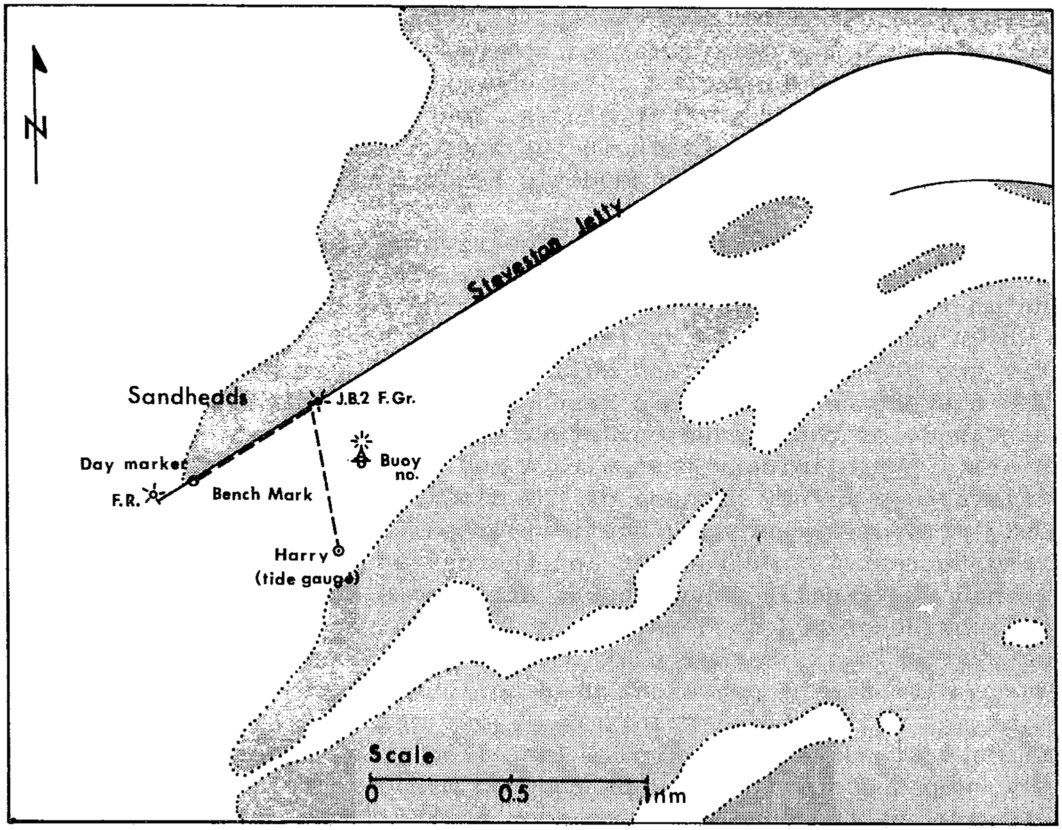
As Figure 13 illustrates, the location of the gauge at Sandheads made conventional levelling over land impossible. It had been decided not to build the gauge on the Steveston Jetty because of the very strong river velocities nearby, which would result in variable pressure heads in the tidal records. Therefore, the pressure gauge was built on a pile just outside the main river flow. The nearest bench mark had been established earlier by the Geodetic Survey of Canada on the Steveston Jetty, 3000 feet away across the mouth of the Main Arm. A preliminary test with a red laser to level across the water was unsatisfactory mainly because of the difficulty in designing an instrument which could project a perfectly horizontal beam over 3000 feet, even without considering terrestrial refraction and the curvature of the earth. An alternative method was finally found which, after some further tests and refinements, will be described in detail in a separate paper. Briefly, the procedure was as follows: Rather than a laser beam directed at a levelling rod from a large distance, a pen-light held against the rod at night provided a very bright and well defined point, no larger than the smallest division (0.01 ft) on a standard survey rod. Installed in a target, this pen-light was slowly moved up and down the rod by the rod man, and followed through the level telescope by the observer. As soon as the light point crossed the horizontal crosshair of the telescope, the observer instructed the rod man by radio to read the rod. A series of observations was made, with the target moving in opposite directions an equal number of times, to cancel out errors due to human response. Similar sightings were subsequently taken on a rod on the other survey mark and the means of both sets of sightings computed to obtain the difference in elevations between the reference points.

The effect of the earth's curvature and refraction is quite significant for distances over 1000 feet. The amount varies as the square of the distance and is roughly 0.18 feet for 3000 feet. It is therefore important that the foresight and backsight are exactly equal, which cancels this error as well as the instrument's collimation error.

Before attempting to level across the Fraser in this fashion, tests were carried out with an automatic level along a one mile stretch of beach near Victoria. An error of 0.02 feet was found over this distance; in other words, the method was of second order precision, a result which was confirmed by field tests at a later date.

The 3000 foot long sights to the bench mark and tide gauge were taken during a cool September night, shortly after midnight in excellent visibility. The observer's position on the jetty was located by marking off equal backsights and foresights on the chart.

Fig.14 NIGHT LEVELLING AT SANDHEADS

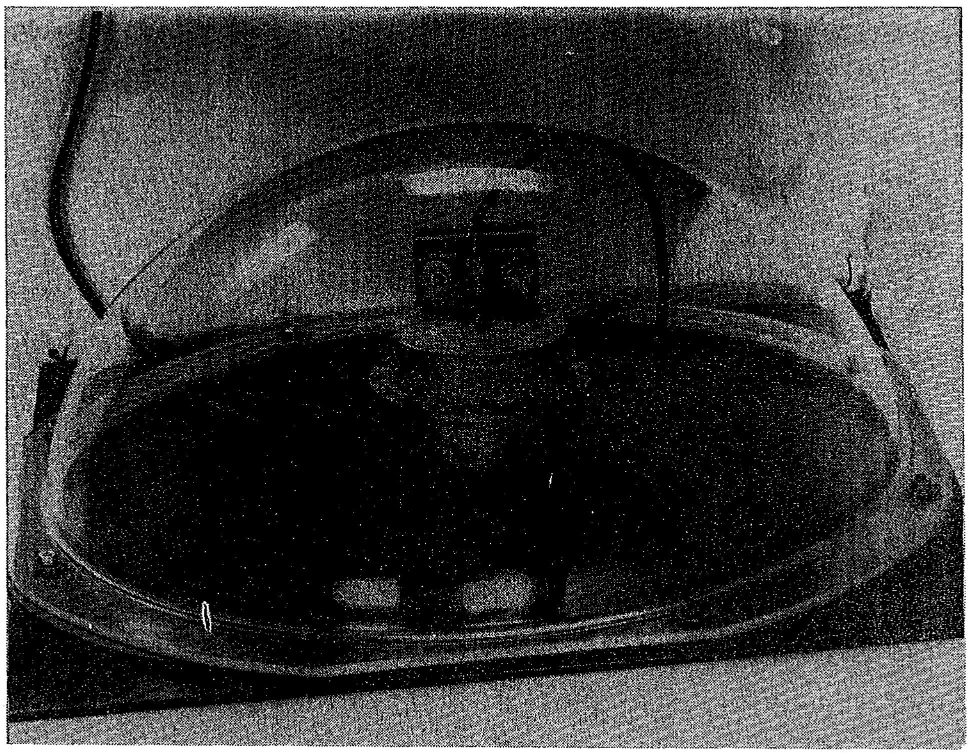


To ensure that both sightings would be taken over water and would presumably be equally affected by refraction, they were taken at high tide when almost the entire jetty between the observer and the bench mark was flooded.

The two sets of sightings were completed within an hour; the instrument used was a Zeiss N12. As Figure 14 shows, only one set-up was possible for this particular problem; therefore, the results could be verified only by repeated observations. If we assume second order levelling, the error in the observed Sandheads tides used for the model would not have exceeded 0.02 feet.

The gauges at Middle Arm and Canoe Passage were established in more convenient sites. However, the gauge in North Arm also required some improvisation. The recorder was put on a pile on the North Arm Jetty, but the pressure unit had to be placed on the river bed. To prevent the diaphragm from becoming clogged up by sand, it was built in an eight inch high plastic dome, weighted by 1/4 inch steel plate, and raised about one inch above the plate. Small 1/16 inch holes in the top of the dome exposed the diaphragm to the ambient water pressure. Although this design kept the pressure unit free from sand for several months and also provided it with a stable base (and consequently the tidal records with a constant reference level), it had an important disadvantage: it measured the total pressure, whereas the static pressure was required for the boundary condition.

Fig. 15 PRESSURE UNIT OF NORTH ARM TIDE GAUGE



The dynamic pressure (about 3/2 ρU^2, where U is the water velocity above the dome) at the top of the dome would cause significant errors in the tide gauge records during the freshet. Therefore, it was necessary to place the pressure unit of the gauge near the shore in slower moving water. However, in this location, anchored log booms would press the unit deeper into the sand at very low tides, thus changing the reference level of the records. Despite frequent surveillance with a launch stationed in Richmond, several days of records were lost due to log booms.

The pressure gauges were set for salt water at a specific gravity of 1.025. Such a gauge, operating in ten feet of fresh water, would record 9.75, i.e. 0.25 feet too low, a significant discrepancy for the boundary conditions. To examine the density distribution in the water column above the gauge, several salinity measurements were taken with a Beckman portable RS 5-3 salinometer within a few feet of the gauge positions. These measurements indicated that during the freshet the salt water intrusion at all four sites was negligible. Since the initial calibration was carried out during the freshet, we assumed that the records of all four temporary delta gauges contained negative errors varying from zero feet at low tide to -0.2 feet at high tide. However, frequent comparisons in the field with tide staffs, particularly at the important Sandheads and North Arm gauges, made it possible to adjust the field data to compensate for this error.

In addition to spot measurements of salinities and temperatures near the sites of the tide gauges, a number of cruises were made in the delta to determine the limit of salt water intrusion, as it varies with tides and river discharge. These observations, which have been published as a data record (2), were reconnaissances.

The field program primarily considered the vertical tidal movement. A detailed study of currents and the behaviour of the salinity wedge was deferred to a later date, when we may have made sufficient progress in numerical techniques to develop a useful stratified model.

CALIBRATION

After the schematization of the river, and the development of the computer program, the model was calibrated.

The tidal curves produced by the model at the sites of the tide gauges along the river were compared with the curves recorded by the gauges. The friction coefficient C in the equation of motion was subsequently adjusted throughout the river until the model output finally agreed satisfactorily with the prototype data, a trial and error procedure analogous to the calibration of a physical model with friction elements.

The model was run with, as boundary conditions, the measured discharges at Hope (adjusted for Chilliwack), and the actual tides recorded by the temporary tide gauges at the entrances of the four distributaries. Three consecutive days, 16-18 July 1969, were selected for the first calibration. During this period, there was a spring tide in the Strait of Georgia, which provided a large tidal range (11 feet or 3.35 m); the discharge (150,000 cfs or 4200 m^3/sec at Hope) was high enough to virtually eliminate salt water intrusion and its effect upon the consistency of the recorded tidal heights; the winds at Sandheads were light easterly, averaging six mph, so that the effect of wind upon the tides could be neglected; and finally, all tide gauges performed well during this period.

Both model-produced and observed heights were referred to geodetic datum for all locations.

To adjust the friction coefficients, the program was run about a dozen times until the model-produced and observed tide curves agreed within acceptable limits (in most cases 0.5 feet or 15 cm). The friction coefficients were assigned to blocks of segments rather than to individual segments, which would have been more representative of the prototype flow but would have involved a very large number of tests at an unwarranted cost.

After the schematization had been extended to the Strait of Georgia, and the boundary conditions transferred to the two permanent gauges at Point Atkinson and Tsawwassen (Figure 13), the model was verified by comparing the output with the gauge records for other dates in 1969 (freshet and non-freshet). The discrepancies were in the order of 0.5 to 1.0 feet in height and one-half to one hour in time.

The calibration was carried out for a relatively low freshet in 1969. The model's validity had yet to be established for an unusually high discharge, not only because of changes in the schematization of the cross-sectional areas due to flooding, but also because of a hydrodynamic consideration:

In the equation of motion:

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} = -\,g\,\frac{\partial h}{\partial x} - g\,\frac{|u|u}{C^2 d}\ ,$$

we can manipulate only the last term (the friction term) to align the tide curves produced by the model with the curves recorded by the gauges. We thus "tune" the model by adjusting the friction coefficient C.

The significance of the friction term increases as the square of the water velocity, i.e. with the discharge. Consequently, a model which has been calibrated at a discharge of 150,000 cfs, may not respond realistically to a discharge of 400,000 cfs, but would be valid for discharges below 150,000 cfs. The year 1969, with a peak discharge at Hope of less than 300,000 cfs, was obviously not a good year to calibrate the Fraser model.

It was not until 1972, with a peak discharge at Hope of 450,000 cfs (12,700 m^3/sec) that this concept could be further examined.

The program was run and recalibrated for observed tides at Point Atkinson and Tsawwassen with a very high range of 15 feet, and for observed discharges at Hope increasing from 385,000 to 400,000 cfs.

As Table I illustrates, the original friction coefficients established for a discharge at Hope of 150,000 cfs induced large height discrepancies at extreme discharges. The program run with these same friction coefficients for a low discharge of 51,000 cfs produced minor discrepancies of the same order as for the calibration discharge of 150,000 cfs. However, the friction coefficients determined at a discharge of 398,000 cfs also apply well to the discharges of 150,000 cfs and 51,000 cfs. The table confirms the suggestion that the model's height predictions are reliable only for discharges at or below that for which it was calibrated (in our case, 398,000 cfs).

A similar comparison of time differences between model-produced and observed high and low waters (Table II) is much less conclusive because the exact times of high and low waters during a freshet are difficult to identify.

As mentioned earlier in this section, another potential weakness in the application of a river model at high discharges is the change in the schematization of cross-sections due to flooding. However, most of the abrupt changes in cross-sectional areas in the Fraser occur at low discharges, and can be schematized from detailed charts. The crest of the dykes along the Fraser is set at two feet above the highest known water level (i.e. the 1894 flood). We may therefore assume that the river flow will be contained by the dykes, and that the schematized cross-sections are not altered by flooding.

Figure 16 shows the distribution of the friction coefficients resulting from the final calibration at 398,000 cfs (11,300 m^3/sec).

The friction coefficients determined at 150,000 cfs are also shown to illustrate the importance of the discharge when calibrating the model.

MODEL CALIBRATION AT HIGH AND EXTREME DISCHARGES

TABLE I: HEIGHT ERRORS IN FEET (PREDICTED - OBSERVED)

Q_{HOPE} used for calibration (cfs)	TIDE	MISSION	PT. COQ.	N. WEST.	FRAS. ST.	DEAS	STEVESTON
		DATES RUN: Nov. 12-14/71 (Q_{HOPE} = 51,000 cfs)					
150,000	HI	- 0.9	- 0.3	- 0.6	- 0.1	- 0.5	- 0.3
	LO	- 0.7	- 0.3	+ 0.3	+ 0.2	- 0.5	+ 0.2
398,000	HI	- 0.9	- 0.3	- 0.5	- 0.1	- 0.3	- 0.1
	LO	- 0.9	- 0.3	+ 0.3	+ 0.2	+ 0.4	+ 0.4
		DATES RUN: June 11-13/72 (Q_{HOPE} = 398,000 cfs)					
150,000	HI	+ 1.8	+ 0.6	+ 0.3	- 0.2	- 0.2	+ 0.2
	LO	+ 1.9	+ 0.8	+ 1.8	+ 1.6	+ 1.6	+ 0.8
398,000	HI	+ 0.4	- 0.5	- 0.6	- 0.1	- 0.3	+ 0.1
	LO	+ 0.4	- 0.5	+ 0.6	+ 0.6	+ 0.2	+ 0.2
		DATES RUN: July 16-18/69 (Q_{HOPE} = 150,000 cfs)					
150,000	HI	0	- 0.1	- 0.3	- 0.2	- 0.4	- 0.2
	LO	- 0.3	- 0.4	- 0.2	+ 0.2	- 0.2	- 0.3

TABLE II: TIME ERRORS IN MINUTES (PREDICTED - OBSERVED)

Q_{HOPE} used for calibration (cfs)	TIDE	MISSION	PT. COQ.	N. WEST.	FRAS. ST.	DEAS	STEVESTON
		DATES RUN: Nov. 12-14/71 (Q_{HOPE} = 51,000 cfs)					
150,000	HI	- 30	0	+ 15	- 23	- 23	+ 8
	LO	- 23	- 8	- 8	0	+ 15	- 1
398,000	HI	- 30	+ 8	- 8	- 30	- 30	+ 8
	LO	- 23	- 15	+ 8	- 8	+ 15	- 1
		DATES RUN: June 11-13/72 (Q_{HOPE} = 398,000 cfs)					
150,000	HI	- 75	- 15	0	- 15	0	- 104
	LO	- 8	- 23	+ 30	+ 45	- 15	- 122
398,000	HI	- 105	- 38	- 15	- 15	- 8	- 3
	LO	0	- 23	+ 38	+ 30	- 23	- 23
		DATES RUN: July 16-18/69 (Q_{HOPE} = 150,000 cfs)					
150,000	HI	+ 68	- 8	+ 15	- 30	0	+ 11
	LO	- 45	- 15	- 23	- 15	- 30	- 19

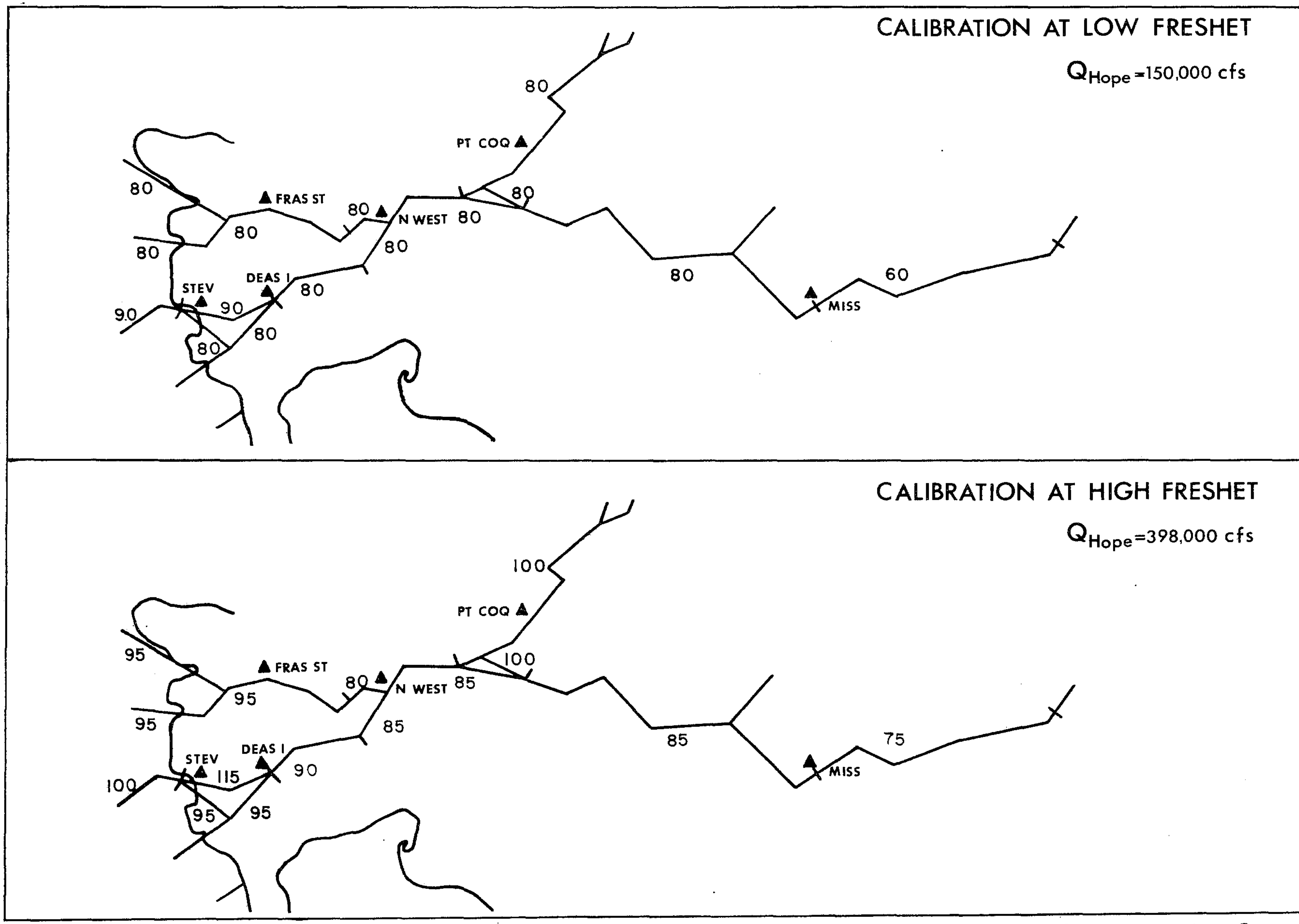


Fig. 16 DISTRIBUTION OF FRICTION COEFFICIENTS

RESULTS

The principal objective of the Fraser model was to compute water surface elevations for the tidal portion of the river, as a function of the tides in the Strait of Georgia, and the river discharges. An obvious application is the prediction of heights and times of high and low waters in the navigable part of the river, by relating them to the corresponding high and low waters at Point Atkinson, and the discharges at Hope. The times and heights of maximum and minimum water levels at Point Atkinson are predictable and are tabulated in the Canadian Tide and Current Tables. Short-term discharge estimates can be made, based on measurements of the previous days and the weather forecast for the Hope area; the program can be adjusted easily in the case of unexpected changes.

The model was run for several tidal cycles with ranges between lower low and higher high waters varying from 8 to 16 feet observed at Point Atkinson and Tsawwassen during the following periods: June 25-July 2, 1969; August 28-September 4, 1969 and January 11-18, 1969; and for seven discharges at Hope between 25,000 and 300,000 cfs. Of course, very few of these discharges actually occurred during any of the eight-day periods. Eight locations along the Fraser were selected for tidal predictions: Steveston, Deas Island Tunnel, Middle Arm, Fraser Street Bridge, New Westminster, Port Mann, Port Coquitlam and Mission. Heights and time differences of a total of 42 predicted extrema per station per discharge were plotted against the heights of the corresponding extrema at Point Atkinson. Figures 17 and 18 are the height and time lag plots for New Westminster. Only the higher high and the lower low waters were considered, and the first day of each run was ignored. The least-squares best-fit curves of 2nd order ($y = ax^2 + bx + c$) were subsequently plotted for each case, Figures 19-34.

Since the daily higher high and lower low waters at Point Atkinson do not occur near geodetic datum (approximately mean sea level), the central, dashed portions of the curves are estimates. The curves are best-fit curves and do not necessarily represent the true hydrodynamic relationship between the river extrema and those at Point Atkinson.

If we could develop an expression for the water surface elevation H at an upstream point x, as a series of simple-harmonic functions of time, we could set $\partial H/\partial t = 0$ for maximum and minimum elevations, solve for t and $H_{ext(rema)}$, and obtain an exact relationship between the extrema at any point x along the river and at Point Atkinson ($x = 0$). However, it is impossible to solve the equations of motion and continuity analytically, and therefore we have to content ourselves with a best-fit curve (an approximation) through data points obtained by a numerical method (another approximation).

The height prediction curves clearly reflect the interaction between tides and discharges: the spread of the curves for New Westminster (Figure 19) compared with that for Steveston (Figure 21) demonstrates the increasing contribution of the discharge to the rise and fall of the water surface elevation as we move upstream. The slope of each individual curve (∂H_{ext} Atkinson/∂H_{ext} Fraser) decreases as the height at Point Atkinson increases, indicating the decreasing influence of the discharge upon the local river heights as the tides in the Strait of Georgia become higher. The height prediction curve for Mission at 300,000 cfs (see Figure 33) is a straight vertical line showing there is no noticeable tidal influence at Mission for that discharge, which would occur at the peak of an average freshet.

To verify these model-produced heights at New Westminster, similar computer plots were generated for four years of observed data (1970-1973). Figures 35-41 show the actual heights of the extrema at New Westminster corresponding to the higher high and lower low waters at Point Atkinson for various discharges. Figure 42 displays the least-squares best-fit curves of 2nd order for these plots, and supports the model results of Figure 19.

Unlike the height comparisons between Point Atkinson and the Fraser gauges, the best-fit curves for the predicted time differences were plotted without distinguishing among the discharges. As was mentioned in the section on calibration, the choice of the culmination points has much more influence upon the time differences than upon the heights.

In other words, if we slightly misjudge the exact location of a culmination point on the tide curve (in the prototype by visual inspection, in the model predictions by a programming technique), the time would be much more in error than the height. This would particularly be the case at an upstream station during the freshet, where at high tide the change in water surface elevation over several hours might be imperceptible.

Figure 18 illustrates the difficulty of determining a separate time lag curve for each discharge; the time differences of the predicted higher high and lower low waters at New Westminster are plotted against the observed higher highs and lower lows at Point Atkinson. Although there is a definite envelope of maxima and minima, the clustering of the data points, particularly at high waters, makes it impossible to establish a family of discharge curves. Therefore, a single least-squares best-fit curve of second order was calculated over all discharges for each upstream location (e.g. Figure 20 for New Westminster).

Time lags at New Westminster were plotted for four years (1970-1973) of observed data to check the model predictions. Figures 43, 44 and 45 represent the actual time lags between each higher high or lower low water at Point Atkinson and the corresponding high and low at New Westminster for several discharge ranges, and Figure 46 shows the overall best-fit curve. These actual data plots agree closely with the results of the model. For sea water levels at Point Atkinson below mean sea level (i.e. low waters), the time lag curves have a distinct negative slope, which reverses above mean sea level, but only slightly. This reversal is particularly evident at low discharges (Figure 43). The time lag curves for other locations along the river show a similar trend.

Although a detailed interpretation of the complex water motion in the delta is outside the scope of this report, some general comments on the shape of the time lag curves may be enlightening:

At any location x along the river, we may express the vertical displacement of one of the tidal components with respect to mean level as:

$$\eta_{xt} = Ae^{-\mu x} \cos (\omega t - \kappa x), \text{ where}$$

A = amplitude at the entrance of the river

ω = frequency of the component (i.e. $\frac{2\pi}{T}$, T = period)

κ = wave number (i.e. $\frac{2\pi}{L}$, L = wave length)

μ = damping modulus, normally evaluated from tidal records (19). This modulus accounts for tidal friction and, consequently, the decrease in amplitude with x.

At any location x,

$$\frac{\partial \eta}{\partial t} = - A\omega e^{-\mu x} \sin (\omega t - \kappa x),$$

from which we deduce that the tide rises and falls faster at Sandheads (x = 0) than at New Westminster (x = 18 n.m.).

In an average water depth of about 30 feet, the tidal wave would travel from Sandheads to New Westminster in slightly less than one hour, if there was no friction. In that case, low water at New Westminster would occur approximately one hour after the corresponding low water at Sandheads. However, due to friction ($e^{-\mu x}$), the tide will fall more slowly at New Westminster than at Sandheads. One hour after low water at Sandheads, the tide at New Westminster will still be falling, and will continue to do so until the steady-state hydraulic grade line has been re-established. At this point, the tide at Sandheads has started to rise. (The hydraulic grade line between New Westminster and Sandheads is identical to the water surface, and has a steady-state drop of about five feet during the freshet, and of about 0.5 feet during non-freshet conditions, see Figures 48 and 49.) The lower the low water is at Sandheads, the longer it will take the tide at New Westminster to fall to the hydraulic grade line. Therefore, the time lags between low waters at Sandheads and New Westminster will increase with the displacement of the low waters at Sandheads from mean sea level.

Conversely, the tide also rises faster at Sandheads than at New Westminster. At a high tide in the Strait of Georgia, the gradient is close to zero and equilibrium between New Westminster and Sandheads is quickly attained. A very high tide at Sandheads reverses the gradient between Sandheads and New Westminster at low discharges; one hour after high tide at Sandheads, the tide at New Westminster will still be rising since equilibrium has not yet been reached. The higher the high tide is at Sandheads, the longer it will take the rising tide at New Westminster (and the falling tide at Sandheads) to reach equilibrium. Figure 43 indeed shows a trend for the time lags to increase with higher maxima at Sandheads. This trend is not so pronounced as for low waters because the (negative) gradient between New Westminster and Sandheads is much smaller at high tides than the (positive) gradient is at low tides (Figures 48 and 49).

The plotted time differences for high waters at New Westminster (model-predictions: Figure 18; observations: Figure 43) show irregularities at low discharges, which are too large to be caused by friction alone, or by the ambiguity of the location of the culmination point. The complex flow régime at the trifurcation may well be responsible for these isolated points.

(Text continues p. 72)

Fig. 17 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER

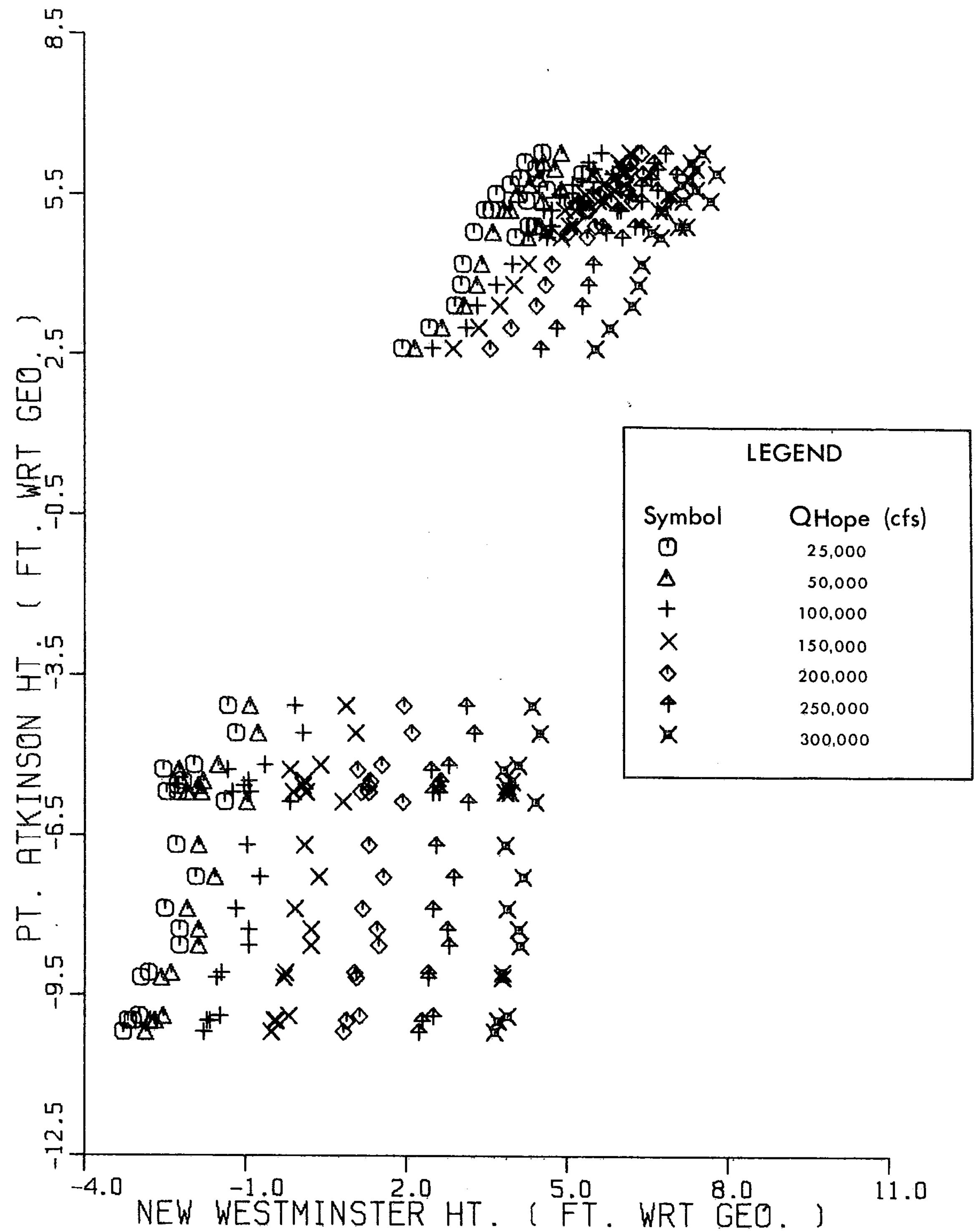


Fig. 18 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER

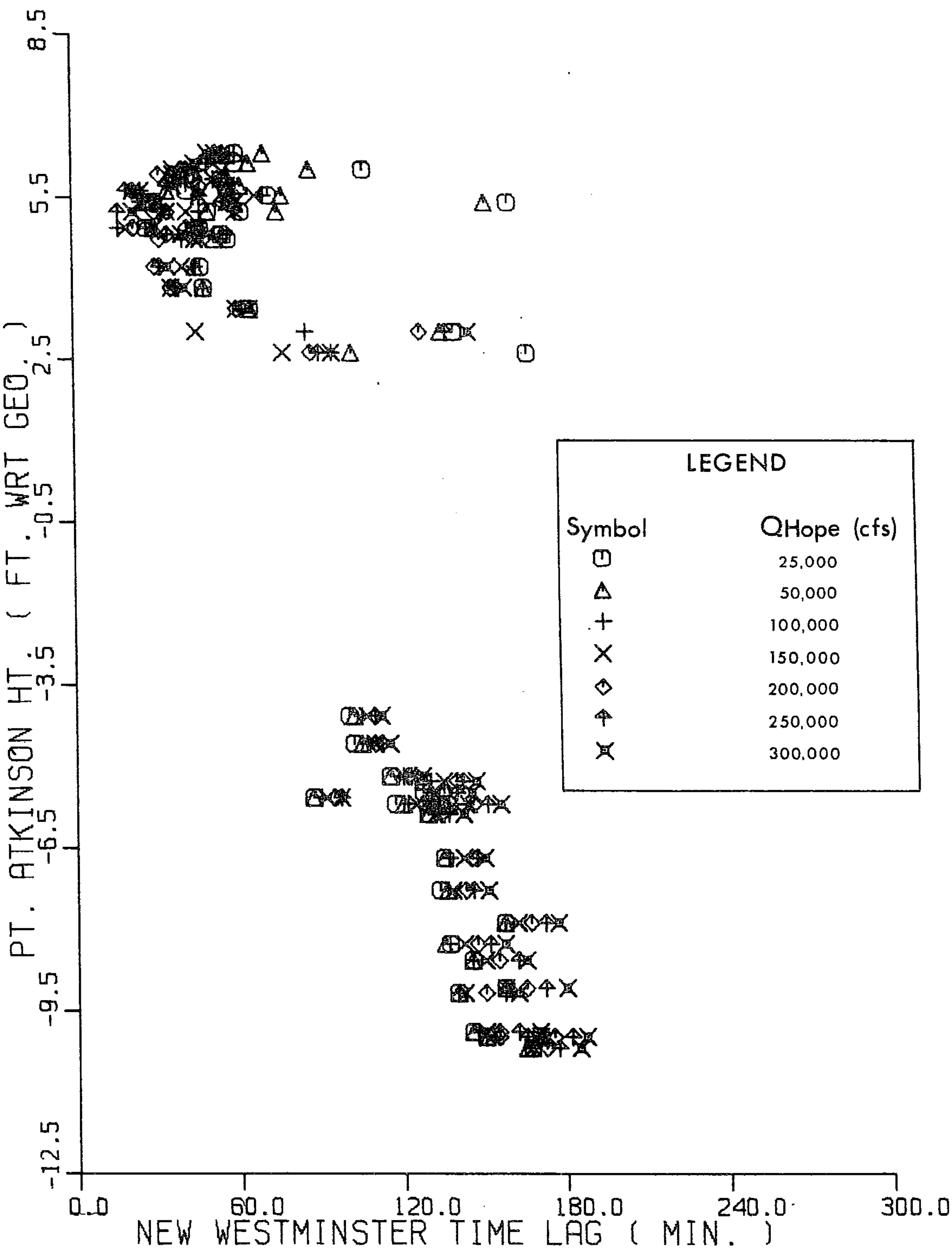


Fig.19 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER

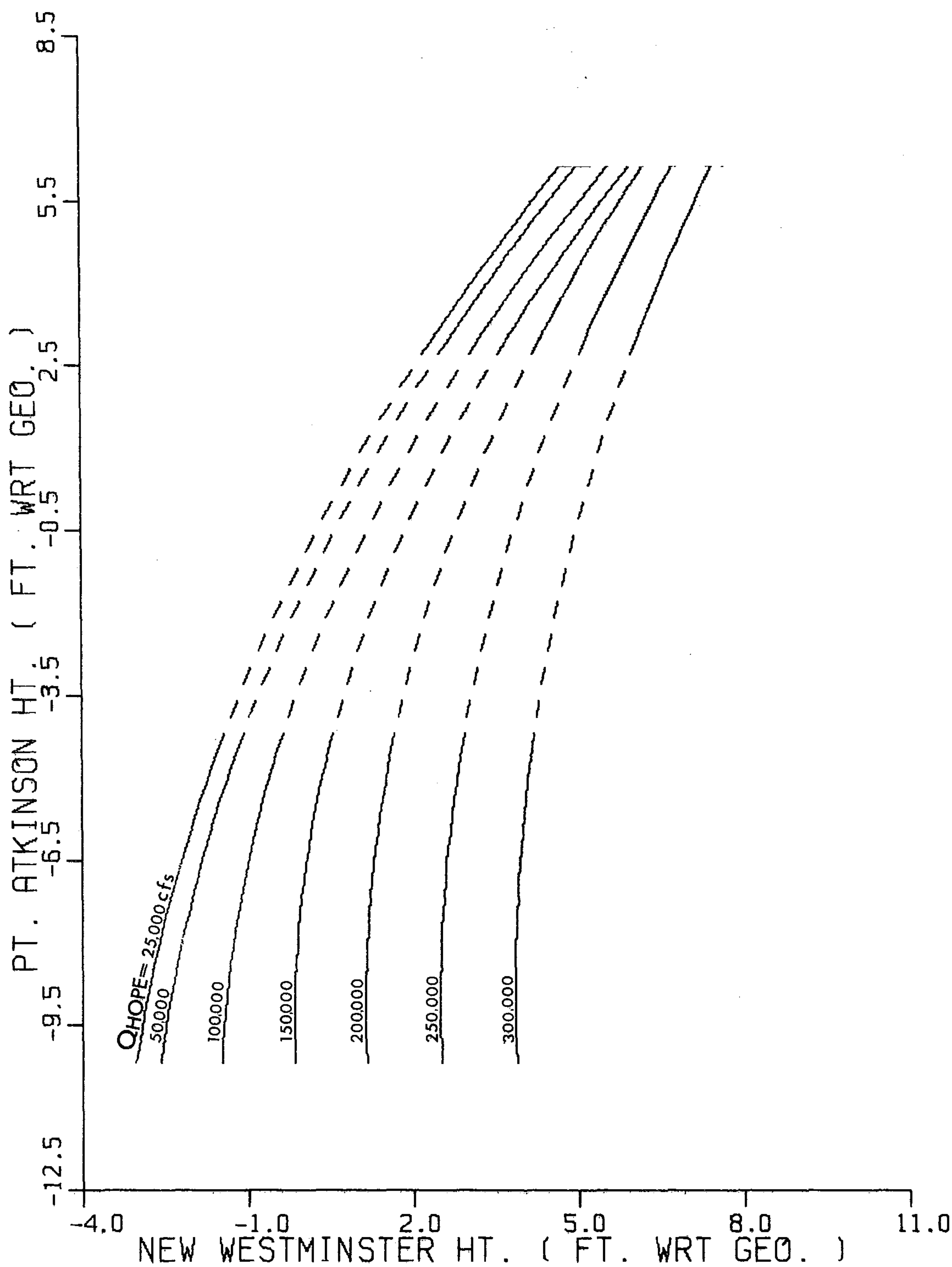


Fig.20 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER

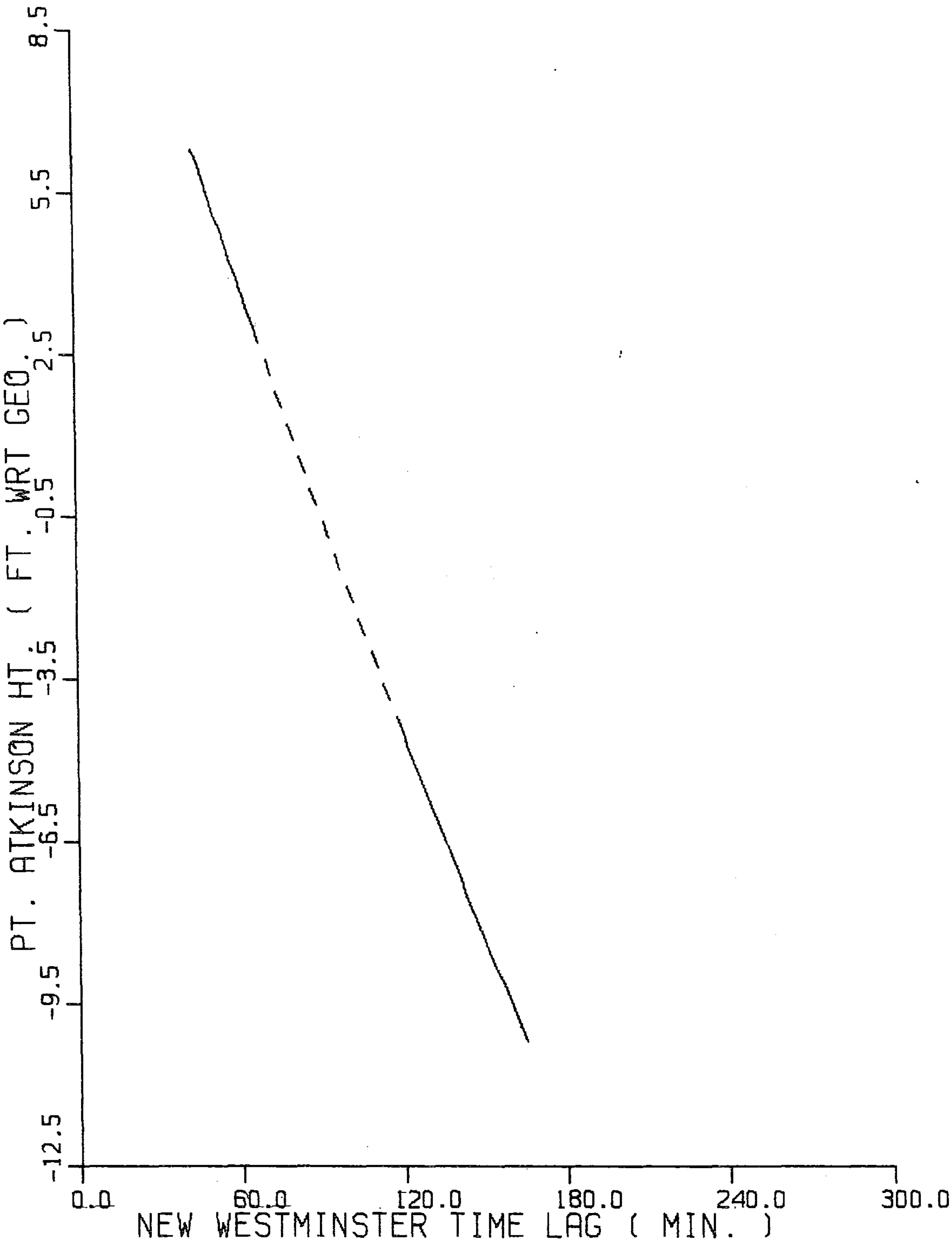


Fig. 21 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND STEVESTON

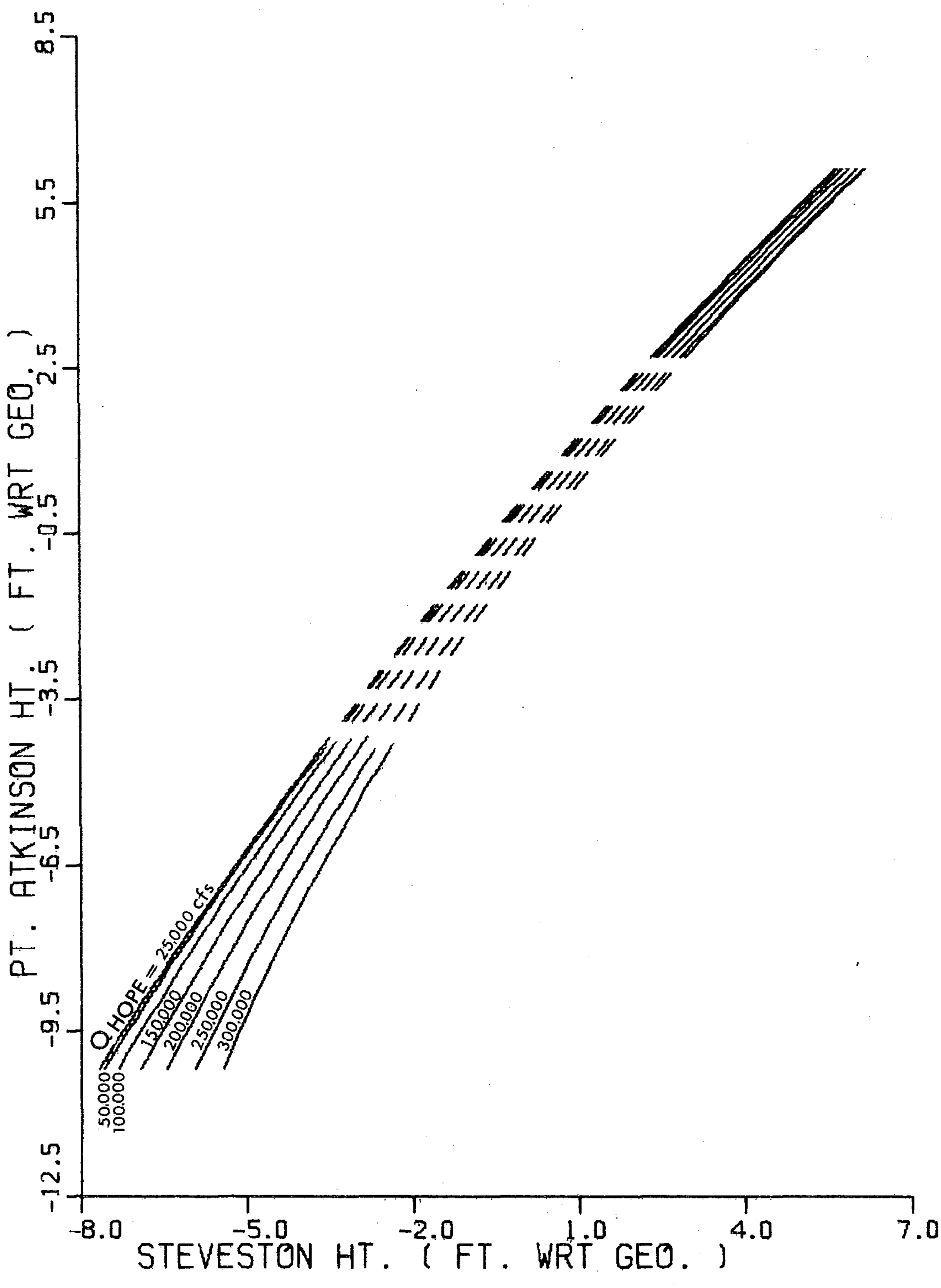


Fig.22 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND STEVESTON

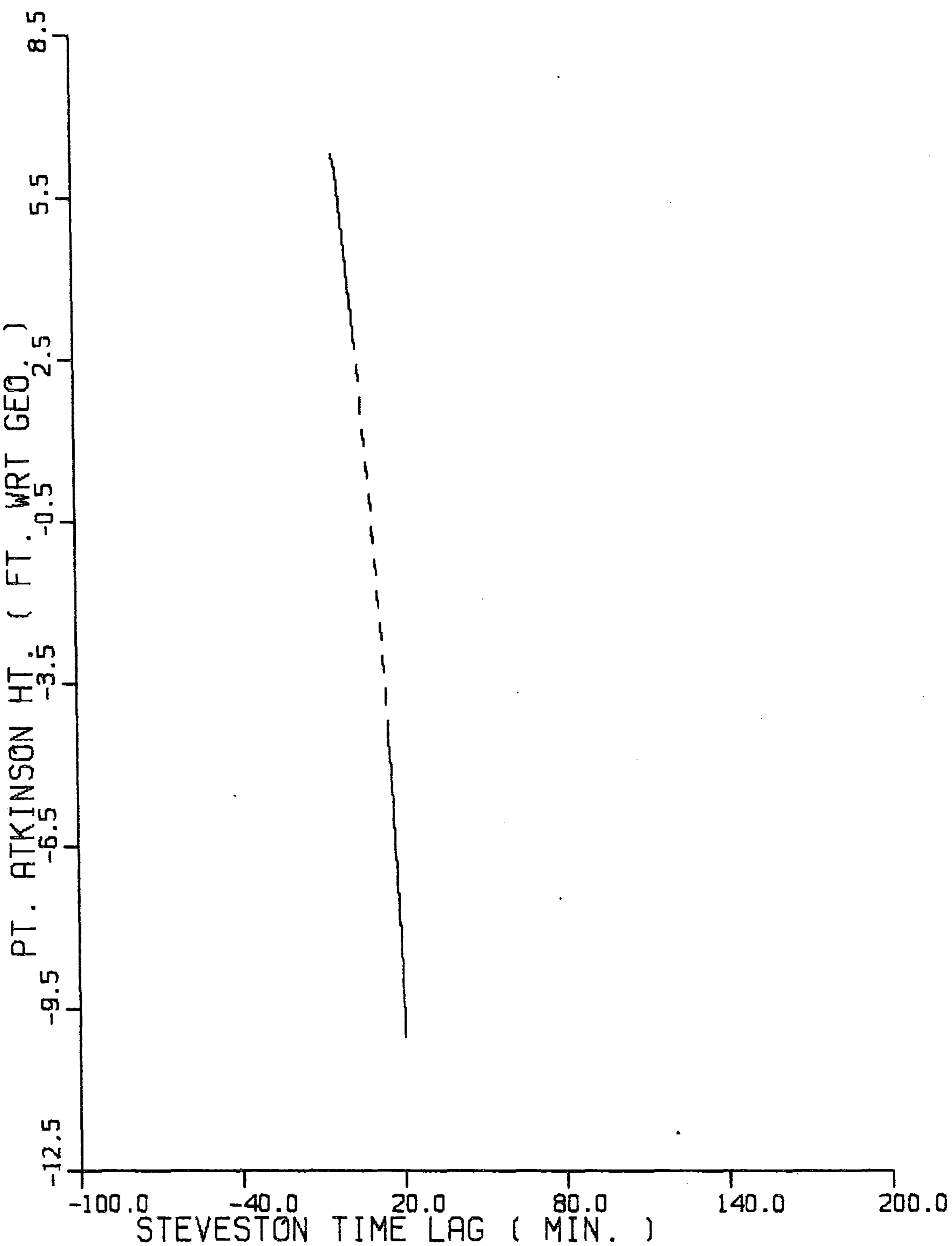


Fig. 23 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND DEAS ISLAND

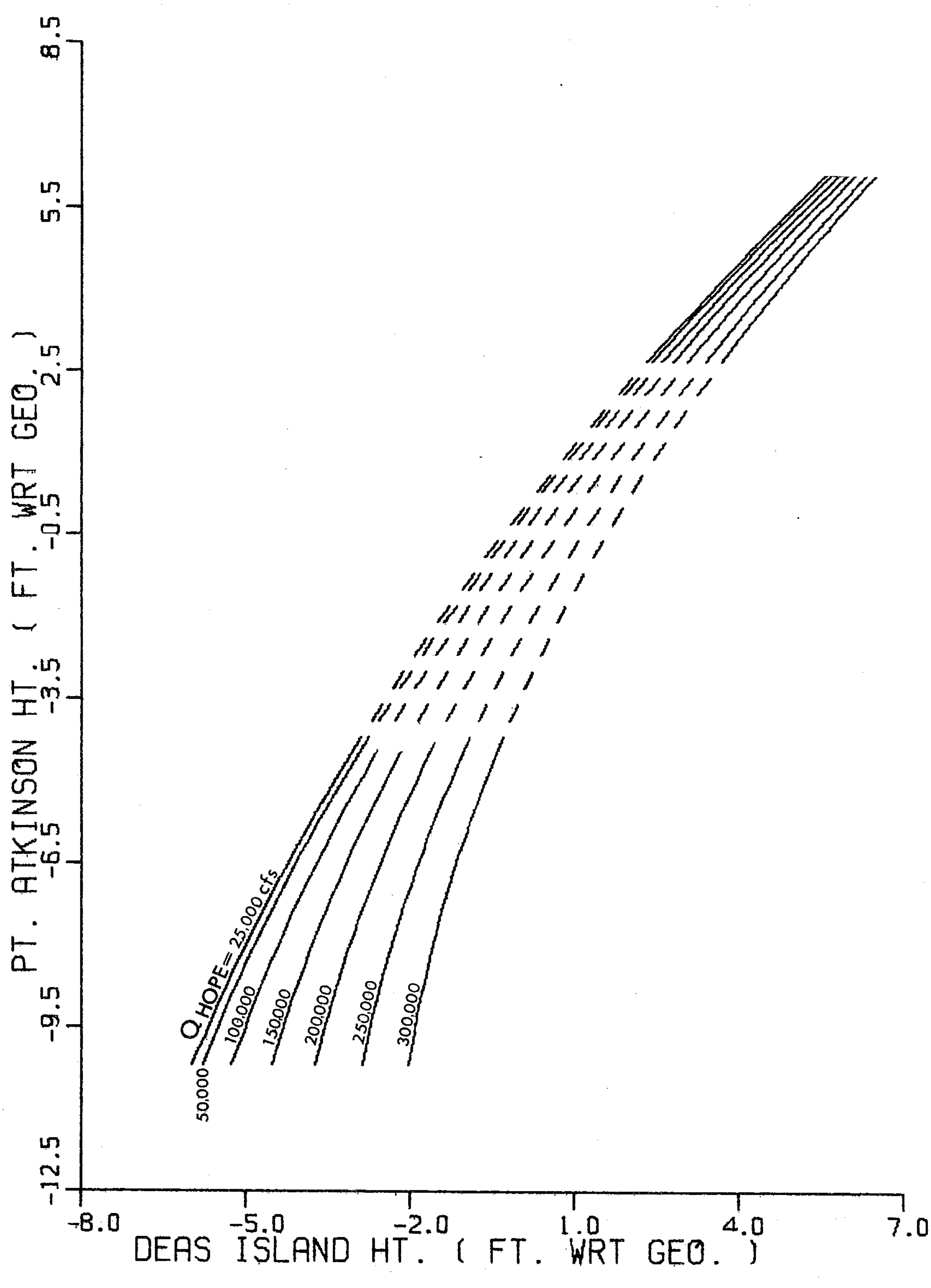


Fig.24 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND DEAS ISLAND

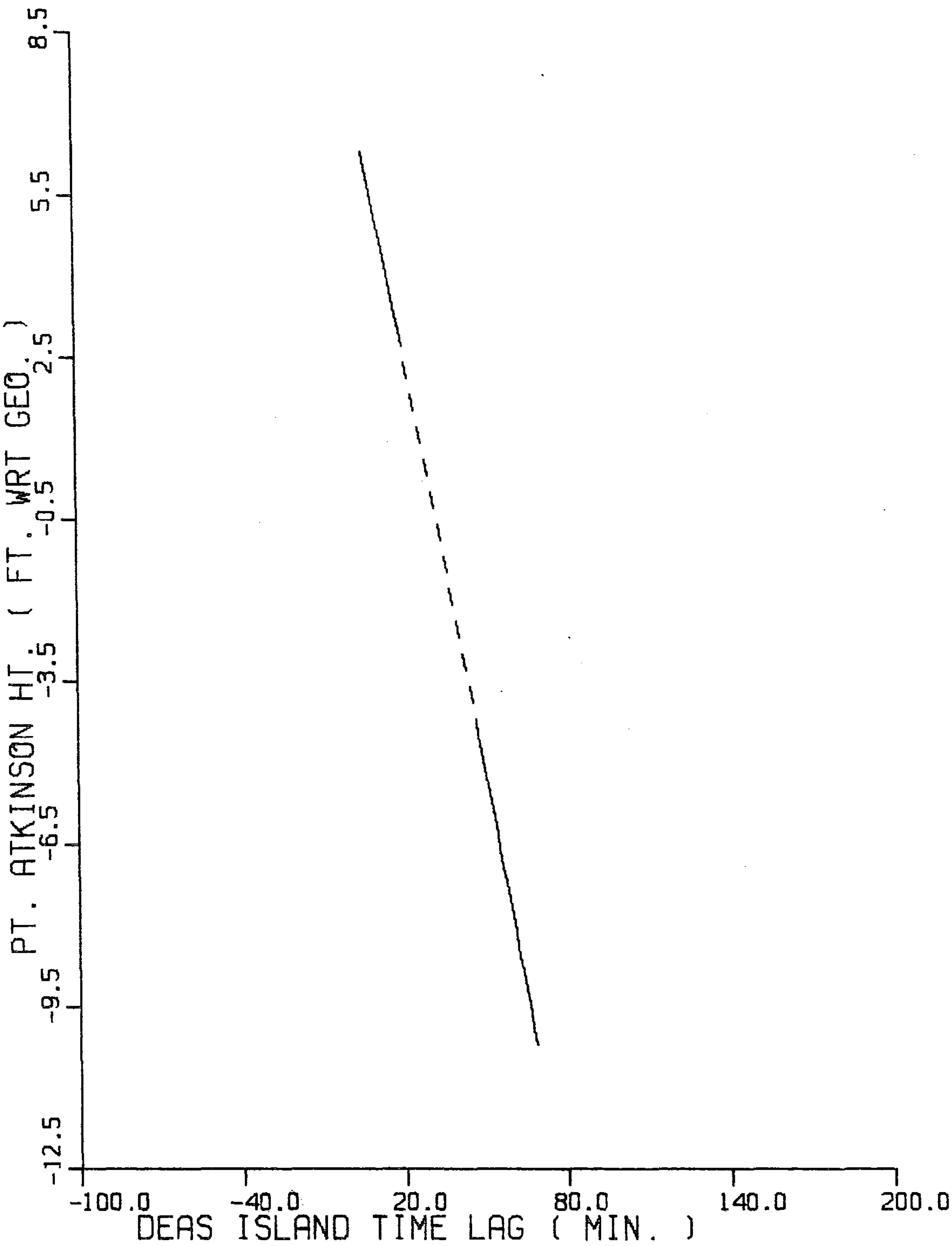


Fig.25 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND MIDDLE ARM

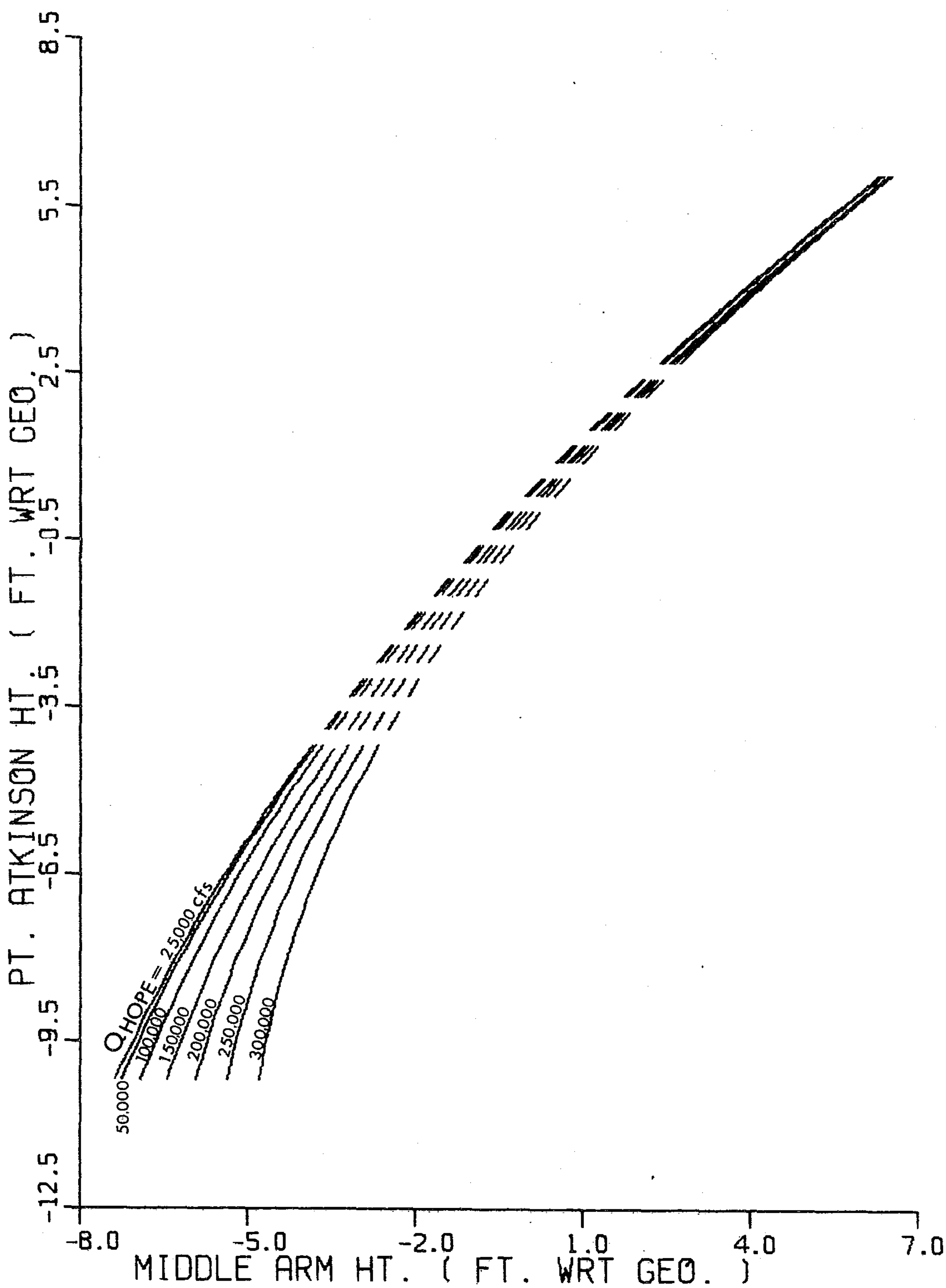


Fig. 26 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND MIDDLE ARM

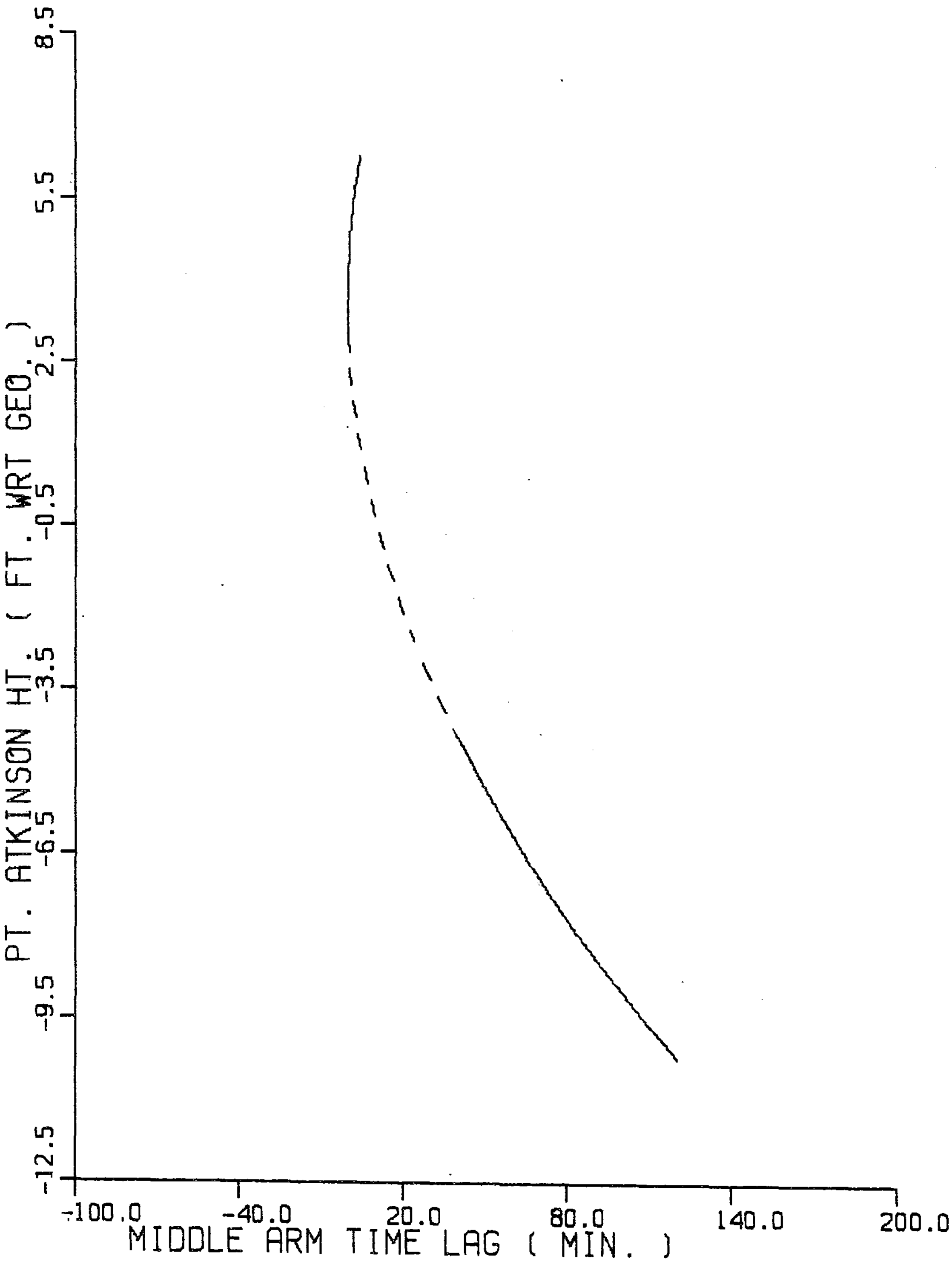


Fig. 27 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND FRASER STREET

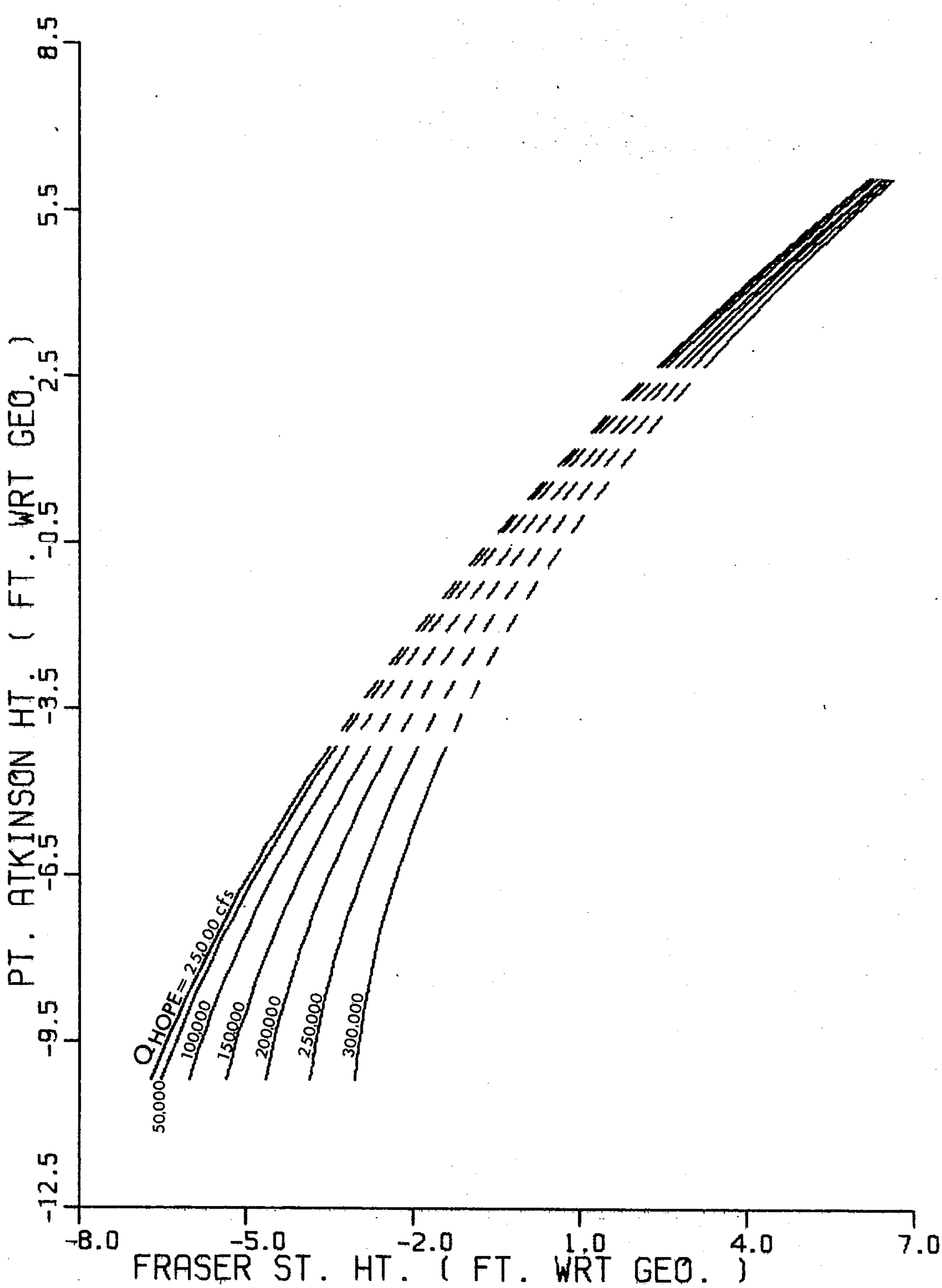


Fig. 28 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND FRASER STREET

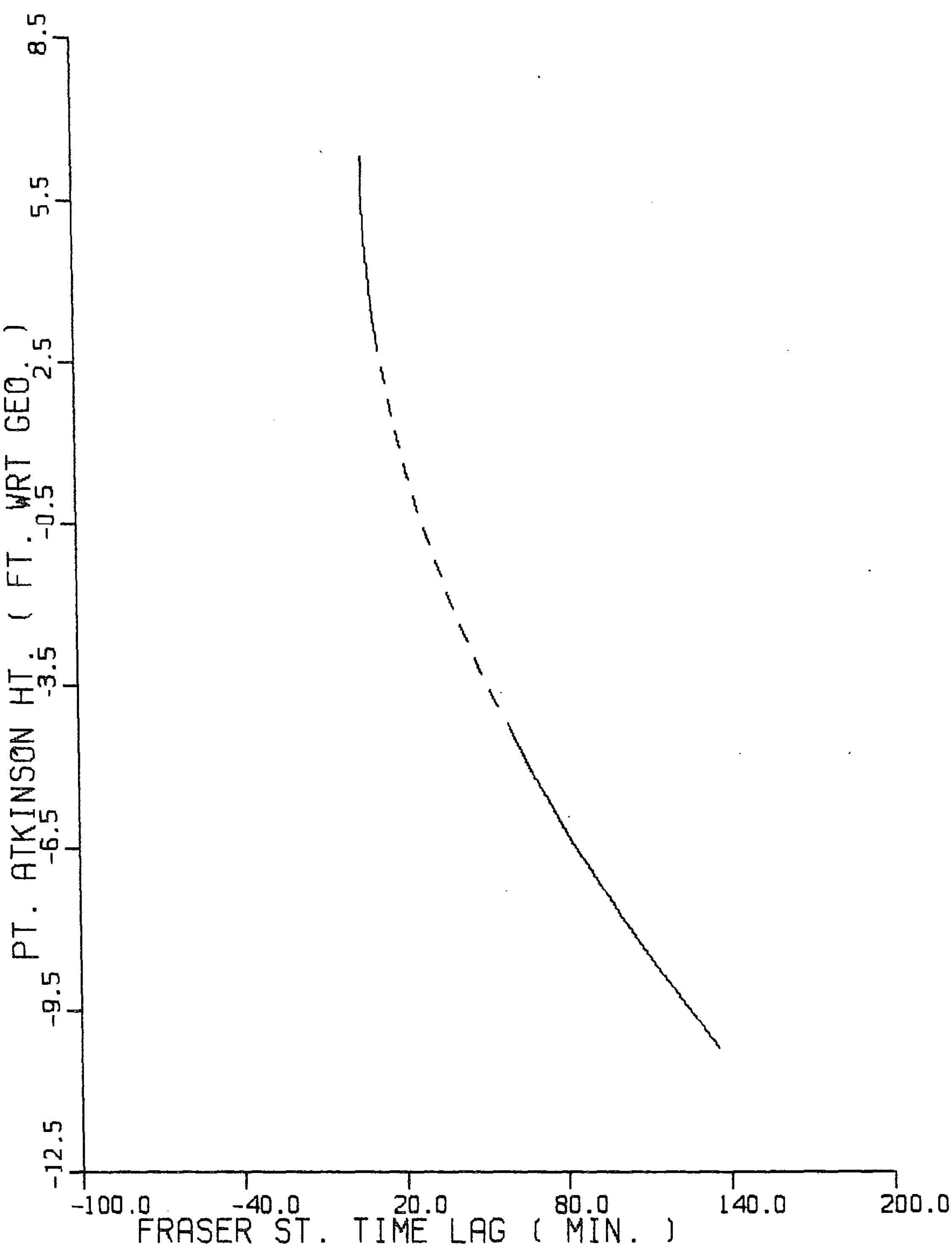


Fig. 29 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND PORT MANN

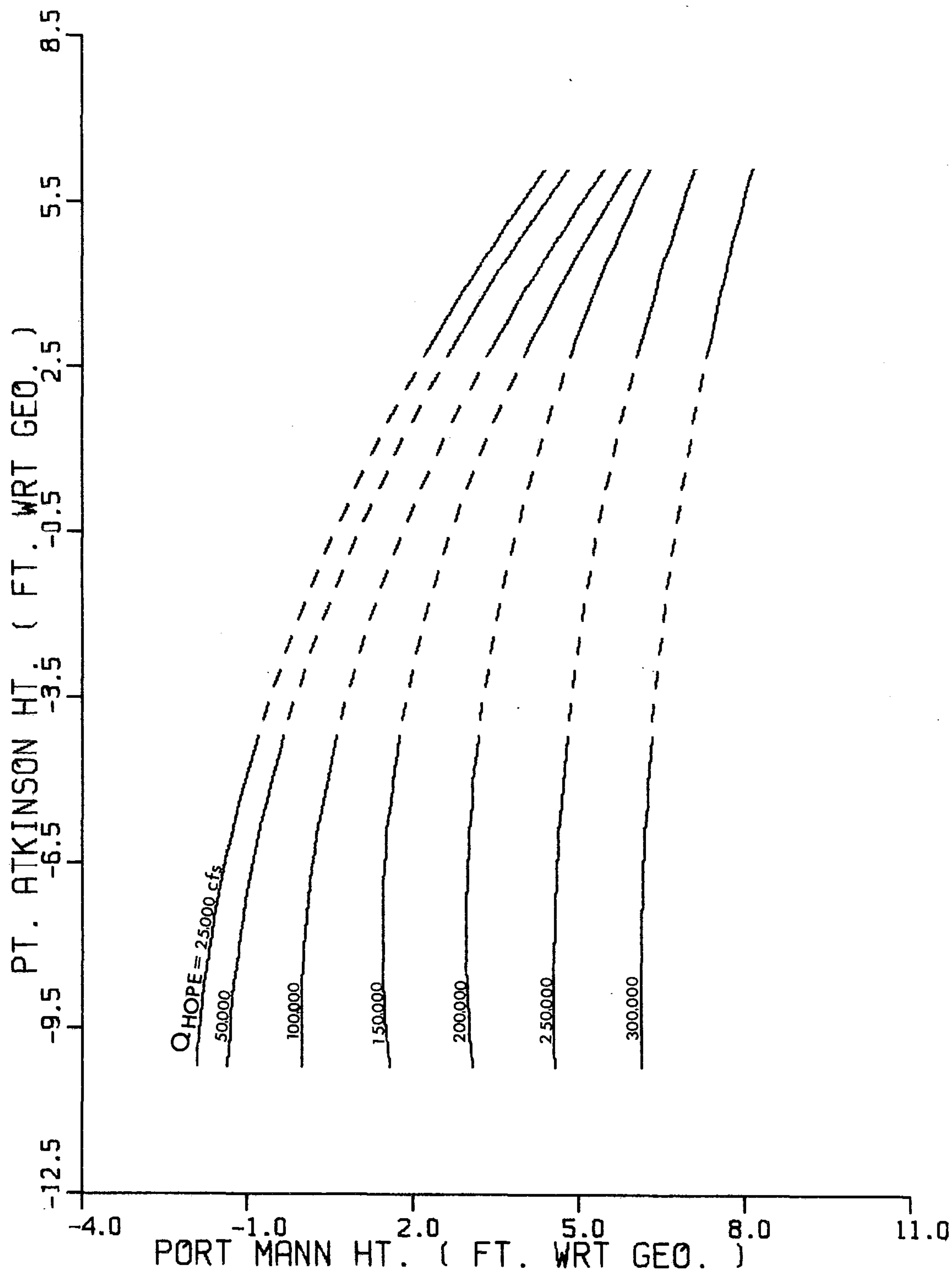


Fig. 30 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND PORT MANN

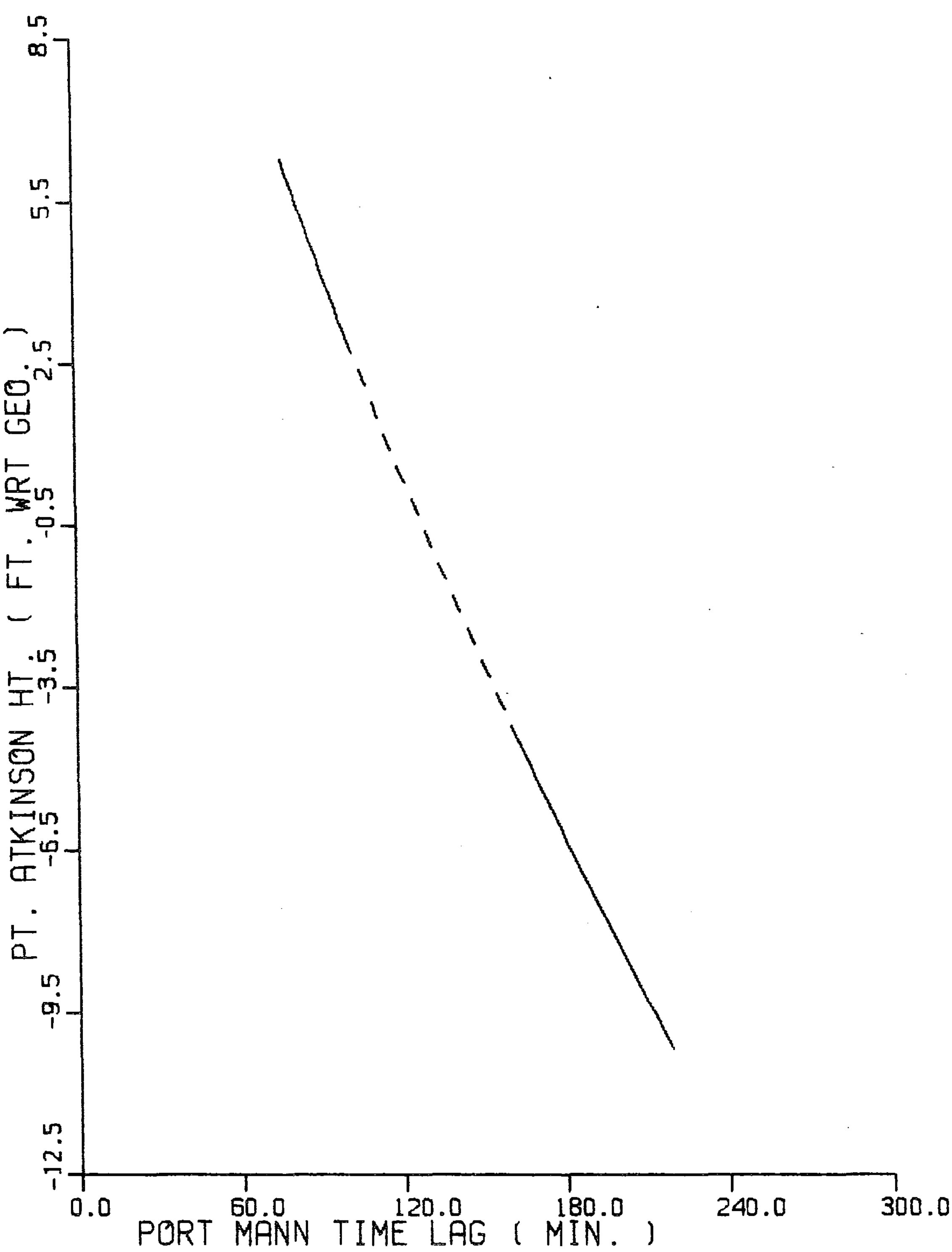


Fig. 31 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND PORT COQUITLAM

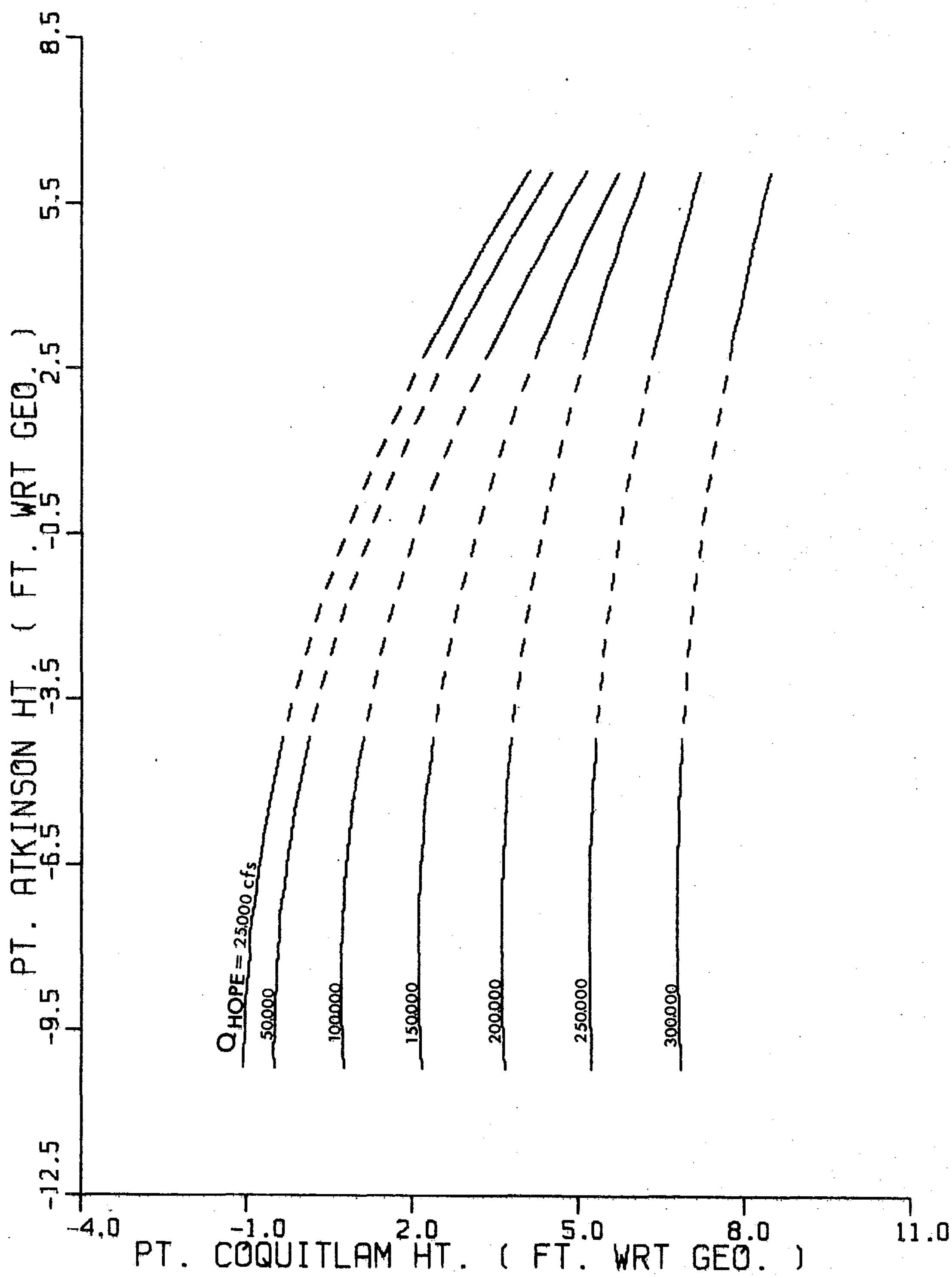


Fig. 32 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND PORT COQUITLAM

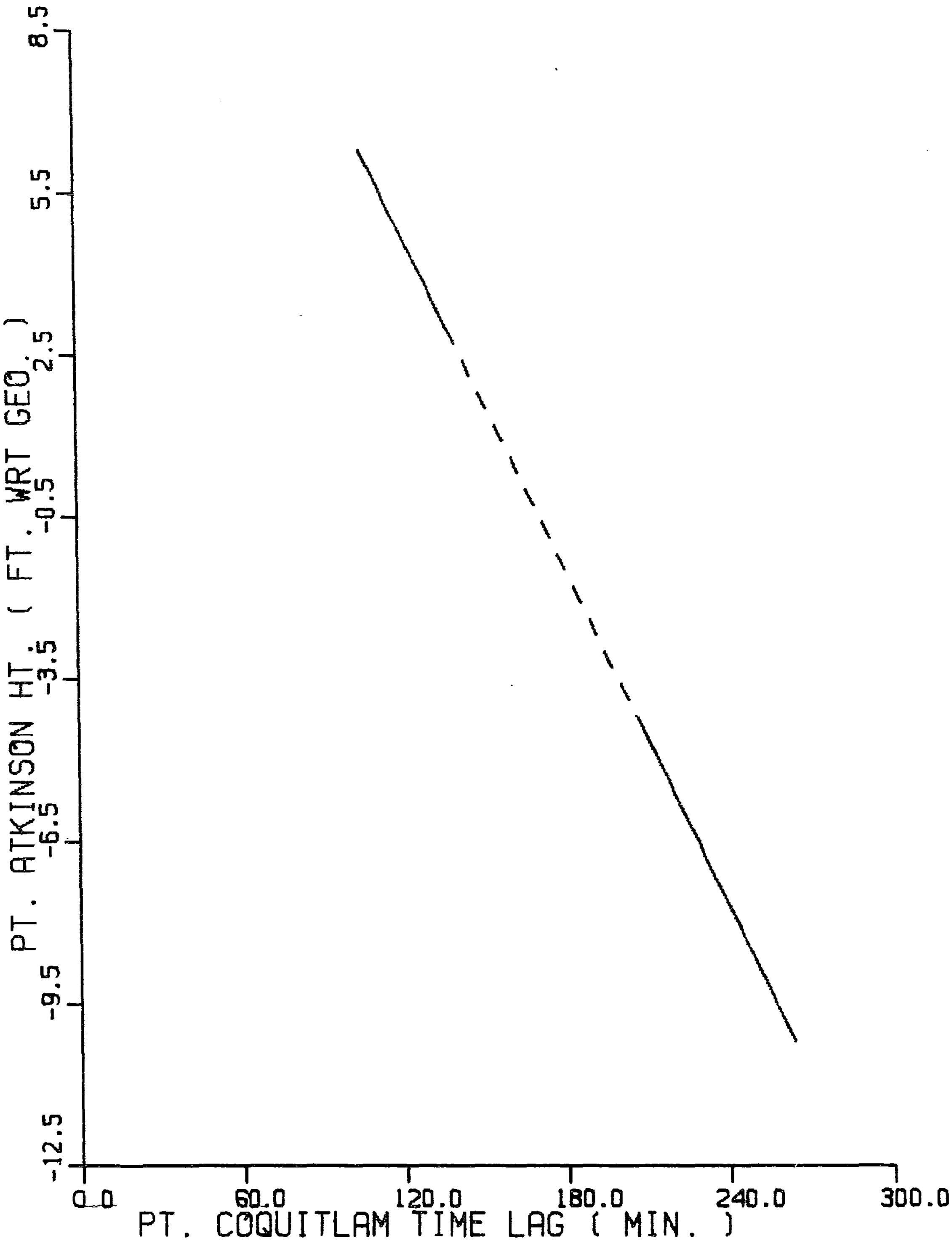


Fig. 33 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND MISSION

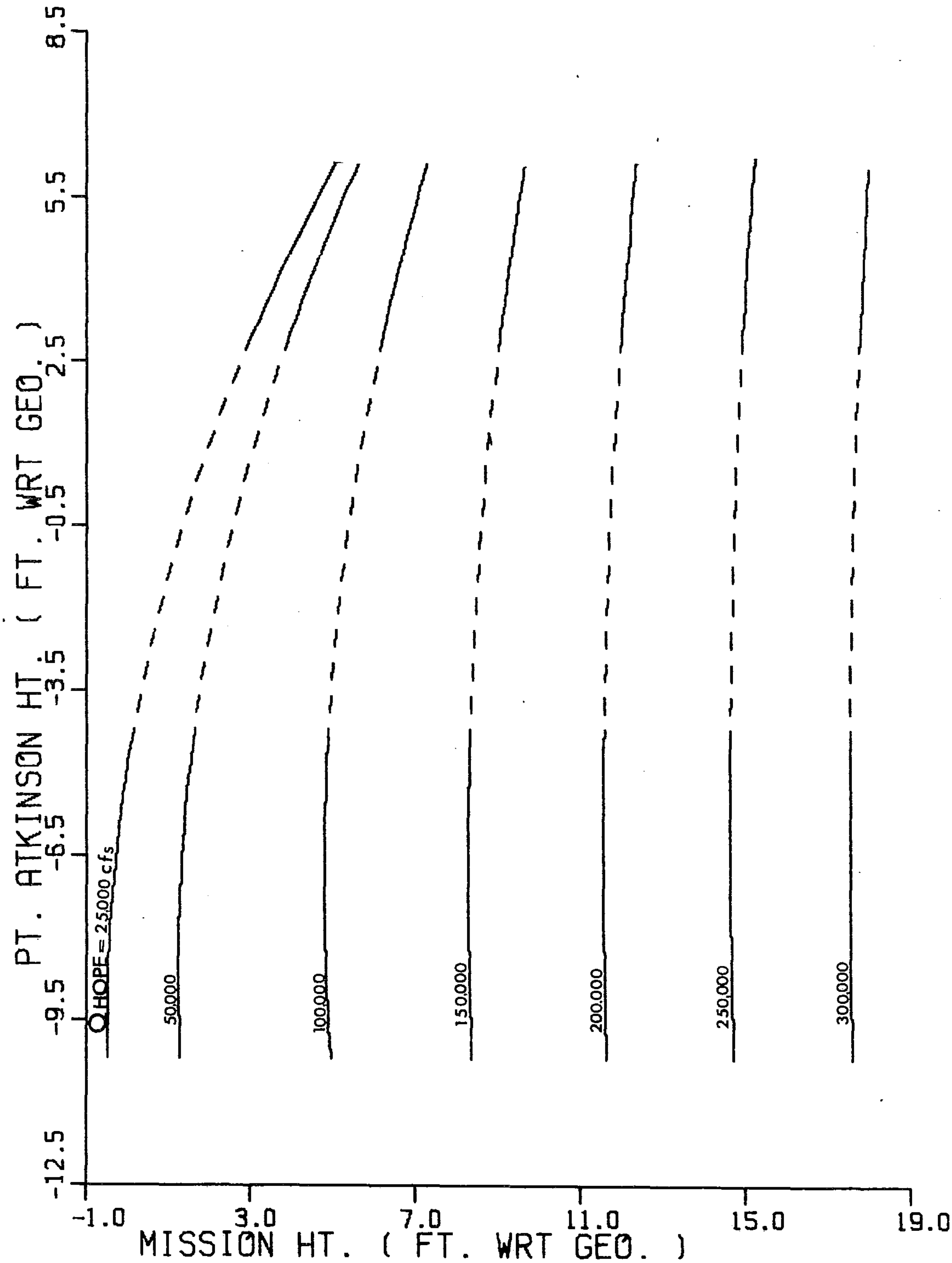


Fig. 34 MODEL-PREDICTED CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND MISSION

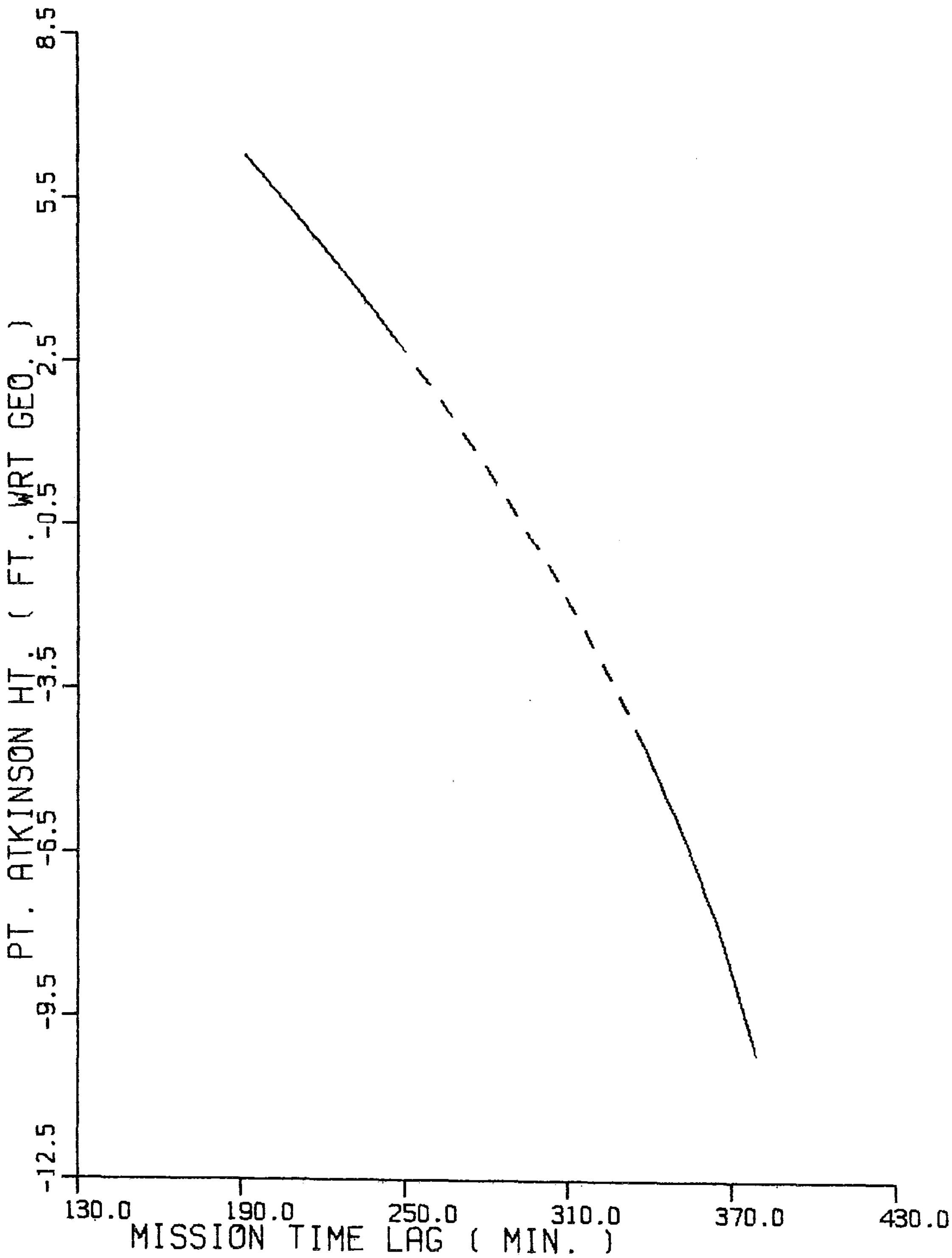


Fig. 35 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

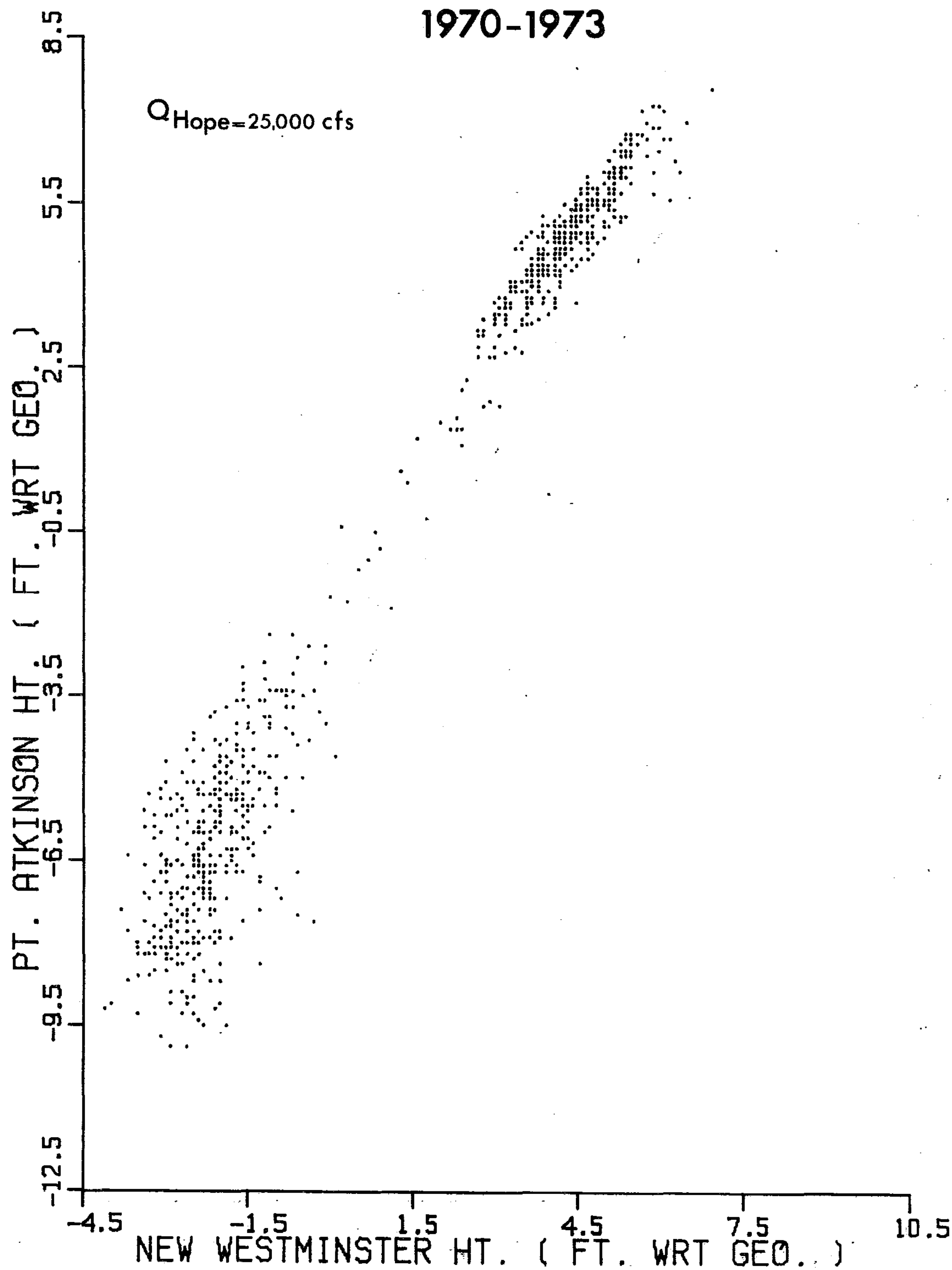


Fig. 36 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

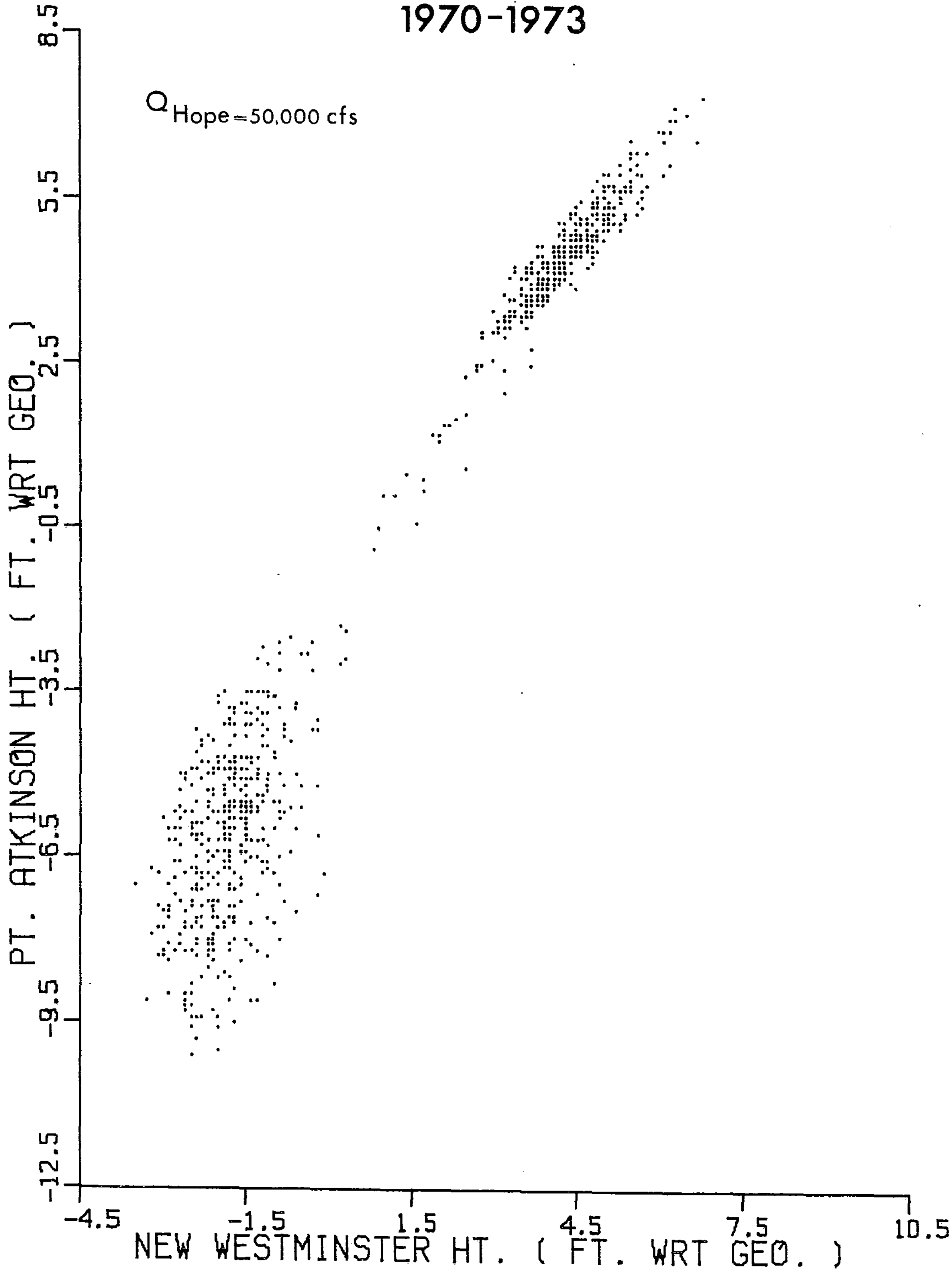


Fig.37 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

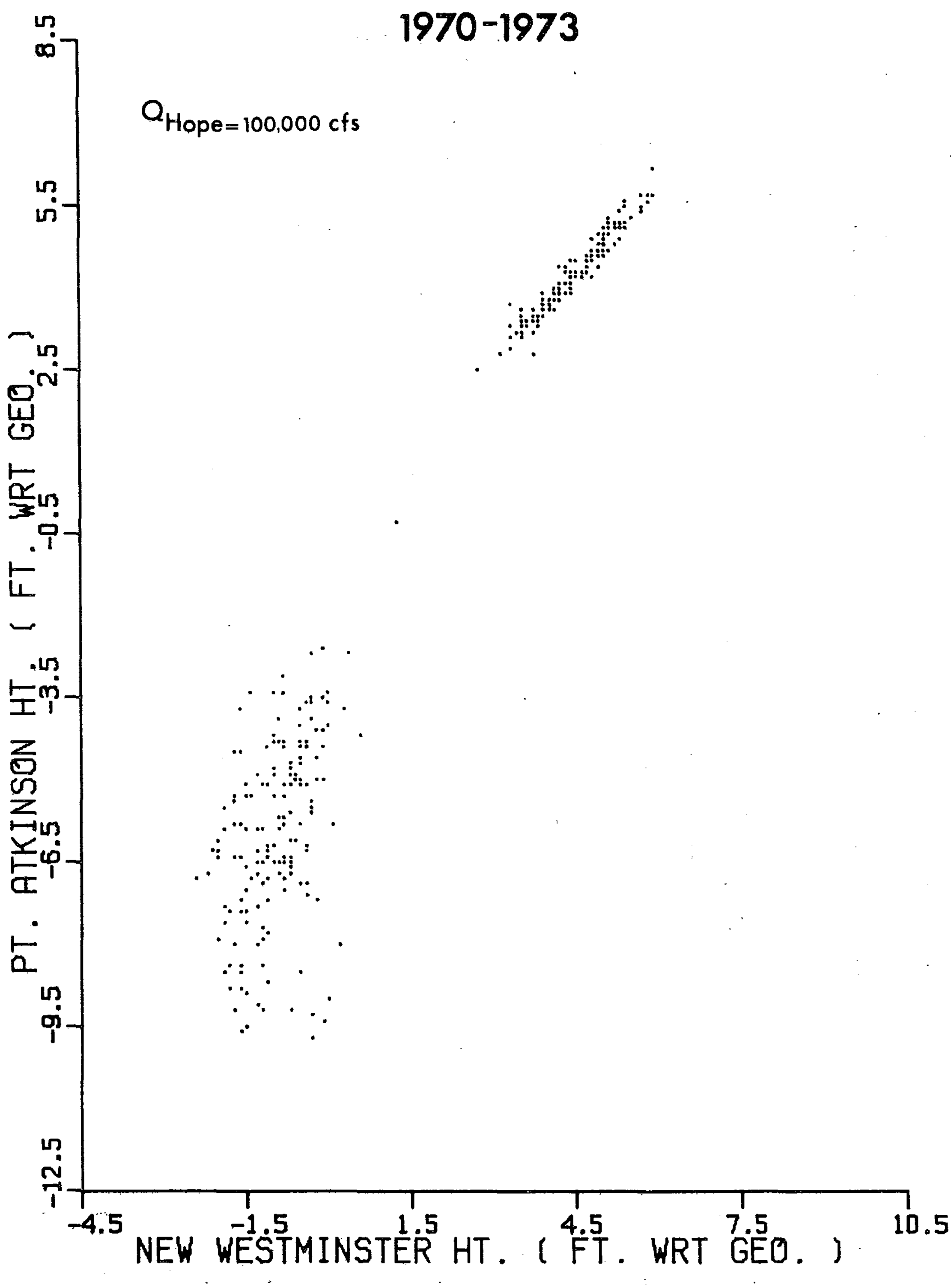


Fig. 38 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

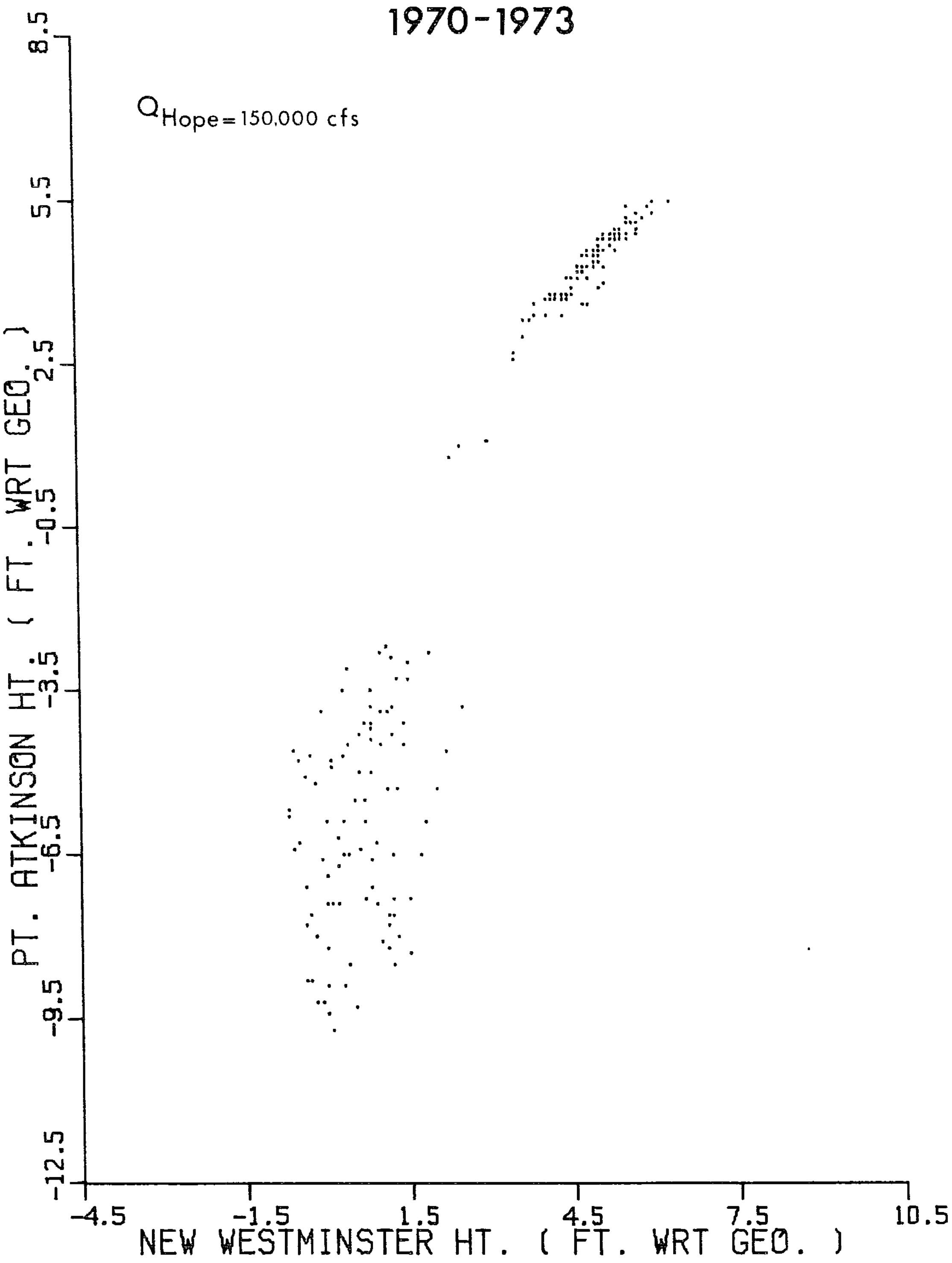


Fig. 39 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

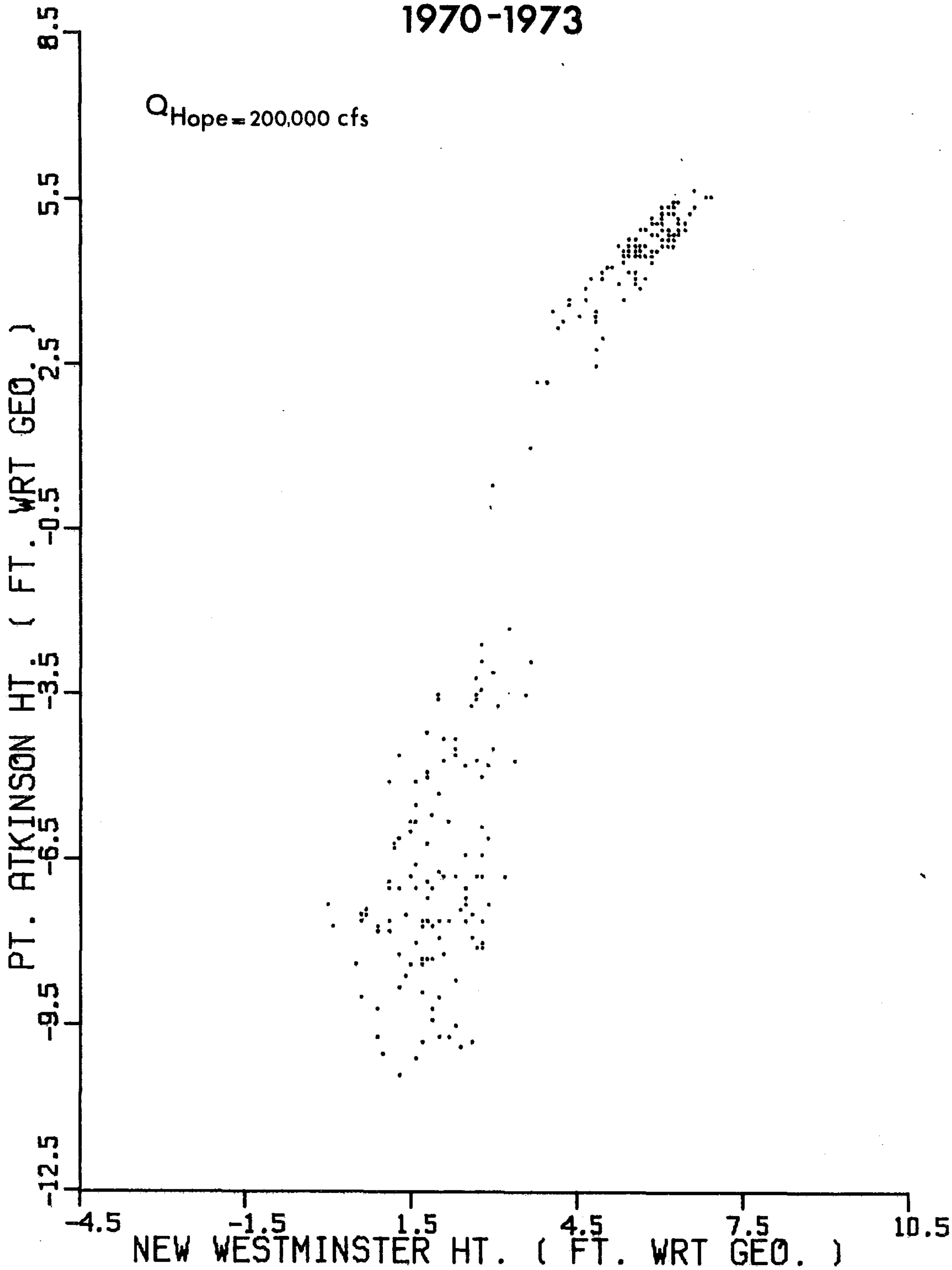


Fig. 40 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

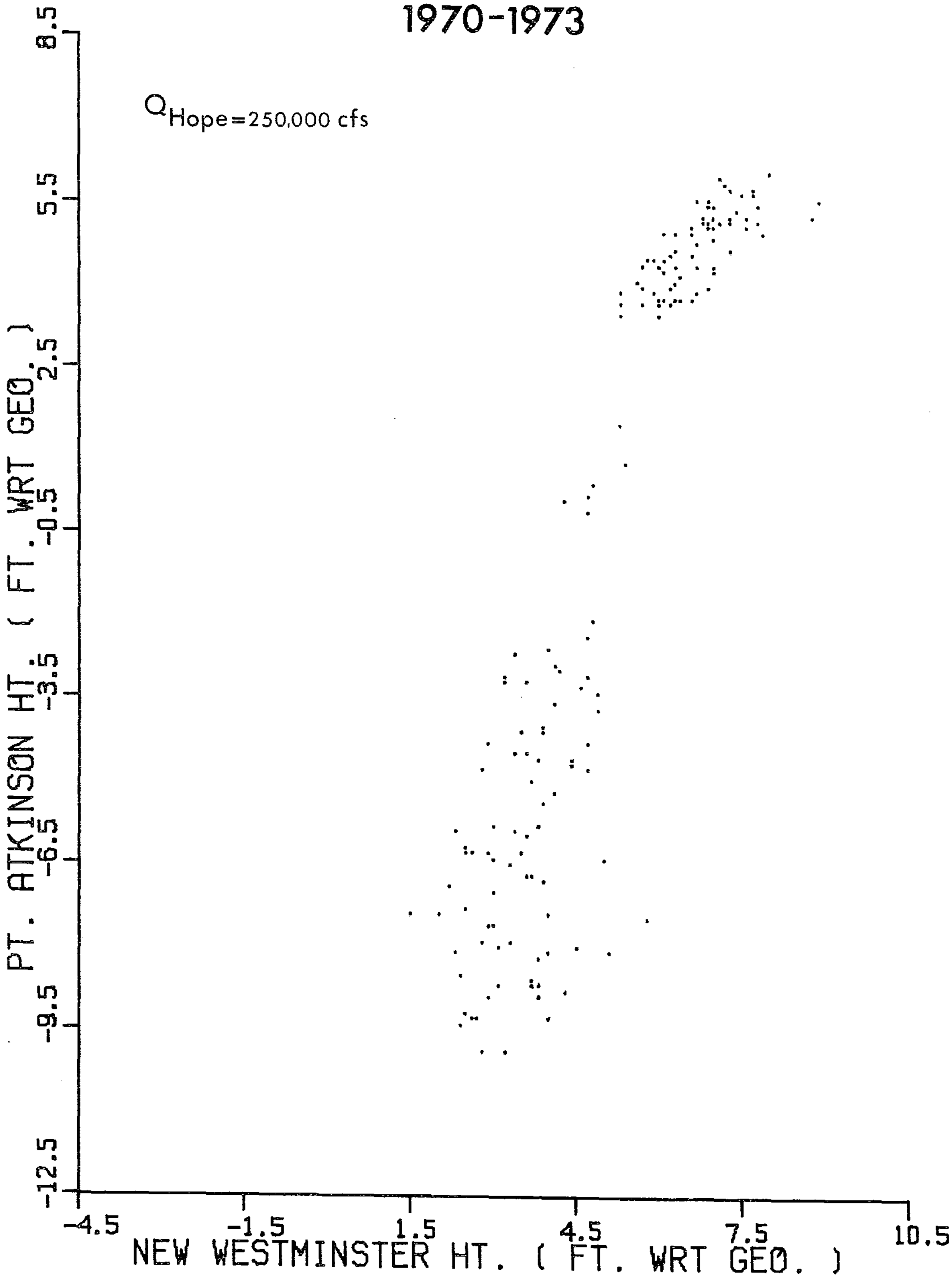


Fig. 41 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

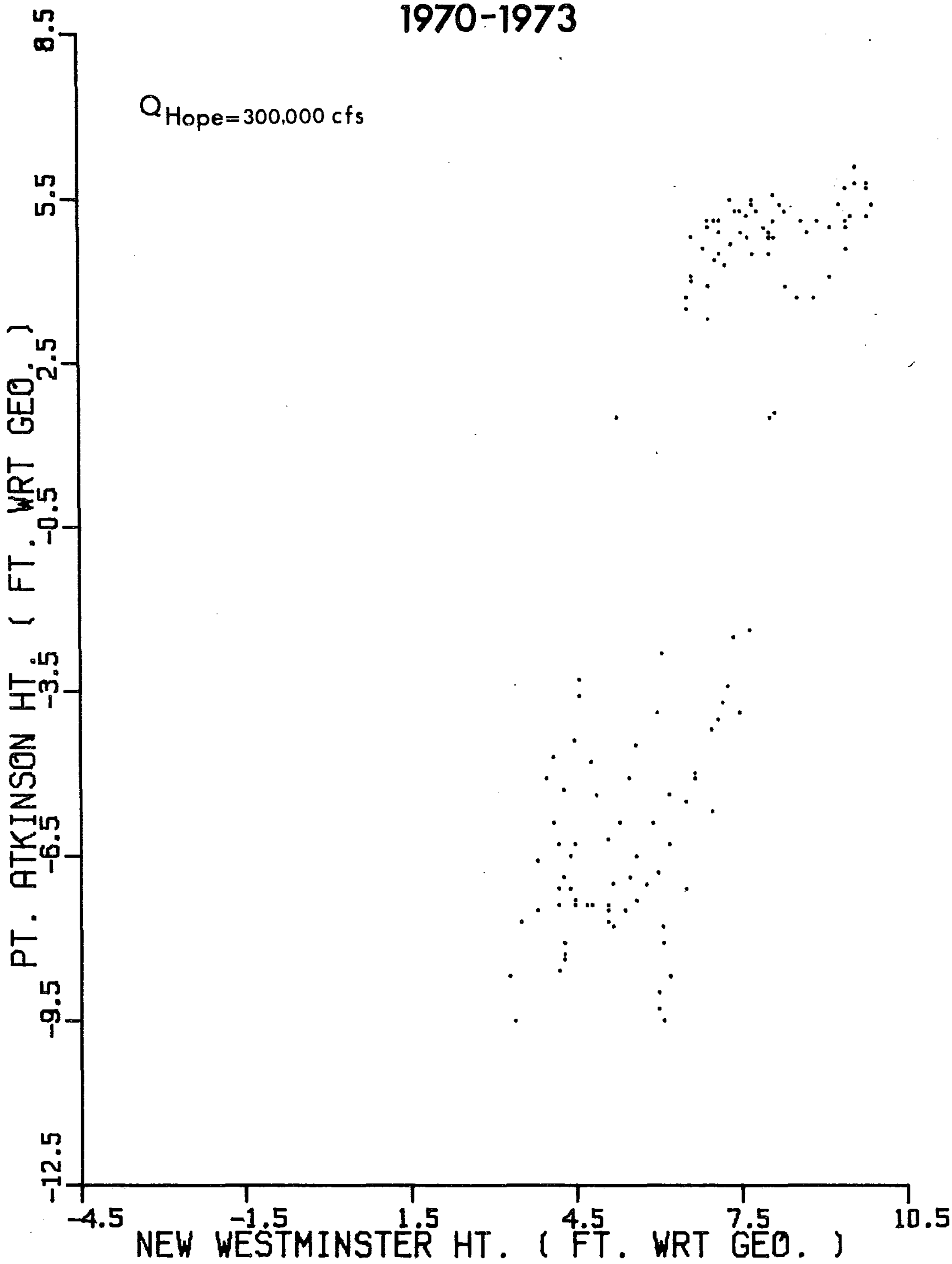


Fig. 42 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

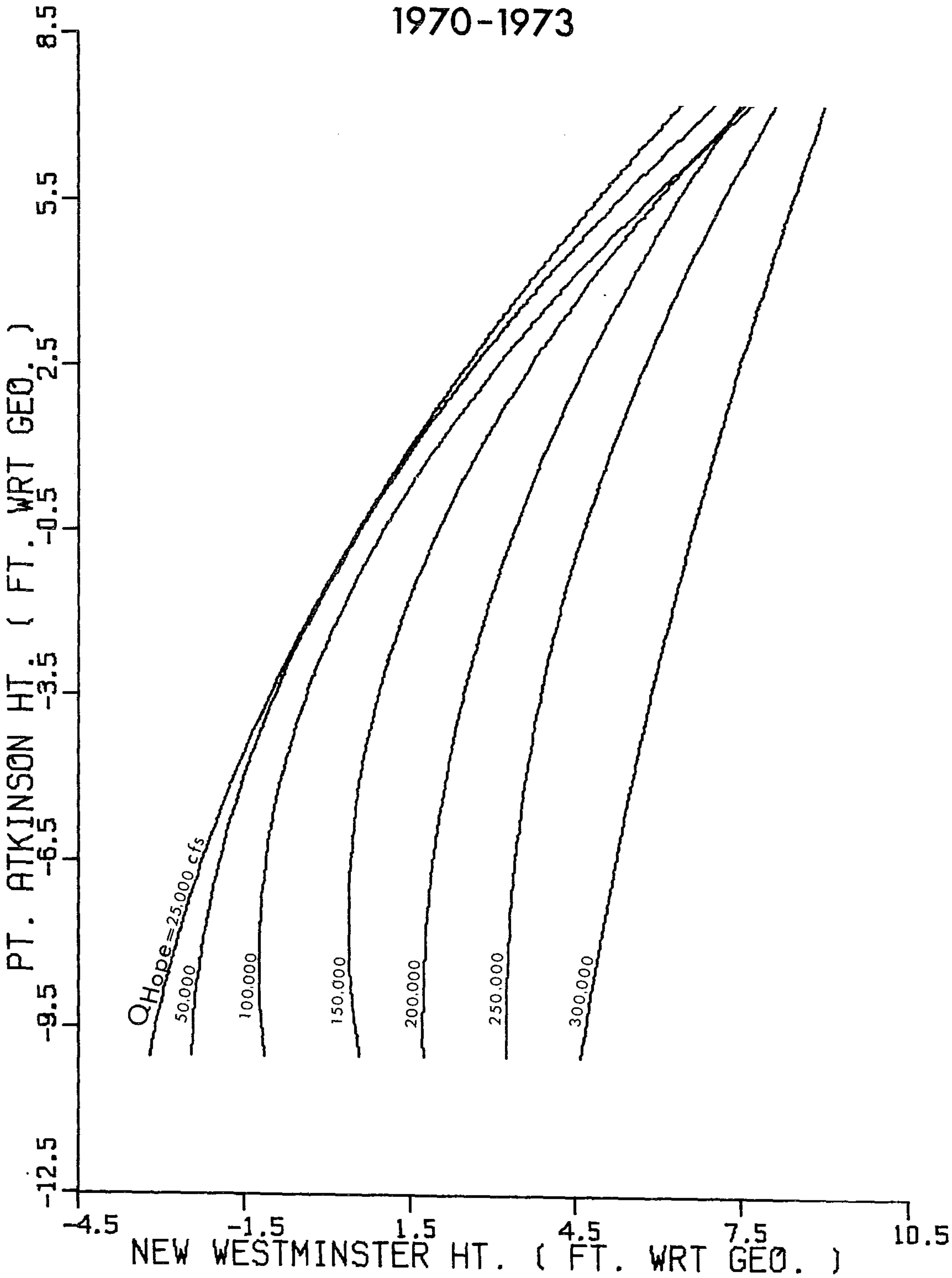


Fig.43 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

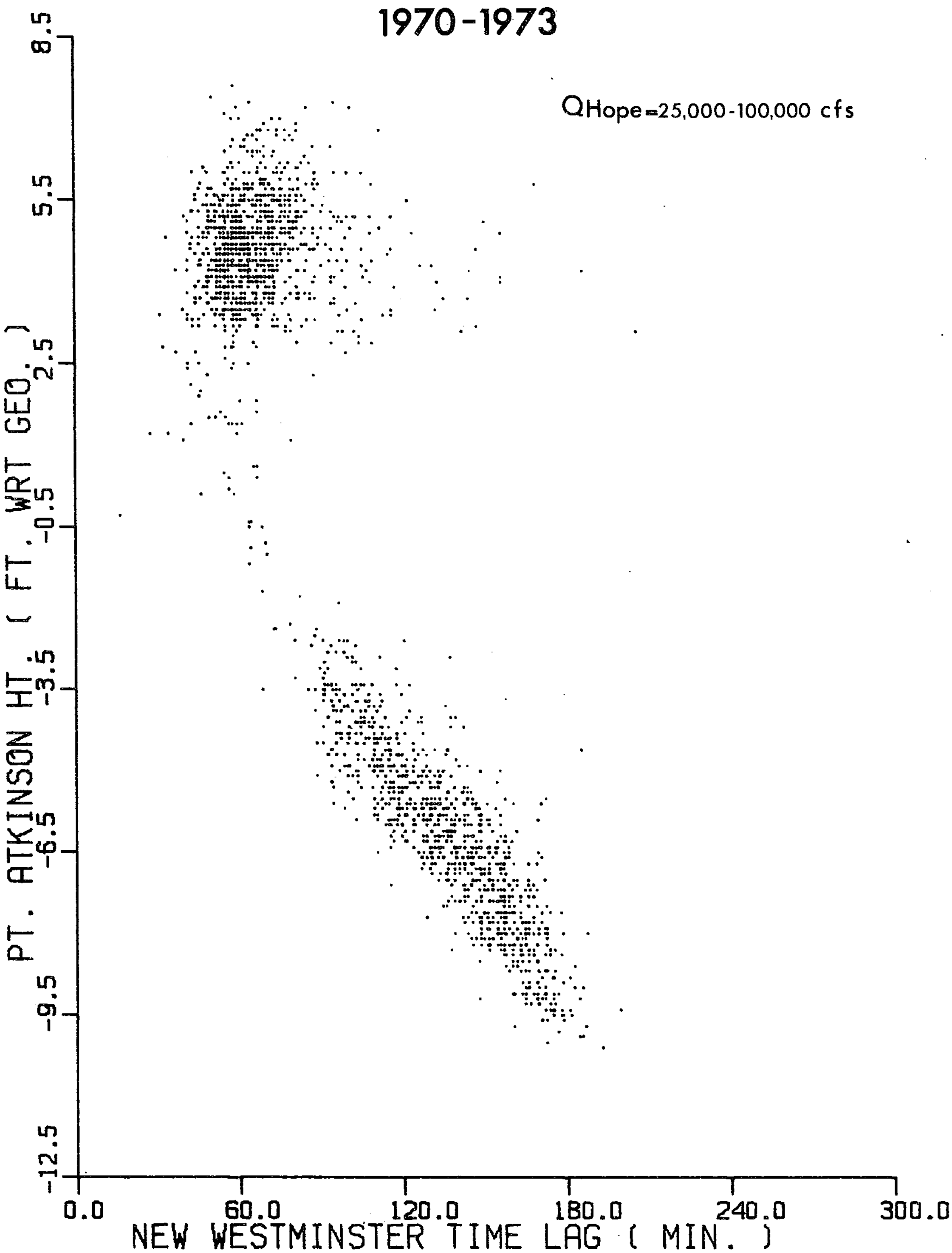


Fig.44 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

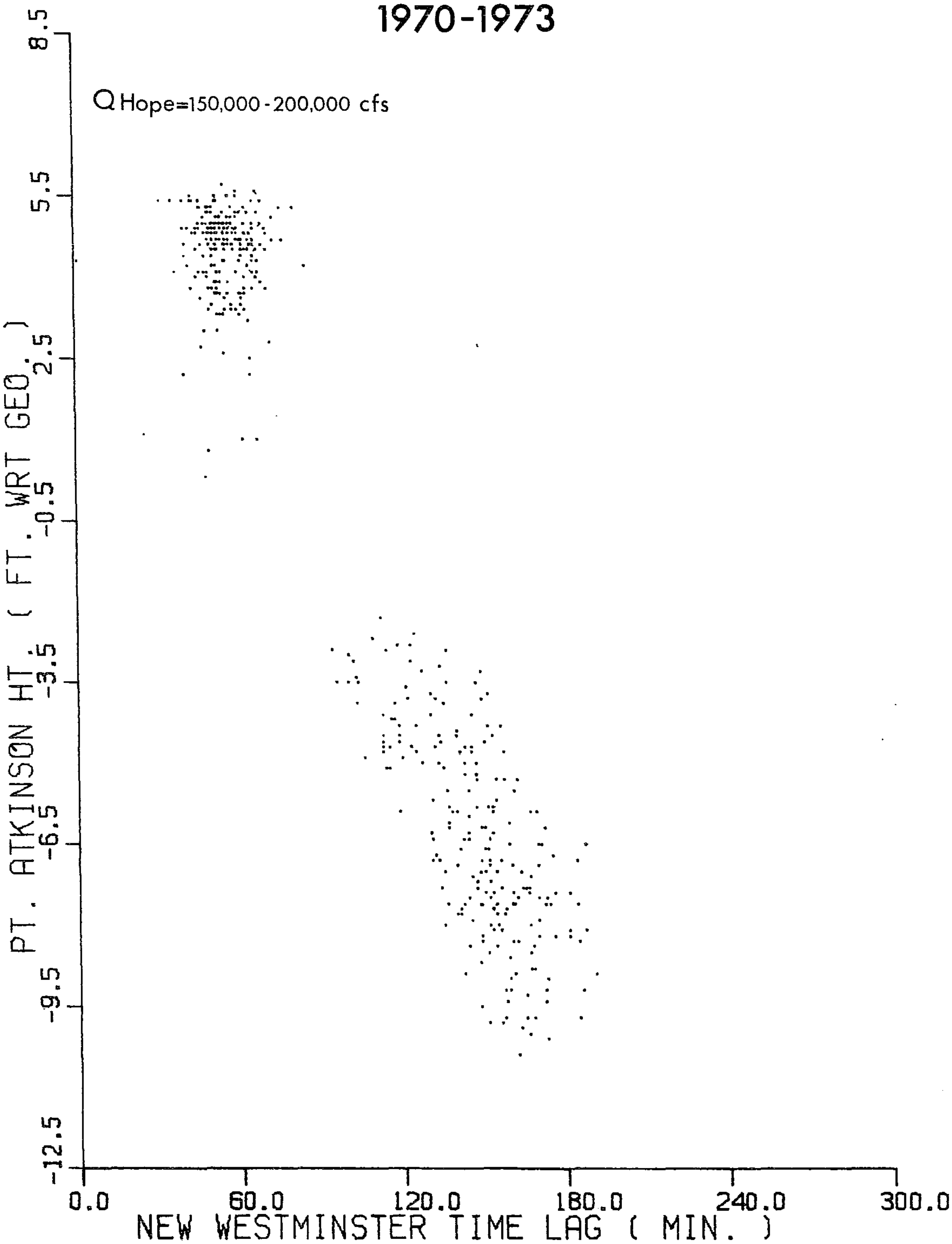


Fig. 45 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973

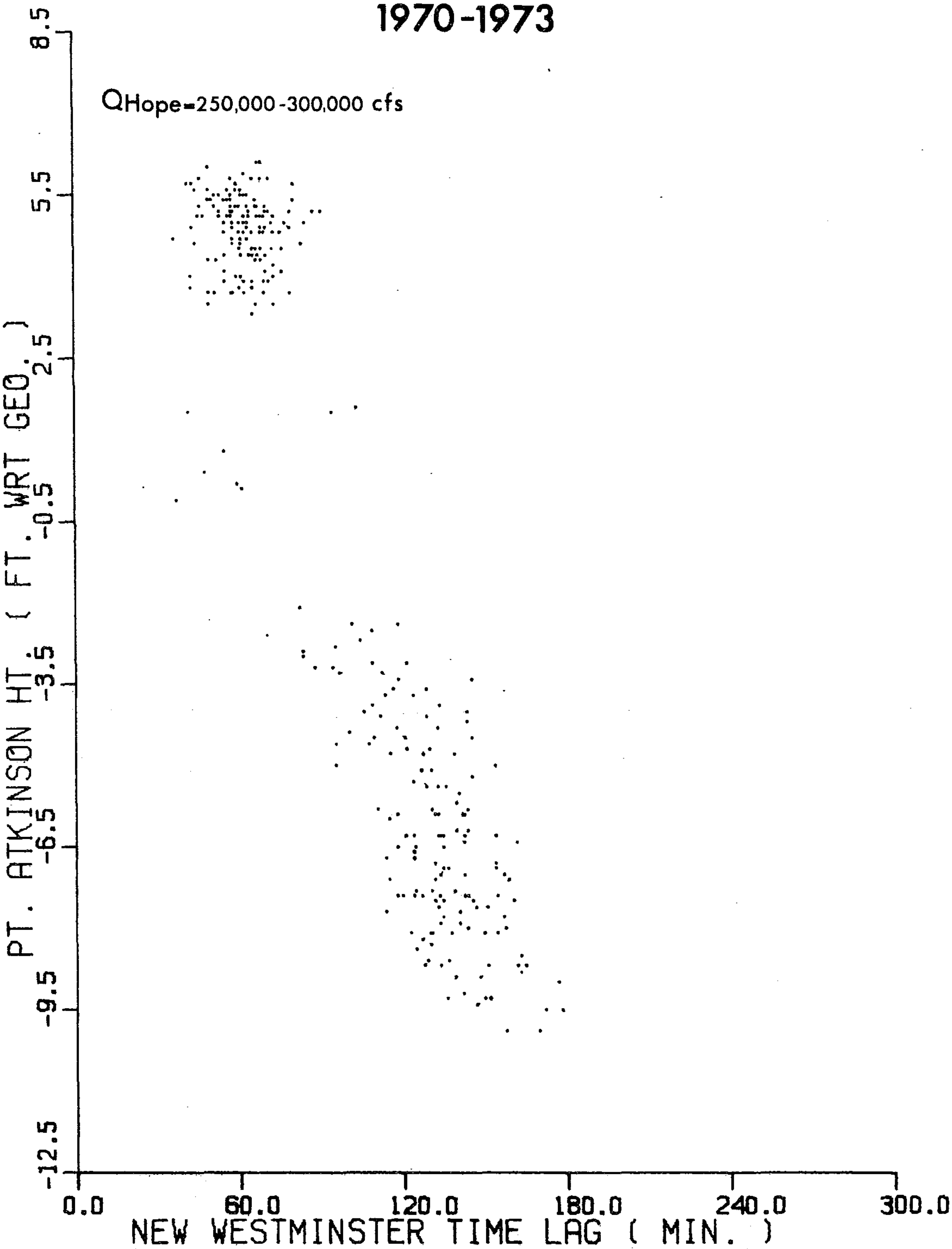
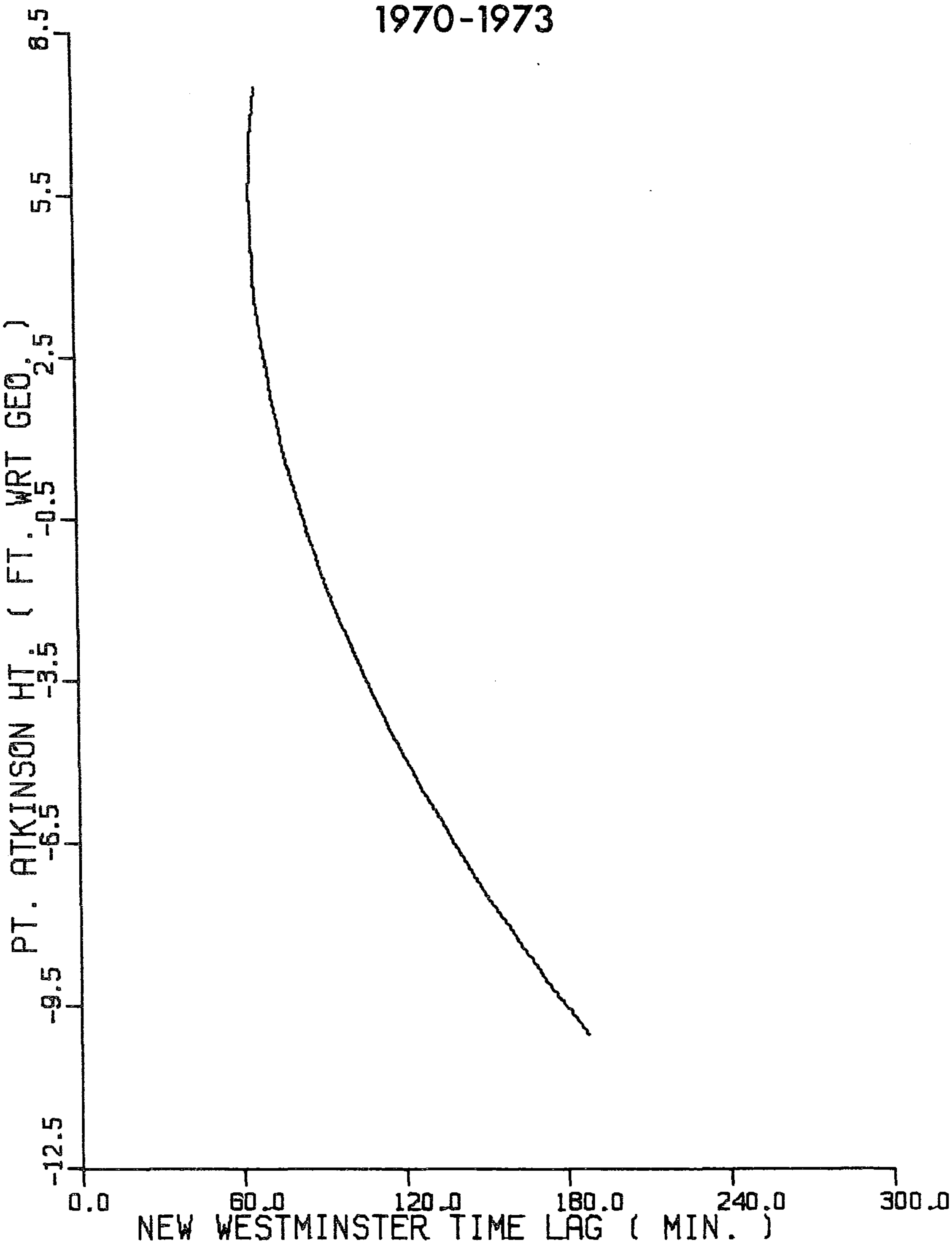


Fig. 46 ACTUAL CORRESPONDENCE BETWEEN HIGHS AND LOWS AT POINT ATKINSON AND NEW WESTMINSTER 1970-1973



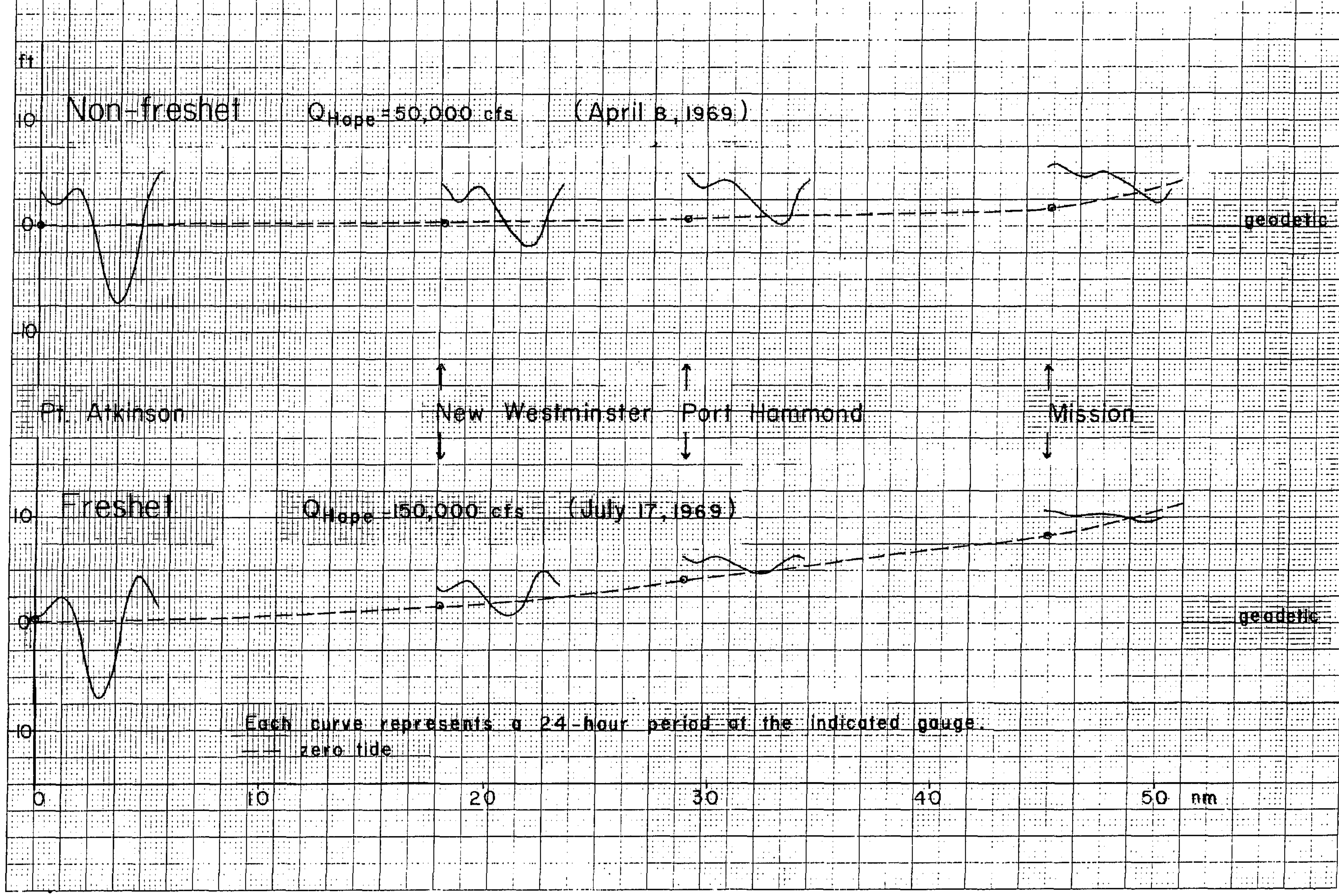
In general, we may conclude that the high waters at New Westminster occur about one hour later than those at Point Atkinson, the low waters about two hours later. This large discrepancy cannot be due to the difference in depth as is occasionally suggested in the literature. An upstream tidal range of six feet would hardly affect the propagation speed $c = \sqrt{gh}$ in an average minimum depth of 30 feet.

The prediction curves for Steveston, Deas Island and New Westminster were converted into tables for publication in the Canadian Tide and Current Tables for 1976.

Figure 47 illustrates the progression of a tidal wave in the Fraser River under freshet and non-freshet conditions, with typical changes in the range and shape of the tide curve along the river. Observed tidal curves for 24 hours are superimposed upon a "river flow only" curve which was produced by the model. Of course, a "river flow only" situation does not exist in the prototype.

Figures 48 and 49 show the model-produced maximum and minimum water levels generated by a spring tide for non-freshet and freshet conditions. The observed tides in the Strait of Georgia on June 28, 1969 were used as the downstream boundary conditions for both cases. The figures demonstrate how the point of convergence of the low water and high water lines, that is, the point where the daily tidal fluctuations cease to exist, moves westward as the discharge increases.

Fig 47 PROGRESSION OF A TIDAL WAVE IN THE FRASER RIVER



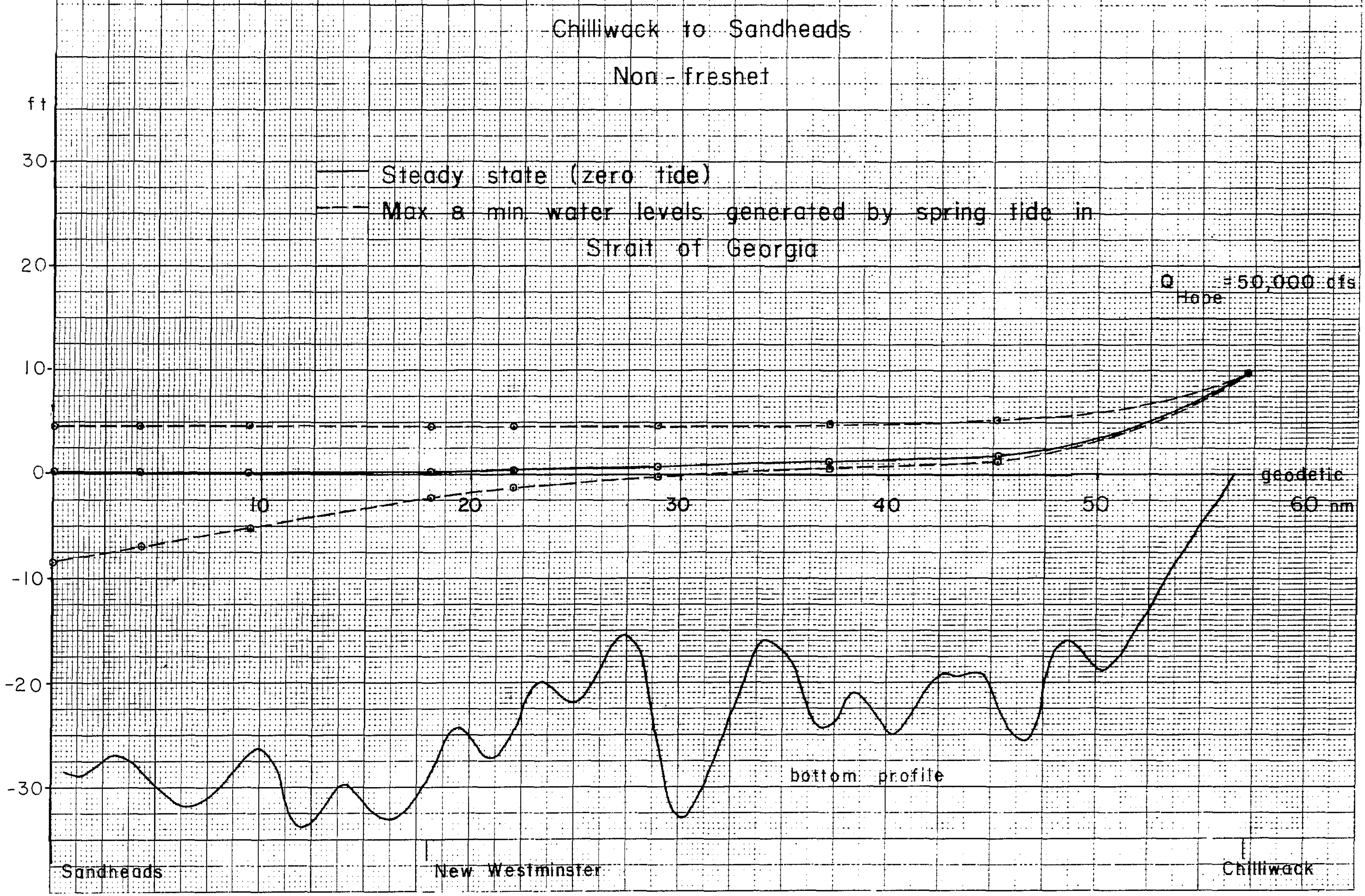
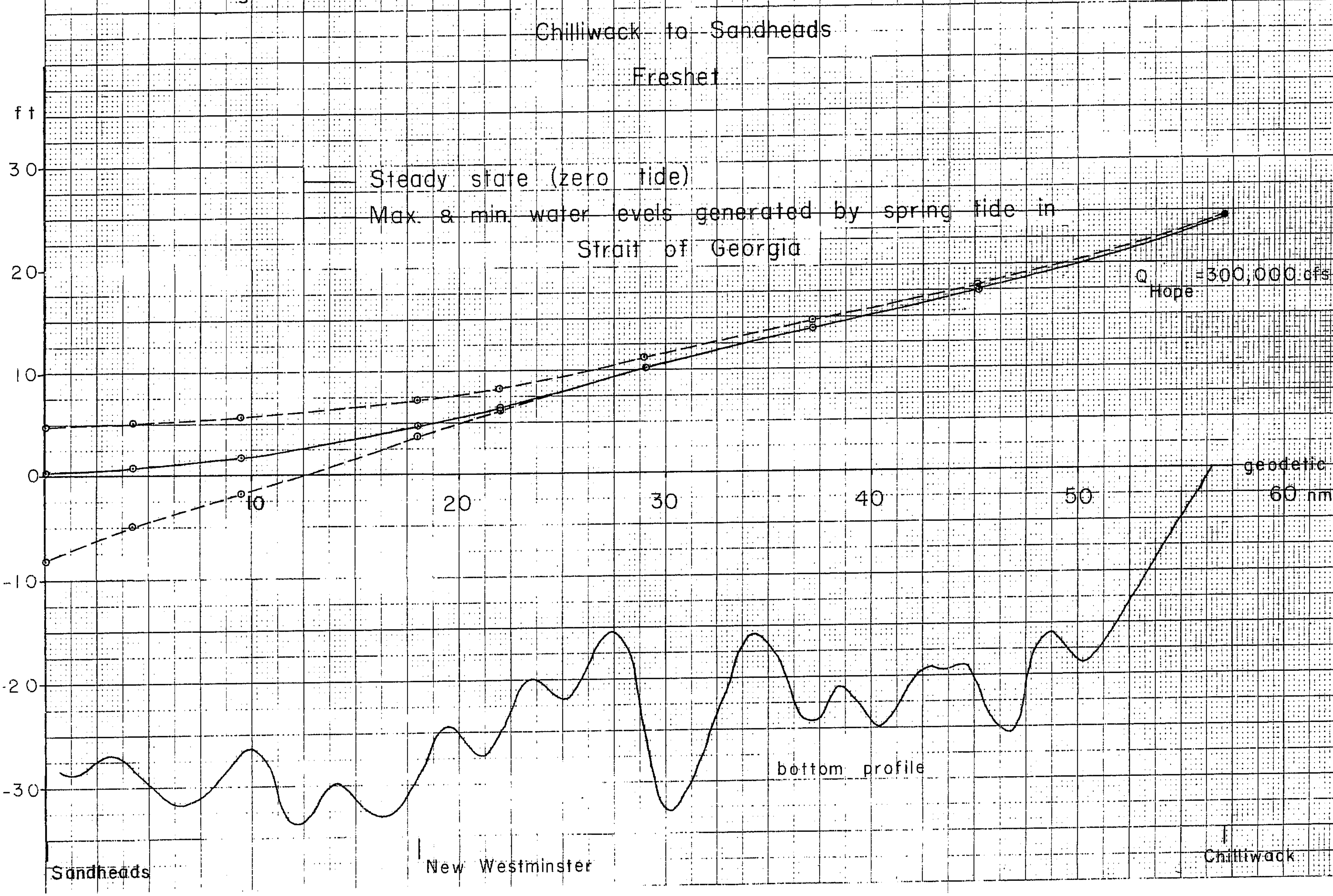


Fig. 48 MODEL-PRODUCED PROFILES OF FRASER RIVER

Fig 49 MODEL PRODUCED PROFILES OF FRASER RIVER



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OTHER APPLICATIONS

A. Energy Dissipation

Considering the area between high and low water lines in Figures 48 and 49 to be a measure of energy losses, most of the tidal energy at low discharges, and almost all of it at high discharges, is dissipated between Sandheads and New Westminster.

A more quantitative evaluation of these energy losses may be obtained from the model by isolating the friction term in the equation of motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = - g\frac{\partial h}{\partial x} - g\frac{|u|u}{C^2 d} .$$

All four terms in their original form are forces per unit mass; the term a $|u|u/C^2d$ representing the friction. The work done by friction on one model segment per unit time would be $(g\,|u|u/C^2d)\ (Q\rho)\ \Delta x$, where Q is the average volume of water passing through the segment per unit time, ρ is the density of the water, and Δx is the length of the segment. During the model's progress, this term is normally evaluated for two segments at a time, with Q (= uA) calculated at the centre section (see Figure 7). The sum of these friction terms for all segments of the schematized estuary over a complete tidal cycle would represent the energy dissipation due to friction during a tidal cycle, in foot-pounds or in ergs, i.e. in finite-difference form:

$$\text{Energy Dissipation} = \sum^{X}\sum^{T}\left(\frac{g\left|\frac{Q}{A}\right|\frac{Q}{A}}{C^2 d}\right)(Q\rho)\Delta x\ \Delta t$$

where X is the number of segments and T is the number of time steps.

The total energy dissipated between Chilliwack and the Fraser mouth (including all four distributaries) was thus computed and averaged over a tidal cycle for both freshet and non-freshet conditions.

For a discharge of 213,000 cfs at Hope (June 20, 1969), the total energy dissipated was found to be 3.17×10^8 foot-pounds/second (0.4298×10^{16} ergs/sec); for a discharge of 29,700 cfs (March 11, 1969), it was 0.47×10^8 foot-pounds/second (0.0637×10^{16} ergs/sec). Per unit surface area, these values were respectively 0.19 foot-pounds/foot^2/second (2760 ergs/cm^2/sec) and 0.04 foot-pounds/foot^2/second (521 ergs/cm^2/sec).

It would be useful to compare the average rate of energy dissipation obtained by this method, with the average rate of energy entering through the boundaries of the model. Generally, for a given time interval, the change in energy in the system should balance the sum of the work on the upstream and downstream boundaries, the work done by the friction, the work done by the wind and the work done by the atmospheric pressure. The effects of wind and atmospheric pressure have been ignored in the model. We assume the net change in energy over a complete tidal cycle to be zero.

G.I. Taylor (20) has developed an expression for the work done per second on the boundaries of a tidal basin.

The average rate at which work is done on one of the boundaries is, in general terms:

$$W_B = \int^{S} \tfrac{1}{2}\rho u dt \left\{g\ (D + h)^2 + gh^2 - gD^2 + u^2\ (D + h)\right\} ds$$

where D is the distance between the bottom and the datum (GB in the Fraser model, see Figure 6), h is the distance between the water level and the datum (H in the Fraser model), and S is the surface width (W in the Fraser model). For each computer time step, W_B can be evaluated at each of the six boundaries of the model. The resulting values are summed over all boundaries over a complete tidal cycle, and the average rate determined.

Applying Taylor's expression for W_B to the same tidal cycles used for the energy dissipation, the energy input into the system was calculated to be 4.93×10^8 foot-pounds/second (0.6683×10^{16} ergs/sec) for Q_{HOPE} = 213,000 cfs and 0.70×10^8 foot-pounds/second (0.0951×10^{16} ergs/sec) for Q_{HOPE} = 29,700 cfs.

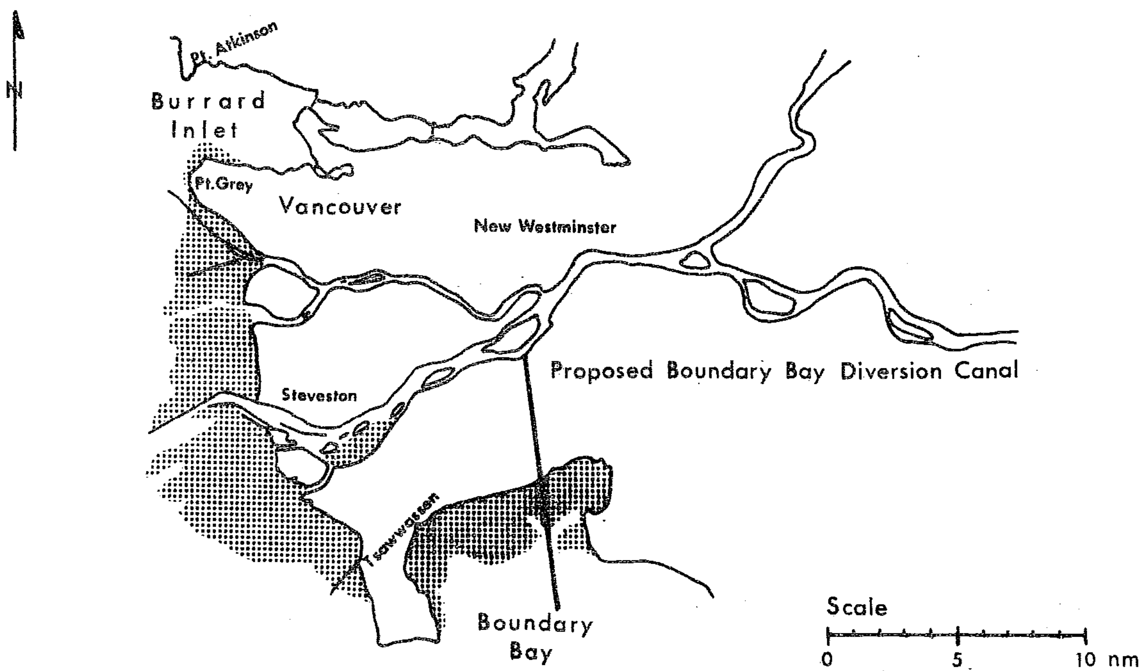
In his paper on the tidal friction in the Irish Sea, Taylor used $k\rho u^3$ for the amount of energy dissipated/cm^2/second, and obtained a value of 0.089 foot-pounds/foot^2/second (1300 ergs/cm^2/sec) for a spring tide in the Irish Sea. In this expression, u is the water velocity, and k is the friction coefficient calculated by Bazin's formula $k = 0.0013\ (1 + M/\sqrt{R})$. M is determined by the nature of the bottom, and R, the hydraulic radius, may be assumed to be equal to the depth in the case of a stream which is very broad compared with its depth. Bazin's M varies from 0.1 for smooth surfaces to 3.2 for rough channels. In the case of the Irish Sea, with a depth of 80 metres, the choice of M is not significant. Using M = 0.85 for a "clean stony bottom", Taylor calculated a value for k of 0.002, observing that this value was very nearly the same as the one obtained for large rivers. However, a value of M = 0.85 appears to be quite low for the Fraser, where bed waves, anchored log booms and training walls would indicate a value of 3.17 (suggested by Bazin for exceptionally rough channels with weeds and boulders). For R = 10 metres, we may put k = 0.0026. Obtaining the average value of u over a complete tidal cycle from the model for all segments, the energy dissipation for a freshet of 213,000 cfs was calculated to be 0.115 foot-pounds/foot^2/second (1680 ergs/cm^2/sec), and it was 0.012 foot-pounds/foot^2/second (179 ergs/cm^2/sec) for a low discharge of 29,700 cfs. Both values are well below those determined by the method using the friction term, and compared poorly with the calculated energy input; however, they are based on much broader assumptions than the previous method.

B. Hydraulic Structures

As we mentioned in the first section, a proposal to construct a diversion canal from Annacis Island to Boundary Bay was examined in 1966 by the National Research Council at Ottawa in a hydraulic and a numerical model (11,12). The canal was intended to alleviate the flood danger of the Lower Fraser River at very high freshets, by diverting about half of the water from the Main Arm. The proposed (but not accepted) canal was 5.3 miles long, 1000 feet wide and had a depth of about 35 feet below geodetic datum. For a catastrophic discharge of 536,000 cfs at Hope, the NRC models predicted decreases in high water levels varying from 0.5 feet at Steveston to 3.5 feet at New

Westminster. Similar results were obtained from the numerical model presented in this report. The canal between Annacis Island and Boundary Bay was schematized in 10 sections, with a depth (GB) of 35.14 feet, and the tides at Tsawwassen as a boundary condition. Only a few program modifications were needed to include the canal in the model, a distinct advantage of a numerical approach. However, numerical methods have not yet been developed to simulate the important scouring effects created by the very high velocities expected at the northern entrance of the canal. A moveable-bed hydraulic model might be more suitable in this respect.

Fig. 50 PROPOSED BOUNDARY BAY DIVERSION CANAL



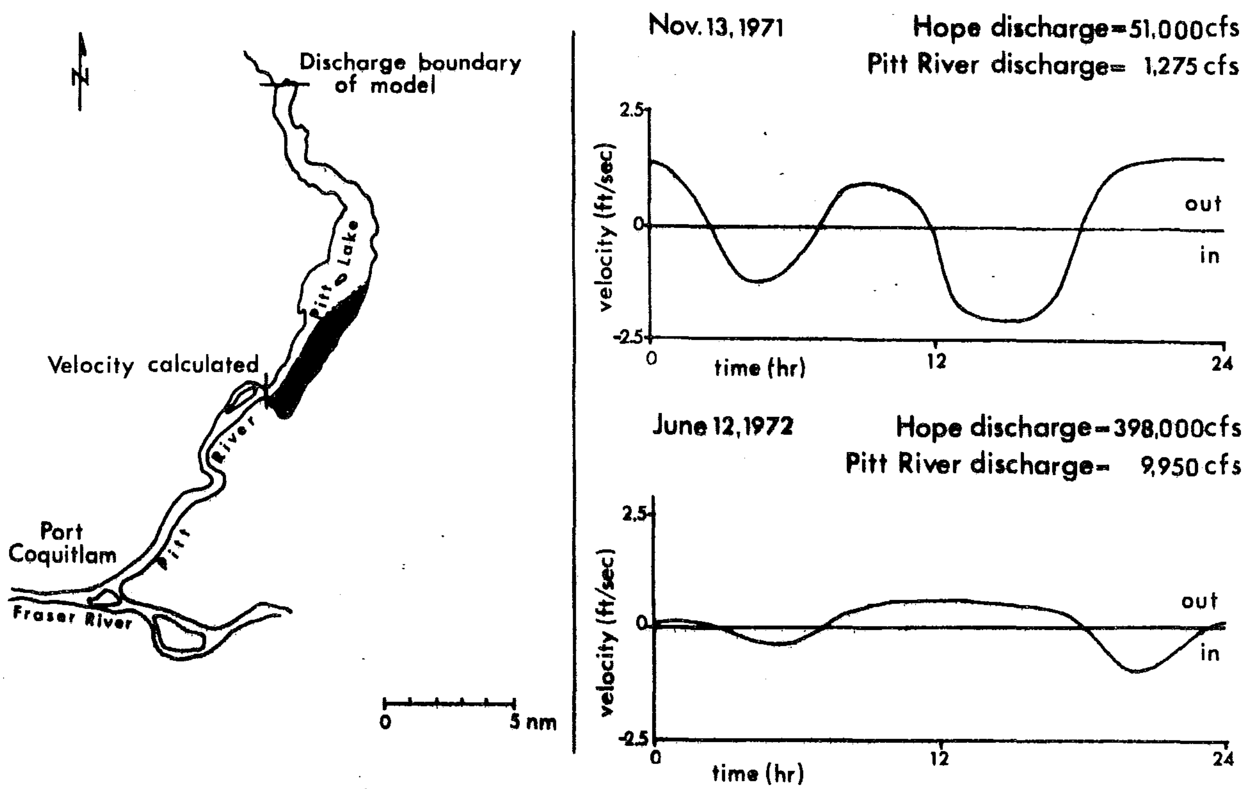
Other proposed hydraulic structures such as the Moran Dam (Figure 1) could be considered in the model simply by adjusting the upstream boundary condition at Chilliwack for the regulated discharge.

C. Sedimentation of Pitt Lake

The rather unusual "negative" delta formation near the southern entrance to Pitt Lake was examined briefly by running the model for two discharges and plotting the calculated river velocities for a 24 hour period as shown in Figure 51.

In both cases, the inward flow is of shorter duration but has a higher maximum than the outward flow. The important factor in this sedimentation process seems to be the magnitude rather than the duration of the current. The inward flow with its higher peak velocity is capable of carrying into the lake sediment particles which are too heavy to be carried out by the weaker outward flow, even though the outflow is of longer duration. Hence, a net inward movement of sediment occurs.

Fig. 51 MODEL-PREDICTED VELOCITIES IN PITT LAKE



D. Movement of Salinity Wedge

During an incoming tide on August 23, 1971, a field study of the behaviour of the salinity wedge in the Main Arm near Steveston measured the speed of the salinity wedge at 1.1 knots. The discharge at Hope for that date was 103,000 cfs. The speed of the wedge was determined by noting the times when the first traces of salinity ($S = 1°/_{oo}$) appeared at two points along the centre line of the river, 1.6 nautical miles apart.

The model-produced water velocity based on the actual boundary conditions was 0.4 knots in the upstream direction, much less than that of the salinity wedge. However, this velocity was averaged over a cross-section and would be much lower than that in the central part of the river.

THE ACCURACY OF THE MODEL

A valuable feature of the Fraser model was the large number of data available to test the accuracy of the predictions.

Figures 52 and 53 compare four years of observed daily higher high and lower low waters at New Westminster with the model-produced values, both for height and time. The model-produced values were computed from the equations of the best-fit curves (Figures 19-34).

The histograms for the height errors show that about 76% of the computed water surface elevations are too low (67% within one foot). This trend is an advantage since the model's principal purpose is to predict tidal heights for shipping, and an underestimated depth is a desirable safety precaution.

Most (92%) of the computed times of high and low waters were in error by less than 30 minutes.

The model was assumed to be calibrated only after a large number of repetitions with a variety of friction coefficients, which were assigned to blocks of segments. To refine the model further, in other words to simulate nature more closely, friction coefficients would have to be assigned to each segment individually, requiring a much larger number of computer runs. This procedure would be extremely costly and might only be warranted if the other parameters used in the model were exact. Unfortunately, the essential parameters, the boundary conditions, are still beset by imperfections in our measuring techniques and instrumentation.

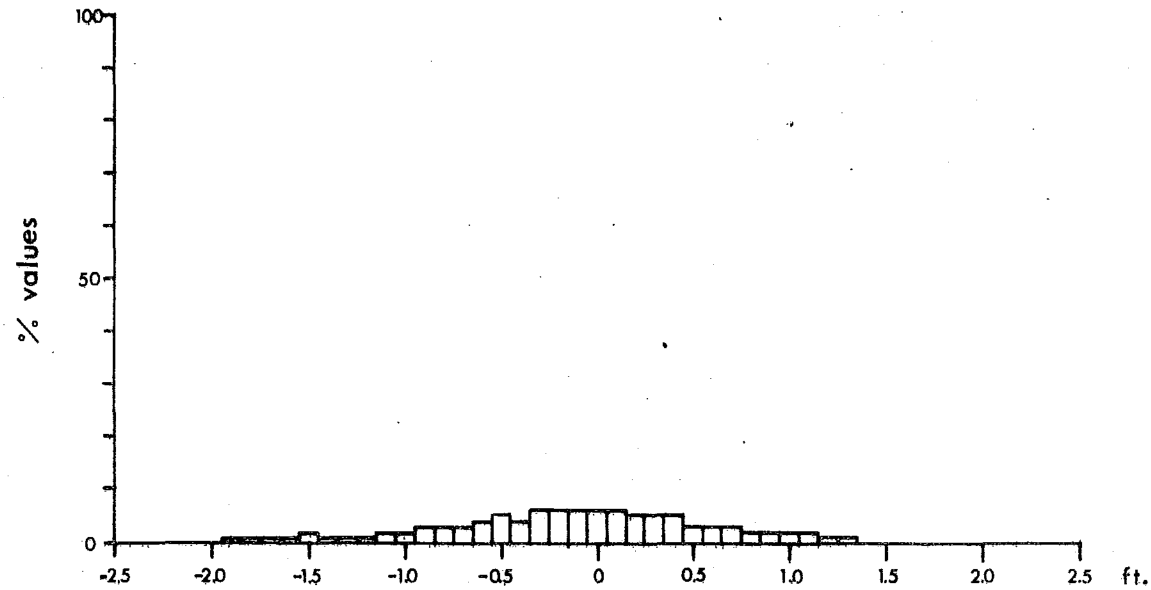
The field data for the upstream boundary condition at Chilliwack are the discharge records of the Fraser River at Hope, computed from point measurements with a non-directional Price current meter. At very high discharges in excess of 350,000 cfs, the current can be measured only near the surface, because the 300 lb current meter is swept away by the current (in the order of 17 feet per second) as soon as it enters the water. A multiplication factor is used to convert the measured surface velocity to mean velocity, which is then multiplied by the cross-sectional area (calculated from depth soundings at low flow) to obtain the discharge (21).

The discharge data derived by this technique may be in error at high discharges, because it is difficult to estimate an accurate multiplication factor by extrapolation from low-flow measurements, and it is virtually impossible to verify this factor.

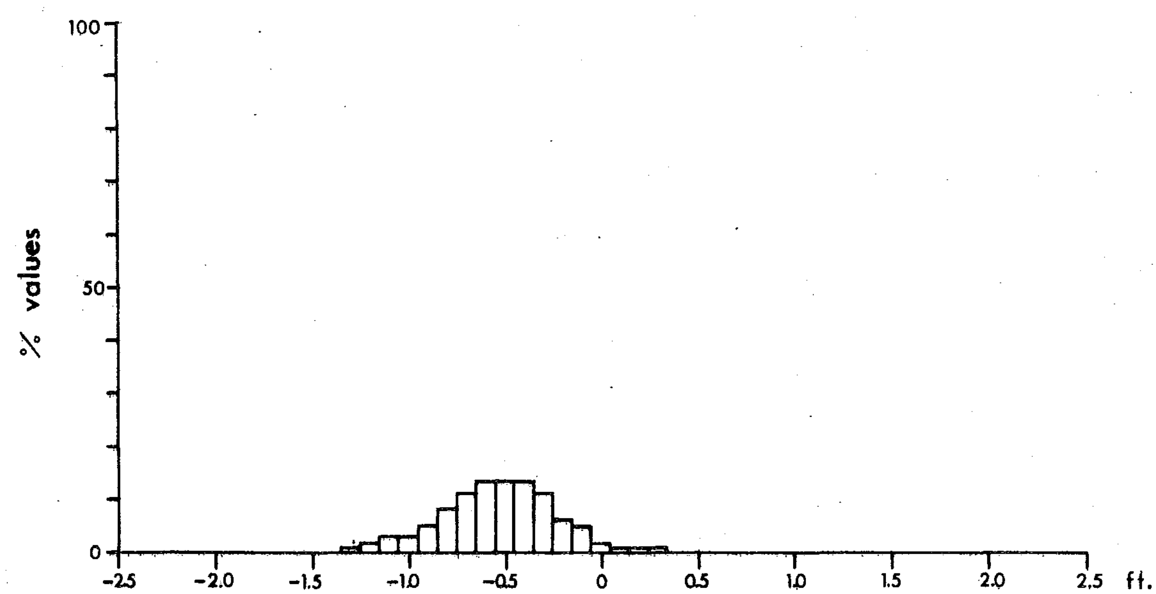
Figure 54 examines the influence of a 5% error in discharge measurements of 398,000 cfs at Hope (June 12, 1972) on the tidal heights at Mission and Deas Island. At this high discharge, the resulting change at Mission is about one foot, at Deas Island 0.2 feet. At low discharges, these changes are considerably reduced.

Although no discharge records exist for the head of Pitt Lake, and hence the boundary condition at that location is an estimate, the flow is small relative to the discharge of the Fraser; any errors introduced into the system by this approximation can be ignored.

Fig. 52 COMPARISON OF MODEL-PREDICTED AND OBSERVED EXTREMA AT NEW WESTMINSTER 1970 - 1973 (Error = predicted-observed)

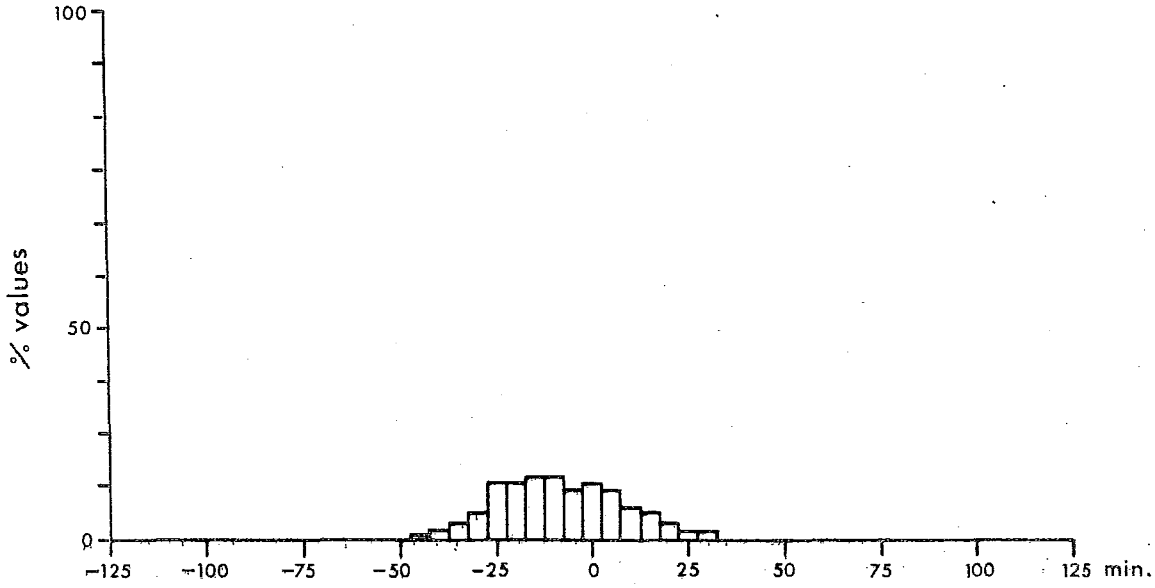


Height error at lower low water

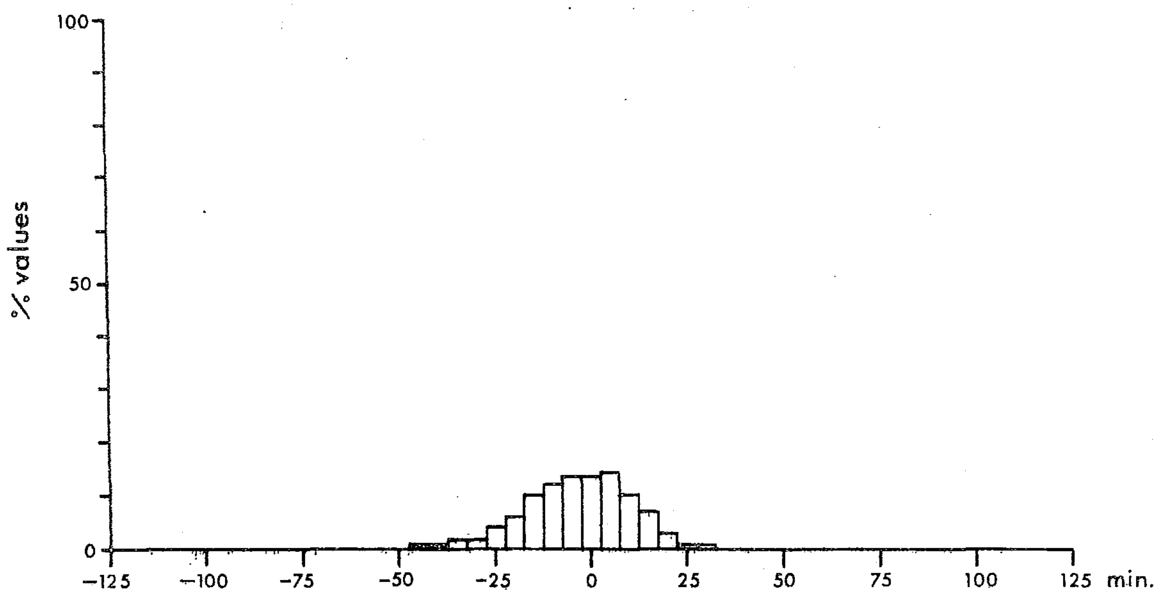


Height error at higher high water

Fig.53 COMPARISON OF MODEL-PREDICTED AND OBSERVED EXTREMA AT NEW WESTMINSTER 1970-1973 (Error=predicted-observed)

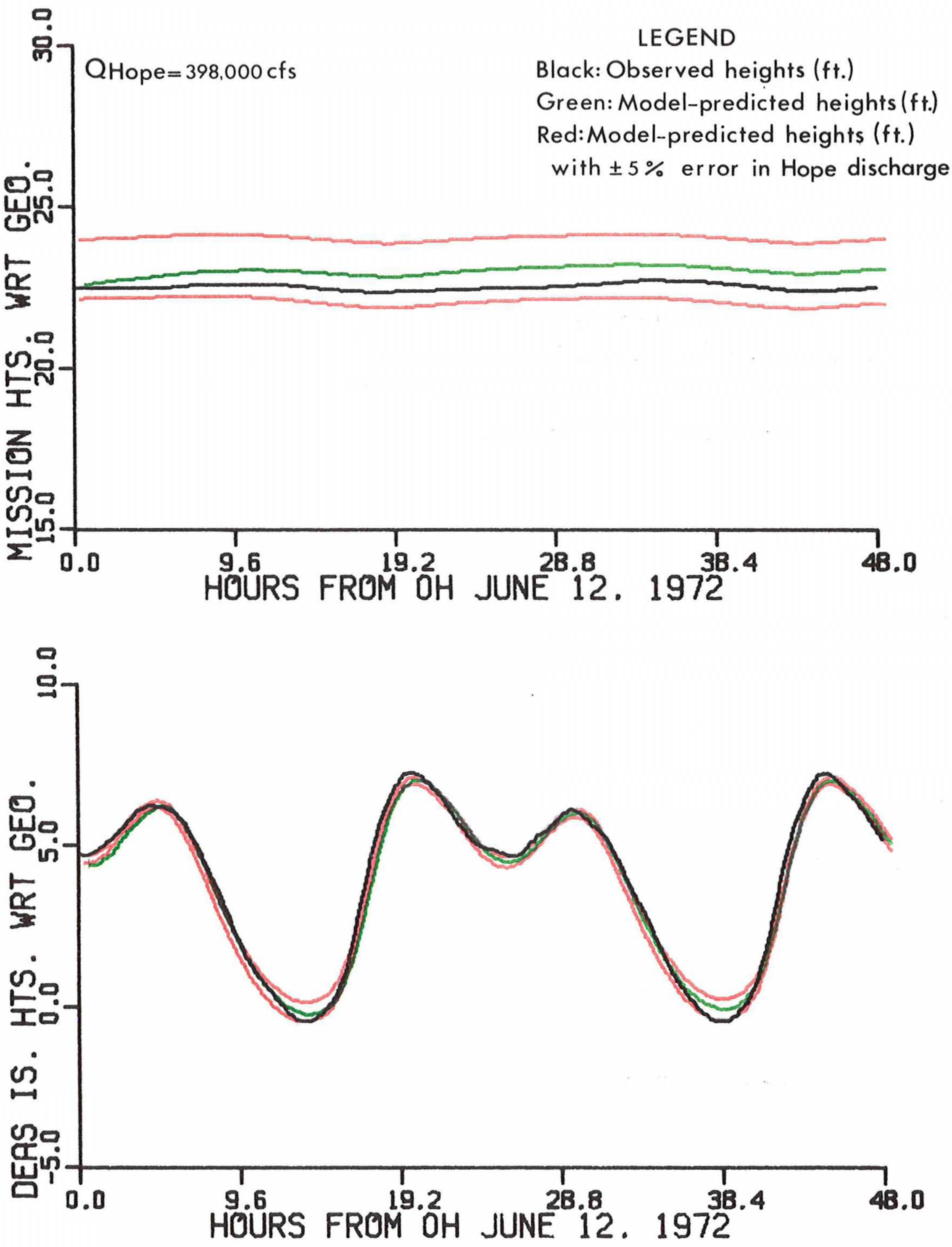


Time error at lower low water



Time error at higher high water

Fig.54 EFFECT OF ERRORS IN UPSTREAM BOUNDARY CONDITIONS (DISCHARGES) UPON MODEL-PREDICTED HEIGHTS



The observed and predicted tidal heights at Point Atkinson and Tsawwassen form the model's downstream input. The observed heights are used for calibrating the model (and for "hindcasting"). They are measured by float gauges, and the recorded values are verified once every two days using a steel tape and a staff with a precision of 0.01 feet. The resulting tide graphs are digitized at hourly intervals, with a precision of 0.01 feet (the thickness of the pen line). The data required for the model are then interpolated to 15 minute intervals using Fourier Series, and subsequently interpolated linearly to 150 second intervals (two time steps). When the Point Atkinson and Tsawwassen tides are used as boundary conditions at the mouth of the Fraser, a small height correction is applied to account for the fresh water outflow (see page 28). This correction was obtained from a comparison between mean sea levels at Point Atkinson, Tsawwassen and Sandheads over a relatively short period (April-September, 1969). The lack of sufficient data makes it difficult to estimate the precision of this correction, but, on the basis of the available values, it seems reasonable to assume an error not exceeding 0.1 feet. Neglecting possible flaws in the interpolation technique, we conclude that a total error in the order of 0.1 feet is accumulated in the processing of observed data.

The predicted heights do not consider the influence of barometric pressure, wind, density, etc. and therefore should be examined more critically:

The Point Atkinson and Tsawwassen tidal predictions for the Fraser model are obtained by a harmonic method using 50 constituents. The computer program used for this method is a simplified version of the program developed by the Marine Environmental Data Service for the Canadian Tide Tables. These two programs were found to be of similar accuracy when their computed maxima and minima were compared with actual data over a total of 56 days of selected spring and neap tides (217 values). This comparison is illustrated by the histograms for Point Atkinson in Figures 55 and 56 (the Tsawwassen histograms are similar, and therefore are not shown).

Although the purpose of these histograms was to confirm the adequacy of a simplified program, they also indicated significant errors in the height predictions of both programs.

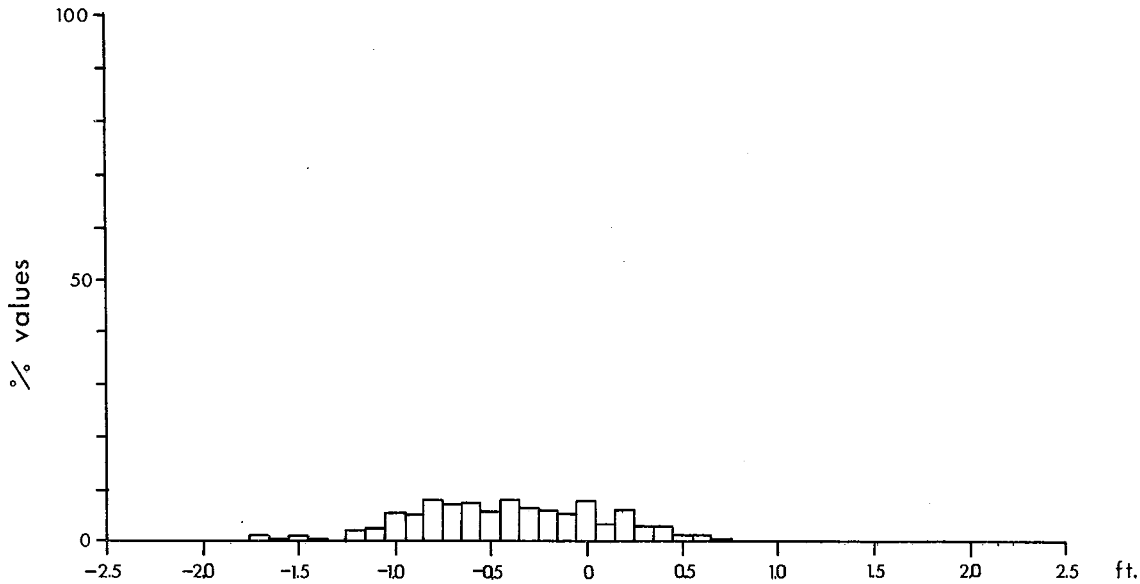
To examine the error distribution more closely, a much larger sample was considered. Figure 57 compares four years (1970-1973) of predicted (from the Tide Tables) and observed higher high and lower low waters at Point Atkinson. Of the predicted heights, 24% were in error by more than 0.5 feet. However, the times agreed surprisingly well: 80% of the predicted times were within ten minutes of the observed values.

The 24% probability that the boundary conditions are in error by more than 0.5 feet, seriously weakens the model's predictive capability, particularly as the calibration itself only aims at an accuracy of 0.5 feet.

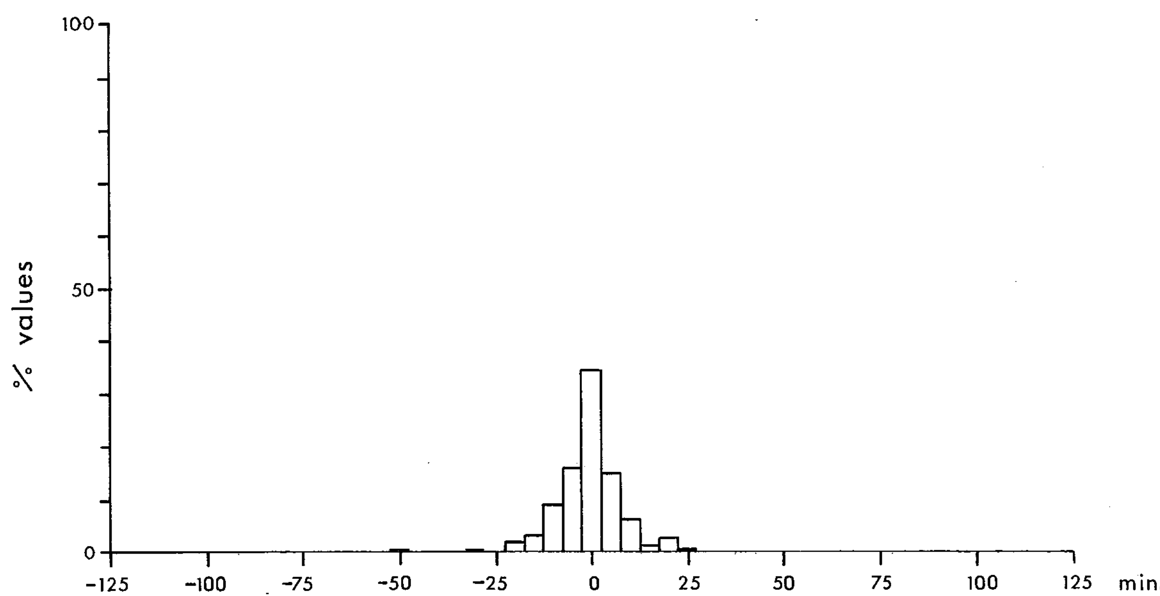
It has been suggested (21) that many of the anomalies in the tidal predictions at Point Atkinson, and other stations in the general vicinity of the Strait of Georgia, are caused by atmospheric pressure fields. Until a method is developed which reproduces these anomalies, the tidal predictions in the Fraser cannot be expected to be more accurate than 0.5 feet, regardless of the level of calibration.

Fig.55 COMPARISON OVER 56 DAYS OF OBSERVED EXTREMA AT POINT ATKINSON WITH THOSE OBTAINED BY HARMONIC METHOD

(Error=predicted−observed)



Height error at Point Atkinson



Time error at Point Atkinson

Fig. 56 COMPARISON OVER 56 DAYS OF OBSERVED EXTREMA AT POINT ATKINSON WITH THOSE LISTED IN TIDE TABLES

(Error = predicted − observed)

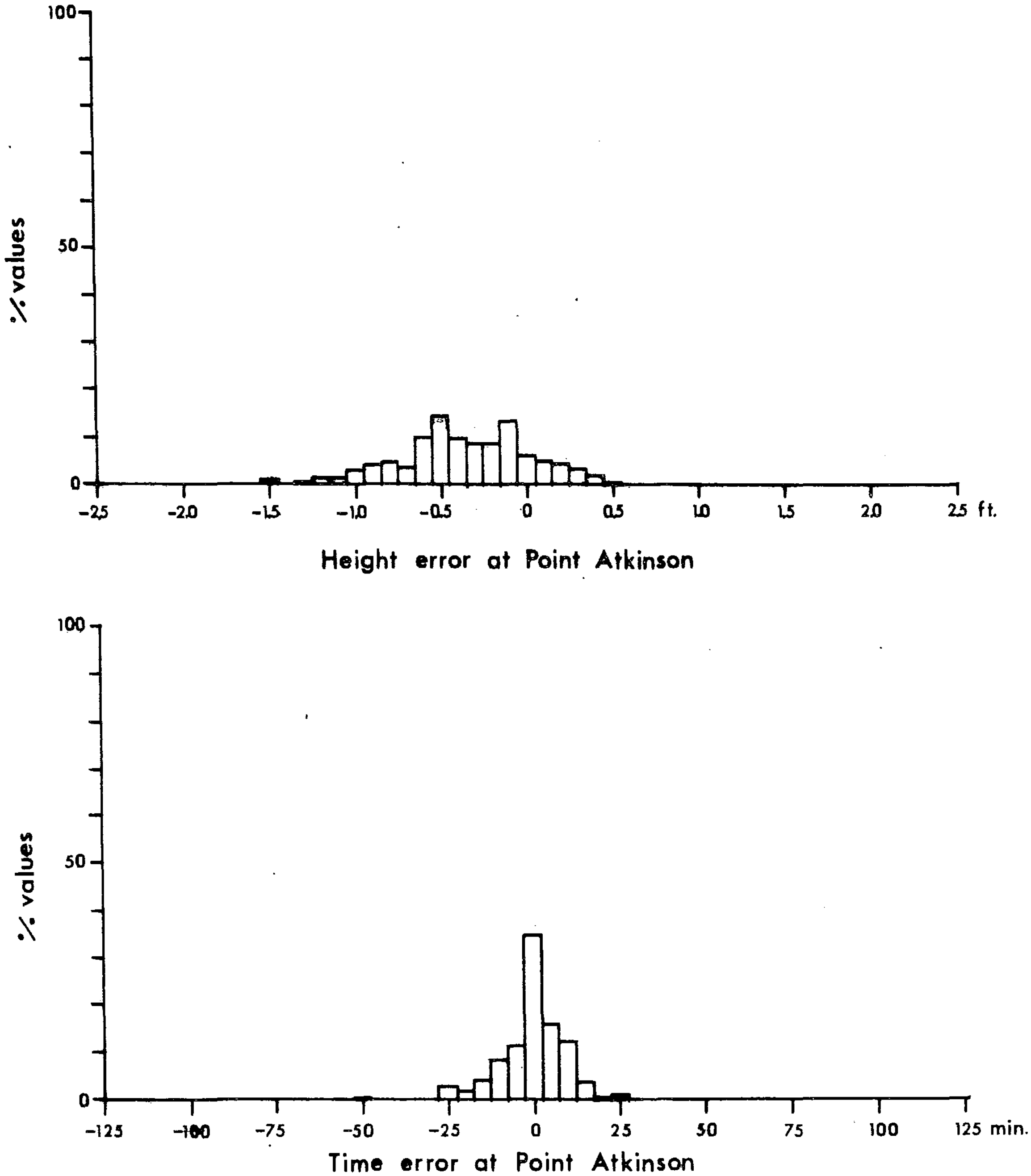
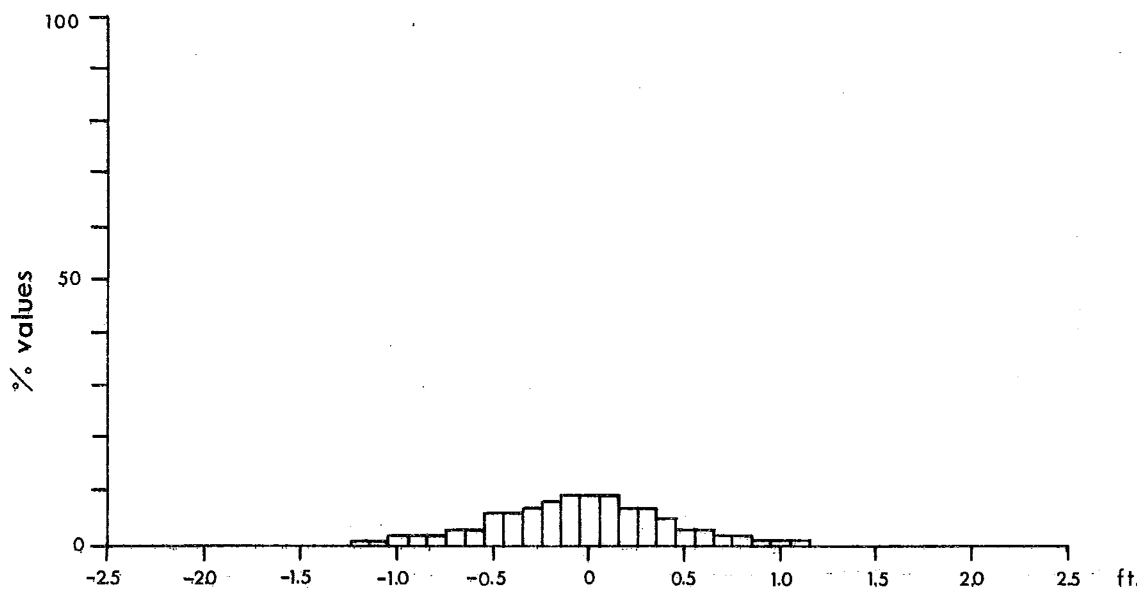
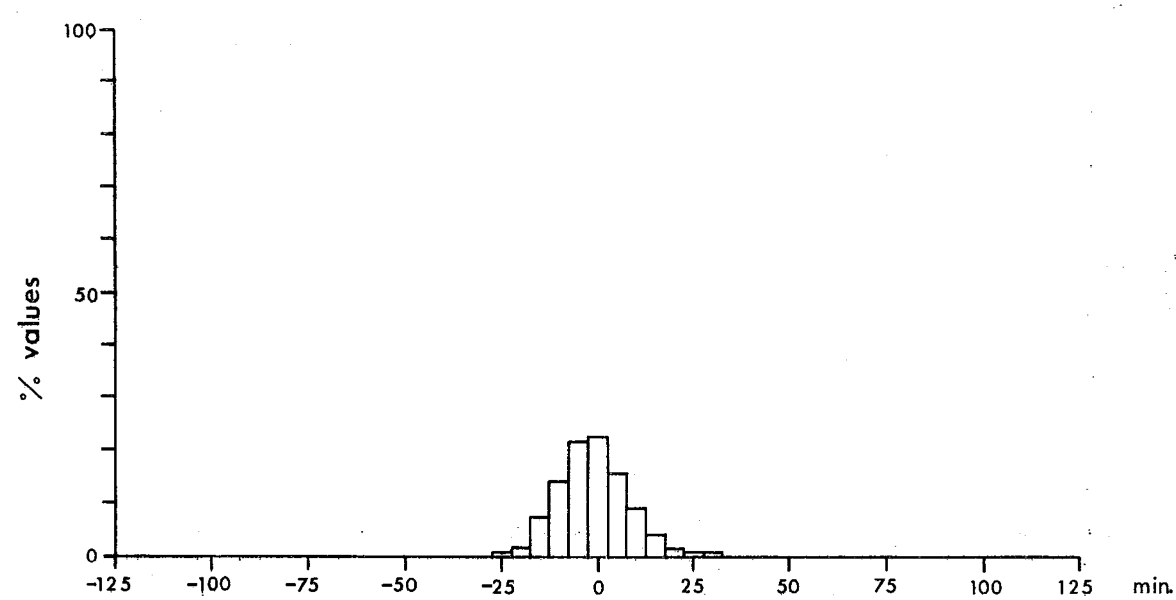


Fig.57 COMPARISON OF TIDE TABLE AND OBSERVED HIGHER HIGH AND LOWER LOW WATERS AT POINT ATKINSON 1970-1973 (Error = predicted - observed)



Height error at Point Atkinson



Time error at Point Atkinson

The effect of an error in height of 0.5 feet in the downstream boundary conditions upon the predicted water surface elevations at Deas Island and Mission is illustrated in Figure 58. A discharge of 51,000 cfs (November 13, 1971) was chosen because the tidal influence at Mission is more pronounced at low discharges.

It is interesting to note that the actual tide (used for the model-predictions in Figure 58) at Point Atkinson on November 13, 1971 was an average of 0.5 feet higher than predicted. The barometric pressure at Vancouver at 1000 hrs PST on this date was 1009.9 mb, 15.7 mb below the average annual pressure, and corresponding to a rise in sea water level of about 15 cm, or 0.5 feet above normal. (Figure 58 indicates the change in predicted levels that would result from such an error.)

Another possible source of errors is the assumption that the flow is homogeneous.

The model was calibrated for freshet conditions, because the model's behaviour is then more sensitive to adjustments of the friction coefficients. Moreover, as Figure 5 shows, even a low freshet tends to keep the salinity wedge outside the delta, thus confirming the supposition of homogeneous flow.

Comparisons between predicted and observed water levels at New Westminster at low discharges suggest that the effect of the saline wedge upon the river heights is negligible, although the exact relationship would be the concern of a two-layer model.

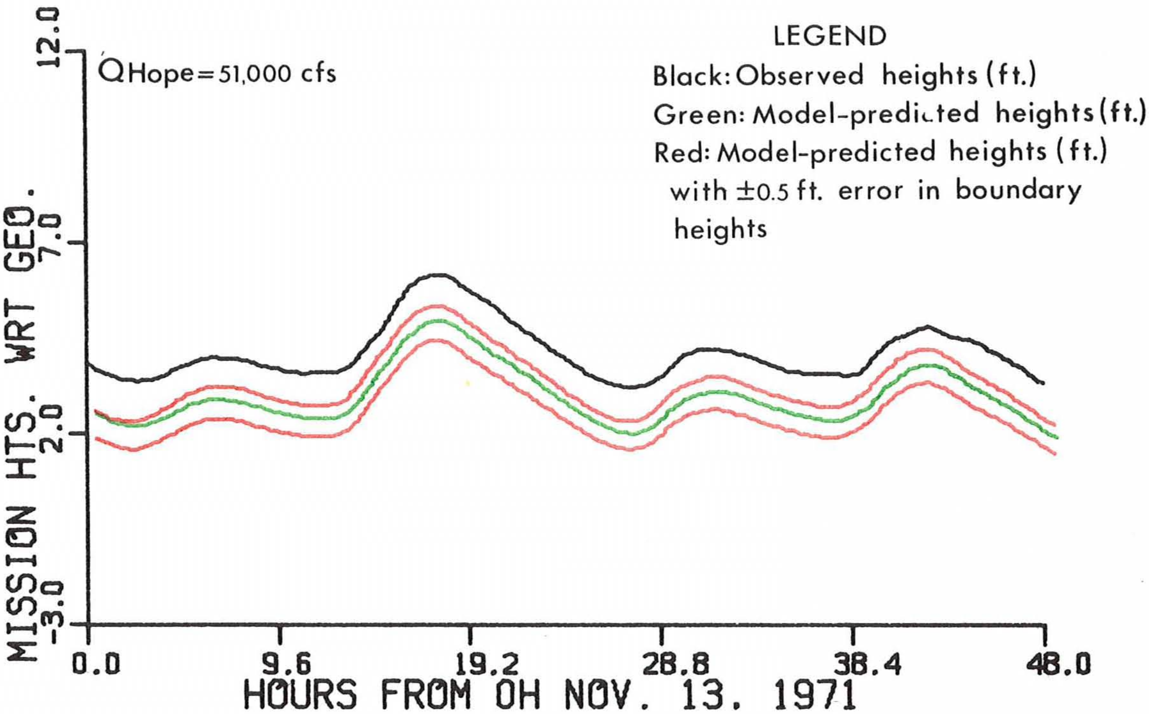
In the schematization, certain simplifications were made which were examined more closely upon the completion of the model:

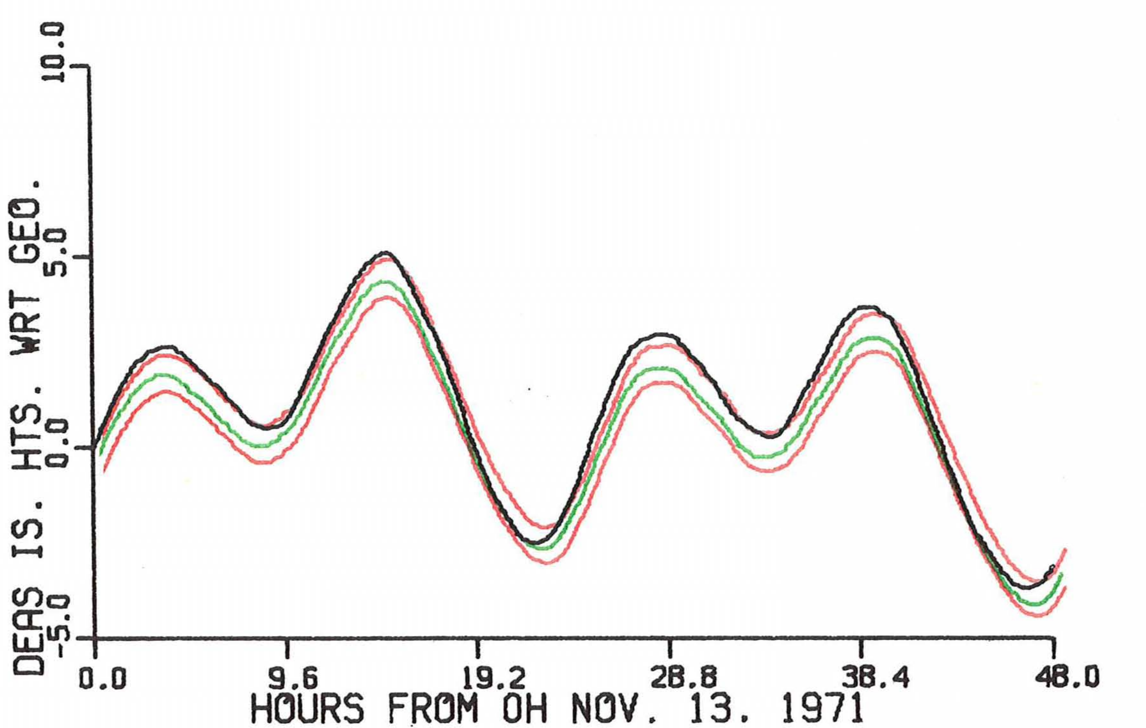
Local run-off, being difficult to assess, was neglected. To determine the consequences of this omission, a flash flood of two days' duration and a peak flow of 800 cfs was introduced into the model at Mission. With a discharge at Hope of 39,000 cfs, the maximum resulting change in the predicted river elevations was only 0.03 feet.

Annacis Channel, a four mile long arm west of New Westminster, was omitted from the schematization because no soundings were shown on the chart, and the channel is bypassed by the main flow, due to a causeway. Inclusion of Annacis Channel (conservatively estimating the depth as 15 feet) produced a discrepancy at New Westminster of -0.06 feet at high tide, and -0.23 feet at low tide, for a freshet discharge of 213,000 cfs at Hope; and of + 0.03 feet and -0.08 feet respectively for a discharge of 30,000 cfs.

The schematization was based on nautical charts which show soundings for normal conditions. However, the model was calibrated for freshet conditions, when the cross-sectional area of the Fraser River channel undergoes changes due to sedimentation. To determine the error in the schematization due to sedimentation, the amount of bed-load dredged annually from the delta (i.e. the amount of sediment deposited during the freshet) was compared with the river volume of the delta at zero tide and a discharge at Hope of 150,000 cfs. The average annual amount dredged between New Westminster and the Strait of Georgia (over all four distributaries) was calculated by the Department of Public Works to be about 4 x 10^6 tons, or 10^8 ft^3 of bed load. With a river

Fig.58 EFFECT OF ERRORS IN DOWNSTREAM BOUNDARY CONDITIONS (TIDAL HEIGHTS) UPON MODEL-PREDICTED HEIGHTS





volume between New Westminster and Georgia Strait of 120×10^8 ft^3, the percentage error in the schematization due to neglecting sedimentation is 0.8%. This percentage is only an overall figure and does not account for local sedimentation (bed waves) which would affect the schematization considerably but which would be difficult to estimate.

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REMARKS

Program Notes

For more efficient use of computer storage space, the matrix of the traditional "leap-frog" scheme shown in Figure 8 was not stored in full by the program. Instead, at the end of the calculations for the time step k + 1, H (k,m) and Q (k + 1,m) are retained in the QH (m) array, and the value of H (k,m) - H (k - 2,m) is retained in the ΔH (m) array, for all appropriate m. These are the only variables which are necessary for calculations at the next time step. Any values which are required for subsequent analysis can be stored elsewhere for later reference. Thus, the 2-dimensional array QH (n,m) can be replaced by the two 1-dimensional arrays QH (m) and ΔH (m), drastically reducing the amount of storage space required since these arrays are independent of the simulation time.

The program was written in Fortran and requires 20 K (20,000) words of storage for execution. Fast Fourier Transform routines were used for the interpolation of tidal data. The program was executed on the Univac 1108 operated by Computer Sciences Canada, Ltd. at Calgary, Alberta. Input/output was performed on both a conversational teletype terminal and a batch terminal interfaced with a card reader and a line printer. Plotting instructions were written on magnetic tape by various routines developed on the IBM 370/168 at the University of British Columbia in Vancouver. The plots were subsequently produced on a local Calcomp 563 plotter interfaced with a Hewlett-Packard 2114A mini-computer.

For an average run of three days simulated time, with height predictions printed out at fifteen minute intervals for six locations, the model requires about 70 seconds of CPU (Central Processing Unit) time.

Least-Squares Polynomial Approximation

Given (n + 1) pairs of values (x_o,y_o), (x_1,y_1), ..., (x_n,y_n), where only the y-values are experimentally produced, we require a polynomial y of degree m

$$y = a_o + a_1x + \dots + a_mx^m$$

which fits the given points as well as possible. When $m < n$, the coefficients $a_o, a_1, \dots a_m$ are determined by minimizing

$$S = \sum_{j=o}^{n} (a_o + a_1x_j + \dots + a_mx_j^m - y_j)^2.$$

Fourier Series Interpolation

Fourier series were used to interpolate tidal heights for the model boundary conditions from hourly intervals to 15 minute intervals. The two subroutines used were written by J.R. Wilson of the Institute of Oceanography at the University of British Columbia. For the given data points, the first call to the subroutines performs a Fourier analysis, calculating the coefficients of the Fourier series. The second reference to the subroutines applies a Fourier synthesis to these coefficients, producing data points at the required intervals.

Harmonic Method of Tidal Prediction

The tidal predictions for Point Atkinson and Tsawwassen follow the "Manual of Harmonic Analysis and Prediction of Tides" of the U.S. Coast and Geodetic Survey, using the general equation

$$h = H_o + \sum fH \cos [at + \text{Greenwich } (V_o + u) - g]$$

where h = height of tide at time t;

H_o = mean height of water level above datum;

H = mean amplitude of constituent;

f = factor for reducing mean amplitude H to year of prediction;

a = speed of constituent

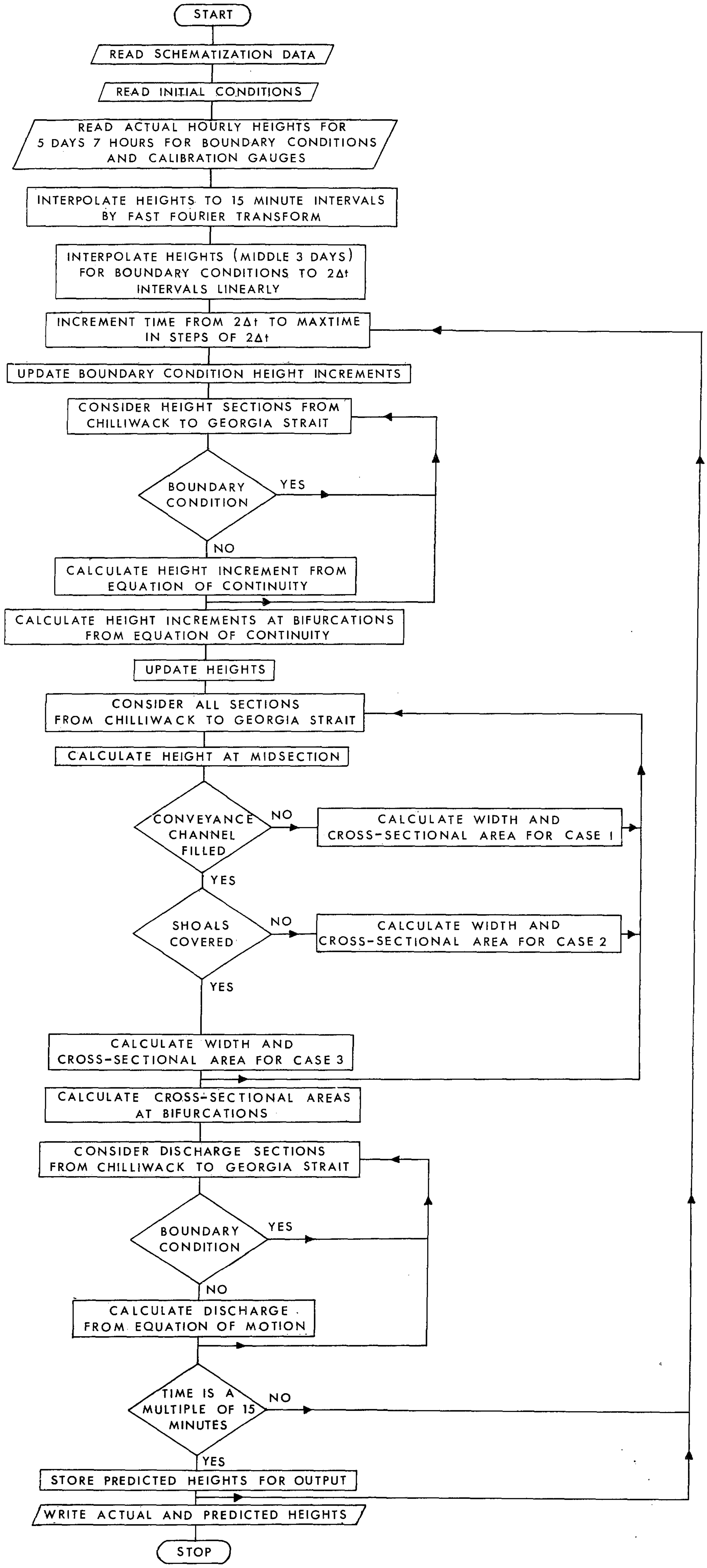
Greenwich $(V_o + u)$ = value of equilibrium argument at Greenwich when $t = o$;

g = modified epoch (or phase lag) of constituent.

A total of 50 constituents were used for the prediction of tidal heights for boundary conditions in the model. This total included 23 shallow water constituents.

The amplitude and modified epoch for each constituent vary with location, and were obtained from "Harmonic Constants and Associated Data for Canadian Tidal Waters", Tides and Water Levels, D.O.E. The speed (or frequency) of each constituent is listed in Appendix 2 of "The Analysis of Tides" by Gabriel Godin. The factor and equilibrium argument vary with the date and were calculated by program ASTRO, developed by Godin. The program uses Doodson numbers which are given in Appendix 1 of "The Analysis of Tides". Various other ratios required as data have also been developed by Godin from the information given in the appendices to his book.

Fig. 59 FLOWCHART



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TERMINOLOGY

Since several disciplines are involved in the project, some definitions of the terms used in this report may be informative.

Barotropic:

the density does not vary along isobaric surfaces in the estuary; more specifically, a barotropic model of an estuary ignores the salt wedge.

Bed load:

coarse material which rolls along the bottom of the river.

Bed wave:

sand dunes on the bed of the river which migrate slowly downstream.

Collimation error (vertical):

error in a surveyor's level due to the line of sight not being parallel to the bubble tube axis. This error is eliminated if the distances of backsight and foresight are equal.

Diurnal tide:

one complete tidal oscillation per day (one high, one low water).

Explicit scheme in a one-dimensional model:

during each computation, one unknown (Q or h) is calculated from a set of previously obtained values. The result is subsequently used to calculate the next Q or h in distance or time. (An implicit scheme computes all values of Q and h at time step t + Δt from the known ones at step t, requiring a large number of simultaneous equations.)

Dynamic pressure head:

the velocity term $v^2/2g$ in Bernouilli's equation. A high local water velocity would "depress" the water surface at the tide gauge significantly, decreasing the static pressure head measured by the tide gauge. In such a case, the gauge readings do not truly represent the tidal heights in the general vicinity.

Geodetic datum:

based on mean sea level prior to 1929 and computed from gauge readings at Caulfeild Cove (Pt. Atkinson).

Geopotential anomaly:

defined as $\Delta D = \int_{p_1}^{p_2} \delta dp$, where δ is the specific volume anomaly (a function of temperature, salinity and pressure) and p_1, p_2 represent the isobaric surfaces. In the Strait of Georgia near the mouth of the Fraser, we put $\Delta D = \int_{328\ ft}^{0} \delta dp$, assuming 328 feet, 100 metres (or about 100 decibars) as the depth of no motion. Different anomalies at two stations outside the mouth of the Fraser indicate a slope of the sea surface (0 decibars).

Freshet:

in this report, a freshet is defined as a discharge at Hope exceeding 100,000 cfs.

Harmonic analysis:

the observed tidal data are separated into a number of harmonic constituents. The analysis leads to amplitudes and phase relations, called harmonic constants, which subsequently are used for "harmonic prediction".

Hydraulic radius:

the cross-sectional area of the channel divided by the wetted perimeter (the portion of the perimeter where the wall is in contact with the fluid).

Hydrograph:

a graphical record of the daily discharge measurements.

Neap tide:

occurs shortly after a first or a third quarter of the moon and has the smallest range in half a lunar month.

Saltation load:

sediment which is transported by bouncing along the bed.

Semi-diurnal tide:

two complete tidal oscillations per day.

Spring tide:

occurs shortly after full or new moon and has the largest range in half a lunar month.

Suspended load:

sediment particles of a size comparable to those in the bed load, but which are kept in the flow area by turbulence, and occasionally fall to the bed.

Wash load:

very fine particles which do not tend to settle out of suspension.

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