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# AN EVALUATION OF THE TRISPONDER POSITIONING SYSTEM

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BY

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# A B S T R A C T

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Trisponder Positioning System

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A series of tests made to evaluate the Trisponder Positioning System for use in hydrographic survey is described. The accuracy, range and operational characteristics are assessed. The various antenna configurations available for use with this system are described.

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# ILLUSTRATIONS

Diagram

## INTRODUCTION

The Trisponder Positioning System, Model 202A, is a line of sight range-range, positioning system, operating in the X-band range of frequencies. The system was loaned by ComDev Marine (C.D.C.) to the Development Group, Pacific Region, Canadian Hydrographic Service, to be evaluated for hydrographic use. The Development Group assessed the accuracy and operational range of the system, the effect of radar interference on the signal stability, and the operational suitability of the system for inshore hydrographic surveys.

The Development Group had the use of the Trisponder system from the 12th July to the 9th August and from the 9th November to the 21st December 1971. The second period of evaluation was necessary as the transponders had not operated efficiently during the summer, also two new antennae were available for testing. All tests were made in the Victoria area. (See diagram #13)

In these evaluations, only one position line generated by the Trisponder was considered; the accuracy of a position depending on the geometry in which the system is deployed.

## THE EQUIPMENT

The Trisponder Positioning System consists of a distance measuring unit, a base unit and up to four transponders. Other peripheral equipment, such as printers and repeaters are also available.

## THE DISTANCE MEASURING UNIT (D.M.U.)

This unit is a light, portable, splash-proof box (16 x 12 x 8 1/2 inches). Ranges from two transponders are displayed on seven-bar digital display tubes in kilometres and metres, to a resolution of ten metres or one metre. The controls for the system are on the face of the D.M.U. It is powered by two 12 volt batteries with a consumption of 1.5 amps at 24 volts.

The functions of the electronics inside the D.M.U. are to:

1. establish an R.F. link with a transponder;
2. determine if the received range signal is valid by checking its pulse repetition frequency (P.R.F.);
3. time the round trip of a pulse with a 29.971 mega hertz (mhz) clock, and account for transponder delay;
4. accumulate ten valid range readings for one metre resolution;
5. reinitiate the sequence if ten valid readings are received, or if fifty readings are rejected;
6. display accumulated data;
7. repeat for a second transponder.

All tests were conducted using one metre resolution on the display. The D.M.U. functioned well during the tests, however, some salt water corrosion necessitated the replacement of three of the range display tubes.

THE BASE UNIT

The base unit is a transceiver operating at the control of the D.M.U. It is powered from the D.M.U.; the two units being connected by co-axial cable. A short length of waveguide connects the transceiver to an omni antenna. By placing the transceiver in the same unit as the antenna, long lengths of waveguide are eliminated and the system's portability is increased. The base unit was mounted on a mast giving an antenna height of 9 ft. for launch operations. No trouble was experienced from this unit for the period of the tests.

THE TRANSPONDERS

Up to four transponders can be deployed at known shore stations. Any two of the four can be interrogated simultaneously from the D.M.U. These units are composed of an antenna, a transceiver and decoder. The decoder will accept only signals with a precisely controlled P.R.F. The use of a precise P.R.F. enables the system to distinguish between transponders and to eliminate much radar interference.

Power is supplied to the transponder from two 12 volt batteries. Power consumption is at the rate of 1.2 amps. At this rate of consumption, two 90 amp/hour batteries could theoretically maintain the transponders for 2 1/2 days operation in the field. However, it was found that two days unattended operation of the transponders was all that was practical, without excessively discharging the batteries. If a transponder site is to be occupied for any length of time, it may be practical to use thermoelectric generators to keep the batteries charged.

The range and accuracy obtained from the Trisponder positioning system depends upon the efficient operation of the transponders. During the test period, five transponders were used. Only two of the five transponders maintained efficient operation for any length of time. These two transponders, after careful "tuning" by a C.D.C. engineer, remained in efficient operation for four weeks until the end of the tests. During the tests in July and August, the transponders were functioning at low efficiency. Therefore, the maximum range for reception of a stable signal was as low as 12 kilometres (kms). The tests at the beginning of November gave similar results. After the transponders had been "set up" by a C.D.C. engineer, the maximum range for reception of a stable signal was 20 kms in the centre of the antenna beam pattern.

New transponders are being developed for the Trisponder system and these new units may prove to be more reliable than those currently in use.

BASE UNIT ANTENNA (See diagram #9)

This antenna is a circular waveguide slotted array. It has a pattern of 360° in the horizontal plane, and 15°, to half power points, in the vertical plane. It has a gain of 6 decibels (db). The radiation pattern (vertical plane) for this antenna is shown on diagram #14. This antenna is mounted above the base unit on the launch.

For launch use, the 15° vertical beam width of the omni antenna allows the vessel to roll to less than 7 1/2° at long ranges before signal degradation takes place. To measure the effect of the omni antenna moving in a vertical plane, tests were made ashore at ranges of 15 kms and 1.2 kms. At 15 kms, moving the antenna from the vertical to 7° off vertical (with no horizontal movement) these tests showed an increase in instability from ±1.2 metres to ±5.2 metres, and an increase in the mean measured distance of +12 metres. At a range of 1.2 kms it was possible to move the antenna more than 7° off vertical without degrading signal. The data from both these tests is listed on diagram #21.

An omni antenna with a 30° vertical beam width is at present under development for the Trisponder system. This new antenna should provide a better quality signal in a moving launch. The new antenna's gain is reported to be 6 db; therefore, the overall range of the system should not be affected.

## TRANSPONDER ANTENNAE

### 45° Antenna (See diagram #10)

This antenna is a small directional horn antenna with a gain of 16 db. It has a beam of 45° to half power points, in both the horizontal and vertical planes. The radiation pattern (horizontal plane) for this antenna is shown on diagram #15. The wide vertical beam pattern makes this antenna suitable for use in helicopter operations, but much of the signal strength is wasted when using a surface vehicle.

The horizontal beam pattern of the 45° antenna has been defined for stability when used in conjunction with a launch (see diagram #1). The tests to define the usable area of coverage of this antenna were limited by both low antenna heights, and by inefficiency of transponder operation. The diagram, therefore, presents a conservative estimate of the system's capability when using this antenna.

### 60° Antenna (See diagram #11)

This antenna is a large directional horn antenna with a gain of 13 db. It has a horizontal beam width of 60° to half power points and a vertical beam width of 12° to half power points. The radiation pattern (horizontal plane) for this antenna is shown on diagram #16. It is a cumbersome antenna, about 18 inches (ins.) long and up to 10 ins. in height. During the tests, the signal from this antenna was lost well before line of sight was reached. The area of coverage of a usable signal from this antenna is shown on diagram 2. The maximum range for the system, using the 60° antenna, was found to be 13 kms.

84° Antenna (See diagram #12)

This antenna is a rectangular slotted waveguide array, with a gain of 17 db. To half power points, it has a horizontal beam width of 84° and a vertical beam width of 5°. The radiation pattern (horizontal plane) for this antenna is shown on diagram #17. The narrow vertical beam is adequate for surface use if the transponder is level.

Tests were made for accuracy and stability of the system throughout the beam pattern of the 84° antenna. Diagram numbers 3 and 4 illustrate this coverage. Because of its wide horizontal beam width and higher gain, this antenna appears to be the best suited, of the three antennae tested, for use in inshore hydrographic survey.

## INSTALLATION AND OPERATION

The Trisponder system is very portable and easily installed. The base unit and the transponders can be mounted on standard tripod fittings. The only consideration to be made when setting up these units is that they be placed so as to be well clear of any obstructions.

The operation of the D.M.U. is simple and all controls are clearly labelled and their functions obvious.

Calibration of the system is achieved by comparing the measurement given by the Trisponder over a short baseline to the accurate known measurement for that base. The range displays are then adjusted to show the correct distance. Residual errors from calibration are small when compared to other errors in the system.

## METHOD OF TESTS

### Range and Stability

To establish the operational stability of the Trisponder system at various ranges, a launch was steered along an arc of constant range from a transponder. The observed range was noted at approximately five second intervals. As a launch cannot be steered exactly along an arc, and because of fluctuations in the range indicated by the Trisponder, it was necessary to fit a curve to the data to estimate the launch's actual movement. The differences between the estimated launch line and the observed Trisponder readings were taken as a measure of signal stability. (This technique was used for the evaluation of the Motorola Range Positioning system.) For the tests of the 45° horn antenna, the differences were found graphically. For the 60° and 84° antennae, a very close series of linear regressions were computed from the raw Trisponder data to estimate the launch's movement. The residuals from these regressions were taken as a measure of the instrument's stability. The algorithm for the computer program to process stability data is given on diagram #18. An example, from the computer print-out, of a launch line estimated from observed Trisponder data is shown on diagram #19. When the graphical and computed methods were compared, little difference was found in the results. In both cases, for the purpose of computing the estimated launch line only, radar interference and other large erroneous readings were filtered out.

The stability tests were made by running launch lines over short arcs at the centre of the antenna beam pattern for various transponder heights. These heights ranged from 18 feet to 130 feet, with a launch antenna height of 9 feet. For tests at ranges greater than 21 kms, a launch antenna height of 15 feet was used. The data obtained from these tests is tabulated on diagram #5.

The limits of the beam pattern generated by the 45° horn antenna were defined for stability with this method. In this case, stability lines were run throughout the beam pattern. Diagram #1 shows a ±5 metre contour for stability of the system. Beyond the ±5 metre contour, the stability of the signal decays rapidly.

The stability of the beam pattern generated by the 84° and the 60° horizontal beam width antennae are shown on diagrams #2 and #4. The contours on these diagrams were developed from stability lines run at various ranges, and from data obtained on tests ashore. Tests ashore were made over four base lines. A set of Trisponder observations was taken with transponder antenna pointing at the receiver. The transponder antenna was then turned through 30° increments, sets of readings being taken at each increment, until the signal was lost. For each set of readings, the standard deviation was calculated. The angle and range at which there was a marked increase in the standard deviation of the sets was taken as the limit of stable signal for the beam pattern.

Precision

To assess the precision of the Trisponder Positioning System, sets of readings were taken over ten base lines. The base lines had been previously measured by tellurometer. These measurements were made at the centre of the beam pattern of the 45° antenna. The median, mean and standard deviation of each set of observations, for both transponders, were calculated. The differences between the tellurometer distances and the Trisponder data are tabulated on diagram #6. One set of observations at 21 kms showed the effect of extreme pulse decay. Another set, at 15 kms, also seems to indicate attenuation due to the proximity of the bases to line of sight or to transponder inefficiency. A set of observations at 3 kms was affected by intense radar activity. The root mean square (R.M.S.) error for the observations (excluding the one set suffering extreme pulse decay) was $\pm 3.3$ metres. The R.M.S. error for all observations not affected by pulse decay or radar interference was $\pm 2.8$ metres.

To establish if the precision found at the centre of the antenna beam was maintained throughout the beam pattern, further tests were made.

For the 45° horn antenna, these tests were conducted from a launch. At several ranges from the transponders, sets of simultaneous observations were made with both transponders pointing at the launch. One transponder was then turned to 22° and 45°, and sets of simultaneous observations were taken to compare the signal from the offset transponder to the signal at centre beam. The data obtained from these tests is tabulated on diagram #7. The observations were corrected to eliminate any errors arising from differences in calibration of the two transponders and for the short distance separating the two transponders.

For the 84° horizontal beam width antenna, the tests to define the precision throughout the beam pattern were conducted over four base lines ashore. Sets of Trisponder distances were taken at 15, 12, 9 and 5 kms. At each range, up to seven sets were observed by turning the transponder from centre beam through 30° increments, to cover the beam pattern. The results are tabulated on diagram #7, and an example of the computer print-out for the precision tests is shown on diagram #20. Similar data was also obtained for the 60° horn antenna.

Precision tests, for the three transponder antennae, indicate that systematic errors exist in observed Trisponder ranges. These errors are negative and increase away from the centre of the beam until the effect of pulse decay becomes appreciable at the edges of the beam. At the edges of the beam, where pulse decay takes over, the error becomes positive and large. Also, the indicated range is very unstable at the edges of the beam. Diagram #3 shows contours for these errors. They have been treated as being random, as it is at present impractical to carry out lengthy calibration procedures or to allow for these errors when using the system in the field.

Repeatability

To check the repeatability of the Trisponder, the system was taken at approximately weekly intervals to the same short base line (1196.6 metres), where sets of observed ranges were taken. If necessary, a calibration adjustment was then made and another set of readings noted. The difference between the after-calibration readings and the before-calibration readings over the seven day interval were taken as a measure of the system's repeatability. This difference, on two occasions, exceeded one metre. When the tests were completed, it became apparent that if the drift exceeded one metre, then the transponders' efficiency had decreased considerably. The transponders did, in fact, drift as much as 2.7 metres.

## Radar Interference

The approaches to Victoria, where the Trisponder tests were conducted, are subject to intense radar activity. At short and medium ranges, radar interference is easily detected by continuous monitoring of the display. Radar interference was observed to cause large errors in up to 4% of the displayed ranges. At the edges of an antenna's beam pattern, radar interference cannot be isolated from the effects of attenuation.

Two instances of radar interference are given to exemplify the problem:

1. When comparing the Trisponder to a tellurometer measured base line (3.0 kms), at least six large naval vessels were operating in the immediate area. The standard deviation of the observed ranges, obtained in this test, was double that usually found at this short range.

2. When running a short launch line to assess the system's stability at 12 kms from the transponders, a destroyer escort was operating in close proximity to the launch. Radar interference, assumed to be from this source, increased the system's instability to ±10 metres, from ±1 metre found on another occasion at this range.

## RESULTS

### Range and Stability

There is a marked breakdown in the stability obtained from the Trisponder positioning system as:

1. line of sight is approached.
2. the edges of the antenna beam pattern are reached.
3. the system's maximum range is reached.

The range at which a stable signal can be received appears to be about 10% less than the range derived from the formula

$$\text{Range} = 1.22 \left( \sqrt{\text{Height}_1} + \sqrt{\text{Height}_2} \right)$$

(Heights are for the transponders and base unit antenna
Range in nautical miles, heights in feet)

It is not unusual to obtain a stability of signal of ±2 metres well within the beam. Towards the edges of the pattern, the instability increases to ±5 metres. From there on out the signal rapidly becomes unusable.

The system has a maximum advertised range of 24 kms. Stability data obtained at 24 kms shows an instability of ±10 metres, at the centre of the antenna beam. At 20 kms the instability was found to be ±3 metres. There is a marked decline of stability between 20 and 24 kms, which suggests that the maximum operational range for launch use is about 20 kms.

The R.M.S. stability for all tests made within 20 kms of the transponder, including those affected by radar interference was ±3.8 metres ($E_1$). When comparing the three transponder antennae, range is directly affected by antenna gain. The 60° horizontal beam width antenna was found to have a maximum range of about 12 kms. The 84° and 45° antennae have ranges to the power limits of the system.

## Precision

By combining the results of precision tests at all ranges and angles within stable signal range of a transponder, an estimate of the Trisponder system's precision can be made. An R.M.S. error of ±3.1 ($E_2$) metres was found from averaged observations for all parts of the beam pattern. There is no significant difference in the results of the precision tests for the three transponder antennae.

## Repeatability

If calibration checks are made frequently (weekly), and if the transponders remain "in tune", the drift with time of the Trisponder system should not exceed ±1 metre ($E_3$).

## Overall Accuracy

By summing the results of the tests for stability, precision and repeatability, an estimate of the Trisponder system's accuracy can be made. If

$$\text{Total Error} = \pm\sqrt{E_1^2 + E_2^2 + E_3^2}$$

then, for one position line developed by this system, an R.M.S. error of ±5 metres can be expected. At the fringes of an antenna's beam pattern, the error can be expected to increase to ±7 metres before the signal becomes unusable. Diagram #8 shows contours derived from the combined tests for the accuracy of the Trisponder system, for one position line only.

## CONCLUSIONS

1. The transponders used with the Trisponder system were the only major source of trouble encountered during the tests. Their record for reliability is poor. However, a new transponder is being developed which may prove to be more satisfactory. At all times, for efficient operation of the system, the transponders must be kept in tune.

2. Of the antennae used with the transponders, the 84° horizontal beam width antenna is the most suitable for hydrographic survey.

   The 15° vertical beam width omni antenna used on the launch limits the system's stability at long ranges. A 30° vertical beam width omni antenna, at present being developed, should prove more suitable if it has a 6 db gain.

3. The D.M.U. and the base unit operated efficiently throughout the tests. The D.M.U. provides a good display and simple controls. The absence of waveguide makes these two units easily portable.

4. The power consumption of the transponders is rather high. For field use, the transponders would have to be serviced every two days, which may prove to be operationally inconvenient.

5. The Trisponder system is capable of displaying ranges from any two of four transponders. This feature could have advantages for use on specialized surveys.

6. Radar interference did not appear to be a major problem during the tests, although at times it was necessary to suspend operations until the source of the interference was out of the working area.

7. The Trisponder Positioning System will provide position lines with an accuracy of ±5 metres within the beam pattern of the antenna used, and of ±7 metres at the fringes of the beam pattern. For these accuracies it has a maximum range of 20 kms.

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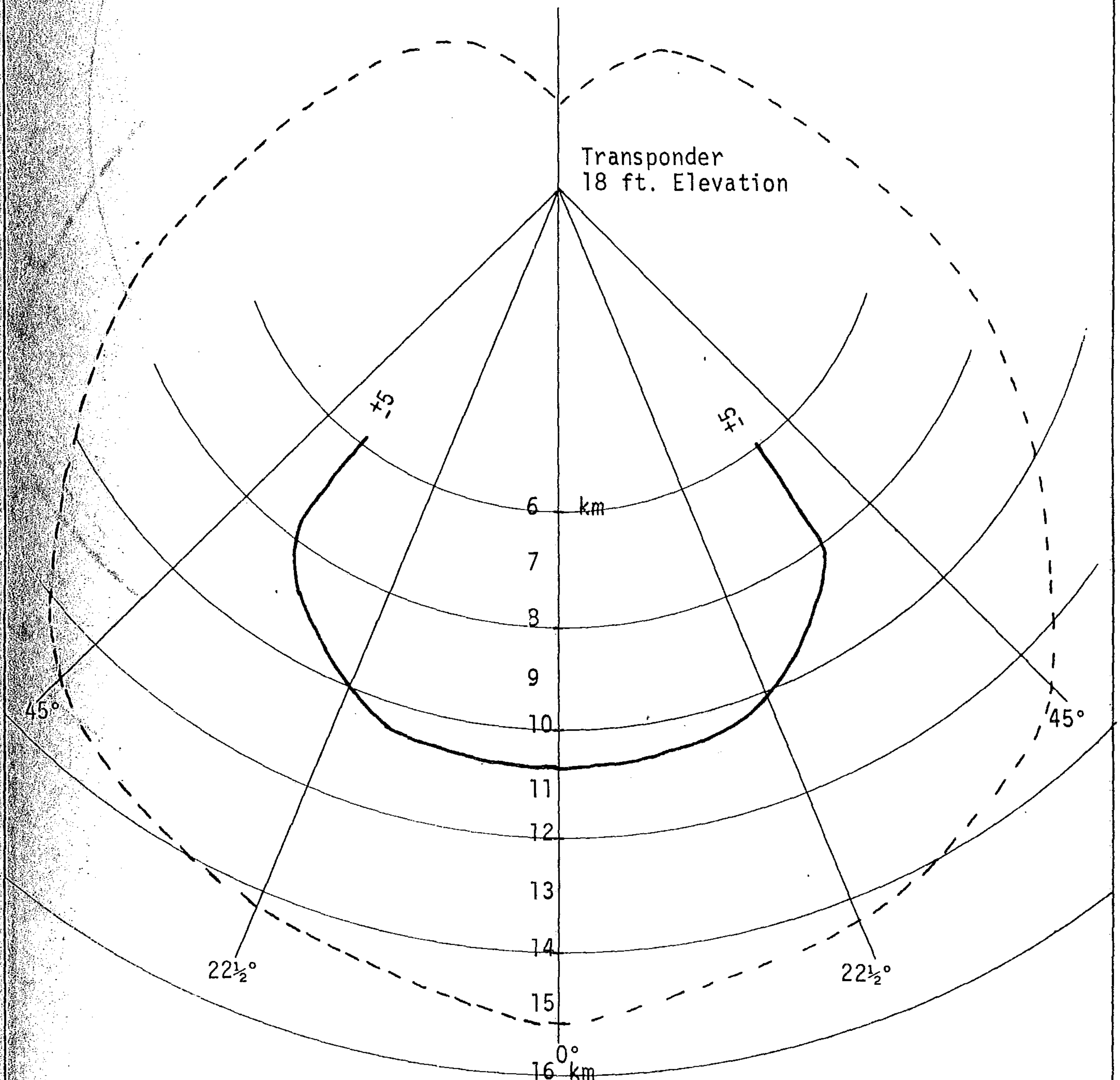
# I L L U S T R A T I O N S




Transmitter Antenna Height 9 ft.

Range of Signal Loss ---

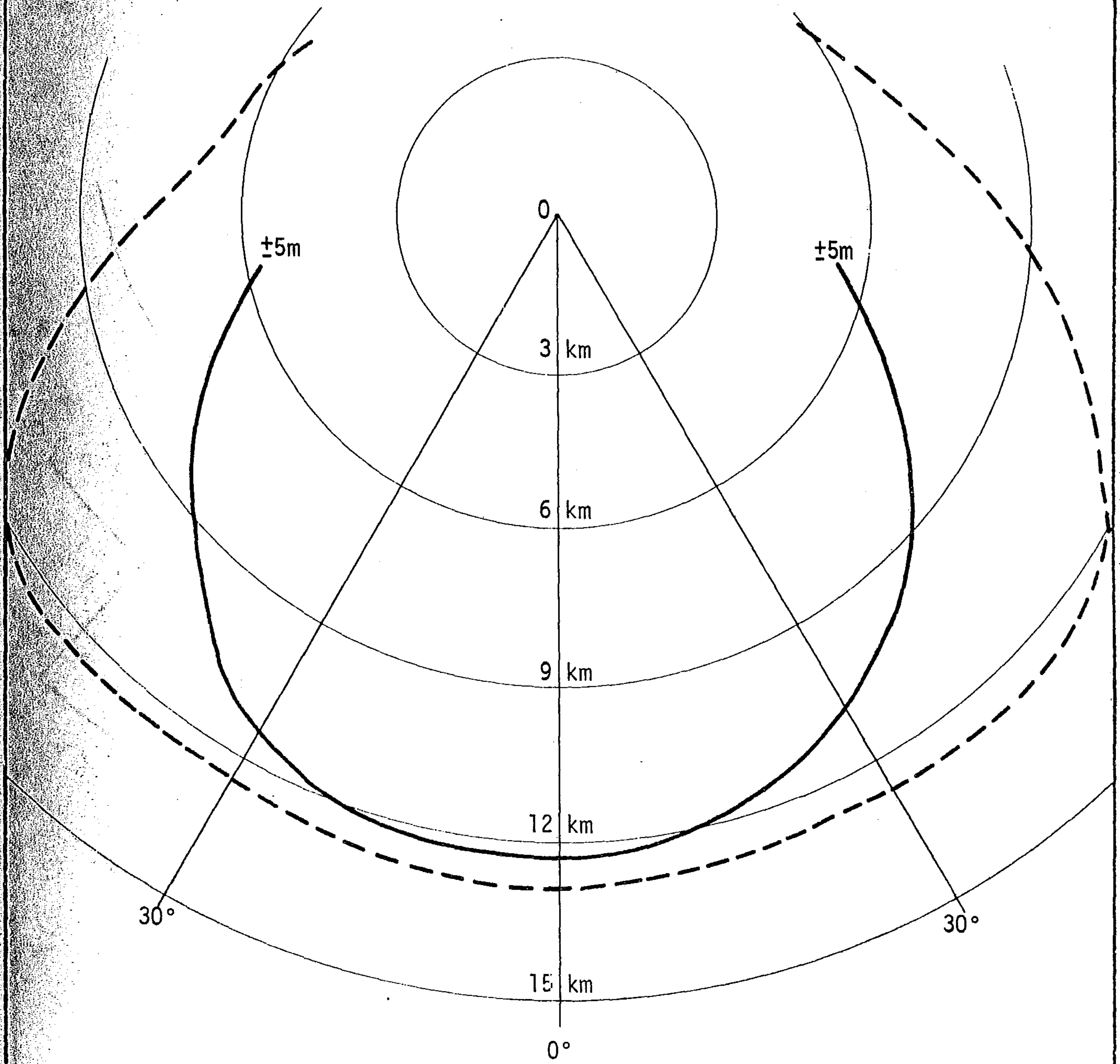
Range of Stable Signal ——

Trisponder

Beam Pattern

45° Horn Antenna

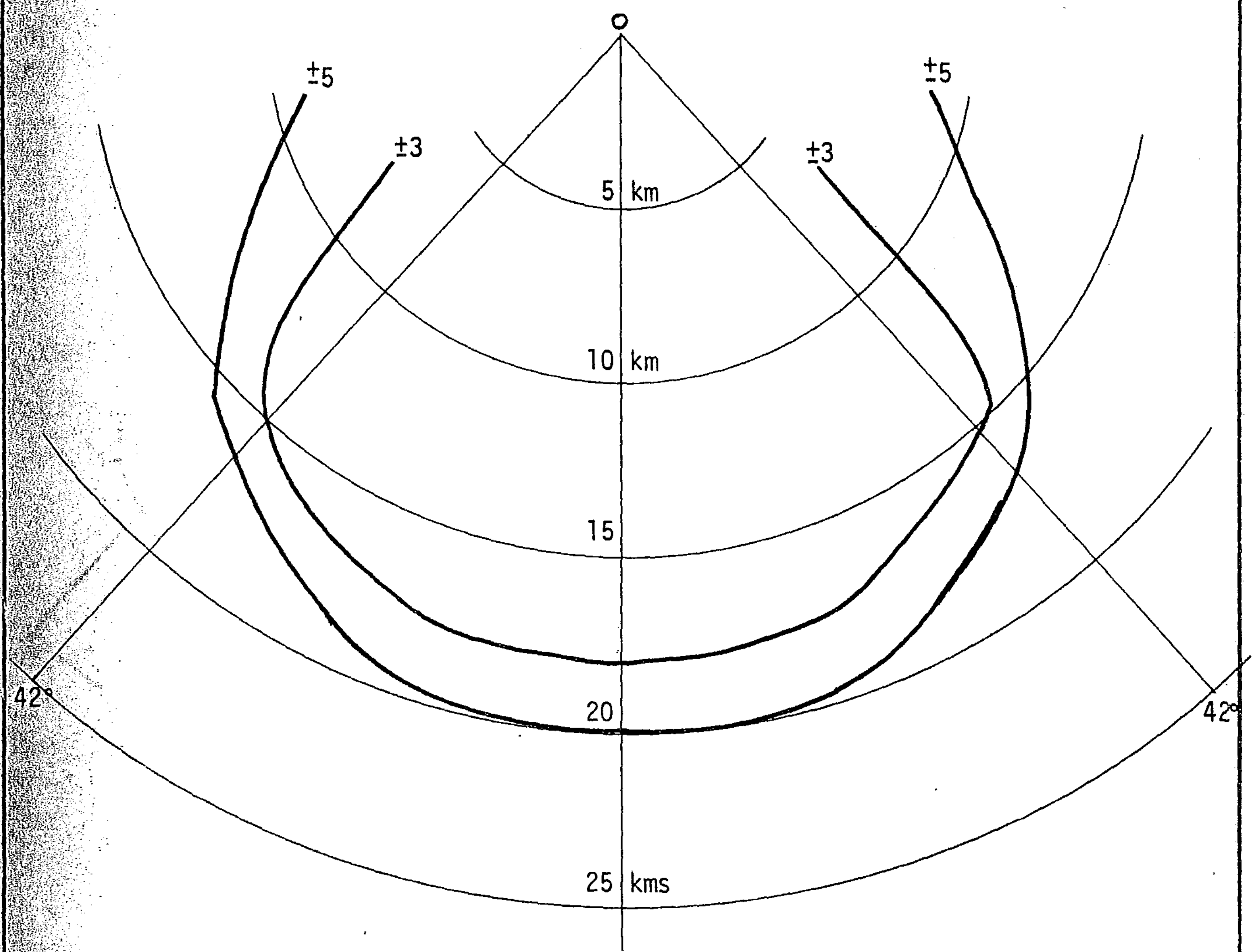
Diagram #1




TRISPONDER
STABILITY
60° ANTENNA

Transponder Antenna Height <130 ft.
Range of Signal Loss ------
Range of Stable Signal ———
Transmitter Antenna Height 9 ft.
Stability Contour in Metres
Range in Kilometres

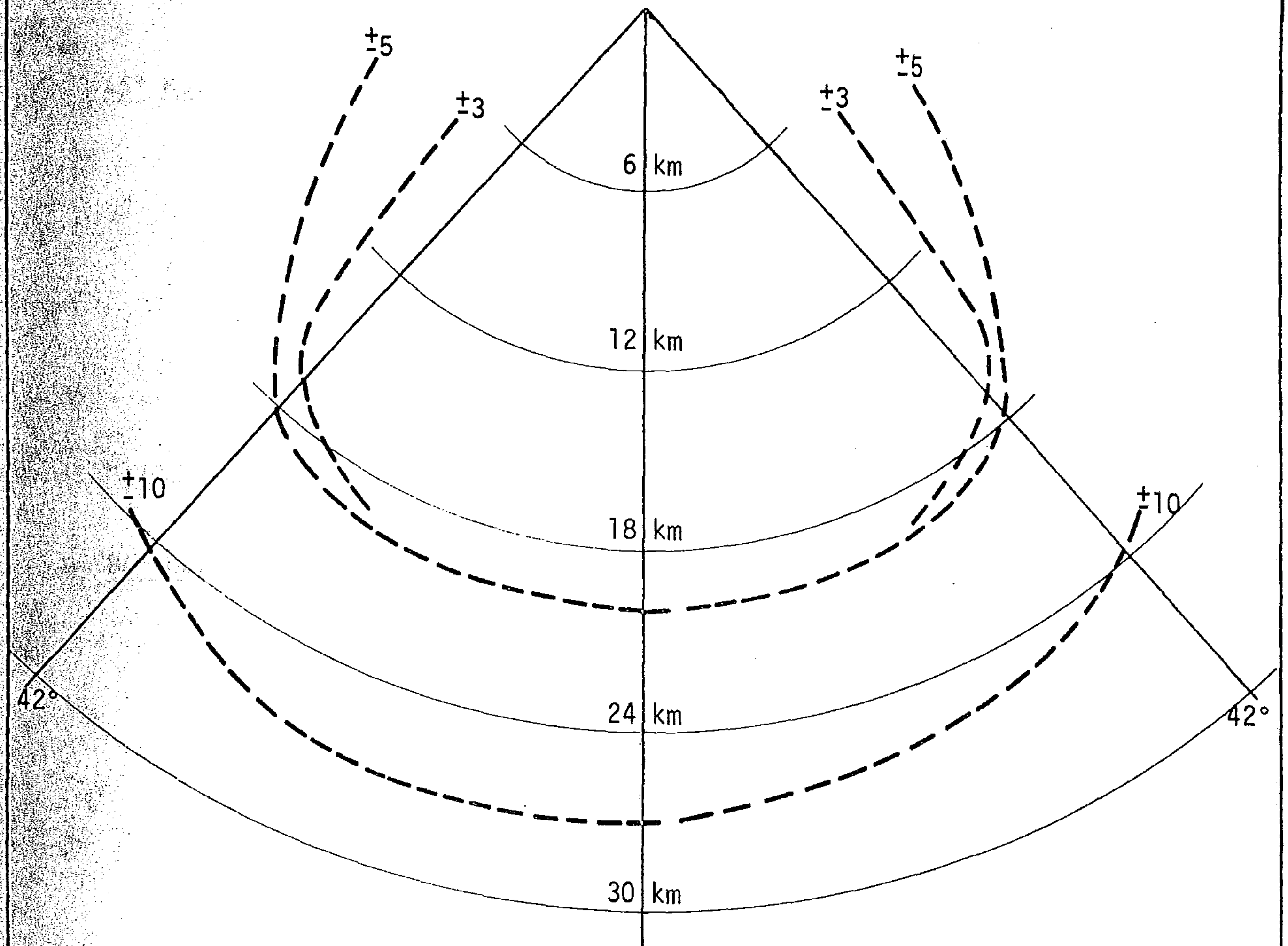
Diagram #2




TRISPONDER
PRECISION
OF AVERAGED READINGS

Transponder Antenna Heights < 130 ft.
Contours in Metres
Range in Kilometres
Transmitter Antenna Height 9 ft.
84° Antenna

Diagram #3




TRISPONDER
STABILITY
84° ANTENNA

Transponder Antenna Heights 130 ft.
Stability Contours in Metres
Range in Kilometres
Transmitter Antenna Height 15 ft.

Diagram #4

## PRECISION TESTS

Comparison of Trisponder to Tellurometer Measurements.

### Transponder 'A'

| Range (km) | Median Difference (m) | Stnd. Dev. (m) | |
|---|---|---|---|
| 0.9 | +2.63 | ±1.65 | |
| 2.1 | +4.10 | ±1.46 | |
| 2.9 | +5.67 | ±2.83 | # |
| 4.9 | +2.69 | ±0.85 | |
| 8.9 | +2.26 | ±1.37 | |
| 9.8 | -1.22 | ±1.95 | |
| 12.0 | -4.23 | ±1.03 | |
| 14.8 | +1.35 | ±6.57 | * |
| 17.0 | -1.18 | ±1.41 | |
| 21.5 | +21.58 | ±5.87 | * |

### Transponder 'B'

| Range (km) | Median Difference (m) | Stnd. Dev. (m) |
|---|---|---|
| 0.9 | +0.08 | ±1.88 |
| 2.1 | +1.62 | ±1.19 |
| 2.9 | +1.05 | ±1.41 |
| 4.9 | +0.66 | ±0.94 |
| 8.9 | -5.43 | ±1.58 |
| 9.8 | -2.98 | ±1.50 |
| 12.0 | +2.75 | ±1.26 |
| 14.8 | -3.30 | ±1.30 |
| 17.0 | -5.87 | ±2.35 |
| 21.5 | +3.40 | ±2.31 |

# = Radar Interference          * = Attenuation

N.B. All observations made at the centre of the beam pattern.

Diagram #6

BEAM PATTERN TESTS

The mean differences between sets of observations at centre beam and at various angles throughout the beam are tabulated in metres.

45° Horizontal Beam Width Antenna

| Range km | Angle from Centre Beam 22° | 45° |
|---|---|---|
| 5 | - | -2.1 |
| 7 | -2.7 | +20.2 |
| 9 | -4.7 | +6.7 |
| 10 | -2.4 | +18.2 |
| 11 | +6.3 | |

84° Horizontal Beam Width Antenna

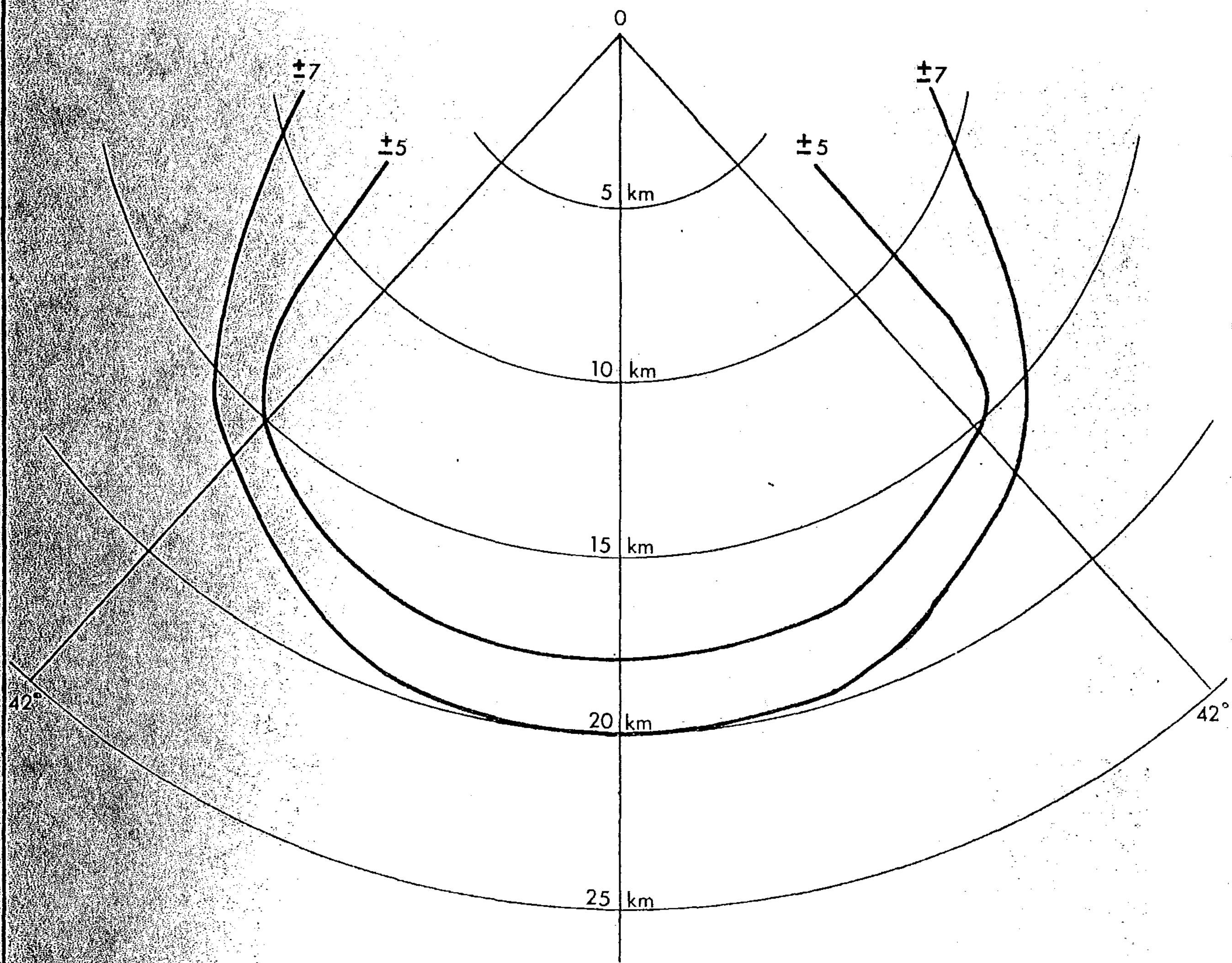
| Range km | Angle from Centre Beam 30° | 60° | 90° | 120° | 150° | 180° |
|---|---|---|---|---|---|---|
| 5 | -0.2 | -0.4 | - | -1.6 | -2.4 | -6.2 |
| 9 | -2.2 | -5.3 | -5.1 | -4.6 | (+3.6 at 135°) | |
| 12 | -2.0 | -5.3 | -3.3 | +8.8 | | |
| 15 | +0.6 | +18.5 | | | | |

60° Horizontal Beam Width Antenna

| Range km | Angle from Centre Beam 30° | 60° | 90° | 120° |
|---|---|---|---|---|
| 5 | -0.1 | -2.9 | -5.5 | +5.6 |
| 9 | -2.5 | +0.6 | (+17.6 at 75°) | |
| 12 | -4.3 | 0 | | |

N.B. All observations were made with a transponder height of 18 ft.

Diagram #7



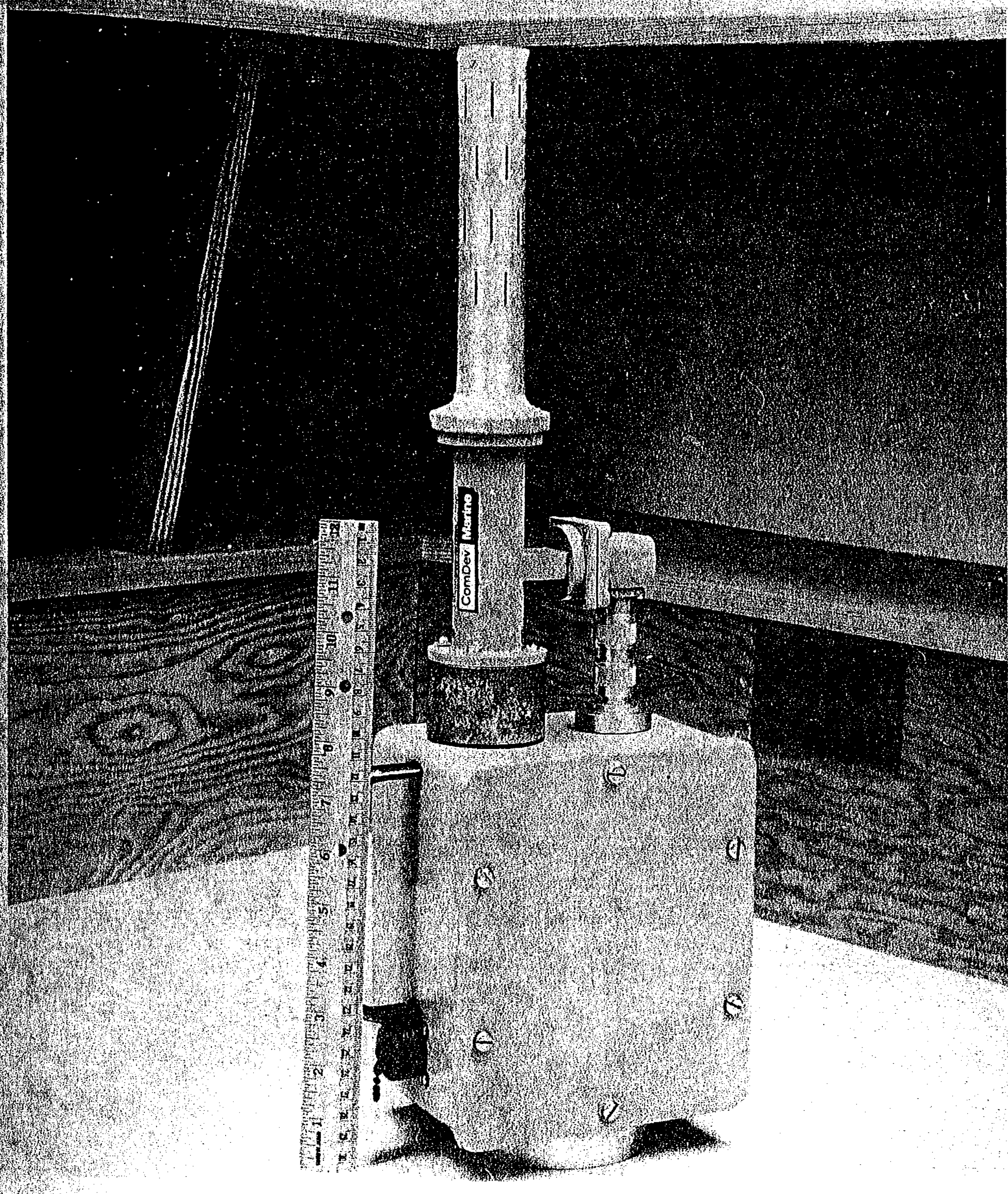

TRISPONDER
OVERALL ACCURACY

(derived from stability, precision and repeatability data)

Transponder Antenna Heights 130 ft.
Contours in Metres
Range in Kilometres
Transmitter Antenna Height 9 ft.
84° Antenna

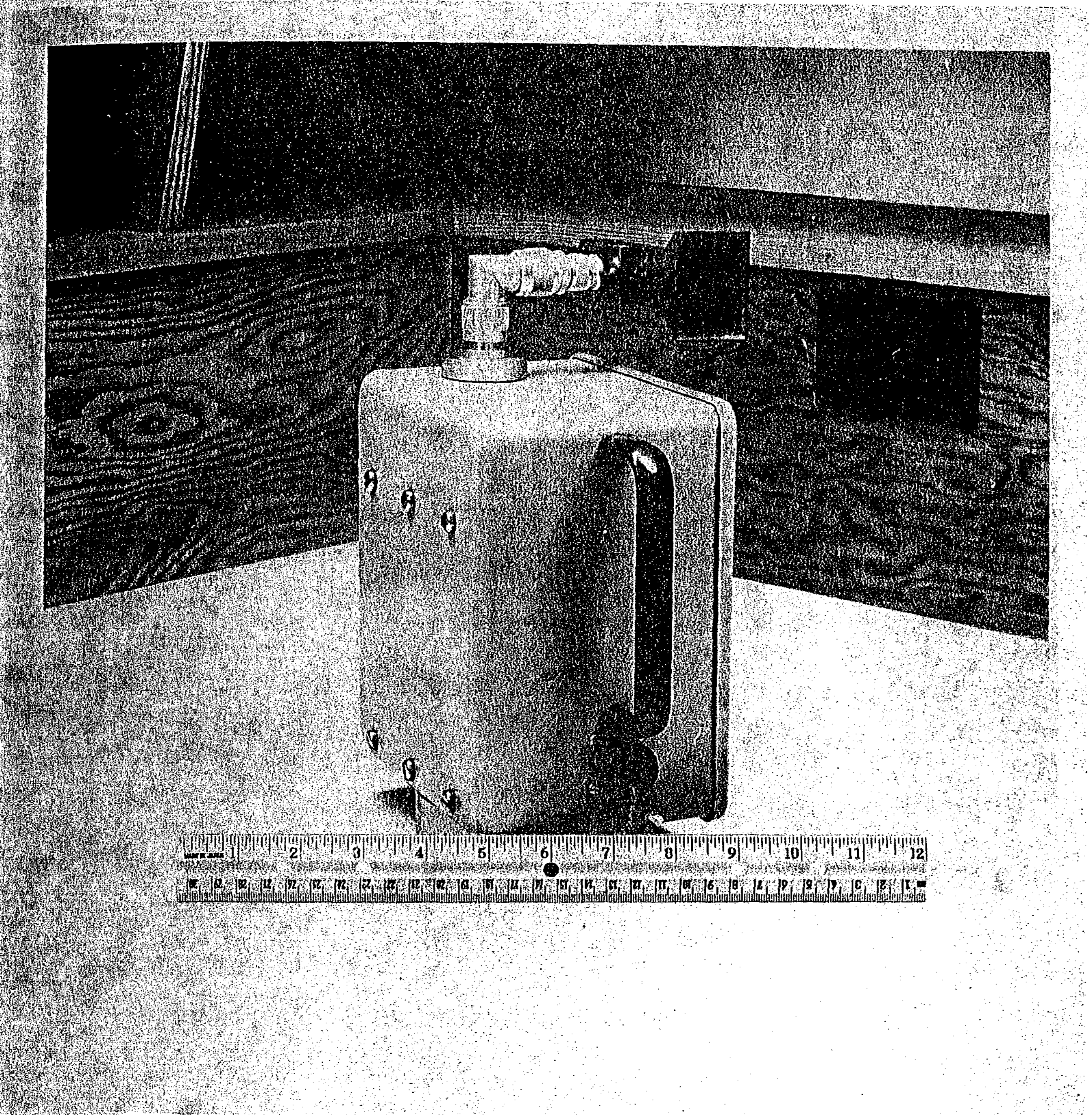
For One Position Line Only

Diagram #8



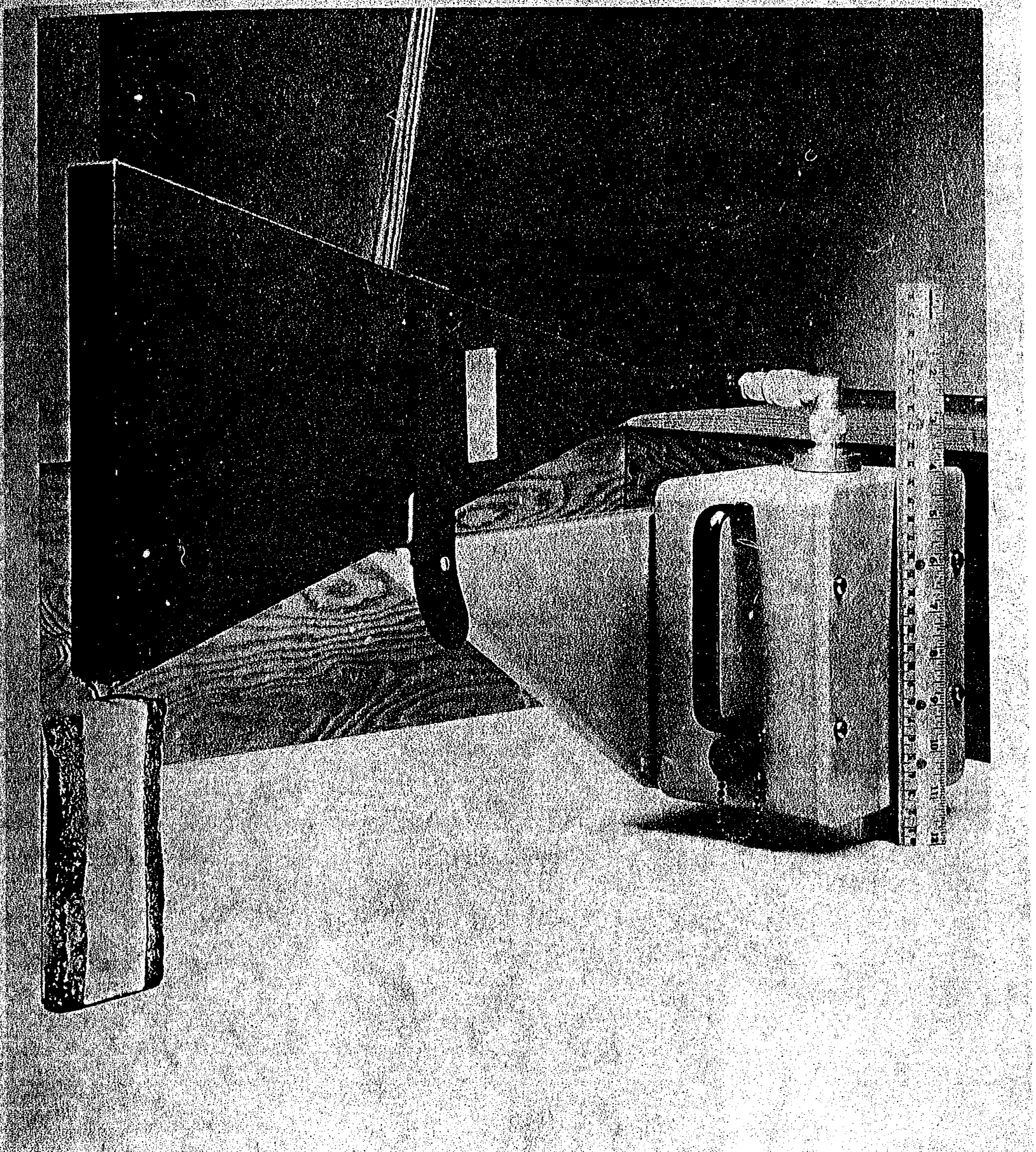

OMNI ANTENNA
with the base unit

Diagram #9



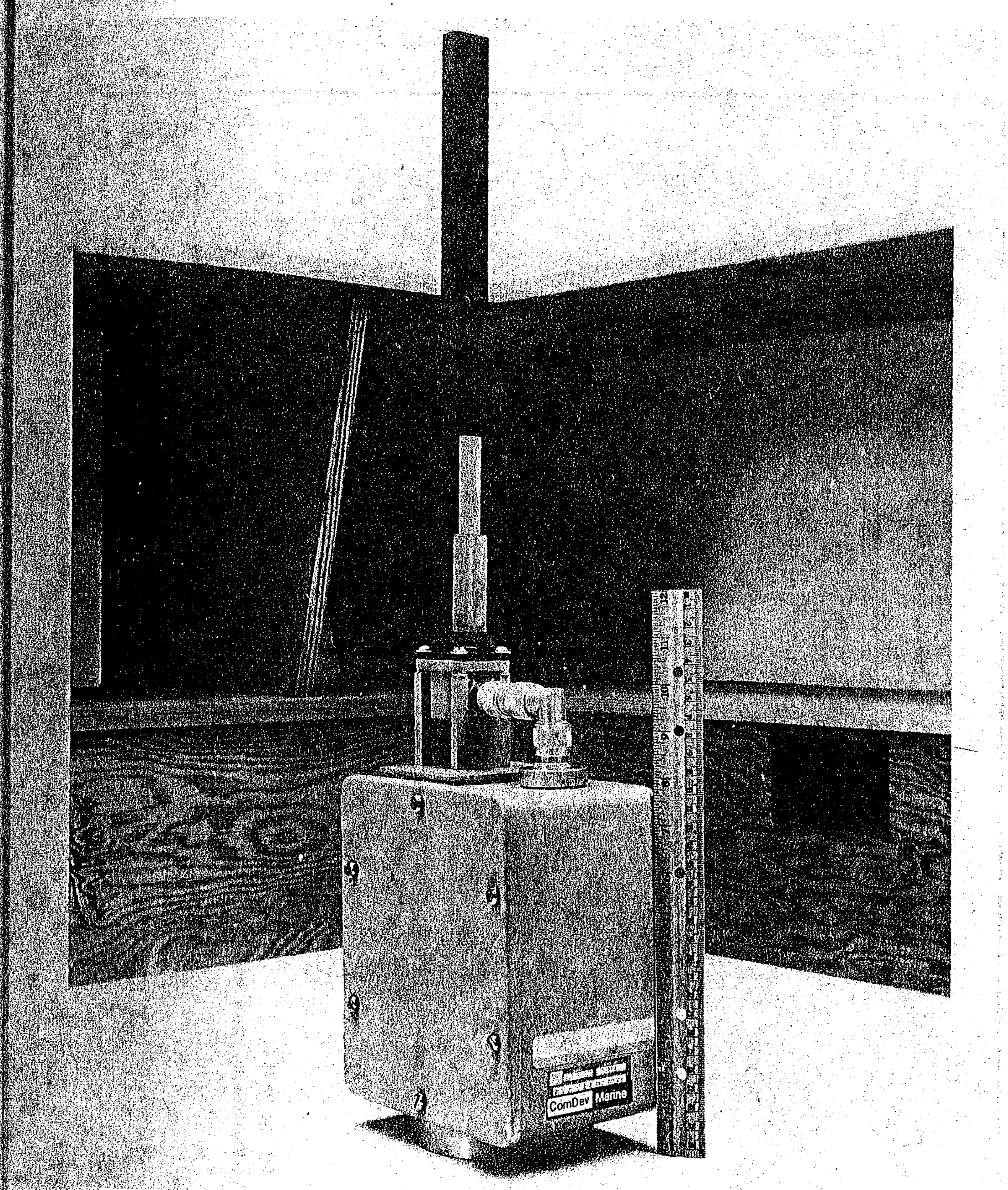
45° ANTENNA
with a transponder

Diagram #10



60° ANTENNA
with a transponder

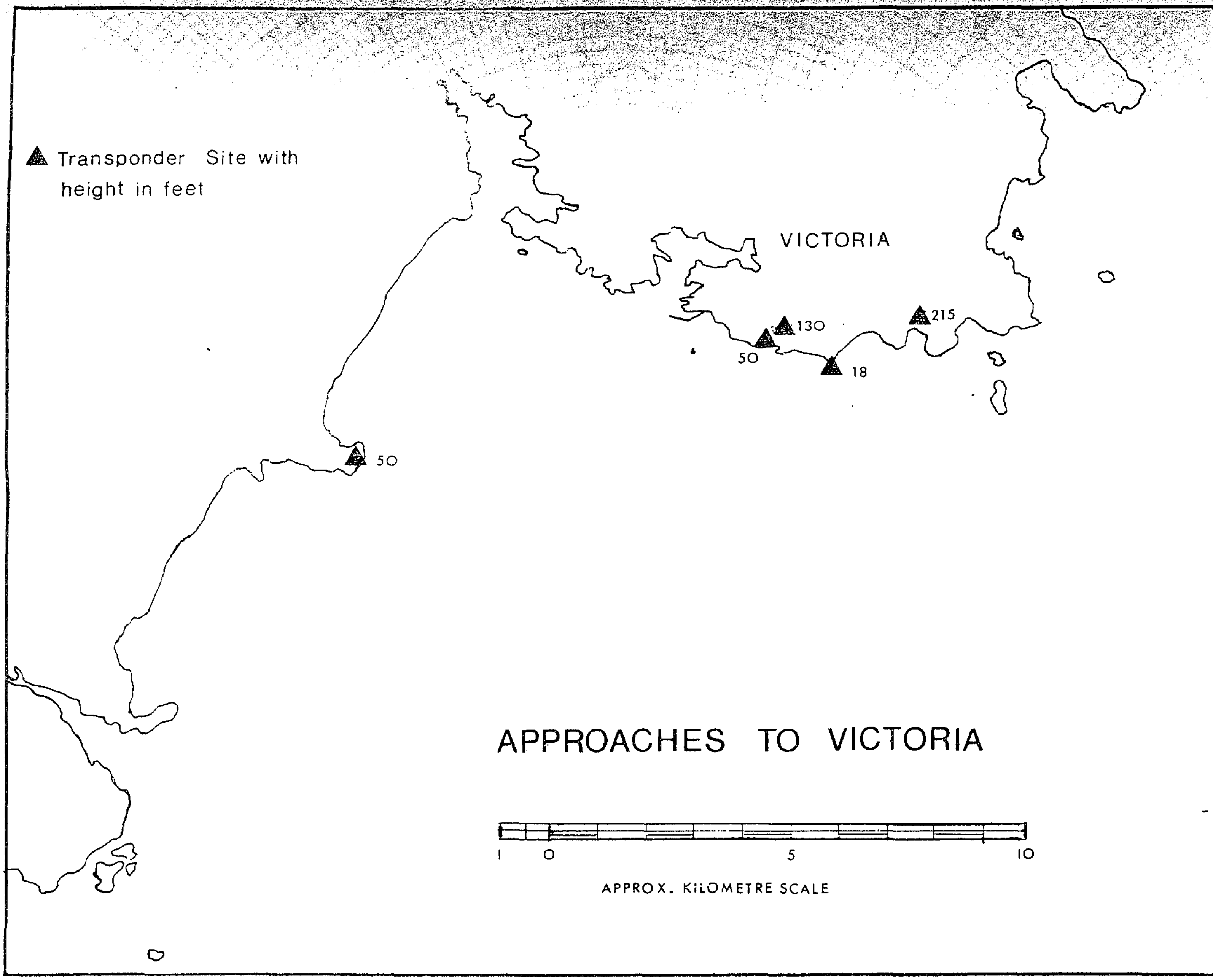
Diagram #11

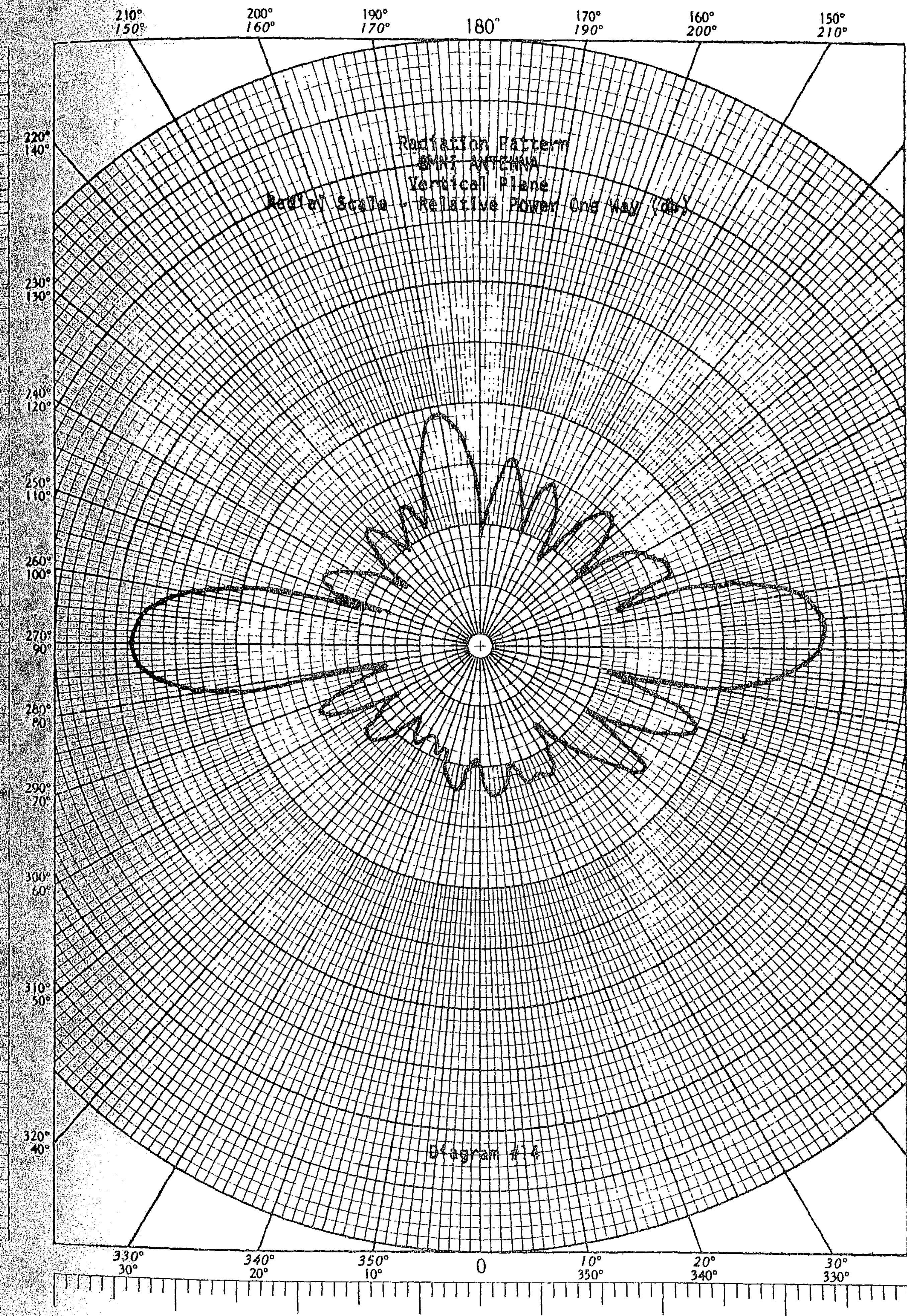


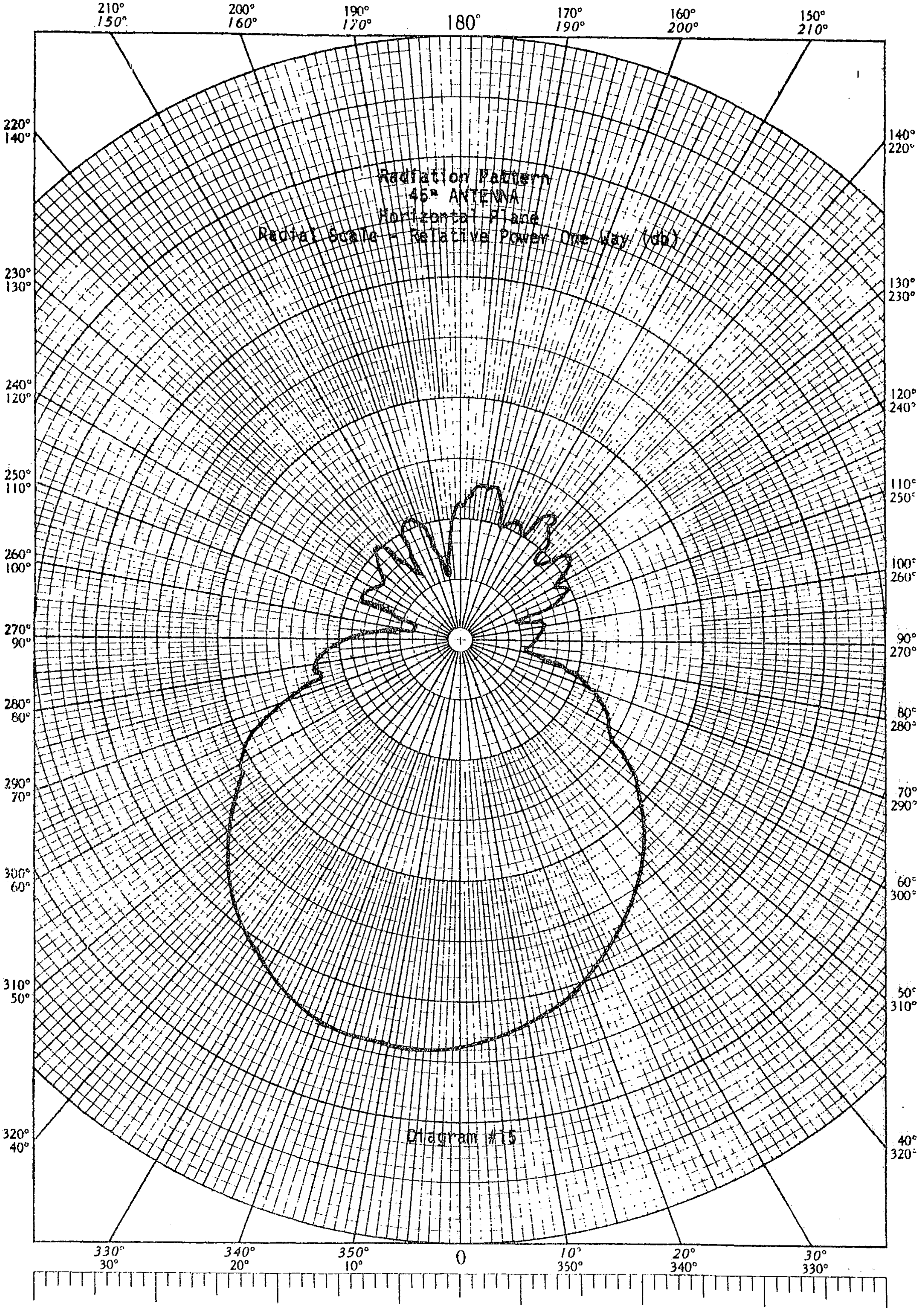

84° ANTENNA
with a transponder

Diagram #12

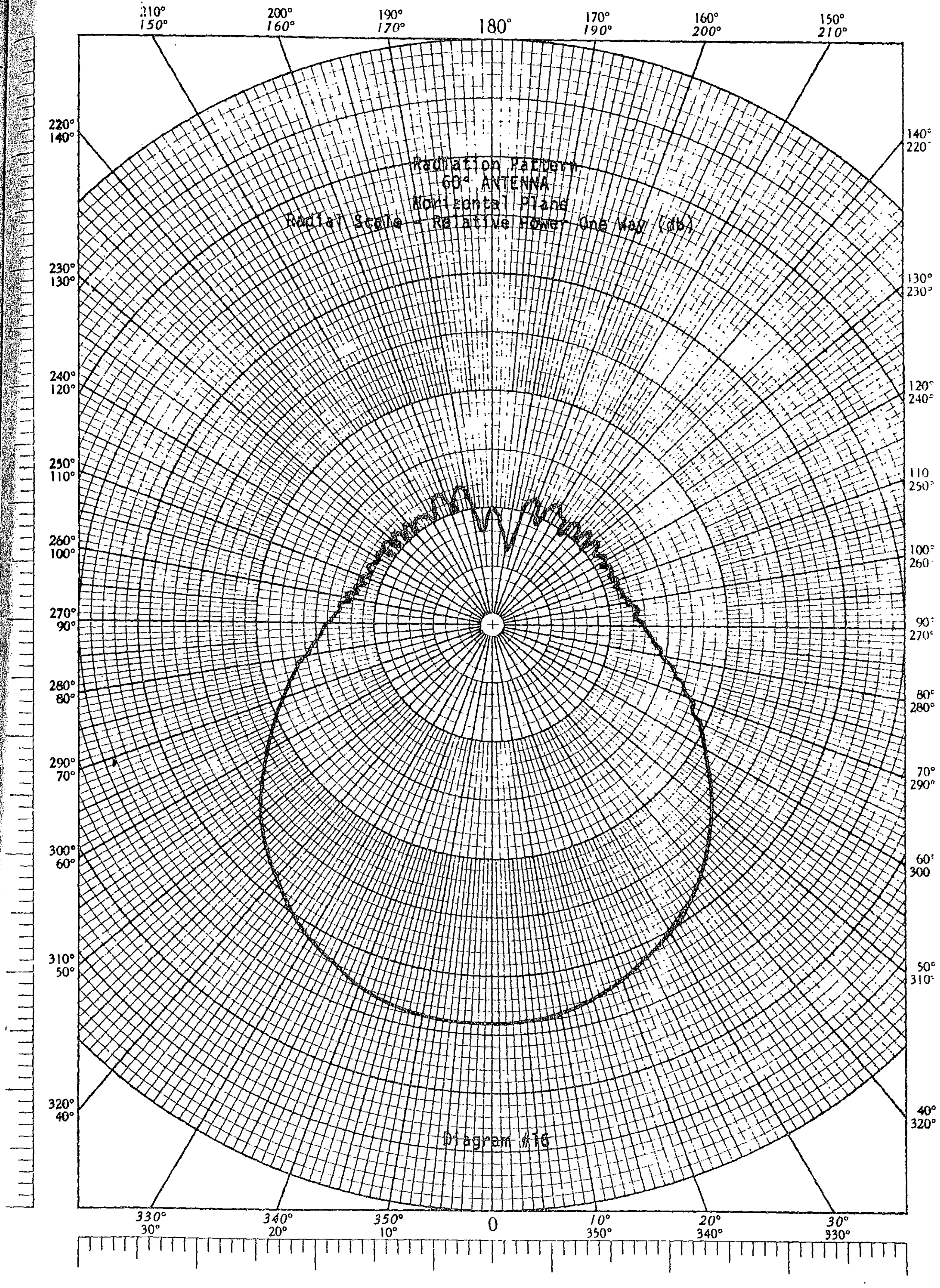
Diagram #13

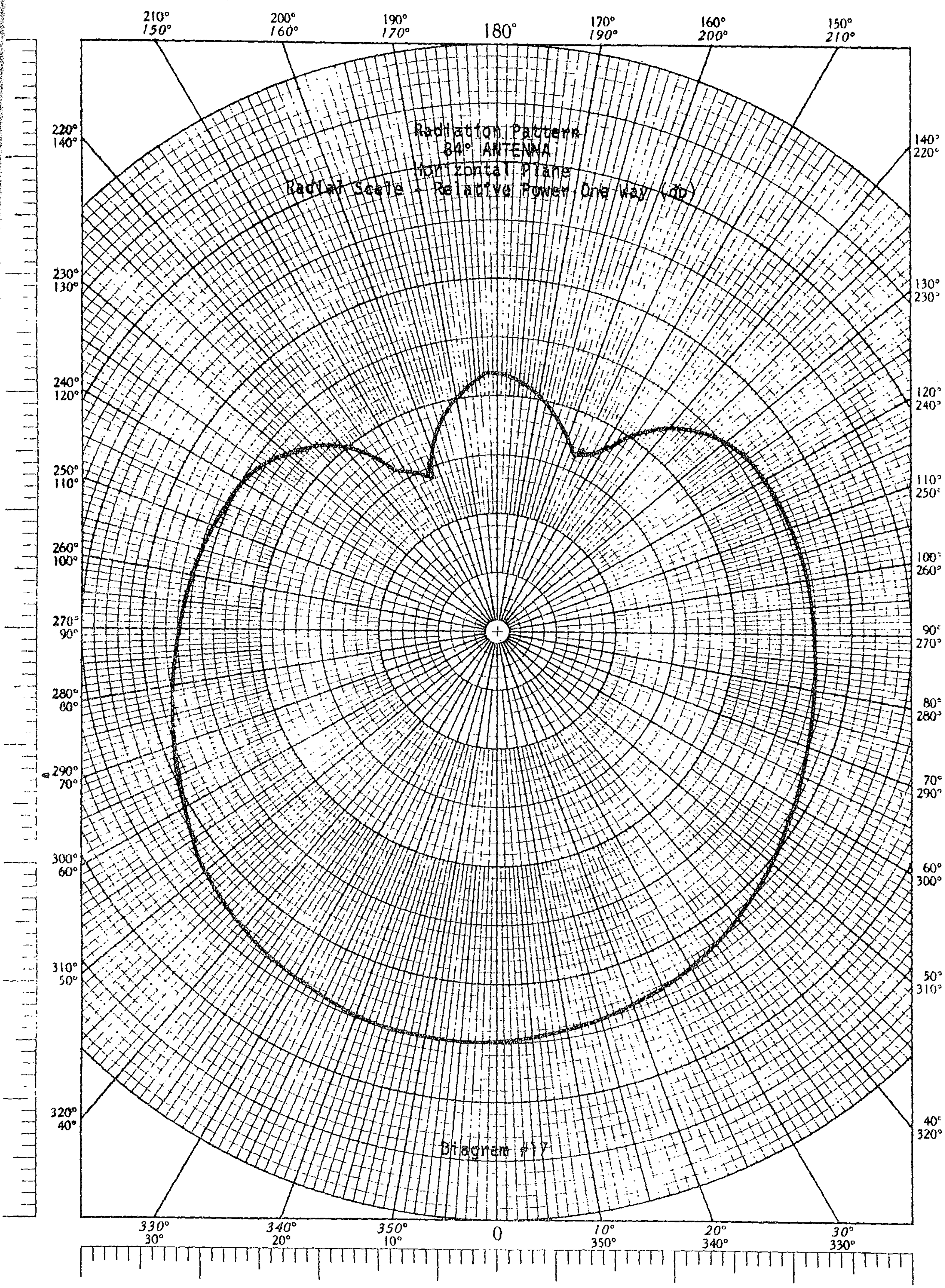

Transponder Site with height in feet
VICTORIA
130
215
50
18
50
APPROACHES TO VICTORIA
1 0 5 10
APPROX. KILOMETRE SCALE


Radiation Pattern
OMNI ANTENNA
Vertical Plane
Radial Scale - Relative Power One Way (db)
180°
0
Diagram #14



Radiation Pattern
45° ANTENNA
Horizontal Plane
Radial Scale - Relative Power One Way (db)
Diagram #15
180°
0


Radiation Pattern
60° ANTENNA
Horizontal Plane
Radial Scale - Relative Power One Way (db)
Diagram #16
180°
0


Radiation Pattern
84° ANTENNA
Horizontal Plane
Radial Scale - Relative Power One Way (db)
Diagram #17
180°
0

Algorithm for the program to process stability data

$r_i \in \{R : \text{Observed Ranges}\}$

$s_i \in \{S : \text{Calculated Ranges}\}$

$d_i \in \{D : \text{Difference of Calc. and Obs. Ranges}\}$

#A To initialize the calculation

$$s_{(1,2,3)} = \frac{1}{3}(r_1 + r_2 + r_3)$$

and $$s_{(n-2,n-1,n)} = \frac{1}{3}(r_{n-2} + r_{n-1} + r_n)$$

#B To filter interference

$$c_i = r_i$$

If $|c_i - c_{i-1}| > 10$ AND $|c_i - c_{i+1}| > 10$

then $c_i = \frac{1}{2}(c_{i-1} + c_{i+1})$

and $c_i \in \{c : \text{filtered observed ranges}\}$

#C To calculate the linear regressions - consider the filtered observed ranges $c_{i-2}$, ---, $c_{i+2}$ where i is the index at the centre of the interval.

$$b_i = \frac{\sum_{i-2}^{i+2} c_i i - \frac{1}{5}\left(\sum_{i-2}^{i+2} i \sum_{i-2}^{i+2} c_i\right)}{\sum_{i-2}^{i+2} c_i^2 - \left(\frac{1}{5}\left(\sum_{i-2}^{i+2} c_i\right)^2\right)}$$

and $a_i = \frac{1}{5}\sum_{i-2}^{i+2} i - b_i\left(\frac{1}{5}\sum_{i-2}^{i+2} c_i\right)$

then $s_i = a_i + b_i c_i$

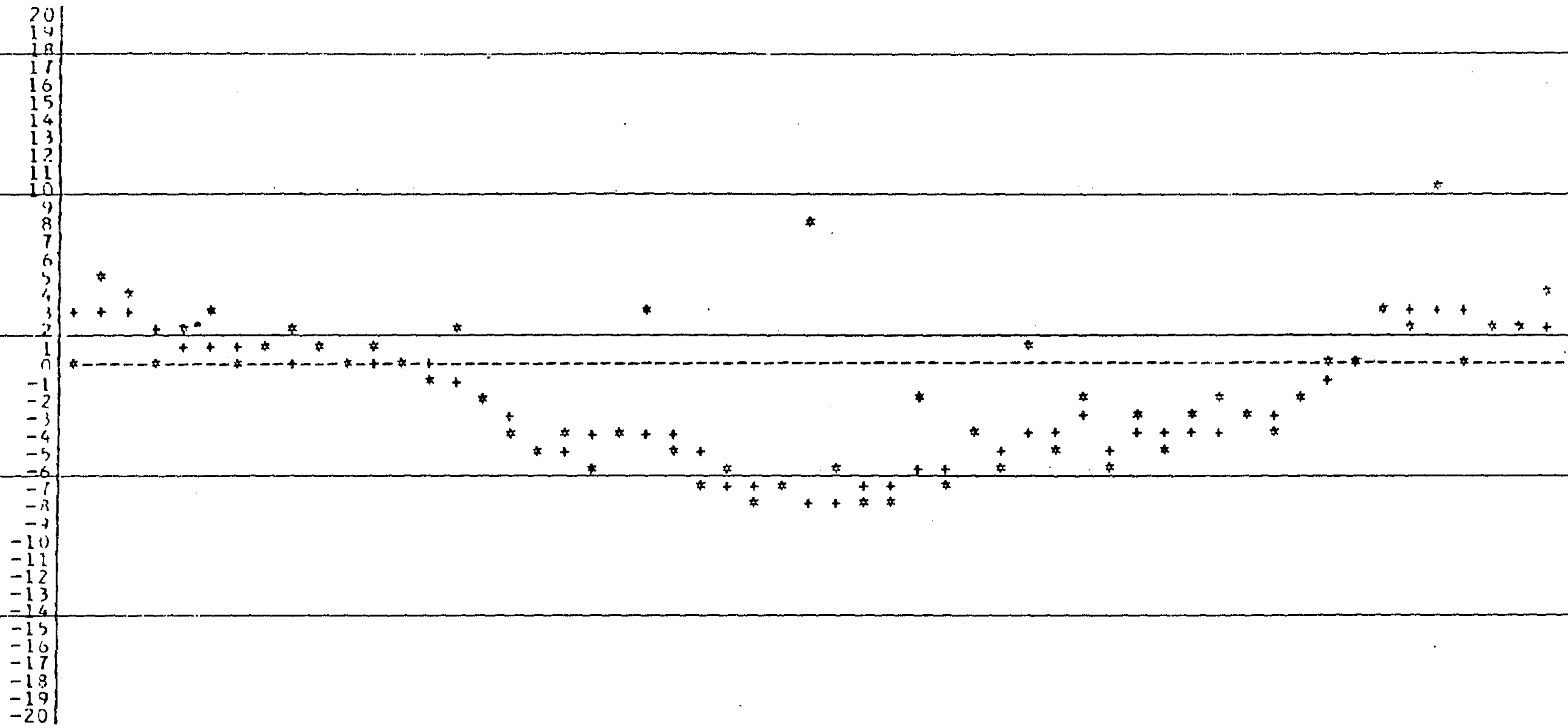
and $d_i = r_i - s_i$

For the next value of **S** $i = i + 1$

Diagram #18

OBSERVED DATA AND ESTIMATED LAUNCH LINE

RANGE 20000. ANTENNA HEIGHT 50. FEET



OBSERVED DATA * CALCULATED DATA +

An example of the computer print-out
from the program used to process
STABILITY DATA

Diagram #19

RANGE 15 KM   ANGLE FROM CENTRE BEAM   30 DEG.

TELLUROMETER DISTANCE   14840.7

TRISPONDER DATA

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 14839. | 14837. | 14838. | 14837. | 14835. | 14837. | 14838. | 14838. | 14838. | 14837. | 14837. |
| 14837. | 14838. | 14836. | 14838. | 14839. | 14837. | 14838. | 14837. | 14838. | 14840. | 14839. |
| 14839. | 14837. | 14838. | 14838. | 14836. | 14839. | 14838. | 14838. | 14839. | 14837. | 14838. |
| 14836. | 14838. | 14839. | 14837. | 14837. | 14839. | 14836. | 14836. | 14837. | 14839. | 14839. |
| 14838. | 14838. | 14841. | 14837. | 14839. | 14836. | 14839. | 14838. | 14838. | 14851. | 14836. |

CALIBRATION   0.0

DIFFERENCE TELLUROMETER & TRISPONDER

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| -1.70 | -3.70 | -2.70 | -3.70 | -5.70 | -3.70 | -2.70 | -2.70 | -2.70 | -3.70 | -3.70 |
| -3.70 | -2.70 | -4.70 | -2.70 | -1.70 | -3.70 | -2.70 | -3.70 | -2.70 | -0.70 | -1.70 |
| -1.70 | -3.70 | -2.70 | -2.70 | -4.70 | -1.70 | -2.70 | -2.70 | -1.70 | -3.70 | -2.70 |
| -4.70 | -2.70 | -1.70 | -3.70 | -3.70 | -1.70 | -4.70 | -4.70 | -3.70 | -1.70 | -1.70 |
| -2.70 | -2.70 | 0.30 | -3.70 | -1.70 | -4.70 | -1.70 | -2.70 | -2.70 | 10.30 | -4.70 |

MEAN DIFFERENCE   -2.72   MEDIAN DIFFERENCE   -2.70   STANDARD DEVIATION   3.47

DIFFERENCE TRISPONDER AND TELLUROMETER

```
+10 | *
    |
    |
 +5 |
    |
    |
  0 | **--------------------------------------------------------
    | ************
    | ******************
    | **************
    | *******
 -5 | *
    |
    |
-10 |
```

An example of the computer print-out
from the program used to process
PRECISION DATA

Diagram #20

THE EFFECT OF THE MOVEMENT OF THE OMNI ANTENNA
IN THE VERTICAL PLANE

<u>Example 1</u>

Tellurometer distance 14840.7 metres

Transponder 'A'

| | Angle from the vertical | | |
|---|---|---|---|
| | 0° | 4° | 7° |
| Mean difference | -2.7 | +1.3 | +9.3 |
| Stnd. deviation | +1.2 | ±3.2 | ±5.2 |

Transponder 'B'

| | Angle from the vertical | | |
|---|---|---|---|
| | 0° | 4° | 7° |
| Mean difference | +4.3 | +16.3 | +39.3 |
| Stnd. deviation | ±3.0 | ±6.9 | ±6.7 |

<u>Example 2</u>

Tellurometer distance 1196.6 metres

Transponder 'A'

| | Angle from the vertical | | | |
|---|---|---|---|---|
| | 0° | 5° | 8° | 16° |
| Mean difference | -0.5 | -0.8 | -0.7 | +2.8 |
| Stnd. deviation | ±1.7 | ±0.6 | ±0.7 | ±1.2 |

Note: All measurements in metres at the centre of the transponder antenna beam pattern.

Diagram #21