CANADIAN TIDAL MANUAL

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Wake behind Turret Island (locally called Tremble Island) at maximum ebb tidal stream in Nakwakto Rapids at the entrance to Seymour and Belize Inlets, British Columbia (51°06′N, 127°30′W). Maximum ebb stream is about 16 knots, while maximum flood stream is about 13 knots in the opposite direction. (Photo by M.J. Woodward, Canadian Hydrographic Service, 1982.)

CANADIAN TIDAL MANUAL

Prepared under contract by
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DEPARTMENT OF FISHERIES AND OCEANS
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PREFACE

Fluctuations in the water level along the shores of any body of water are of obvious interest and concern to those who inhabit those shores - interest in the cause and the nature of the fluctuations, and concern over the possibility of flooding, dried out jetties, exposed water intakes, increased erosion, reduced irrigation, etc. People who work on or in the water or travel upon it are also concerned with fluctuations in water level, although they may think of them more as fluctuations in depth, being involved in such tasks as navigating a vessel in shallow water, drilling from an oil rig, or setting a lobster pot. Of equal importance to some is the horizontal flow, or current, the fluctuations in which are frequently related to those in the water level.

Along ocean coasts and in bays and estuaries connected to the ocean the tide is usually the major cause of fluctuations in water levels and currents, but non-tidal phenomena such as wind stress, storm surges, and atmospheric pressure may play important roles as well. In lakes, even in large lakes such as the Great Lakes, the tide has no significant effect on the water levels. There is, however, one place in the Great Lakes where the current is significantly influenced by the tide. In Little Current Channel, the narrow and shallow connection between Georgian Bay and the North Channel of Lake Huron, a reversing tidal stream of more than one knot in amplitude may be observed in the absence of wind and other non-tidal disturbances. This is an exceptional situation, and usually in lakes and rivers the fluctuations in currents, as well as in water levels, result from variations in precipitation, evaporation, runoff, atmospheric pressure and wind, and from basin oscillations called seiches. In some inland systems the water level and flow may, of course, be controlled by dams, or temporarily backed up by ice or log jams.

The hydrographer's interest in water level fluctuations relates to his responsibility to provide accurate depth information on charts. Since the actual depths at a particular time depend on the water level at that time as well as on the bathymetry, the depths shown on a chart must be referred to a standard reference surface, or datum. This chart datum is chosen as a surface below which the water level will seldom fall, so that only rarely could the actual depth be less than the charted depth. Choice of a suitable chart datum requires a knowledge of the nature of the water level fluctuations over the region being charted: this knowledge can usually be obtained only from lengthy observations of the water level. During the hydrographic sounding survey the height of the water surface above the chart datum must be determined, to permit each sounding to be reduced by that amount to provide the chart depth below chart datum. After the chart is produced and in service. mariners, surveyors, or engineers may still require to know the water level above chart datum so they may add it to the charted depths to obtain the actual depths.

Water level information for sounding reduction is usually provided from temporary gauges established in the immediate vicinity of the survey by the hydrographic field party. Permanent water level gauges are maintained at major ports and other selected sites around the coast and on major inland waterways to provide continuous water level information for these localities. The data accumulated over the years from the permanent and temporary gauges provide the basis for interpretation of the water level characteristics, selection of chart datums, and prediction or estimation of future water levels. These data are available to the public in a variety of formats through a central data bank at the Marine Environmental Data Service (MEDS), through various bulletins and publications (see Bibliography under Canadian Hydrographic Service), and in some cases as real-time data through direct telephone communication with teleannouncing gauges. For tidal waters, predicted times and heights of high and low water are provided annually in the Canadian Tide and Current Tables. A tidal block on the navigation chart tabulates the extreme and average heights of high and low water for various locations on the chart. On navigation charts of non-tidal waters a hydrograph is usually provided to show the average and the extreme monthly mean water levels that have been observed. No predictions are made for water levels in non-tidal waters, but a Monthly Water Level Bulletin for the Great Lakes and Montreal Harbour gives, along with statistics of past observations, a forecast of the monthly mean water levels for the next 6 months within an envelope of error.

The hydrographer is also required to provide information on currents where they may be of concern to navigation, particularly in restricted and difficult passages. The gathering and publication of information are more complicated and difficult for currents than for water levels. This is because currents may vary in direction as well as intensity, they may change in a very short distance under the influence of the bathymetry, they may differ considerably between the surface and the bottom, and their observation usually involves offshore moorings. Current information is provided on the navigation charts where this is feasible. For many important tidal passages the times and speeds at maximum flood and ebb and the times of slack water are predicted and published annually in the Canadian Tide and Current Tables. Current information is also published in Sailing Directions and Small Craft Guides in more descriptive form, particularly when the information is difficult to quantify or is based only on superficial observations or reports. In regions where the currents display great temporal and spatial variation, and where these variations are understood, publication of a Current Atlas may be required to represent the information adequately.

The material contained in this Tidal Manual is designed to provide the theoretical background (Part I) and the technical instruction (Part II) necessary for the effective performance of the tasks involved in gathering and using tide, current, and water level information on hydrographic field surveys. In treating instrumentation and techniques the emphasis is mainly on principles, with reference to manuals or other documentation for the specifics of particular instruments or routines. It is hoped that this will retard the advent of the Manual's obsolescence in the face of advancing technology. A minimum of mathematics has been employed, and an attempt has been made to discuss the results in simple terms following any mathematical development. A reader who is not comfortable with mathematics is advised to read between the formulae and to try to absorb the basic ideas, but not to skip the sections completely.

PART I
Theory and Concepts

CHAPTER 1

TIDES AS WAVES

1.1 What is the Tide?

Every reader of this book will have some notion of what is meant by the word "tide" as applied to the ocean. Some will think of the daily or twice-daily rise and fall of the water on the face of a cliff or around the pilings of a pier, others of the advance and retreat of the water over a shallow foreshore, and still others may think of the variable horizontal flow of the water that carries their ship off course, sometimes in one direction, sometimes in another. The tide is all of these things, but more generally we will define the ocean tide as “the response of the ocean to the periodic fluctuations in the tide-raising forces of the moon and the sun”. This response is in the form of long waves that are generated throughout the ocean. They propagate from place to place, are reflected, refracted, and dissipated just as are other long waves. Thus it is that the tide observed at a particular place is not locally generated, but is the sum of tide waves arriving from all over the ocean, each modified by its experiences along the way. To better understand the tide it will therefore be desirable to consider the characteristics of long waves as well as those of the tide-raising forces that produce them.

1.2. Waves

Wave motion in or along a medium is characterized by:

(a) periodic vibration but no net transport of the particles in the medium,

(b) propagation of energy along or through the medium. and

(c) a restoring force that opposes the displacement of the particles of the medium

When a sound wave travels through air, the particles experience a to-and-fro vibration. and the restoring force is provided by the pressure gradient. When a sound wave travels through a solid, the particles also experience a to-and-fro vibration, and the restoring force is provided by the elasticity of the material. When a wave travels along a taut string, the particles experience a transverse vibration, and the restoring force is provided by the tension in the string. When a wave travels along the surface of a body of water, the particles experience both a to-and-fro and an up-and-down vibration, and the restoring force is provided by a combination of gravity (acting through the hydrostatic pressure) and surface tension. Surface tension is the dominant restoring force only for ripples with 2 cm or less between crests, and these are called "capillary waves." For all longer water waves the dominant restoring torce is gravity, and for this reason they are called "gravity waves." Surface chop, sea, swell, tsunamis, and tides are all gravity waves.

The terminology used to describe waves is illustrated in Fig. I a and Fig. I b. In Fig. I a the wave form is viewed perpendicular to its direction of travel at an instant in time.

Fig. I b depicts the variation in water level at a fixed location over an interval of time as the wave

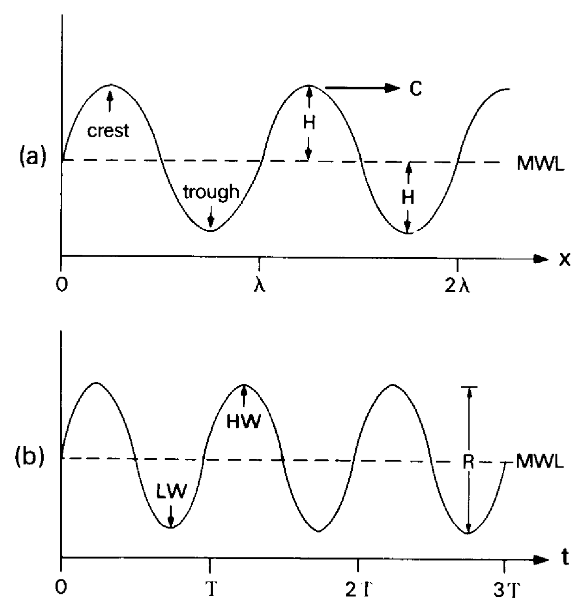


Fig. 1. Sinusoidal wave form as seen (a) in space at an instant of time and (b) at a fixed location over an interval of time.

passes. The wavelength (λ) is the distance between successive crests or successive troughs. The range (R) is the vertical distance of the crest above the trough, or of the high water (HW) above the low water (LW). The terms crest and trough are more commonly used in connection with waves that are short enough to reveal their wave form to the eye. The terms HW and LW are more commonly used only in connection with the tide waves, which are much too long to reveal their torm to the eye. The amplitude (H) is one half of the range. The period (T) is the interval between the passage of two successive crests, or between the occurrence of two HWs: successive troughs or any other identifiable parts of the wave form could equally well be used to define the period. The frequency (f) is the number of periods (or cycles) occurring per unit time; therefore $f = 1/T$. The wave speed (c) is the horizontal rate of advance of all parts of the wave form (crests, troughs, etc.). Since a travelling wave advances one wavelength in one period, $c = \lambda/T$.

A sinusoidal wave form, such as that in Fig. 1, can be generated as the product of the amplitude times the sine or cosine of a continuously increasing angle, called the phase. The angle by which the phase of a wave lags behind the phase of a reference wave is called the phaselag. In tidal work, the cosine form is most commonly used, so that in Fig. I b the height above mean water level (MWL) would be expressed as:

$$h(t) = H \cos(2\pi ft - [\pi/2])$$

With respect to a wave with phase $2\pi ft$, h(t) would be said to have a phaselag of $\pi/2$. The rate at which the phase increases is called the angular speed (ω), and $\omega = 2\pi f$ radians per unit time. In tidal literature the angular speed is usually quoted in degrees per hour and given the symbol “n”. The wave number (k) is the rate at which the phase changes with distance, and $k = 2\pi/\lambda$ radians per unit distance .

1.3. Surface Gravity Waves

It would admittedly be a rare occasion on which the actual sea surface could be adequately represented by a simple sinusoidal wave as in Fig. 1. However, quite complicated sea states may be represented as a composite of many such component waves, each with its own amplitude, wavelength, and direction of propagation. A long swell running on an otherwise calm sea closely resembles a single such component wave. Because the tide can usually be adequately represented as the superposition of a manageable number of these component waves, we will restrict our investigation of surface gravity waves to those of sinusoidal form.

A wave that is moving across the surface as a train of parallel crests and troughs is called a progressive wave . If it is moving in the positive x-direction, the height at distance x and at time t is given by:

(1.3.1) $$h_1(x,t) = H \cos 2\pi([t/T]-[x/\lambda])$$

$$= H \cos (\omega t- kx)$$

This expression may be verified by considering that for an observer travelling with the wave speed $c = \lambda/T$ the phase would remain constant, because the increase due to the increase in t is offset by the decrease due to the increase in x. If the wave train is moving in the negative x-direction, the height is:

(1.3.2) $$h_2(x,t) = H \cos 2\pi([t/T]+[x/\lambda])$$

$$= H \cos (\omega t+ kx)$$

The superposition of two progressive waves that have the same amplitude and frequency but are travelling in opposite directions produces what is called a standing wave. Adding equations (1.3.1) and (1.3.2) and invoking some trigonometric relations give the following expression for the standing wave form:

(1.3.3) $$h_S(x,t) = h_1 + h_2$$

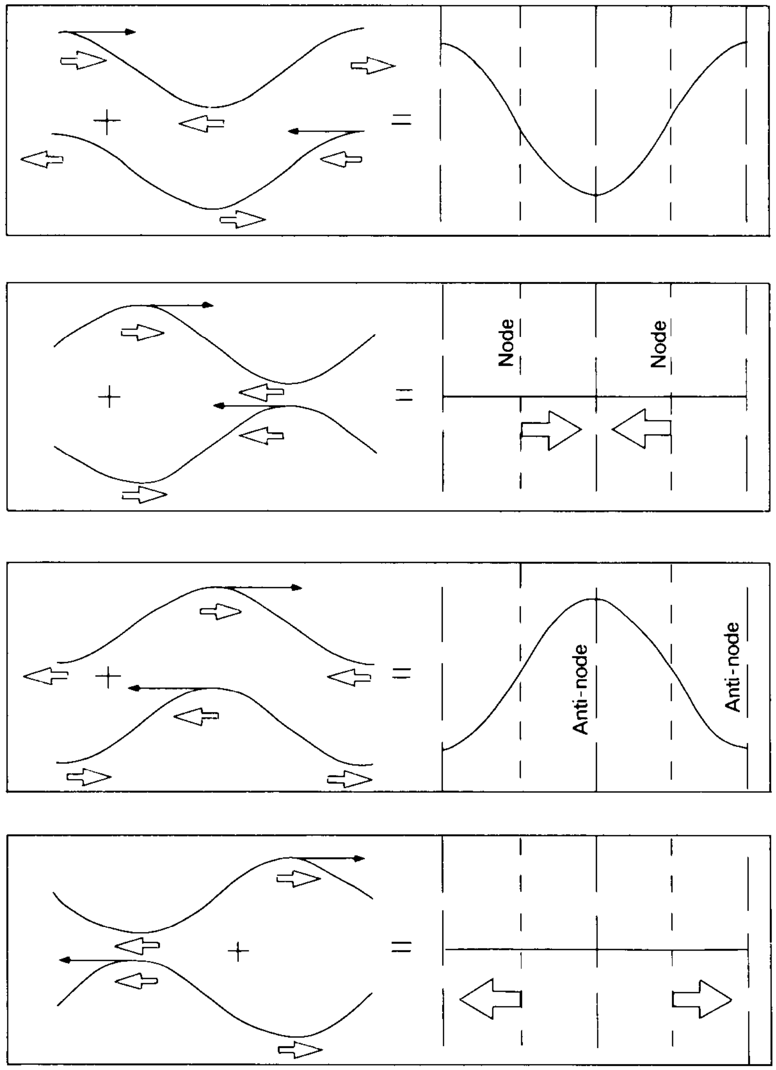


FIG. 2. Formation of a standing wave from two oppositely directed progressive waves. Open arrows show particle velocities; single line arrows show direction of wave propagation.

$= (2H \cos 2\pi\ [X/\lambda])(\cos 2\pi\ [t/T])$

Figure 2 illustrates the formation of a standing wave from two oppositely directed progressive waves. From equation 1.3.3 and Fig. 2 it is clear that the period and wavelength of the standing wave are the same as those of the component progressive waves, that the amplitude of the rise and fall of the surface varies from zero to 2H according to the value of cos kx, and that the phase of the rise and fall is everywhere the same or opposite, according to the sign of cos kx. The places in the standing wave form at which the amplitude is zero are called nodes, and those at which it is maximum are called anti-nodes. The space between nodes is called a loop. Within each loop the phase is the same, but is different by 180° from that in adjacent loops. A standing wave is frequently formed by the reflection of a progressive wave back upon itself, which is why the tide usually displays the character of a standing wave in coastal bays and inlets. In practice we will never encounter a pure progressive or standing wave; every wave will have some of the characteristics of each. The tide in the Strait of Belle Isle is an example of a regime that is neither purely standing nor progressive. The tide propagating out from the Gulf of St. Lawrence combines with the tide propagating in from the Atlantic, but, since the two do not have the same amplitude, only a partially standing wave is formed.

At a true node there should be zero amplitude and a reversal of phase on either side: in the Strait of Belle Isle there is a degenerate node, exhibiting reduced amplitude and rapid spatial change in phase. When a standing wave is formed by reflection, the standing character is most nearly perfect near the reflecting barrier, because away from the barrier the incident wave has a larger amplitude than the reflected wave as a result of attenuation along their paths.

1.4 Long and Short Waves of Small Amplitude

In our theoretical consideration of waves we will implicitly assume that the amplitude is small with respect both to the wavelength and to the depth. The amplitude of a tide wave is always small with respect to its wavelength, but not always with respect to the depth, so we must expect some distortion of our results in shallow water. It can be shown that the wave speed of a sinusoidal wave in water of total depth D is:

(1.4.1) $c = ([g/k] \tan h\ kD)^{1/2}$

where g is the acceleration due to gravity, and that the horizontal particle motion (wave current) and

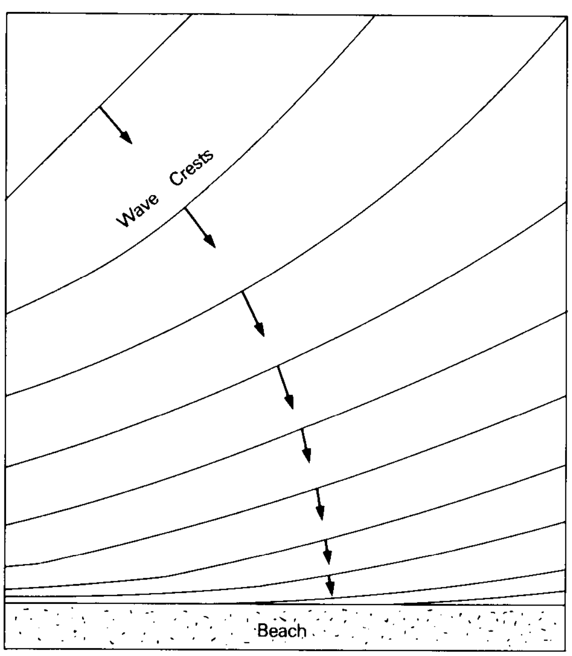


FIG. 3. Orientation of waves parallel to beach, by refraction.

the pressure associated with the passage of the wave both decrease exponentially with the depth by the factor exp (-kz), z being the depth from the surface. The magnitude of kD (= 2 $\pi D\lambda$) thus provides a criterion by which to categorize waves. Short (or deep-water) waves are those for which the wavelength is much less than the depth, and long (shallow-water) waves are those for which the wavelength is much greater than the depth. It must be remembered that the terms are relative, not absolute, and that a short wave may become a long wave on entering shallower water. For short waves kD is very large, and tanh kD is close to unity, so that the wave speed becomes:

$$c_S = (g/k)^{1/2}.$$

For long waves the value of kD is very small, and tan(h) kD is approximately equal to kD, so that the wave speed becomes

$$c_L = (gD)^{1/2}.$$

Because for short waves the speed depends on the wavelength, they experience dispersion, the longer component waves travelling faster and becoming dispersed from the shorter component waves. This is why the long swells (forerunners) from a distant storm arrive first. Long waves do not experience dispersion, their wave speed depending only on the water depth. They do, however, experience refraction if one part of the wave front is travelling in shallower water than the others. The part of the wave front in the shallower water slows down, allowing the rest of the front to pivot around, changing the direction of propagation of the wave. As illustrated in Fig. 3, refraction is responsible for orienting waves parallel to beaches before they break on the shore. Short waves do not experience refraction; but, of course, they may become long waves on entering shallow water and then be refracted as in Fig. 3.

The particle motion and the pressure fluctuation associated with the passage of a short wave decrease rapidly with depth, being only about 4% of their surface values at a depth of half a wave length. The particle motion and pressure fluctuation associated with the passage of a long wave are, however, virtually uniform over the depth (except for the frictional effect near the bottom). These are important facts to consider when planning subsurface pressure or current

measurements.
Since surface tides are long waves even in the deepest parts of the ocean, their signal may be detected by sensors at any depth, whereas the signal from short waves is effectively filtered out below a depth of a half a wavelength. The properties of long and short waves are summarized in Table 1. There are, of course, waves that are intermediate between the long and the short waves, and their wave speed is given by equation 1.4.1. However, since tides are always long waves, we shall confine our further considerations to long waves.

1.5. Particle Motions in Long Waves

In this section we will develop expressions for both the wave speed and the particle speed in a long surface wave. This is being done partly to demonstrate the physical principles, and partly to emphasize the relation between these two speeds and between the particle motion and the wave form. We assume that the particle speed is

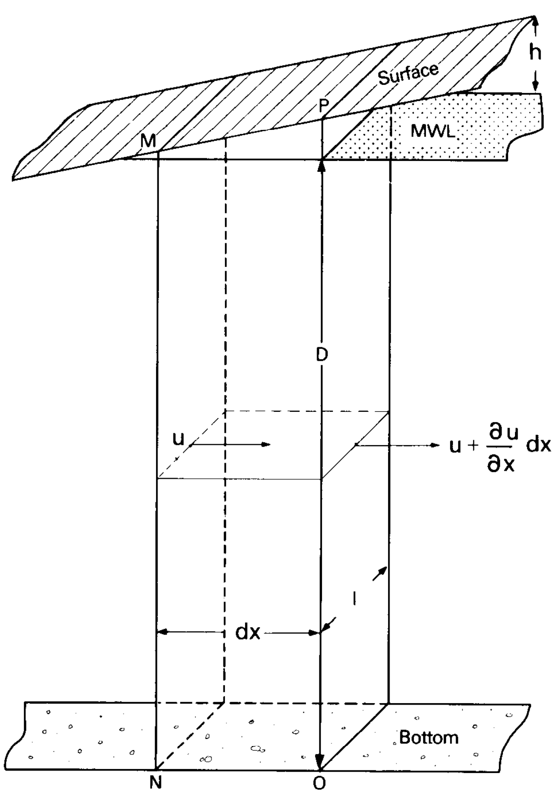


FIG. 4. Diagram to illustrate development of expression for wave speed and particle speed in long progressive surface wave.

uniform over depth and is a to-and-fro motion with the same period, but not necessarily the same phase, as the surface rise and fall. Consider a progressive wave moving from left to right in Fig. 4, with surface amplitude H. Let the particle speed have amplitude U and phase lag with respect to the surface elevation. Therefore

(1.5.1) $h(x,t) = H \cos(\omega t - kx)$
$u(x,t) = U \cos(\omega t - kx - q)$

In Fig. 4, MNOP is one side of a rectangular prism of unit thickness perpendicular to the page, with length dx, and height D + h. Its volume increases at the rate $(\delta h/\delta t)dx$. By the principle of continuity (conservation of matter) this must be equal to the rate at which water is entering minus the rate at which it is leaving through the sides of the prism. Neglecting the small height, h, with respect to the large depth, D, the rate at which water is accumulating inside the prism is

$$D[u(x,t)] - D[u(x,t) + (\delta u/\delta x)dx] = - D(\delta u/\delta x)dx$$

Equating these two rates gives

(1.5.2.) $\delta h/\delta t = - D(\delta u/\delta x)$

Differentiating the expressions in 1.5.1 and substituting in 1.5.2 yields
$-H \omega \sin(\omega t - kx) = - DkU \sin(\omega t — kx - \theta)$
whence

(I.5.3) $\theta = 0$, and $U = (H/D)(\omega/k) = (H/D) c$

Consider now a particle of water on the surface and assume that its acceleration equals the local acceleration, $\delta u/\delta t$ (a reasonable assumption for waves of small amplitude). The force per unit mass acting on the particle is the component of gravity parallel to the surface,
$-g(\delta h/\delta x)$. By Newton's law of motion these two quantities must be equal, whence, upon differentiating the expressions in (1.5.1) and using the relations given in 1.5.3,

$\delta u/\delta t = - g(\delta h/\delta x)$
or
$- \omega(\omega/\delta t)(H/D) \sin(\omega t - kx)$

Table 1. Properties of long and short waves.

	Short waves (deep water)	**Long waves** (shallow water)
Definition	l < D	l > D
Wave speed	$(g/k)^{1/2}$	$(gD)^{1/2}$
Particle motion	Decreases with depth.	Uniform with depth.
Wave pressure	Decreases with depth.	Uniform with depth.
Dispersion	Yes	No
Refraction	No	Yes

$= - gkH \sin(\omega t - kx)$
so

(1.5.4) $(\omega/k)^2 = gD = c^2$

From 1.5.3, since $\theta = 0$, we have the very important result that in a progressive wave the particle motion is in phase with the surface wave form; so that the particle speed is greatest in the direction of the wave travel at the crest, greatest in the direction opposite to the wave travel at the trough, and zero midway between crest and trough. The particle speed beneath any point in the wave is in fact the wave speed multiplied by the ratio of the wave height to the depth. The wave speed is given by 1.5.4 as
$c = (gD)^{1/2}$
as previously deduced from the more accurate expression in 1.4.1.

We will now examine the relation between particle motion and wave form in a standing wave. Just as we obtained the expression 1.3.3 for the form of a standing wave by adding the forms of two oppositely directed progressive waves, we may obtain the expression for the particle motion by adding the particle motions of two oppositely directed progressive waves. The particle speed in a wave travelling to the left is in phase with the wave form, and so must be given a negative sign. The addition gives the standing wave particle motion as

(1.5.5) $u_S,(x,t) = U\cos(\omega t - kx) - U\cos(\omega t + kx)$.

or $u_S,(x,t) = 2U \sin \omega t \sin kx$

From equation 1.3.3 we had the standing wave height as

(1.5.6) $h_S,(x,t) = 2H \cos \omega t \cos kx$

Recalling that the sine of an angle is 90° out of phase with the cosine, we see from a comparison of 1.5.5 and 1.5.6 that in a standing wave the particle speed has maximum amplitude where the surface rise and fall has zero amplitude (i.e. at the nodes) and has zero amplitude at the anti-nodes. We also see that the particle speed achieves its local maximum everywhere when the wave form is flat, and is everywhere zero when the surface has its maximum distortion (i.e. at HW and LW). Figure 2 illustrates the relations between particle motion and wave form in progressive and standing waves.

To demonstrate that tide waves are indeed long waves ($\lambda > D$) and to emphasize the relation between particle speed (tidal stream) and wave speed, Table 2 lists the wavelength, wave speed and particle speed of a tide wave of one-metre amplitude and 12-h period travelling in various depths of water. Comparison of the values in the last two columns shows that the wave speed is everywhere much greater than the particle speed, but that while the wave speed decreases, the particle speed increases with decreasing depth of water. This is one reason that tidal streams are much more evident in coastal waters than in the

Table 2. Characteristics of a tide wave of 12-hour period and 1 metre amplitude in various depths.

Depth (m)	Wavelength (km)	Wave speed (m/s)	Particle speed amplitude (m/s)
5,000	9,600	220	0.04
500	3,000	70	0.14
50	960	22	0.44
* 5	* 300	* 7	* 1.40

* Since this depth is not very large w.r.t. the wave amplitude, the wave would be distorted, and these values

open ocean.

1.6. Basin Oscillations

Almost every physical system has a natural frequency at which it will oscillate when disturbed from its rest position or shape, until friction brings it once more to rest. The most obvious example is the pendulum (or the hair-spring or the quartz crystal) in a clock, whose natural period of oscillation is the time unit that is summed by the clock to record the passage of time. If a system is left undisturbed to oscillate at its natural frequency, it is said to be in tree oscillation; if it is forced to oscillate at the frequency of an imposed force, it is said to be in forced oscillation. When the frequency of the driving force is equal to the natural frequency of the system, a large amplitude response may be obtained with the input of very little energy. This phenomenon is called resonance, and is explained by the fact that the driving force and the restoring force within the system act mostly in unison at forcing frequencies close to the natural frequency of the system, and mostly in opposition to each other at forcing frequencies far from the natural frequency. A simple example of a resonant system is a child seated on a swing that is being pushed by a friend; since the swing is pushed only at the end of each cycle, the frequency of the driving force is automatically matched to the natural frequency of the swing. The clock pendulum is another resonant system, operating on the same principle as the swing. A person singing in the shower may notice that a particular note causes a delightful reverberation; this is because the column of air in the shower stall is resonant at the frequency of that note.

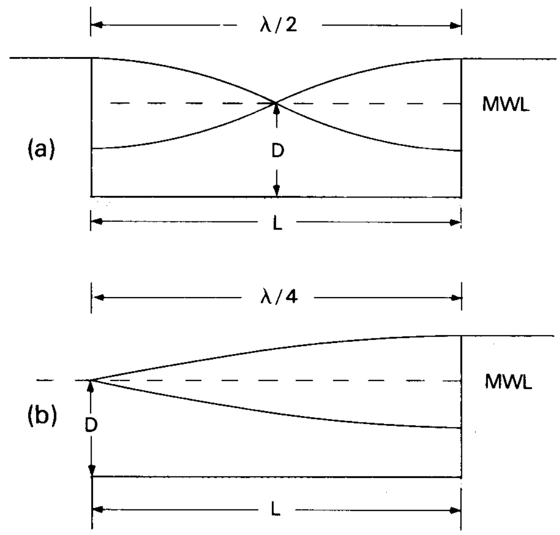


FIG. 5. Free oscillation, or *seiche*, in (a) a closed basin and (b) a basin open at one end.

The free oscillation of the water in a closed basin (bathtub, lake, etc.) takes the form of a standing wave with an anti-node at each end of the basin and one or more nodes between (Fig. 5a). If there is only one node, the length of the basin is half a wavelength, and the natural period of oscillation is given approximately as

(I.6.1) $T_n = (2L)/c = (2L)/(gD)^{1/2}$

where L is the length of the basin and D is an average depth. Although it is less common, a closed basin could oscillate across its width as well as along its length. The free oscillation of the water in a basin open at one end (harbour, bay, inlet, etc .) takes the form of a standing wave with a node at the open end and an anti-node at the closed end (Fig. 5b). If there are no other nodes between the ends, the length of the open basin is a quarter wavelength, and the natural period of oscillation is given approximately as

(1.6.2) $T_n = (4L)/c = (4L)/(gD)^{1/2}$

Free oscillations of water in basins (open or closed) are called seiches. Much of the early study of seiches was done on lakes in Switzerland, and the equations 1.6.1 and 1.6.2 are called Merian's formulae after one of the Swiss workers in this field.

Figure 6 illustrates how the tide at the entrance to an inlet off a large body of water drives a forced oscillation in the inlet in the form of a standing wave with an anti-node at the head of the inlet. If, as is usually the case, the inlet is shorter than a quarter of the tidal wavelength, the standing wave will have a virtual node outside the entrance. It is apparent that the amplitude of the tidal oscillation at the head of the inlet is greater than at the entrance, and that the amplification would be greatest if the node fell right at the entrance; the latter situation corresponds to the condition for resonance. If L is the length of the inlet, D its mean depth, $c = (gD)^{1/2}$ the wave speed in the inlet, and T the tidal period, then the tidal wavelength is cT and the portion of a wavelength within the inlet is L/cT. This represents a phase angle along the x-axis from the head of the inlet of $kx = 2\pi L/cT$. Thus, if H_2 is the amplitude at the head and H_1 that at the entrance of the inlet, by 1.5.6,

(1.6.3) $H_1 = H_2 \cos (2\pi L)/cT$

or $H_2/H_1 = \sec (2\pi L)/cT$

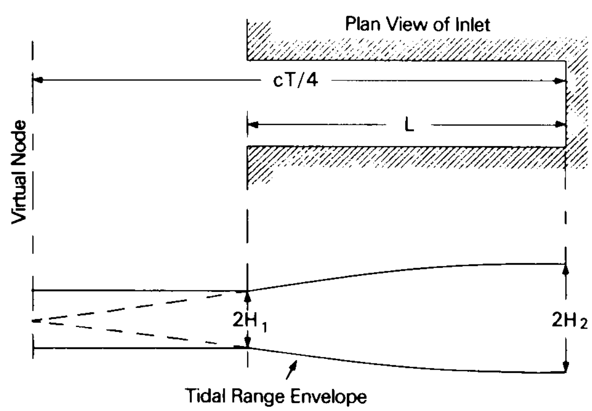


FIG. 6. Amplification of tide in an inlet, driven as a forced oscillation by the tide in a large body of water at entrance.

The amplification factor, H_2/H_1, in 1.6.3 is seen to be infinite for 4L = cT, which is the resonance condition (tidal period, T, equal natural period, 4L/c). However, friction, which we have neglected, becomes very important near resonance, and formula 1.6.3 should not be used for systems near resonance. The Saguenay fjord provides an example of tidal amplification in a system that is not near resonance. The length from the entrance off the St. Lawrence Estuary at Tadoussac to the head of the fjord at Port Alfred is 95 km (L), the mean value of the long-wave speed in the fjord is 40 m/s (c), and the tidal period is 12.4 h (T). From this, 1.6.3 gives the amplification factor as

H_2/H_1, = sec(0.33 rad.) = sec 19° = 1.06.

The actual amplification of the tide range at Port Alfred over that at Tadoussac is 1.16. The extra amplification over that predicted is probably caused by shoaling (see section I.12) of the tide wave in the shallow water near the head. An example of a system that is nearly in resonance with the semidiurnal (T = I/2-d) tide is the system comprising the Gulf of Maine and the Bay of Fundy.

The ocean basins themselves have natural periods of oscillation, but their modes of oscillation are much too complicated to be revealed by the simple considerations above. However, calculation of the natural period of east-west oscillation of the Atlantic and Pacific oceans from Merian's formula, 1.6.1, provides the interesting information that the Atlantic Ocean is more closely tuned to the semidiurnal and the Pacific Ocean more closely tuned to the diurnal tidal frequencies. Taking eastwest widths of 4,500 and 8,000 km, respectively, for the Atlantic and Pacific and a mean depth of 4,000 m for both, 1.6.1 gives 12.6 and 22.3 h, respectively, as the natural periods of the Atlantic and Pacific for east-west oscillation. Pacific tides are indeed observed to have much more diurnal character in general than the Atlantic tides.

1.7. Internal Waves

These are waves that occur below the surface at the interfaces between layers of water of different densities (i.e. in “stratified” water). They may exist independently from any surface wave, but are sometimes induced as a secondary effect of surface waves. Internal tides are frequently formed by the partial reflection of a surface tide wave at a sudden rise in bottom topography. The simplest case to consider is that of a wave at the interface in a two-layer system as shown in Fig. 7. The subscripts, 1 and 2, refer to the upper and lower layers, respectively, and p is the density, h the layer thickness, and u the particle velocity (with amplitude U). The wave form at the interface is that of a long progressive wave traveling from left to right with wave speed c_i. The restoring force in this internal wave is not the full force of gravity, g, per unit mass, but is the buoyancy force $g\Delta p/p$, where Δp is the density difference $p_2 - p_1$ and p is the mean density.

By reasoning that is just a little more difficult than that in section 1.5 for a surface wave. it can be shown that

(l 7 l) $C_i^2 = [g(p_2 - p_1)] / [p_1/h_1 + p_2/h_2]$

(1.7.2) - $U_1 = c_i H_i/h_1$

and

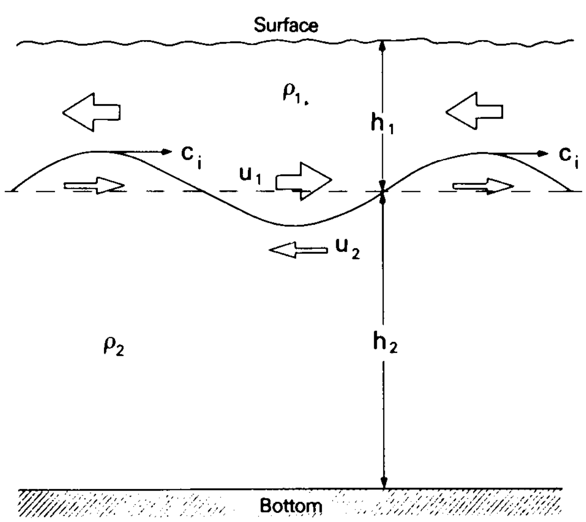


FIG. 7. Internal wave at the interface of a two-layer system. Hollow arrows show particle velocities; single line arrows show direction of wave propagation.

(I 7.3) $U_2 = = c_i H_i/h_2$

where H_i is the amplitude of the internal wave at the interface. If we had wished, we could have treated surface waves as special cases of internal waves at the air-water interface, taking p_1 and p_2 as the densities of air and water, h_1 as the thickness of the atmosphere, and h_2 as the depth of the water. Putting $p_1 < p_2$ and $h_2 < h_1$ to comply with this, reduces 1.7.1 approximately to

$C_i^2 = g(p_2) / [p_2/h_2] = gh_2$

in agreement with our previous expression 1.5.4. If, as is always the case for stratified water, p_2 and p_1 are nearly equal, 1.7.1 simplifies to

(1.7.4) $C_i^2 = g(\Delta p/p)[(h_1 h_2)/(h_1 + h_2)]$

and if it is further assumed that the upper layer is much thinner than the lower layer, as is frequently the case, this simplifies further to

(1 7.5) $C_i^2 = gh_1(\Delta p/p)$

Admittedly the two-layer system of Fig. 7, with its discontinuity in density and particle velocity at the interface, could never occur in a natural body of water. However, the equations 1.7.1, 2 and 3 reveal the following important characteristics of internal waves:

I) Their wave speeds, and hence their wavelengths, are much less than those of surface waves of the same frequency.
2) The particle velocities of internal waves, unlike those of surface waves, may reverse phase and have different amplitudes at different depths.
3) They can exist only in stratified water.
4) They may have very large amplitudes (tens of metres) because the restoring force is so small.
5) Although their vertical amplitude is zero at the free surface, their particle velocities are usually greatest there, because of a thin surface layer of less-dense water.

Internal tides are internal waves of tidal frequency, and these have been observed in the St. Lawrence Estuary. Semidiurnal internal tides were observed in the estuary below Tadoussac with wavelengths of about 60 km. Their presence helped to explain the tidal streams in the area, which could not be satisfactorily accounted for by the surface tide alone. The water column in the estuary can be very crudely represented as two layers with $h_1 = 50$ m, $h_2 = 250$ m, and $\Delta p/p = 0.003$. Substitution of these values, along with $g = 9.8$ m/s^2 into 1.7.4 gives a wave speed of 1.1 m/s, or 4.0 km/h, corresponding to a wavelength of 49 km for the semidiurnal tidal period of 12.4 h.

1.8. Coriolis Acceleration

Newton's classical laws of motion apply only when all measurements are made with respect to an inertial coordinate system, that is, one that is neither accelerating nor rotating. Thus, when measurements are made relative to a coordinate system fixed in the earth, allowance must be made for the rotation of the earth about its axis.

This is done by providing two "fictitious forces," the centrifilgal force and the Coriolis force, in addition to the apparent forces that cause acceleration of a body relative to the surface of the earth. A mass resting on the earth's surface is actually revolving about the earth's axis on a latitude circle once each day, and so is accelerating toward the centre of that circle. The inertia of the mass resists this centripetal acceleration, and, to an earth-bound observer, the mass appears to be pulled away from the axis by what he calls the centrifugal force. Since it varies only with latitude and not with time, the centrifugal force (CF) is conveniently combined with the earth's gravitational attraction (C) in what we know as "gravity" (g). Figure 8 depicts the vector addition of the two forces to give gravity, with the relative size of the centrifugal force vector greatly exaggerated for clarity. The centrifugal force is obviously greatest at the equator and zero at the poles, contributing to the fact that gravity is less at the equator than at the poles.

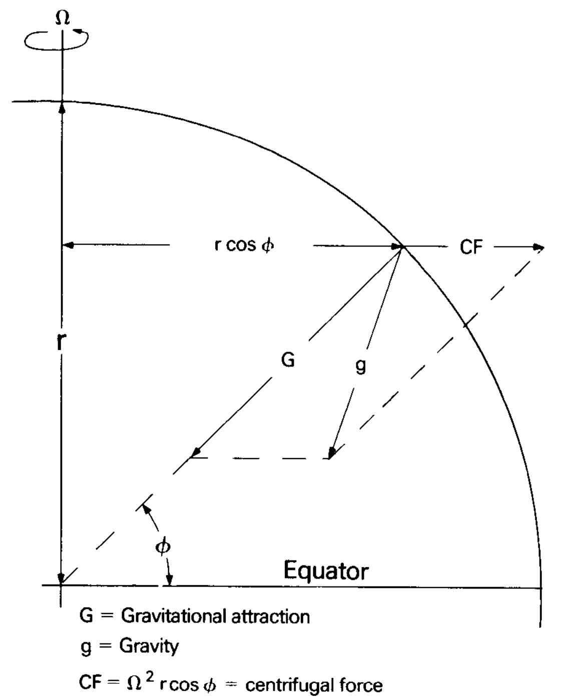


FIG. 8. Vector summation of earth's gravitational attraction (G) and centrifugal force due to earth's rotation (CF) to produce "gravity" (g).

A body in motion relative to the surface of the earth experiences an acceleration to the right of its horizontal direction of travel in the Northern Hemisphere (to the left in the Southern Hemisphere), an acceleration that is proportional to its velocity and to the sine of the latitude. This acceleration is also a result of the earth's rotation, and is allowed for in the Coriolis force. Figure 9 attempts to illustrate the origin of this force. There is a Coriolis force on objects moving vertically and a vertical component of Coriolis force on objects moving horizontally, but we will consider only the horizontal component of the Coriolis force on objects moving horizontally. Imagine the earth to be covered with a frictionless film, the surface of which conforms to that of a level surface, i.e. is everywhere normal to the direction of gravity. N and S are the north and south poles, and Ω is the earth's angular velocity. As a body moves to higher latitude, the easterly velocity of the earth's surface decreases, and so the easterly velocity of the body relative to the earth increases. This is seen as an acceleration to the right in the Northern Hemisphere and to the left in the Southern Hemisphere.

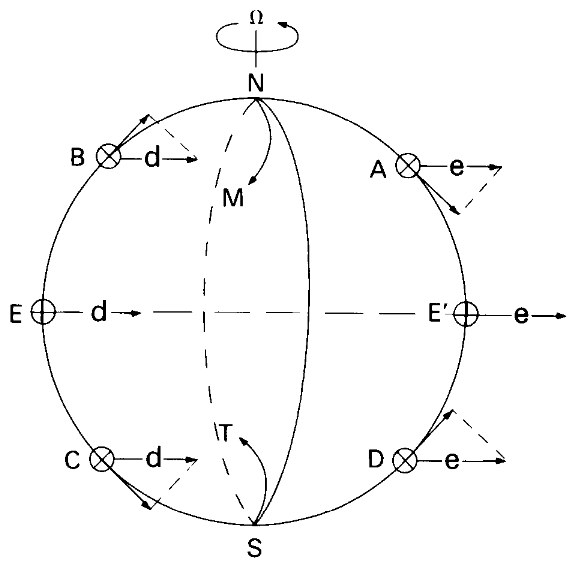


FIG. 9. Diagram to illustrate the origin of the Coriolis force, or acceleration.

Special cases of this are shown for a body projected south from N and for one projected north from S. Viewed from an inertial coordinate system both of these bodies would travel along the great circle NS, with no east-west velocity. Relative to the earth, however, they appear to follow the paths NM and ST, acquiring westerly velocity components as they move to lower latitudes, and so accelerating to the right in the Northern Hemisphere, and to the left in the Southern Hemisphere.

The Coriolis force due to the east-west velocity component arises from the fact that an easterly moving body experiences centrifugal force in excess of that included in gravity, and this excess centrifugal force has a component that accelerates the body along the level surface toward the equator. Similarly, a westerly moving body experiences a centrifugal force less than that in gravity and is accelerated along the level surface toward the pole. These accelerations are again seen to be to the right of the velocity in the Northern Hemisphere and to the left in the Southern Hemisphere. This effect is illustrated at points A and D for easterly velocity and at points B and C for westerly velocity. The velocity vector at each point is directed into the page. The vectors **d** represent a deficit and vectors **e** an excess of centrifugal force over that allowed for in gravity for a body at rest on the surface. Their horizontal components are the horizontal Coriolis forces. Points E and É show that for an east-west velocity at the equator the Coriolis force has only a vertical component. There is no horizontal Coriolis force for a north-south velocity at the equator because the rate of change of the earth's surface velocity with latitude is zero there. This can all be summed up in the statement that the horizontal component of the Coriolis force acting on a body moving with velocity v over the earth's surface acts to the right of the velocity in the Northern Hemisphere and to the left in the Southern Hemisphere, and has magnitude $2\Omega v \sin\varphi$, where φ is the latitude. $2\Omega \sin\varphi$ is called the Coriolis parameter, usually denoted as f.

The Coriolis force is rarely noticeable in laboratory-scale measurements, but is very significant in large-scale geophysical motions such as winds, ocean currents, and tides. It is this force that imparts the cyclonic and anti-cyclonic circulation to the atmosphere around low and high pressure regions and turns the ocean current systems into large circular gyres. It also acts on the tidal streams, changing the direction of propagation and the shape of the tide waves. When the tide propagates as a progressive wave along a channel in the Northern Hemisphere (NH), the range of the tide is observed to be greater on the shore to the right of the direction of propagation. This is because the tidal streams at HW are in the direction of propagation, and the Coriolis force acting on them moves water to the right until a slope of the surface is created to balance it. This raises the HW on the right shore and lowers it on the left. At LW the tidal streams are in the opposite direction, and the surface slope created to balance the Coriolis force lowers the LW on the right shore and raises it on the left. When the channel width is small compared to the tidal wavelength, only insignificant cross-channel tidal streams are required to create the surface slopes referred to above. Most channels are much narrower than the half wavelength of the surface tide required for resonance, but many may have a width comparable to the half wavelength of an internal tide. This is the case in the St. Lawrence Estuary, where the Coriolis force acts on an internal tide propagating seaward to produce strong crosschannel tilting of the interface between density layers, with correspondingly strong cross-channel tidal streams oppositely directed in the two layers.

1.9. Inertial Currents

The Foucault pendulum is one of the few laboratory experiments that can demonstrate the effect of earth rotation (Coriolis force) on a body in motion. It consists of a heavy mass suspended on a long single filament swinging freely through a small arc. The vertical plane of the oscillation is observed to rotate through 360° in a period of ($24/\sin\varphi$) hours, which period is referred to as the pendulum day. It is easiest to visualize this phenomenon for the special case of a pendlllum suspended directly over the North or South Pole and swinging back and forth in a plane fixed in space, while the earth rotates once in 24 h beneath it. lf it were possible to design such a pendulum to have a period of oscillation equal to one pendulum day, it would be observed to travel around in a circle once each half pendulum day. Again it is easiest to visualize this at one of the earth's poles: the pendulum would start tracing a circle at the centre of its swing and complete the circle when it returned again to the centre a half period later; by this time the earth would have rotated 180°, so the circle traced by the pendulum in the next half period would fall on top of the first circle. To explain this circular motion in a coordinate system fixed to the earth it would be necessary to invoke, in addition to gravity, the centrifugal force due to the circular motion and the Coriolis force due to the motion relative to the surface.

The above thoughts are pertinent to the consideration of what are called inertial currents. Water in the ocean that has been set in motion and is now drifting under its own inertia could be expected to keep deflecting to the right (NH) or left (SH) until it is moving in a circle (clockwise in the NH, counterclockwise in the SH) such that the centrifugal force away from the centre of the circle just balances the Coriolis force toward the centre. This is called the inertial circle. The time taken to complete the circle is the inertial period, and will be seen to equal one half pendulum day. lf the water is moving at speed v in a circle of radius r, the centrifugal force away from the centre is v^2/r. Let f be the Coriolis parameter at the latitude φ ($f = 2\Omega \sin\varphi$), so that the Coriolis force toward the centre is fv. The balance of forces is therefore

(1.9.1) $fv = v^2/r$, whence $r = v/f$

The circumference of the inertial circle is thus $2\pi r = 2\pi v/f$, and the time taken to travel around the circumference is the inertial period, T_1, so

(1.9.2) $T_1 = 2\pi(r/v) = 2\pi/f = 2\pi/(2\Omega \sin\varphi)$

Since $\Omega = 2\pi/24$ hours, $T_1 = (12/\sin\varphi)$ hours

The inertial period is seen to be the half pendulum day as anticipated. This is a period that is frequently detected in ocean current measurements. At 45° latitude it is 17 h, at 30° it is 24 hours, and at 75° it is 12.4 hours (the same as the semidiurnal tidal period). From 1.9.1 the radius of the inertial circle is seen to be proportional to the current speed for a given latitude. At 45° latitude the radius for a I km/h current is 2.7 km, and at the pole it is 1.9 km. At the equator the radius is infinite, meaning there are no inertial circles there since the Coriolis force is zero. It should be noted that the motion in an inertial circle is not that of an eddy, and that all parts of the water are moving in the same direction at the same time in inertial motion. For those versed in carpentry a helpful analogy might be that of the movement of an orbital sanding plate (cf. inertial motion) versus the movement of a rotary sanding disk (cf. eddy motion).

1.10 Amphidromic Systems

The word amphidrome is from the Greek for "a round race course," and describes a system in which wave crests propagate like the spokes of a wheel around a central amphidromic point, with wave amplitude increasing outward from zero at the centre. Figure 10 illustrates the formation of an amphidromic system in a coastal embayment by the action of the Coriolis force on what would

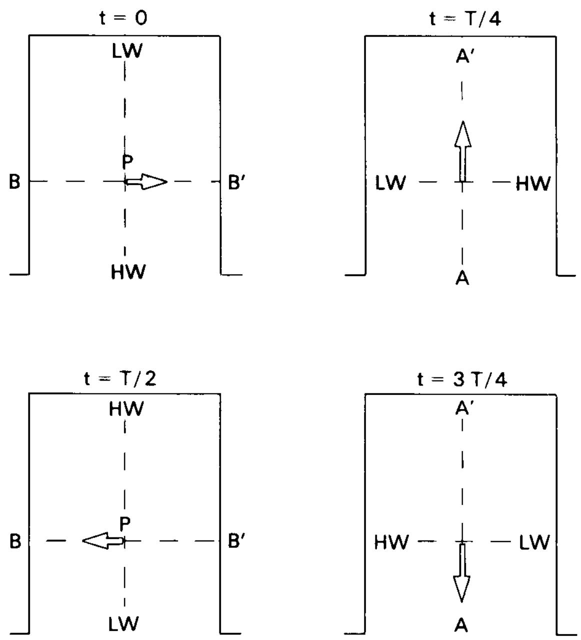


FIG. 10. Amphidromic system in a coastal embayment. Open arrows show the tidal streams (particle velocities).

otherwise be a simple standing wave. Let the embayment be greater than a quarter wavelength long, so that in the absence of earth rotation there would be a nodal line across the embayment at BB', with high water (HW) at A coinciding with low water (LW) at A' and vice versa. Let us now add earth rotation and follow the oscillation through one period (T), starting with HW at A at time t = 0. From t = 0 to t = T/2, there is an axial flow through P from A to A', and the resulting Coriolis force to the right sets up a cross flow component from B to B' sufficient to create a surface slope that balances the Coriolis force. Since the axial flow is greatest at t = T/4, the surface slope across the embayment is also greatest then, giving a HW at B' and a LW at B at t = T/4. At t = T/2 HW is at A ' and LW at A, as in the ordinary standing wave. At t = 3T/4 the axial flow through P from A ' to A is maximum, and HW will be at B with LW at B', to provide the surface slope necessary to balance the Coriolis force on the outflowing water. Thus, in the Northern Hemisphere earth rotation can convert a simple standing wave in a basin into an amphidromic system (or amphidrome), in which the crest travels counterclockwise around the perimeter of the basin about a pivotal point, P, called the amphidromic point. The vertical amplitude is zero at P and the particle velocity reaches its maximum there, but now the particle velocity vector rotates counter-clockwise, tracing out an ellipse. The amplitudes of the wave at B and B' and of the particle velocity across the basin depend on the geometry and size of the basin and the length of the period of oscillation relative to that of the half pendulum day. The origin and nature of amphidromes in the open ocean are less simple than those described above, and sometimes the sense of rotation is opposite to that in an embayment. Figure 29 shows the amphidromic system of the semidiurnal tide wave in the Gulf of St. Lawrence, and Fig. 30 shows an amphidrome of the diurnal tide wave in the Atlantic Ocean off Nova Scotia.

1.11 Tides and Tidal Streams

Since the tide propagates as a set of long waves in the ocean, much of the character of its vertical and horizontal motion has been revealed in the preceding consideration of long waves. The terms defined in section 1.2 to describe the characteristics of a wave are also applied to tides, but some special tidal terms are used as well. The definitions given here conform as closely as possible to common usage in Canadian tidal literature. In a tide wave the horizontal motion, i.e. the particle velocity, is called the tidal stream. The vertical tide is said to rise andfall, and the tidal stream is said to flood and ebb. If the tide is progressive, the flood direction is that of the wave propagation: if the tide is a standing wave, the flood direction is inland or toward the coast, i.e. “upstream.” The flow is the net horizontal motion of the water at a given time from whatever causes. The single word “current” is frequently used synonymously with “flow”,” but the term residual current is used for the portion of the flow not accounted for by the tidal streams. A tidal stream is rectilinear if it flows back and forth in a straight line, and is rotary if its velocity vector traces out an ellipse. Except in restricted coastal passages, most tidal streams are rotary, although the shape of the ellipse and the direction of rotation may vary. The ellipse traced out by a tidal stream vector is called the tidal ellipse. Slack water refers to zero flow in a tidal regime. The stand of the tide is the interval around high or low water in which there is little change of water level: this need not coincide with slack water.

In a purely progressive surface tide, maximum flood occurs at HW, maximum ebb occurs at LW, and slack water occurs at mid-tide rising and falling. In a purely standing surface tide, the slack waters occur at HW and at LW, maximum flood occurs at mid-tide rising, and maximum ebb occurs at mid-tide falling. This follows from the discussion in section 1.5, and is illustrated in Fig. 5 for long waves in general. Except for some frictional effect near the bottom, the tidal streams associated with a surface tide are the same from top to bottom. If tidal streams are observed to vary in speed, phase or direction over the water column, the presence of an internal tide is indicated. The average tidal stream in such a case belongs to the surface tide, and the departures at various depths from this average are the tidal streams belonging to the internal tide. This situation presents the possibility for slack water to occur at different times at different depths. Figure 11 illustrates various flow patterns that may result from the vector addition of a residual current and a rectilinear or a rotary tidal stream. It is seen that a rectilinear tidal stream experiences slack water twice during each period (Fig. 11 a) unless it is accompanied by (1) a residual current in the same direction but with speed greater than the tidal stream amplitude, in which case the flow is unidirectional with varying speed (Fig. 11b), or (2) a residual current in a different direction from that of the tidal stream, in which case the flow changes direction through a small angle (Fig. 11c). It is also apparent from Fig. 11 that a rotary tidal stream rarely experiences slack water, but that its direction changes through 360° in each cycle (Fig. 11d) unless the speed of the residual current exceeds or equals the amplitude of the tidal stream in that direction (Fig. 11e and f). In the latter cases, the direction of the flow swings back and forth through an angle less than or equal to 180°.

Since the observed tide consists not of a single wave, but of the superposition of many tide waves of different frequency and amplitude, it will never fit exactly any of our simple descriptions. Because of this, we cannot expect the heights of successive HWs or of successive LWs to be identical, even when they occur in the same day. Thus, the two HWs and two LWs occurring in the same day are designated as *higher and lower high water* (HHW and LHW), and *higher and lower low water* (HLW and LLW). It is likewise only the tidal stream associated with a single frequency tide wave that traces a perfect tidal ellipse. The composite tidal stream each day traces a path more closely resembling a double spiral, with no two days' patterns identical. Also, no tide is ever a purely progressive or a purely standing wave, so that slack water should not be expected to occur at the same interval before HW or LW at all locations.

1.12. Shallow-water Effects

One of our assumptions in the discussion of long waves of sinusoidal form was that the amplitude was much less than the depth. When a tide propagates into shallow water, this assumption may no longer be valid, and, as might be expected, the wave form is distorted from its sinusoidal form. In such shallow water the crest is found to propagate faster than the trough, producing a steeper rise and a more gradual fall of the water level as the tide wave passes. Figure 12 demonstrates this effect on the St. Lawrence River tide between Neuville and Trois Rivières. The outflow of the river and the bottom friction contribute to the distortion of the wave. The tide in this part of the St. Lawrence River is attenuated by friction as it progresses upstream, and is not reflected to produce a standing wave.

Tides in the open ocean are usually of much smaller amplitude than those along the coast. As mentioned earlier, this is partly due to amplification by reflection and resonance. It is, however, more generally the result of shoaling. as the wave propagates into shallower water, its wave speed decreases and the energy contained between crests is compressed both into a smaller depth and a shorter wavelength. The tide height and the tidal

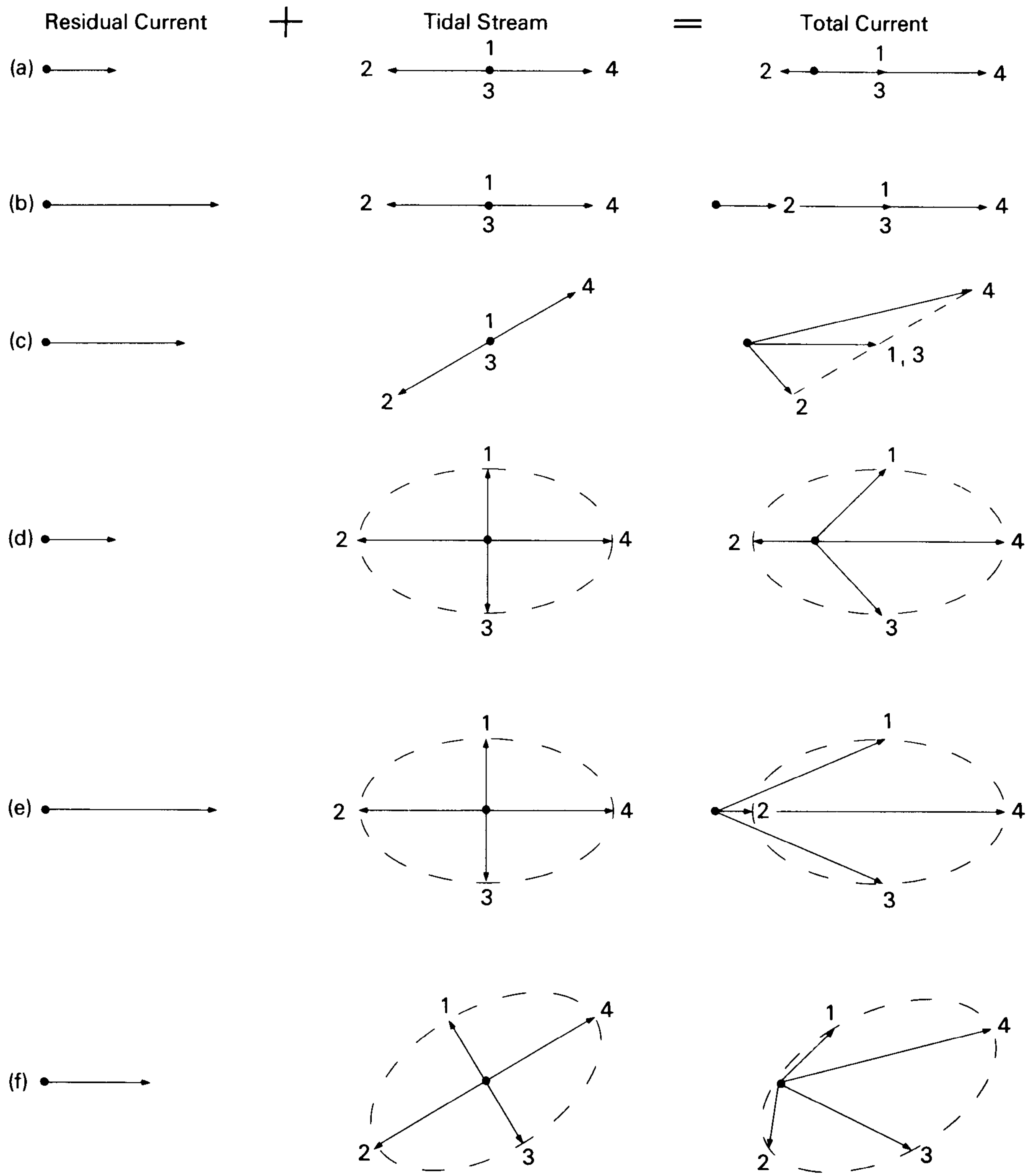


FIG. 11. Flow patterns resulting from combination of various residual currents with rectilinear and rotary tidal streams.

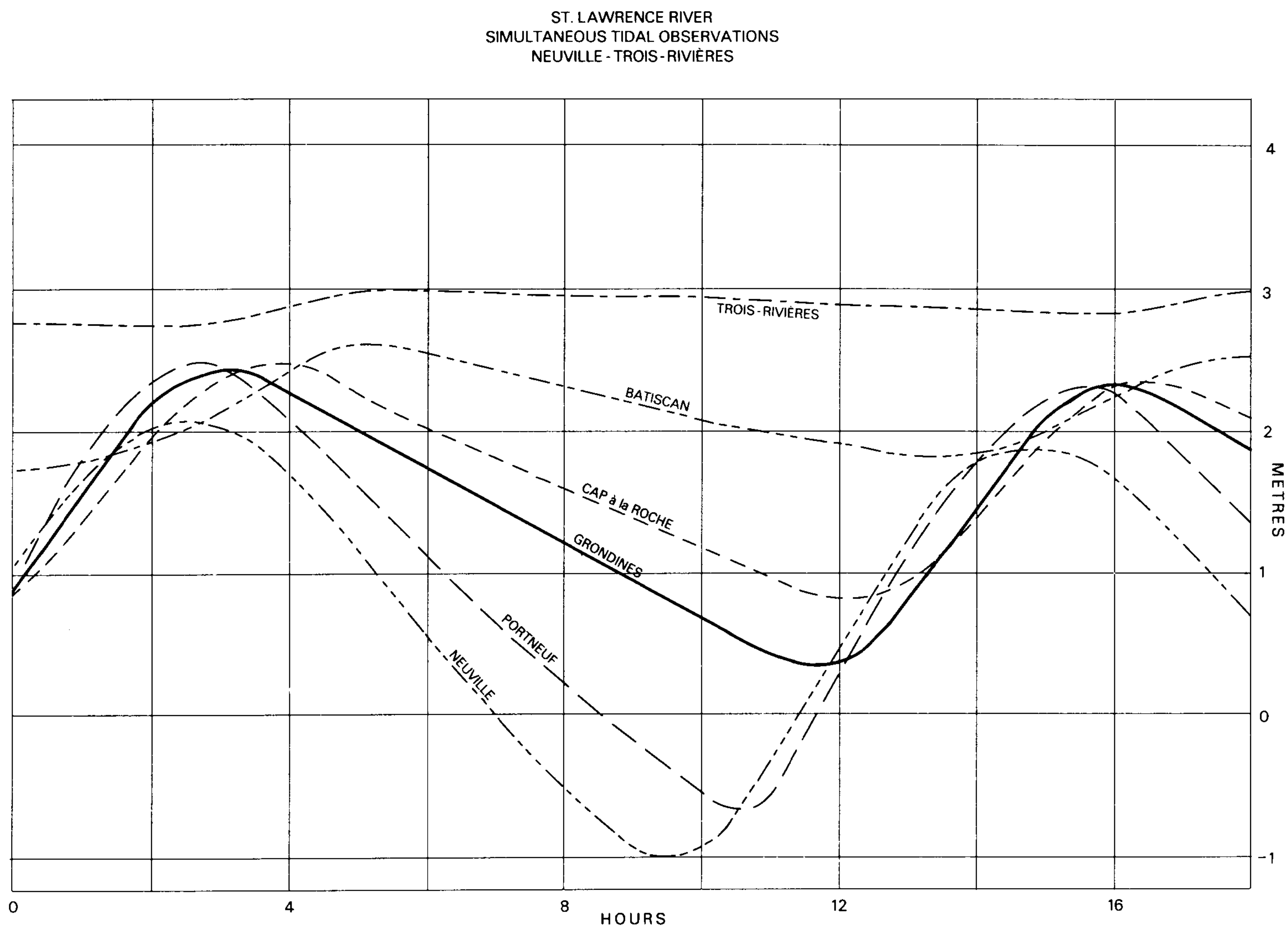


Fig. 12. Shallow water distortion of tide wave in St. Lawrence River, bctween Neuville and Trois Rivières. (from figure 13 of *Tides in Canadian Waters*, by G. Dohler).

stream strength must increase accordingly. If, in addition, the tide propagates into an inlet whose width diminishes toward the head, the wave energy is further compressed laterally. This effect, called funneling, also causes the tide height to increase.

Sometimes the front of the rising tide propagates up a river as a bore, a churning and tumbling wall of water advancing up the river not unlike a breaking surf riding up a beach. Creation of a bore requires a large rise of tide at the mouth of the river, some sandbars, or other restrictions at the entrance to impede the initial advance of the tide, and a shallow and gently sloping river bed. Simply stated, the water cannot spread uniformly over the vast shallow interior area fast enough to match the rapid rise at the entrance. Friction at the base of the advancing front, plus resistance from the last of the ebb flow still leaving the river, causes the top of the advancing front to tumble forward, sometimes giving the bore the appearance of a travelling waterfall. There are spectacular bores a metre or more high in several rivers and estuaries of the world. The best known bore in Canada is that in the Petitcodiac River near Moncton, N.B., but there is another in the Shubenacadie River and in the Salmon river near Truro, N.S., all driven by the large Bay of Fundy tides. These are impressive (about a metre) only at the time of the highest monthly tides, and may be no more than a large ripple during the smallest tides.

The Reversing Falls near the mouth of the St . John River at Saint John, N.B. is also caused by the large Bay of Fundy tides and the configuration of the river. A narrow gorge at Saint John separates the outer harbour from a large inner basin. When the tide is rising most rapidly outside, water cannot pass quickly enough through the gorge to raise the level of the inner basin at the same rate, so on this stage of the tide the water races in through the gorge, dropping several metres over the length of the gorge. When the outside tide is falling most rapidly, the situation is reversed, and the water races out through the gorge in the opposite direction, again dropping several metres in surface elevation. Twice during each tidal cycle, when the water levels inside and out are the same, the water in the gorge is placid and navigable. The surface of the water in the gorge near the peak flows is violently agitated and the velocity of flow is too rapid and turbulent to permit navigation through the gorge. This phenomenon is called a tide race in other less notorious situations.

A tide rip or overfall is an area of breaking waves or violent surface agitation that may occur at certain stages of the tide in the presence of strong tidal flow. They may be caused by a rapid flow over an irregular bottom, by the conjunction of two opposing flows, or by the piling up of waves or swell against an oppositely directed tidal flow. If waves run up against a current, the wave form and the wave energy are compressed into a shorter wavelength, causing a growth and steepening of the waves. If the current is strong enough, the waves may steepen to the point of breaking, and dissipate their energy in a wild fury at sea. Violent tide rips may be formed in this way.

Plate 1.
Hopewell Rocks ("the flowerpots") at Cape Hopewell, New Brunswick, on Chignecto Bay at the inner end of the Bay of Fundy, at low water. The Rocks have been eroded and formed into unusual shapes by water and sand suspended in the strong tidal stream. (Photo courtesy of the Canadian Government Office of Tourism.)

Plate 2. Fishermen checking salmon fishing weir at low water near Saint John, New Brunswick. Fish are carried into and trapped by the weir because of the strong tidal flow: they are then fished out of the weir at low water. (Photo by R. Brooks, NFB Phototeque, 1964.)

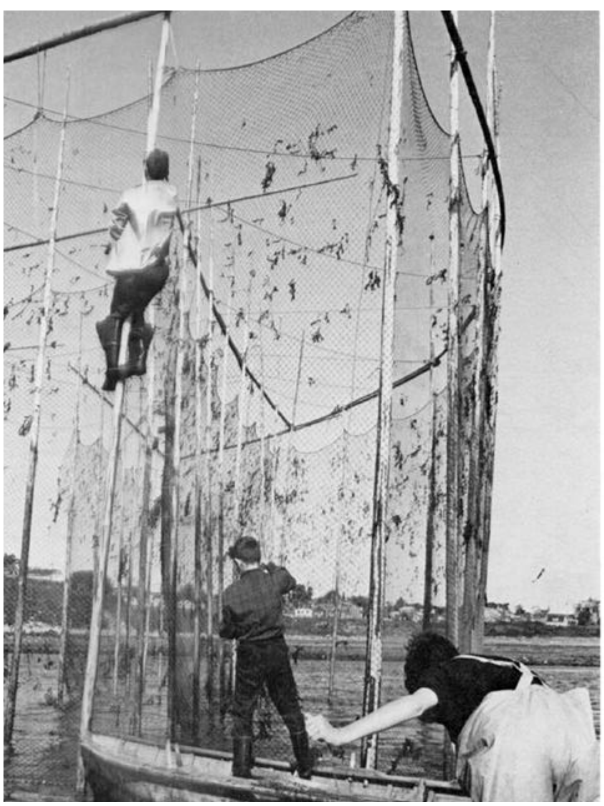


Plate 3. View of jetty and "mattress" at low water, Parrsboro, Nova Scotia, on the north shore of Minas Basin, at the inner end of the Bay of Fundy. (Photo by R. Belanger, Bedford Institute of Oceanography.)

Plate 4. MV *Theta* resting on wooden "mattress" beside a jetty at low water. Parrsboro, Nova Scotia. (Photo by Canadian Hydrographic Service, 1960)

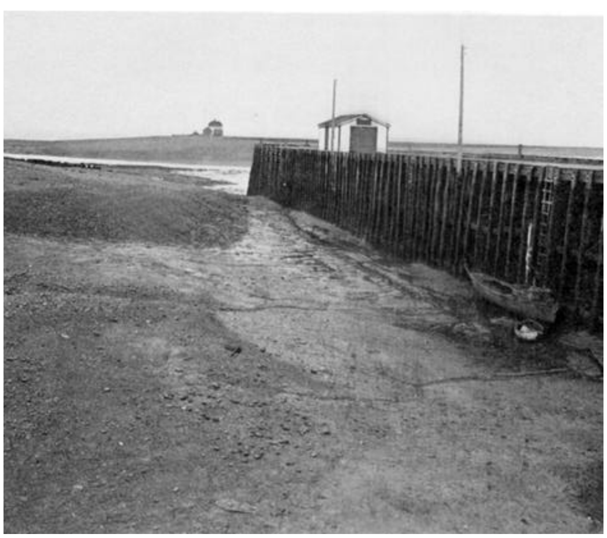
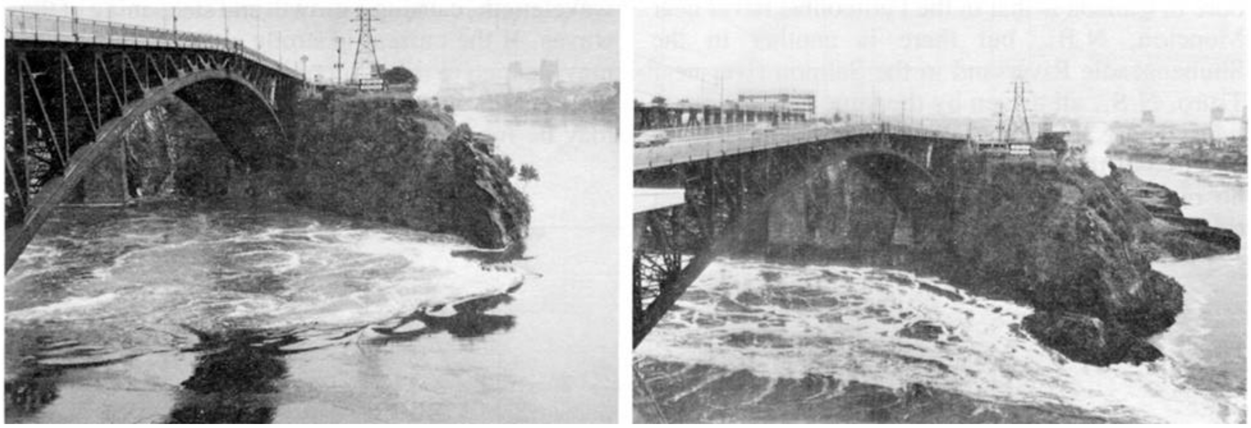


Plate 5. Corresponding views of jetty at Parrsboro, Nova Scotia, at extreme low water (left) and high water (right). (Photos by C. Blouin, NFB Phototeque. 1949.)

Plate 6. The Reversing Falls at Saint John, New Brunswick, at the mouth of the St. John River. The photo is an aerial view at slack water, showing the inner basin, the outer harbour, and the bridge over the gorge that separates them. The recorded extreme high and low waters at Saint John are 9.0 and -0.4 m,respectively, above chart datum, and at these times the flows would have been correspondingly greater. (Photo by Lockwood Survey, NFB Phototeque, 1966.)



(Plate 6.) Left shows the inflow through the gorge at high water in the outer harbour (7.6 m above chart datum at time of photo). Right shows the outflow through the gorge at low water in the outer harbour (0.9 m above chart datum at time of photo). (Photos by D.G. Mitchell, Canadian Hydrographic Service, 1963.)

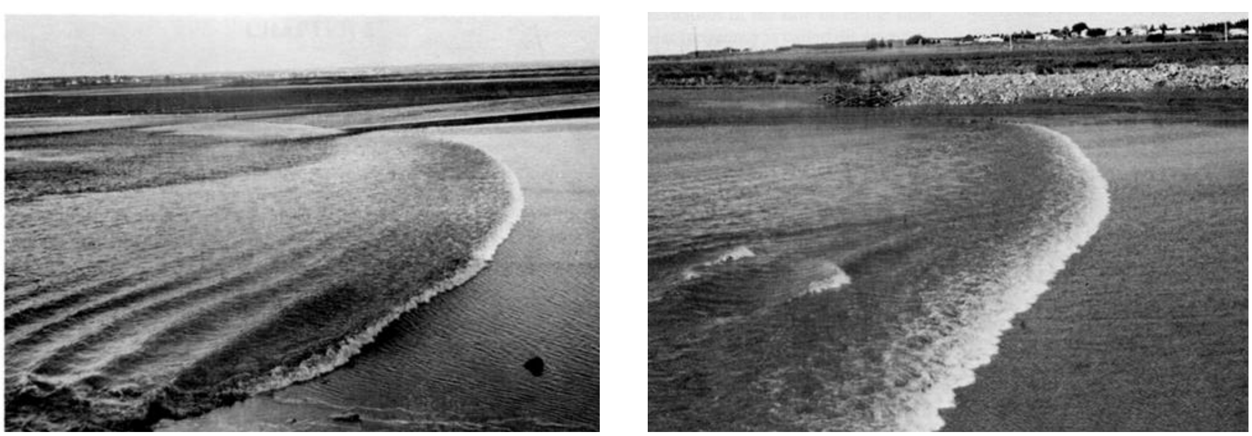


Plate 7. (Left) Tidal bore on the Petitcodiac River at moncton, New Brunswick. (Photo by D.G. Mitchell, Canadian Hydrographic Service, 1960.); (Right) Tidal bore on the Salmon River, near Truro, Nova Scotia. (Photo by F.G. Barber, Ocean Science and Surveys, DFO, 1982.)

CHAPTER 2
THE TIDE-RAISING FORCES

2.1 Introduction

It was explained in Chapter 1 that the local tide results from the superposition of long waves of tidal frequencies generated throughout the ocean by the tide-raising forces of the moon and the sun. It remains to investigate these forces, particularly with a view to determining the frequencies that characterize their fluctuations. It has been reasonably assumed, and later established by experience, that these are also the frequencies of the tide waves generated in the ocean, and so are the main frequencies present in the fluctuations of the local tide. Shallow water distortion, however, may be expected to add multiples and combinations of these frequencies (over-tides) to the spectrum of a coastal tide. Fluctuation in the tide or in the tidal force at a particular frequency is called the harmonic constituent at that frequency. The amplitudes and phaselags of the constituents are the harmonic constants of the tide, the phaselag usually being referred to the phase of the corresponding constituent in the tide-raising force at Greenwich. While it may be expected that the harmonic constituents present in the spectrum of the tide-raising forces will be present in the spectrum of the local tide, it should not be expected that they will be present in the same proportion or with the same phase relation. This is because ocean basins and coastal embayments are more nearly resonant at some tidal frequencies than at others, because nodes and amphidromes occur at different locations for constituents of different frequencies, and because processes such as the transfer of energy from surface tide to internal tide may be frequency selective in different situations.

The tide-raising forces are simply the portions of the moon's and the sun's gravitational attraction that are unbalanced by the centripetal (centrally directed) acceleration of the earth in its orbital motions. At the centre of mass of the earth, and only at this point, there is an exact balance between the gravitational attractions and the centripetal accelerations, this being the condition for orbital motion. Earth gravity, which includes the centrifugal force due to rotation of the earth on its axis, determines the shape of level surfaces and hence the shape of the mean level of the sea; but it does not contribute to the tide-raising forces because it does not vary with time. Although, as we shall see later, the moon has more effect on the tide than does the sun, it will be convenient to consider the sun's contribution first, since the orbital parameters are easier to envisage for the earth-sun system.

2.2. Sun's Tide-Raising Force

In this section we require Newton's laws of motion and of universal gravitation, and an understanding of centripetal acceleration. The law of motion states that the acceleration of a body equals the force acting on it per unit mass, or

(2.2.1) acceleration = force / mass

The law of universal gravitation states that a body of mass M exerts a gravitational attraction on a unit mass at a distance r of

(2.2.2) $F_g = GM / r^2$

in which G is the universal gravitational constant. The centripetal acceleration is the acceleration of a body toward the centre of curvature of the path along which it is moving, and for a body with velocity v along a path with radius of curvature r, it is:

$$A_c = v^2 / r$$

Let us now compare the gravitational attraction of the sun on the earth to that of the moon on the earth. The mass of the sun is 27 million times that of the moon, and the distance of the sun from the earth is 390 times that of the moon. Using this information in equation 2.2.2 gives

$$F_g\ (\text{sun}) / F_g\ (\text{moon}) = 27\text{x}10^6 / 390^2 = 178$$

so the gravitational attraction of the sun on the earth is 178 times that of the moon. This may at first seem surprising since we know the moon to be more effective in producing tides; but it is only the portion of the gravitational force not balanced by the centripetal acceleration in the earth's orbital motion that produces tides. This unbalanced portion will shortly be shown to be proportional to the inverse cube rather than the inverse square of the distance from the earth, but still proportional to the mass as in equation 2.2.2. Thus, the tide-raising forces of the sun are about 178/390 = 0.46 times those of the moon.

Figure 13 depicts a portion of the earth's orbit around the sun, with the cross section through the earth greatly exaggerated with respect to the sun's size and distance. Since the acceleration related to the earth's axial rotation is already accounted for in earth gravity, the earth should be thought of here as maintaining a fixed orientation in space during its revolution about the sun; thus each part of the earth experiences the same centripetal acceleration toward the sun. In particular, the centripetal acceleration at the centre of the earth, O, is exactly equal to the sun's gravitational attraction at that point, this being the condition for orbital motion. The centripetal acceleration, being everywhere constant, is therefore everywhere equal to the gravitational attraction at the centre, GS / r^2, where S is the sun's mass and r its distance from the centre of the earth. At a point such as A, that is closer to the sun, the gravitational attraction is greater than at the centre, O, and so has an unbalanced component that attempts to accelerate a mass at A, away from O and toward the sun. At a point such as A', that is farther from the sun, the gravitational attraction is less than at O, and the unbalanced component attempts to accelerate a mass at A' away from O and away from the sun. At B and B' in Fig. 13, the gravitational attraction has almost the same magnitude as at O, but is directed toward the sun along a slightly different line, so that the unbalanced components are both acting toward O. These unbalanced components of gravitational attraction are the sun's tide-raising forces. At A, A', B, and B' they are vertical, but at intermediate points they are inclined to the vertical . At four of

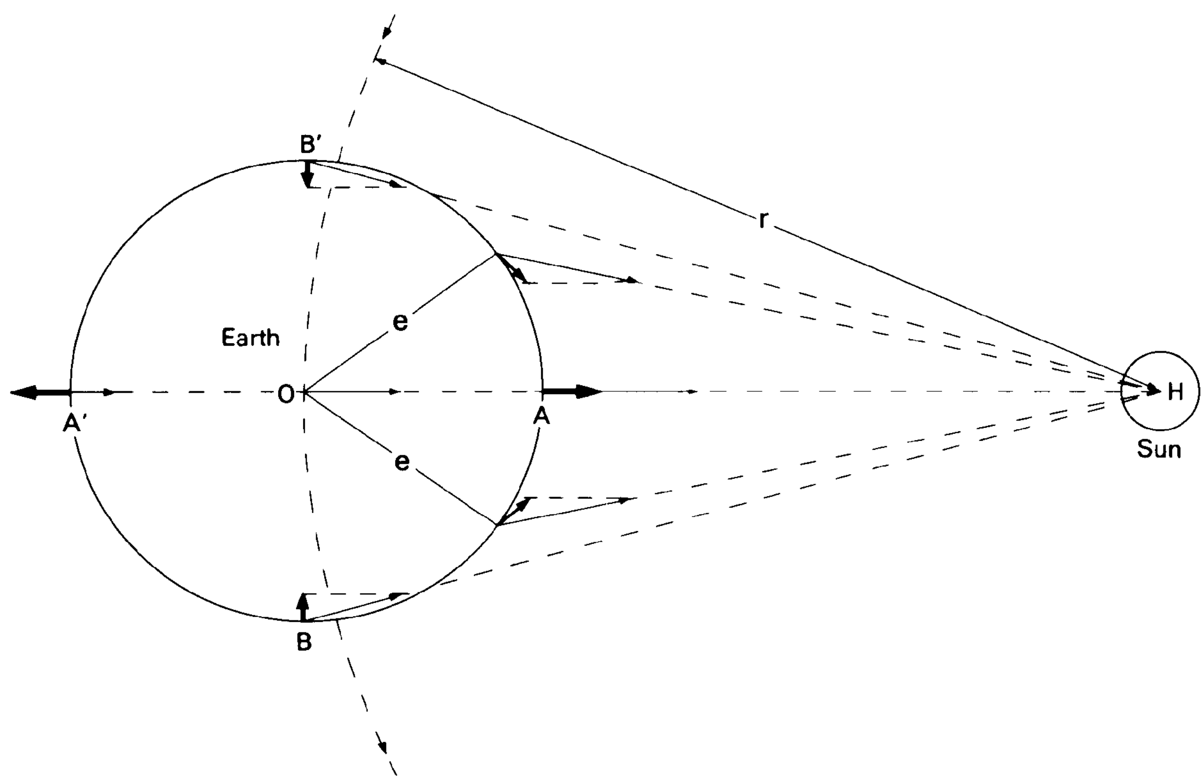


FIG. 13. Origin of sun's tide-raising forces (heavy arrows) as differences between sun's gravitational attraction and earth's centripetal acceleration in solar orbit.

the intermediate points the forces are entirely horizontal. The horizontal components of the tide-raising forces are called the tractive forces since it is they that accelerate water away from B and B' toward A and A' in an attempt to bring the surface everywhere normal to the vector sum of gravity and the tide-raising force. This ideal surface, referenced to the mean sea level defined by gravity alone, is called the equilibrium tide. To picture the sun's equilibrium tide in three dimensions, imagine the shapes traced out by revolving Fig. 13 about the axis AA'. Ocean tides are significant mostly because the water moves relative to the solid surface of the earth. If the earth were sufficiently pliable, it too would change shape to conform to the equilibrium tide surface, and there would be little or no relative movement of the water. The earth is not perfectly rigid, and does change shape slightly in response to the tidal forces, but these earth tides are small enough to neglect in this mostly qualitative discussion.

We will now estimate the magnitude of the tide-raising forces. As already stated, the sun's gravitational attraction at O in Fig. 13 is GS / r^2. At A it is $GS / (r-e)^2$, at A' it is $GS / (r+e)^2$, and at B and B' it is $GS / (r^2+e^2)$, where e is the earth's radius. All the attractions are directed from the point toward the sun's centre H. Since the tide-raising force at a point is the difference between the sun's local attraction and its attraction at the centre of the earth, we have the sun's tide-raising force, F_1, at A as

$$(2.2.3)\quad F_1(A) = [GS / r^2]\,[1 + 2e/r + \ldots -1] = 2GSe / r^3$$

In 2.2.3 and 2.2.4 we use the binomial expansion for $(1 - e/r)^{-2}$ and $(1 + e/r)^{-2}$ and neglect squares and higher powers of e/r, since it is so small. At A',

$$(2.2.4)\quad -F_1(A') = [GS / r^2]\,[1 - 2e/r + \ldots -1] = -2GSe / r^3$$

In 2.2.3 and 4, and in what follows, we have adopted the sign convention that a force directed vertically upward is positive. This explains the minus signs on the left side of 2.2.4 and 2.2.5. At B and B', the vector subtraction of the sun's attraction at O from that at B and B' gives, within the same approximation as above, only a component of the tide-raising force directed toward O, and

$$(2.2.5)\quad -F_1(B) = -F_1(B') = [GS/r^2\,(1 + e^2/r^2)^{-1}]\sin\beta = GSe/r^3$$

where β = angle OHB = angle OHB'. In 2.2.5 we neglected e^2/r^2 and approximated $\sin\beta$ as (e/r).

From the above expressions we see that the tidal forces are proportional to the mass of the sun and to the inverse cube of its distance, and that the compressional forces around the great circle BB', midway between A and A', are one half the strength of the expansional forces at the points A and A', at which the sun is in the zenith and the nadir, respectively.

2.3 Moon's Tide-Raising Force

In the previous section we spoke of the earth as orbiting around the sun, but actually the earth and the sun are both orbiting around a common centre of mass, which is less than 500 km from the centre of the sun. Similarly, the moon and the earth are orbiting about a common centre of mass, which is inside the earth, about 1 700 km beneath the surface. It is the revolution of the earth in this small orbit that is the counterpart of its revolution about the sun, which was considered in section 2.2. With this in mind, and with r as the moon's distance and M, the moon's mass, replacing S, we may apply the logic of section 2.2 directly to the earth-moon system (with H in Fig. 13 now being the moon's centre). This permits us to write down immediately expressions for the moon's tide-raising forces. The expansional forces at the points for which the moon is in the zenith and the nadir are

$$F_1(A) = F_1(A') = 2\ GMe / r^3 \tag{2.3.1}$$

and the compressional forces on the great circle around the earth's surface midway between these two points are

$$F_1(B) = F_1(B') = -\ GMe / r^3 \tag{2.3.2}$$

We have already noted in section 2.2 that the tide-raising forces of the sun are only about a half of those of the moon. It may be of some interest to compare the moon's tide-raising force to the force of gravity at the earth's surface. Neglecting the centrifugal force due to axial rotation, the surface gravity is

$$g = GE / e^2 \quad \text{so} \quad G = ge^2 / E \tag{2.3.3}$$

where E is the earth's mass. The maximum lunar tidal force is that expressed in 2.3.1 which, with the help of 2.3.3 may be rewritten as

$$F_1(A) = 2g(M/E)(e/r)^3 \tag{2.3.4}$$

M/E = 1/80 and e/r = 1/60, which, on substitution into 2.3.4 give the maximum lunar force as 10^{-7}g. So the tidal force is at most one ten-millionth of the earth's surface gravity. These are small forces indeed, but they act on every particle of water throughout the depth of the ocean, accelerating them toward the sublunar (or subsolar) point on the near side of the earth and toward its antipode on the far side. The undulations thus set up in the deep ocean are in fact quite gentle, and only become prominent when their energy is compressed horizontally and vertically as they ride up into shallow and restricted coastal zones.

2.4. Tidal Potential and the Equilibrium Tide

Many force fields can be expressed as the negative gradient of a scalar field, called the potential field. Such force fields are said to be conservative, since the work done against the force in moving from a point A to a point B depends only on the positions of the two points, and not at all upon the path followed in moving between them. This constant amount of work required to move unit mass (or unit charge, etc.) from A to B is the difference in potential between A and B. The earth's gravity field is a conservative field, whose potential is given the name geopotential. The difference in geopotential between points is the work done against gravity in moving a unit mass from one point to the other. Equi-geopotential surfaces are the familiar level surfaces, to which free water surfaces would conform in the absence of forces other than gravity. The lunar and solar tide-raising forces are also conservative, and can be expressed as the negative gradient of the tidal potential. Since the sum of one or more conservative force fields can be expressed as the negative gradient of the sum of their potentials, we may add the tidal potential to the geopotential and interpret equipotential surfaces in the combined field as "level" surfaces in the combined gravity and tidal force fields. In particular, one of these equipotential surfaces would be the surface of the equilibrium tide, the surface to which water would conform if it could respond quickly enough to the changing tidal forces. Because the geopotential does not vary with time and because we are interested in the time-variable tides. we need only consider the tidal potential, and interpret its variations as variations in the total potential at the mean sea level.

The tidal potentials, p_t at the point P (Fig. 14) are given very closely by the expressions

(2.4.1)

$$-p_1(\text{moon}) = (GMa^2 / 2r_m^3)(3\cos^2\alpha_m - I)$$

$$-p_1(\text{sun}) = (GSa^2 / 2r_s^3)(3\cos^2\alpha_s - I)$$

where r_m and r_s are the moon's and the sun's distances from the earth, the angles α_m, and α_s are their zenith angles (co-altitudes), and a is the distance from the centre of the earth to the point P (equals earth's radius, e, if P is at the surface). The other symbols are as previously defined. The minus signs in front are required to conform with the convention that the force is the negative gradient of the potential. Differentiation of 2.4.1 with respect to a gives the vertical component of the tidal force.

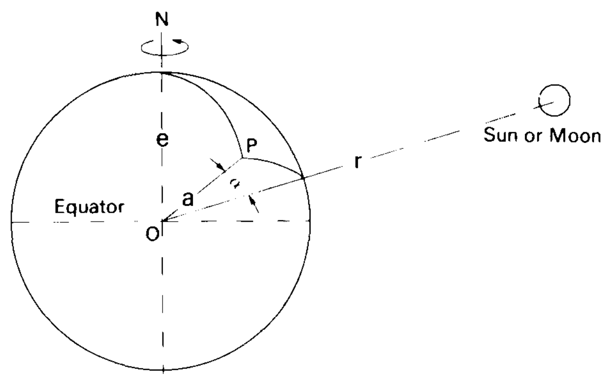


FIG. 14. Spherical triangle formed by north pole, the point on the earth directly beneath the sun (or moon) and the general point P on the earth's surface.

With cos a = 1, this reproduces the expression 2.2.5 for the tidal force at A (Fig. 13), and with cos a = 0, it reproduces the expression 2.2.5 for the tidal force at B.

The equilibrium tide surface must be an equipotential surface in the combined tidal and gravity field, and so any increase in the tidal potential must be matched by a decrease in the geopotential (i.e. a fall in the surface), and any decrease in tidal potential must be matched by an increase in geopotential (i.e. a rise in the surface). Using this fact we can calculate the height of the equilibrium tide above the mean sea level. Let the height of the equilibrium tide be Δh, which corresponds to an increase of $g\,\Delta h$ in geopotential. To maintain an equi-potential surface, this increase must be equal and opposite to the tidal potential, p_t so

(2.4.2) $\Delta h = -\, p_t/g$

Substituting the expressions for G from 2.3.3 and for p_t, from 2.4.1 into 2.4.2 gives the heights of the lunar and solar equilibrium tides as

(2.4 3) $\Delta h(\text{moon}) = (Me^4 / 2Er_m^3)\,(3\cos^2\alpha_m - 1)$

$\Delta h(\text{sun}) = (Se^4 / 2Er_s^3)(3\cos^2\alpha_s - 1)$

In substituting from 2.4.1 we put a equal to e because the equilibrium tide is for points at the earth's surface. The extreme values of Δh occur for $\alpha = 0°$ and $\alpha = 90°$. Using

$e = 6{,}400$ km, $M/E = 0.012$

e/r_m, $= 0.017$, $S/E = 3.3\text{x}10^5$

$e/r_s = 4.3 \times 10^{-5}$,

2.4.3 gives the extreme equilibrium tide heights as

(2.4.4) $\Delta h(\text{moon}) = 0.38$ m, and -0.19 m

$\Delta h(\text{sun}) = 0.17$ m, and -0.08 m.

The ratio of the solar to the lunar values in 2.4.4 is 0.46, the same as the ratio of the extreme solar and lunar tide-raising forces (see section 2.2). In fact, the equilibrium tide reflects all the important characteristics of the tide-raising forces, and, being a scalar instead of a vector, is a much more convenient reference for local tidal observations and predictions.

2.5 Semidiurnal and Diurnal Equilibrium Tides

The equilibrium lunar and solar surfaces defined by the expressions in 2.4.3 are ovals of revolution centred at the earth's centre and with axes directed toward the moon and the sun. They appear to rotate from east to west as the earth rotates daily on its axis with respect to the moon and the sun. The inclination of their axes north and south of the equator changes with the declination of the moon and of the sun, in a monthly cycle for the moon and an annual cycle for the sun. The ovals also change in shape as the orbital distances, r_m and r_s change in monthly and annual cycles, respectively. In formal tidal study the characteristics of the equilibrium tide are determined from mathematical analysis of expression 2.4.3 and the known astronomical parameters. It is, however, useful to obtain an intuitive understanding of how the various tidal harmonic constituents arise, and that is how we will now proceed.

Figure 15 depicts the sun's equilibrium tide superimposed on the equi-geopotential surface of mean sea level, (a) for the sun on the equator, (b) for the sun, at maximum north declination, and (c) for the sun at maximum south declination.

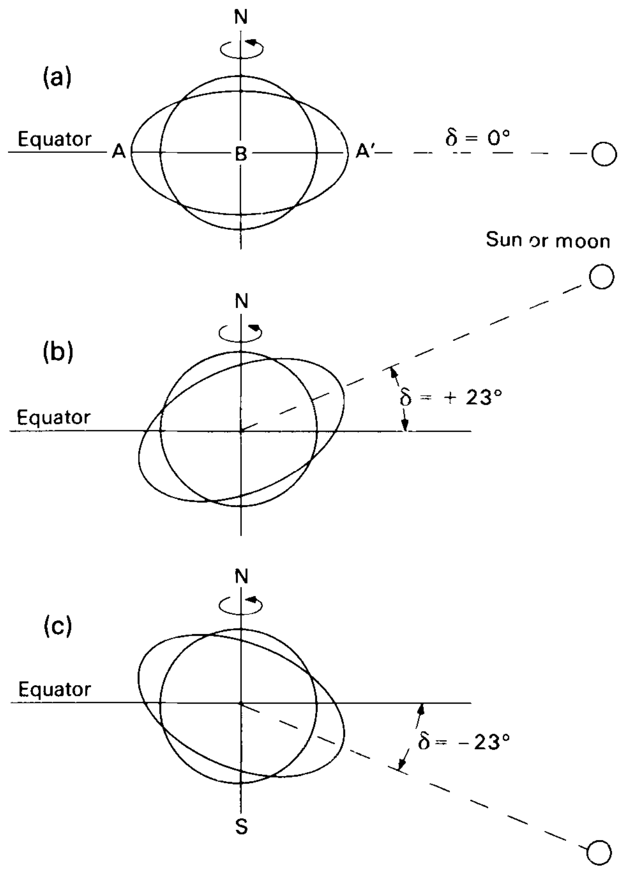


FIG. 15. Equilibrium tidal surface (AA') for sun or moon (a) on equator, (b) north of equator and (c) south of equator.

From (a) it is seen that with the sun at zero declination an observer on the equator rotates with the earth once each solar day with respect to the sun's equilibrium tide, passing through LW at points B and B' (B' is on the opposite side of the earth from B), and through HW at points A and A'. In fact, an observer at any latitude would experience equilibrium LW as his meridian passed through B and B', and HW as it passed through A and A', although the heights of HW would decrease with increasing latitude north or south of the equator. This is the origin of the semidiurnal solar constituent of frequency two cycles per day (30°/h); it is designated S_2. If we simply replace the sun with the moon in the above discussion, we have the explanation for the origin of the semidiurnal lunar constituent (M_2). Its frequency is two cycles per lunar day (28.98°/h). The lunar day is about 50 min longer than the solar day, because the moon advances about 12.5° in its orbit each day with respect to the sun's position.

When the sun is north or south of the equator, one centre of HW for its equilibrium tide is north and the other is south of the equator, as shown in Fig. 15b and c. In these cases, an observer moving around with the earth at the equator would still experience two equal HWs and two equal LWs per day, although the HWs would not be as high as in case (a). An observer at a northern latitude would experience HHW at noon and LHW at midnight in (b) while an observer at a southern latitude would experience HHW at midnight and LHW at noon. In case (c), there would be the same inequality in the two HWs for observers away from the equator, but the northern observer would now experience HHW at midnight, etc. In the equilibrium tide the two LWs would have the same height (but not necessarily so in an actual tide). The difference in height between HHW and LHW is called the diurnal inequality, and it increases with the declination of the sun and with the observer's latitude (north or south) for an equilibrium tide. In fact, if the sun's declination is d, the band of low water around the earth in the equilibrium tide extends no farther north and south than latitude 90° - δ, and observers at higher latitudes than this would see only a distorted diurnal tide, with one true HW and an extended period of low water. A semidiurnal tide with a diurnal inequality can be considered as the sum of a semidiurnal and diurnal tide. This is illustrated in Fig. 16, which shows the combination of the semidiurnal and diurnal contributions to produce the equilibrium tide of the sun at 23° declination, for an observer at latitude (a) 15°, (b) 35°, (c) 55°, and (d) 75°.

Since the diurnal tide must reinforce the noon HW when the sun and the observer are on the same side of the equator, fall to zero when the sun is on the equator, and reinforce the midnight HW when the sun is on the opposite side of the equator, it is clear that more than a single diurnal constituent is required. From trigonometry we have the relation:

$$\cos (n_1 + n_0)t + \cos (n_1 - n_0)t = 2(\cos n_0 t)(\cos n_1 t) \quad (2.5.1)$$

If n_1 is the angular speed 360° per solar day (15°/h) and n_0 is 360° per year (0.04°/h), the right side of 2.5.1 is seen to be a diurnal oscillation of frequency n_1 whose amplitude is modulated at the

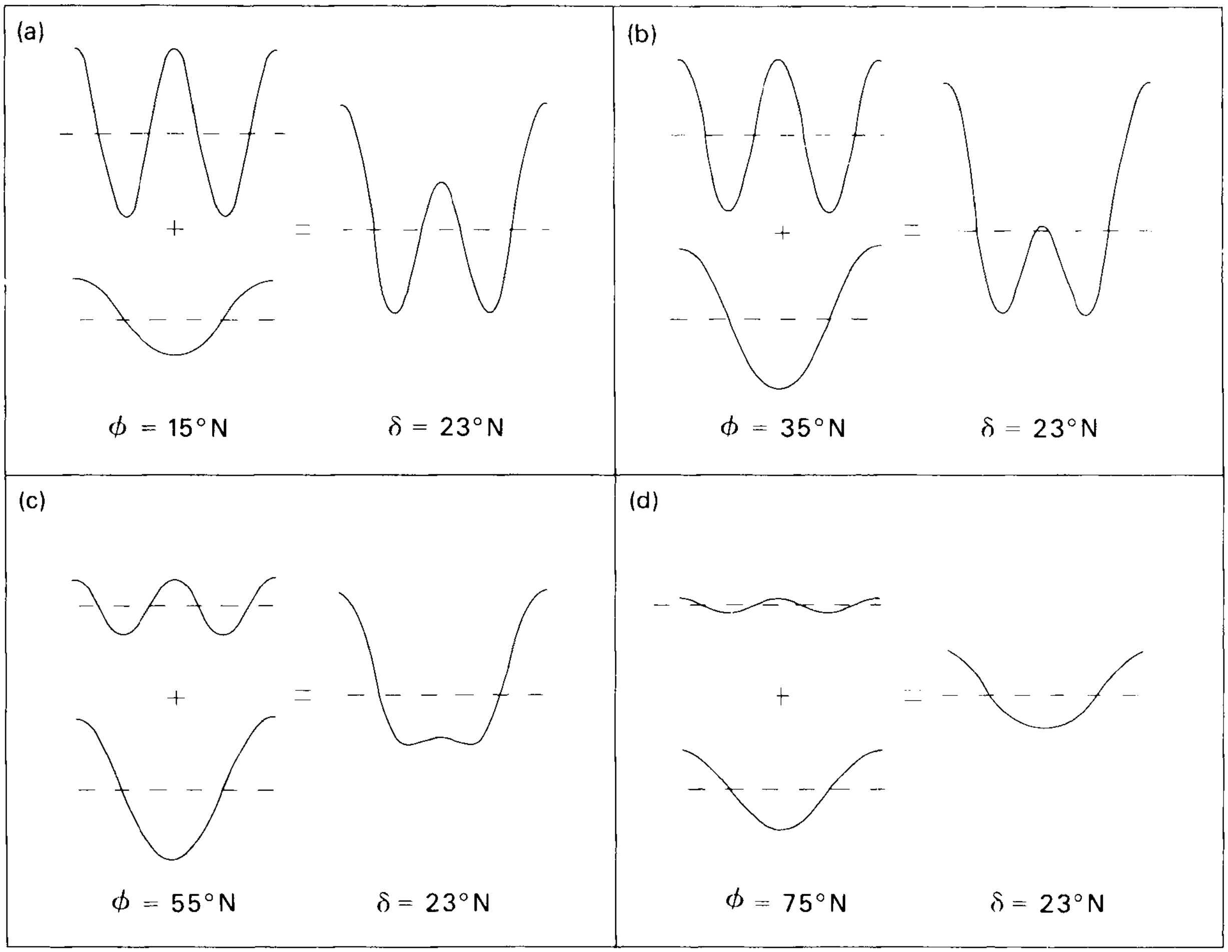


FIG. 16. Representation of solar equilibrium tide as sum of semidiurnal and diurnal contributions for sun at 23° declination, and for latitudes (a) 15°, (b) 35°, (c) 55° and (d) 75°.

annual frequency n_0 falling to zero at n_0t = 90° and 270°, and having maximum amplitude but with opposite phase at n_0t = 0° and 180°. Figure 17 shows a plot of 2.5.1 for a few cycles of n_1, around n_0t = 90° to illustrate the change in amplitude and shift in phase. This is the origin of the two solar declinational diurnal constituents P_1 with frequency 14.96°/h, $(n_1 - n_0)$ and K, with frequency 15.04°/h, $(n_1 + n_0)$. Because the earth's rate of rotation on its axis with respect to the "fixed stars" is equal to its rate of rotation with respect to the sun plus its rate of orbital revolution about the sun, the frequency of K, is seen to be one cycle per sidereal day.

The lunar equilibrium tide changes with the declination of the moon over a period of a month in the same manner as the solar tide changes with the sun's declination over a year. This, then, gives rise to two lunar declinational diurnal constituents with frequencies of one cycle per lunar day plus and minus one cycle per lunar month. The frequency of the earth's rotation with respect to the moon plus the moon's frequency of orbital revolution about the earth is also equal to one cycle per sidereal day, so that one of the moon's constituents has the same frequency as the

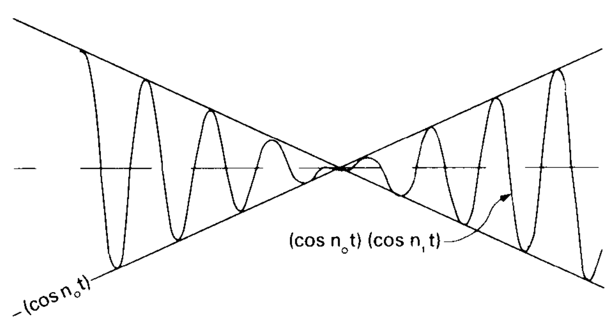


FIG. 17. Plot of $\cos(n_1 + n_0)t + \cos(n_1 - n_0)t = 2(\cos n_0 t)(\cos n_1 t)$ for a few cycles on either side of $n_0 t = 90°$.

corresponding one for the sun, K_1 . Because of this, K_1 serves double duty, and is called the luni-solar declinational diurnal constituent. The other lunar constituent of the pair is O_1. with angular speed 13.94°/h.

The orbits of the moon about the earth and of the earth about the sun are not circles, but are ellipses, with the earth and the sun occupying one of the foci in the respective orbits. Thus, the distances of the moon and the sun from the earth change within the period of the particular orbit, 1 month for the moon and 1 year for the sun. The orbital points at which the moon is closest to and farthest from the earth are called perigee and apogee, respectively. The corresponding points in the earth's orbit about the sun are called perihelion and aphelion. The dependence of the tidal potential on the inverse cube of these distances (r_m, and r_s in 2.4.1) causes the shape of the solar equilibrium tide (see Fig. 15) to lengthen along its axis and compress in the middle at perihelion, and to shorten along its axis and expand in the middle at aphelion. The shape of the lunar equilibrium tide changes similarly at perigee and apogee, respectively, but the change is much more pronounced for the lunar than for the solar tide. The effect of this is to modulate the amplitudes of the solar constituents with a period of a year and of the lunar constituents with a period of a month. But there is a further complication; the earth and the moon do not travel at constant angular velocities around their orbits, but travel faster when they are closer to the central body. The effect of this is to modulate the phases of the lunar constituents over a period of a month, and those of the solar constituents over a period of a year. The combined effect of the amplitude and phase modulations can be imitated mathematically by adding to each constituent two satellite constituents with frequencies equal to that of the main constituent plus and minus the orbital frequency, but with the amplitude of one satellite greater than that of the other. Figure 18 attempts to illustrate how the amplitude and phase modulations are accomplished. Tidal constituents may be regarded as rotating vectors, since they have a fixed amplitude and a uniformly increasing phase angle. A vector sum is obtained graphically by placing all of the participating vectors head to tail and joining the tail of the first to the head of the last. OR is the main constituent, with angular speed n, RS is the first satellite constituent, with speed $n_1 = n + n'$, and ST is the second satellite constituent, with speed $n_2 = n - n'$. The sum of the three constituents is OT, and relative to the rotating vector OR, the point T traces out an ellipse with centre at R. Its semi-major axis equals the sum of the satellite amplitudes, and its semi-minor axis equals their difference. It may be seen from Fig. 18 that as T moves around the ellipse once for each cycle of the main constituent, the amplitude of the vector sum, OT, oscillates between OP and OP', and the phase oscillates about that of OR through the angle QOQ'. If the satellite amplitudes are equal, the ellipse collapses to a line, and there is amplitude modulation only. This is the origin of the larger and smaller lunar elliptic semidiurnal constituents N_2 (speed 28.44°/h) and L_2 (speed 29.53°/h), respectively, and also of the larger and smaller solar elliptic semidiurnal constituents T_2 (speed 29.96°)/h) and R_2 (speed 30.04°/h). R_2 is so small it is usually neglected.

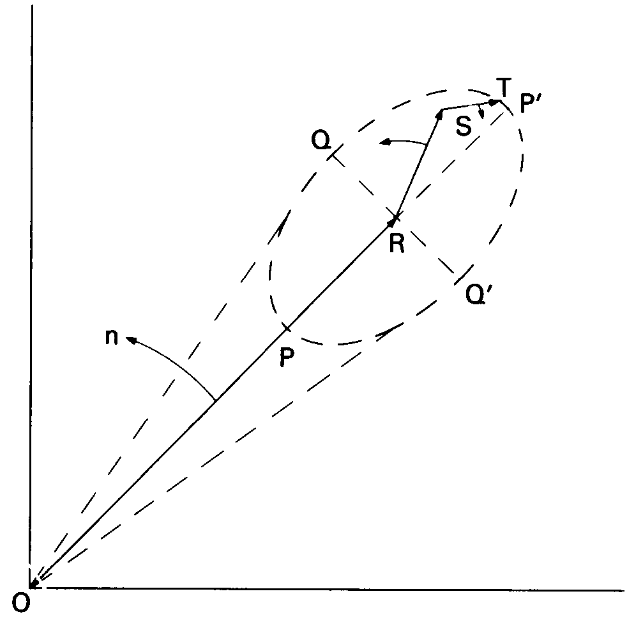


FIG. 18. Amplitude and phase modulation produced by vector sum of main constituent plus two counter-rotating satellite constituents.

2.6 Long-period Equilibrium Tides

Here we will discuss tidal constituents whose periods are comparable to the sun's or the moon's orbital period. It is important to distinguish between a long-period constituent and a long-period modulation of a short-period constituent. The long-period modulation changes the range of the tide over the long period, but does not change the mean water level, whereas the long-period constituent does not change the range of the tide, but introduces a long-period fluctuation in the mean water level. To demonstrate the origin of the long-period tidal constituents we look again at Fig. 15. It is apparent that an observer near the North or South Pole will experience a lower daily average equilibrium tidal elevation when the tide-raising body (sun or moon) is on the equator as in (a) than when it is north or south of the equator as in (b) or (c). Although most easily envisaged for high latitudes, this effect is present at other latitudes as well. It results in the introduction of the lunar fortnightly constituent M_f (speed I .10°/h), into the lunar equilibrium tide, and the solar semi-annual constituent S_{sa} (speed 0.08°/h) into the solar equilibrium tide. M_f and S_{sa} are thus related to the moon's and the sun's cyclic changes in declination. There is also a lunar monthly constituent, M_m (speed 0.54°/h), and a solar annual constituent, S_a(speed 0.04°/h), and these are related to changes in the lunar and solar distance over a month and a year, respectively.

2.7 Mathematical Analysis of the Equilibrium Tide

In the preceding discussions we have considered the equilibrium tide as the envelope of equal tidal potential surrounding the earth at a given time. We will now express it as a time-varying function at a fixed location on the earth. To do this, we must express cos a in 2.4.3 in terms of the local latitude and of the declination and hour angle of the sun and moon. The hour angle of a celestial object is its longitude angle west of the observer's longitude.

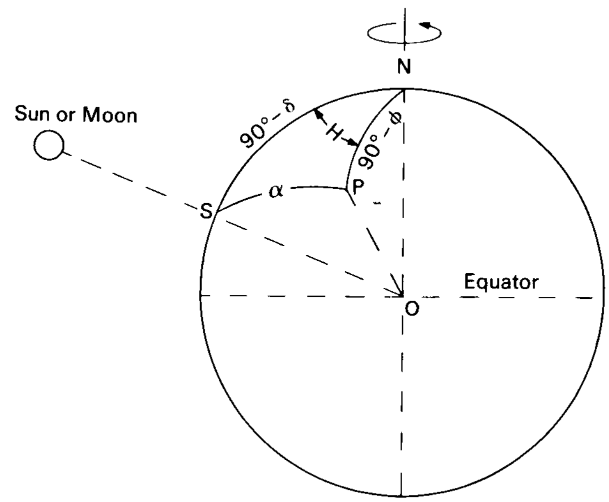


FIG. 19. Projection of the sun (or moon), the earth's polar axis, and a general point on the earth's surface onto a sphere surrounding the earth (the celestial sphere), to form the spherical triangle SNP.

In Fig. 19, PSN is a spherical triangle on a sphere surrounding the earth, and with its centre at the earth's centre, O. P is the projection of the observer's position from the centre of the earth onto the sphere, N is the projection of the North Pole onto the sphere, and S is the intersection with the sphere of the line joining the centre of the earth to that of the celestial object (sun or moon). Angle PON is the co-latitude of P (90° - φ), angle SON is the co-declination of the celestial object (90°- δ), POS is its zenith angle (α) with respect to P, and PNS is its hour angle (H) with respect to P. From a formula of spherical trigonometry we have

(2.7.1) $\cos \alpha = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H$

Substituting 2.7.1 into 2.4.3, and using some trigonometric relations to simplify it, gives the equilibrium tide at P as

(2.7.2) $$\Delta h = BC\ (\cos 2\delta - 1/3) + BC_1 (\sin 2\delta) \cos H + BC_2 (\cos 2\delta + 1) \cos 2H,$$

where $B = 3Me^4 / 2Er_m^3$ for the moon, $3Se^4 / 2Er_s^3$ for the sun,
and

$C_O = 3/8 (\cos 2\varphi - 1/3)$
$C_1 = 1/2 \sin 2\varphi$
$C_2 = 1/8 (\cos 2\varphi + 1)$.

H and δ are the local hour angle and declination of the moon or the sun, as appropriate. The C_i are constants for a given latitude.

In discussing 2.7.2 we will refer only to the moon, but the same logic applies for the sun and its equilibrium tide. The first term on the right of 2.7.2 introduces the long-period species of constituent, since B varies over a period of a month and $\cos 2\delta$ varies over a period of half a month. The second term introduces the diurnal species of constituent since the hour angle (H) advances at a frequency of one cycle per lunar day. Multiplication by $\sin 2\delta$ splits the species into constituents whose frequencies differ by two cycles per month, as shown in 2.5.1 and Fig. 17 from a different approach. The third term on the right of 2.7.2 introduces the semidiurnal species of constituent, since 2H advances at a frequency of two cycles per lunar day. Multiplication by B modulates the species at a frequency of one cycle per month, giving rise to constituents N_2 and L_2 as defined in section 2.5. The factor $\cos 2\delta$ also modulates a portion of the semidiurnal species at a frequency of two cycles per month, introducing a pair of lunar declinational semidiurnal constituents not previously discussed. Their frequencies are two cycles per lunar day plus and minus two cycles per month. The constituent with the higher frequency is also the larger of the pair, and has the same frequency as the corresponding solar constituent, both being equivalent to two cycles per sidereal day. It is thus called the lunisolar declinational semidiurnal constituent or K_2 (speed 30.08°/h). Many other constituents could be discovered by examining the modulation of the declinational constituents by B and by treating the departure of some of the factors from true sinusoidal functions of time. The relative amplitudes of the constituents in the equilibrium tide can also be determined from analysis of 2.7.2 and substitution of the parameter values. The purpose of this section, however, is simply to demonstrate some of the techniques employed in identifying the important tidal frequencies. Numerical analysis on fast electronic computers now provides tools for investigation of the equilibrium tide that were not available during the early development of tidal theory. It is now quite feasible to generate from 2.7.2 the combined equilibrium tide for the sun and the moon as a time series of elevations covering many years, and to analyse it numerically into its constituents, identifying their frequency, phase, and amplitude. In Appendix A are listed a few of the equilibrium tidal constituents, along with their frequencies (as angular speed) and their amplitudes relative to that of M_2.

2.8 Spring and Neap Tides

It cannot be stressed too much that at no place on the earth is the actual tide the same as the equilibrium tide at that place. Nevertheless, many of the characteristics of the two are similar except for magnitude and timing. The equilibrium spring tide occurs on the day that the sun's and the moon's HWs fall on the same meridian, which, as shown in Fig. 20, occurs at new and full moon. The HWs occur near local noon and midnight and are higher than average because of the reinforcement of the two. The two LWs also reinforce, but in the opposite sense, making them lower than average. The result, then, is a larger than average range of equilibrium semidiurnal tide at spring tide. The equilibrium neap tide occurs on the day that the sun's and the moon's HWs most closely coincide with the other's LWs, which, as shown in Fig. 20, occurs at the moon's first and last quarter. The result is a smaller than average range of tide at neap tide. The range of the combined solar and lunar tide does not, of course, change suddenly at spring and neap, but is modulated sinusoidally over the half-month period between successive springs or neaps. From the standpoint of tidal constituents. it is the interaction of M_2 and S_2 as they come in and out phase with each other that produces the springs and neaps. This fortnightly modulation of the semidiurnal tide is prominent in actual tide records as well as in the equilibrium tide, so much so in fact that in many parts of the world HW and LW at springs are taken as standards of extreme high and low waters. This is not invariably the case, however, because in other parts of the world there may be another characteristic tide that dominates over the spring tide. In the Bay of Fundy, for example, the perigean tides (the large semidiurnal

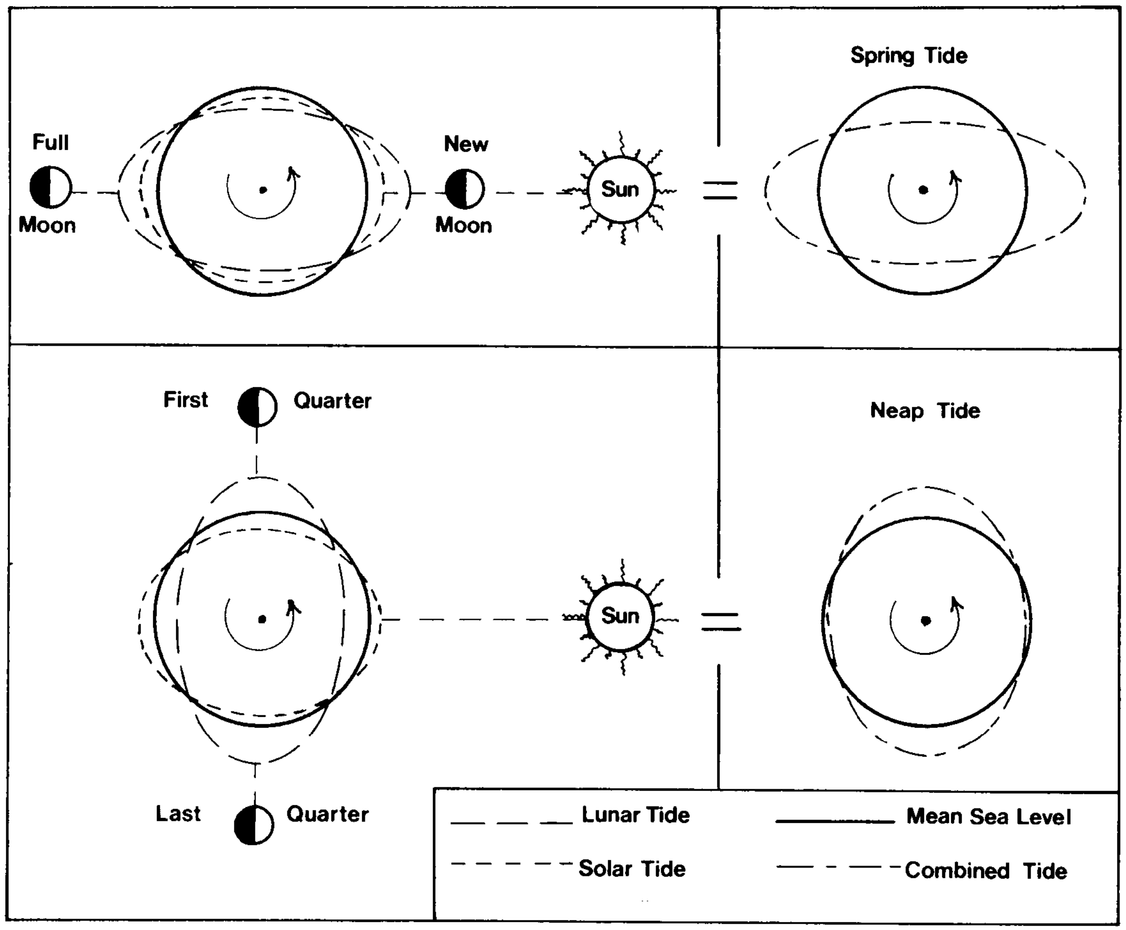


FIG. 20. Combination of lunar and solar equilibrium tides to produce spring tides at new and full moon and neap tides at moon's first and last quarter.

tides associated with the moon's perigee) are equally as significant as the spring tides. In regions such as Canada's Pacific coast and parts of the Gulf of St. Lawrence it is the diurnal inequality that renders the simple spring tide heights unsatisfactory as standards of extreme high and low water.

2.9 Classification of Tides

Tides are frequently classified according to the diurnal inequality they display, as a means of providing a simple description of the character of the tide in various regions. The formal classification is usually made on the basis of the ratio of some combination of the diurnal harmonic constituents over a combination of the semidiurnal constituents. A criterion that is commonly used is the ratio of the amplitude sum of O_1 and K_1 over the amplitude sum of M_1 and S_2.

The ratio that is used in Canadian tidal classification is more complicated than this, but the principle is the same - the larger the numerical value of the ratio, the larger the diurnal inequality. The purpose of defining a ratio is to automate the classification once the constituents are known, avoiding the need to scan long periods of record visually. Regardless of the method used, the intent is to classify tides into four groups, qualitatively described as follows:

Semidiurnal (SD): two nearly equal HWs and two nearly equal LWs approximately uniformly spaced over each lunar day.
Mixed, mainly semidiurnal (MSD): two HWs and two LWs each lunar day, but with marked

inequalities in height and irregularities in spacing.
Mixed, mainly diurnal (MD): sometimes two unequal HWs and LWs at irregular spacing over a lunar day, and sometimes only one HW and one LW in a day.
Diurnal (D): only one HW and one LW each lunar day.

Since the equilibrium tide is the same for all points at the same latitude, the earth could be divided into bands of latitude conforming to the above classification, with equilibrium tides at latitudes less than 10° being SD, those between latitudes 10° and 40° being MSD, those between 40° and 60° being MD, and those at latitudes higher than 60° being D. The actual tides, of course, may reflect the character of tide waves propagated from far away, and should not be expected to conform to the same classifications within latitude bands. Figure 21 shows a sample tidal curves for one month at four Canadian locations to illustrate the four classes defined above. It is noted that all four locations lie within the same three-degree band of latitude. Figure 22 indicates on a map of Canada the regions in which the various types of tide dominate. Although the East Coast is predominantly a region of mainly semidiurnal tide, we find the only examples of the diurnal tide in the Gulf of St. Lawrence. This is because these points lie near an amphidromic point of the semidiurnal tide. We also note that the tide observed in the Arctic archipelago is heavily semidiurnal in character, unlike the equilibrium tide for these latitudes. This is because much of the tide in the Canadian Arctic has propagated in through the passages from the North Atlantic Ocean.

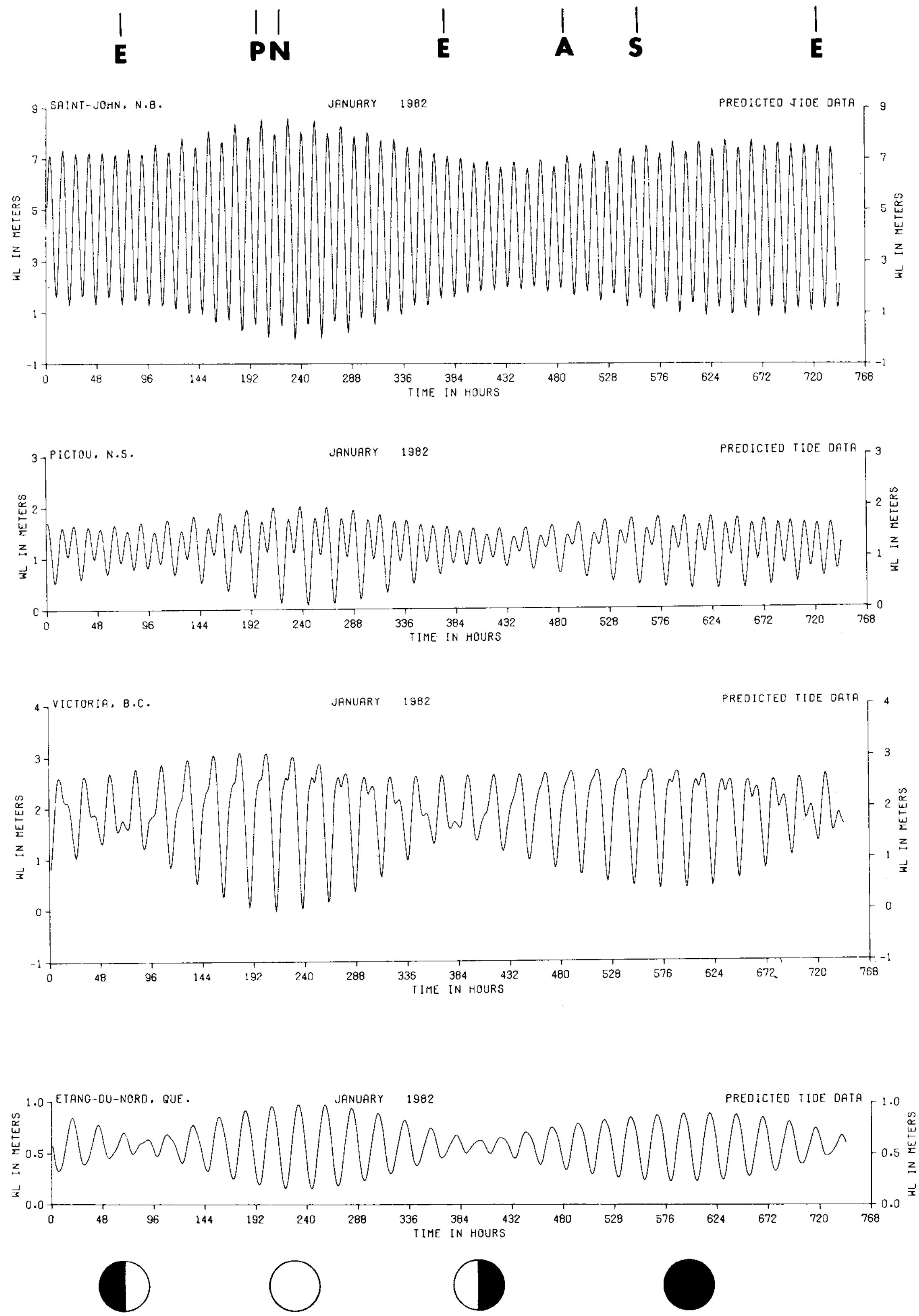


FIG. 21. Typical 1-month tidal curves from four Canadian sites, illustrating classes of tide. (a) is SD, (b) is MSD, (c) is MD and (d) is D. Letters A and P indicate apogee and perigee. E, N, and S indicate the moon is on, north, or south of equator. The circles indicate the moon's phases.

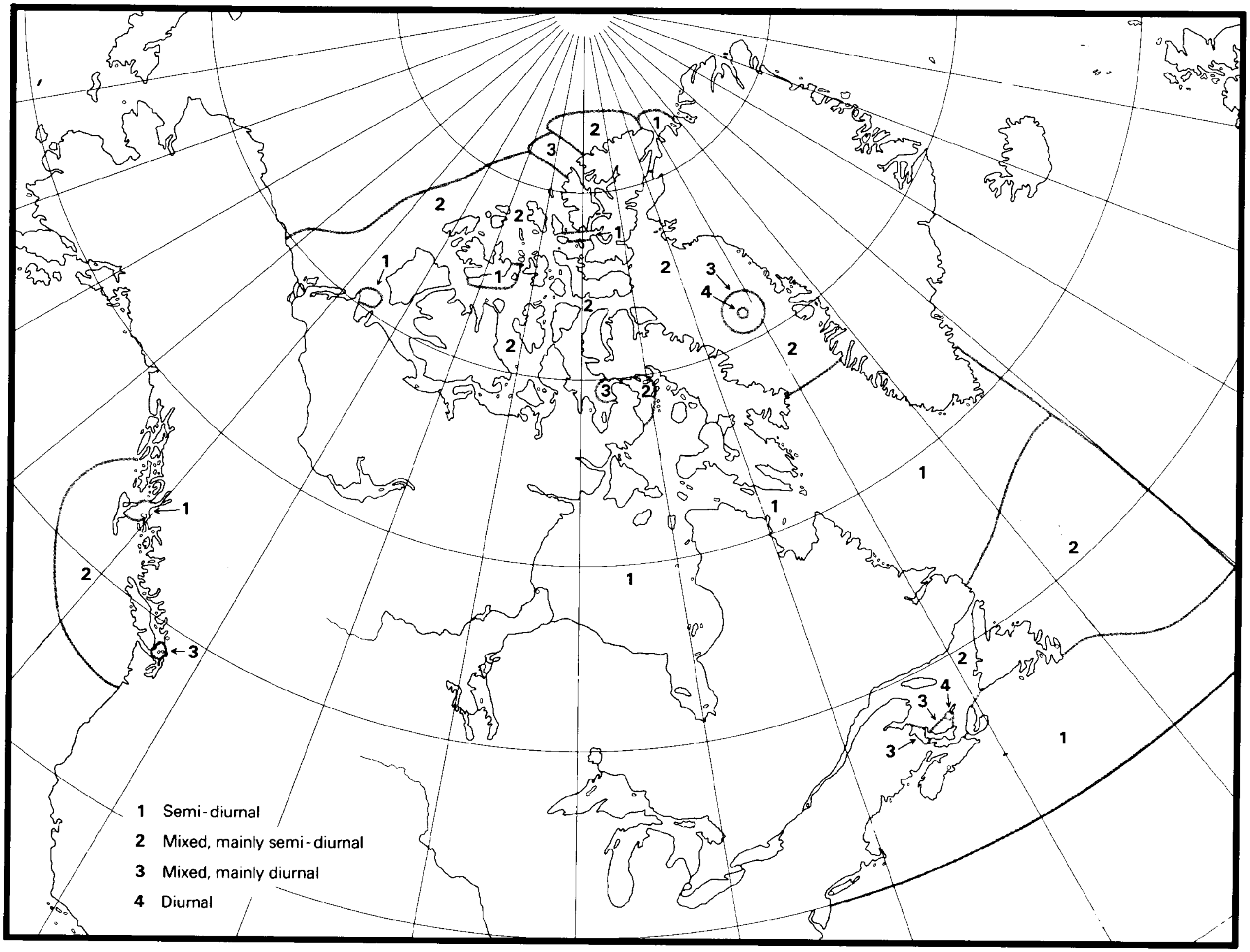


FIG. 22. Classification of tides at locations in Canadian waters.

CHAPTER 3
TIDAL ANALYSIS AND PREDICTION

3.1 Introduction

We have demonstrated in Chapter 2 how to identify the fundamental tidal frequencies present in the spectrum of the equilibrium tide, and have speculated that these should be the fundamental frequencies present in any actual tidal record. Basically, tidal analysis consists of identifying in a tidal record the amplitudes of all the important harmonic constituents and their phaselags with respect to the phases of the corresponding constituents in the equilibrium tide. Tidal prediction consists of recombining these constituents in the proper phase relation to the corresponding constituents in the equilibrium tide at the desired times. Both analysis and prediction require a knowledge of the phases of the harmonic constituents in the equilibrium tide at specified times, which may be extracted from tables or calculated by formulae involving the astronomical parameters. Although it is usually easier to take things apart than it is to put them back together again, the principles of tidal prediction are much simpler than are those of tidal analysis. Nevertheless, we will start with a consideration of tidal analysis, the breaking down of a tidal record into its component parts. Initially, a look at the Fourier theorem will be helpful.

3.2 The Fourier Theorem

This theorem states that if F(t), a function of the variable t, is defined over the range from t = -T/2 to t = T/2, then F(t) can be expressed as a constant plus an infinite series of sinusoids of frequencies (or wave numbers, if t is thought of as a distance) 1/T, 2/T, 3T, . . . i/T . . . etc. The sinusoidal oscillation with frequency 1/T is called the fundamental, and the oscillations whose frequencies are multiples of 1/T are called harmonics of the fundamental. Every harmonic that is added to the series increases the precision to which it reproduces F(t) in the range -T/2 to T/2. Outside this range, the series of sinusoids would produce the same image of F(t) between T/2 and 3T/2, and repeat it again and again for every interval of length T. Unless F(t) is believed to be periodic with period T over all values of t, these repetitions are simply ignored, and the series of sinusoids is used only to reproduce values of F(t) within the defined range -T/2 to T/2. Instructions are also included in the theorem for the evaluation of the terms in the series. The constant term is simply the average value of F(t) over the defined range. Evaluation of the fundamental or one of the harmonics involves multiplying every point in F(t) by the sine and by the cosine of that harmonic frequency times t, and forming the average of these products over T. The Fourier theorem is stated much more compactly in mathematical form as follows:

If F(t) is defined between -T/2 and T/2, then

$$(3.2.1)\quad F(t) = H_o + \sum_{i=1}^{\infty} H_i \cos\left(2\pi i \frac{t}{T} - \theta_i\right)$$

$$\text{where } H_o = \text{avg. over } T \text{ of } F(t),$$

$$H_i \sin\theta_i = \text{avg. over } T \text{ of } 2F(t)\sin 2\pi i \frac{t}{T}$$

$$H_i \cos\theta_i = \text{avg. over } T \text{ of } 2F(t)\cos 2\pi i \frac{t}{T}$$

It would be convenient indeed if the tidal cycle repeated itself exactly at regular intervals such as a month or a year, because then a Fourier series could be formed as described above to provide predictions for all time, with no regard to further tidal theory. Certainly a Fourier series can be formed to reproduce any finite tidal record to any desired accuracy, but since tides do not repeat exactly after any known interval, the series could not be used to predict values for any time outside the record, and would be of little value. So what is the pertinence of the Fourier theorem to tidal analysis? If, in 3.2.1, we allow T to become infinite, the fundamental frequency becomes infinitesimal, and the frequency interval between harmonics also becomes infinitesimal; i.e. all frequencies become candidates for inclusion in the Fourier series. Our knowledge of the frequencies present in the equilibrium tide tells us to look only at these frequencies out of the Fourier spectrum of all possible frequencies. The last two equations of 3.2.1 are then used to estimate the amplitudes and phaselags of the tidal constituents from observed tidal records. The tidal records are not infinitely long, however, and some ingenuity is required in forming meaningful averages for the expressions in 3.2.1 from short tidal records. This is the challenge of tidal harmonic analysis.

3.3 Harmonic Analysis of Tides

The harmonic method of analysis treats every tidal record as consisting of a sum of harmonic constituents of known frequency plus non-tidal "noise." This may be expressed as

$$(3.3.1)\quad h(t) = \sum_{i=0}^{n} H_i' \cos (E_i' - g_i) + \text{"noise"}$$
$$= H_0' + \sum_{i=1}^{n} H_i' (\cos E_i' \cos g_i + \sin E_i' \sin g_i) + \text{"noise"}$$

in which h(t) is the instantaneous height, H_i' is the amplitude of the "i"th constituent, E_i' is the phase of the equilibrium constituent at Greenwich at Greenwich Mean Time (GMT) numerically equal to the local observation time, and g_i is the phaselag of the constituent behind the Greenwich phase, E_i'. It is important to note that the Greenwich phase used here is the actual phase of the equilibrium constituent at Greenwich only if the observations are recorded in GMT. If they are recorded in a time zone z hours west of Greenwich, then E_i' is the phase of the constituent at Greenwich z hours earlier. This may seem confusing at first, but it avoids the need to convert observation times into GMT, and the choice of a reference phase can be quite arbitrary as long as it advances at the proper speed and is applied consistently in all calculations. It is, however, necessary to record the time zone carefully along with the results of any tidal analysis to assure consistency in phase reference. The significance of the primes on H_i' and E_i' will become apparent later. The subscript, i, is simply a counter for the n harmonic constituents considered necessary to represent the tidal signal adequately; H_o' represents the average water level during the record (zero frequency, E_o' = g_o = 0).

The aim of the harmonic analysis is to determine the values of all the H_i' and the g_i. H_o' is simply the average of all the observations, and is usually denoted as Z_o in tidal terminology. If each value of h(t) is multiplied by cos E_i' and the average taken over the length of the record, 3.3.1 gives

$$(3.3.2)\quad \text{avg. } [(\cos E_1')h(t) = H_1' \cos g_1 (\cos^2 E_1')$$
$$+ H_2' \cos g_2 (\cos E_1' \cos E_2')$$
$$+ H_1' \sin g_1 (\cos E_1' \sin E_1')$$
$$+ H_2' \sin g_2 (\cos E_1' \sin E_2')$$
$$+ \text{etc.} + (\cos E_1')(\text{"noise"})]$$

and, if each value of h(t) is multiplied by sin E_1', the average gives

$$(3.3.3)\quad \text{avg. } [(\sin E_1')h(t) = H_1' \sin g_1 (\sin^2 E_1')$$
$$+ H_2' \sin g_2 (\sin E_1' \sin E_2')$$
$$+ H_1' \cos g_2 (\sin E_1' \cos E_1')$$
$$+ H_2' \cos g_2 (\sin E_1' \cos E_2')$$
$$+ \text{etc.} + (\sin E_1')(\text{"noise"})]$$

The "noise" terms are considered to average to zero, their signal being assumed to be random with respect to tidal frequencies. If all n constituents could complete an exact number of cycles in the same length of record, all of the averaged coefficients in 3.3.2 and 3.3.3 would be zero except for $\cos^2 E_1$' and $\sin^2 E_1$', whose average values would be 0.5. This would give the simple Fourier solution of 3.2.1, namely

$$(3.3.4)\quad H_1' \sin g_1 = \text{avg. of } 2h(t) \sin E_1'$$
$$H_1' \cos g_1 = \text{avg. of } 2h(t) \cos E_1'$$

Of course the tidal constituents cannot all complete an exact number of cycles in the same length of record, and we must contend with the residual terms in 3.3.2 and 3.3.3 instead of assuming the enticingly simple solution of 3.3.4. Repeating the above process of multiplication by the sine and cosine of the E_i' and averaging for the other n-l constituents completes a set of 2n equations in the 2n unknowns (the H_i' sin g_i and the H_i' cos g_i). As a matter of interest, these are the same 2n "normal equations" that would have been produced if the problem had been tackled by the "method of least squares," so their solution for the H_i' and g_i gives a best fit to the data in a "least squares" sense.

While the generation of the $4n^2$ coefficients and the solution of the 2n simultaneous equations in 2n unknowns is feasible with today's high-speed, large-memory electronic computers, such was not always so in the days of manual computation.

One simplification that is frequently used for manual computation is to replace the sine and cosine multipliers by the "box-car" functions that equal + I when the corresponding sine or cosine are positive, equal - I when they are negative, and equal zero when the sine or cosine are zero. Figure 23 illustrates how multiplication by the box-car equivalent of the sine or cosine function produces an average contribution of $2/\pi$ times the corresponding (sine or cosine) component of a constituent of the same frequency, zero times the complementary (cosine or sine) component of a constituent of the same frequency, and zero times both the cosine and sine components of a constituent of exactly twice the frequency. The principle by which the pure sine and cosine multipliers sort out the coefficients of 3.3.3 and 3.3.4 is the same as that by which their box-car analogues worked in Fig. 23. The box-car multipliers, however, are most effective in separating one species of tide from another (diurnal, semidiurnal, etc.) but for separation of constituents of the same species, the pure sine and cosine multipliers produce a better-behaved set of coefficients.

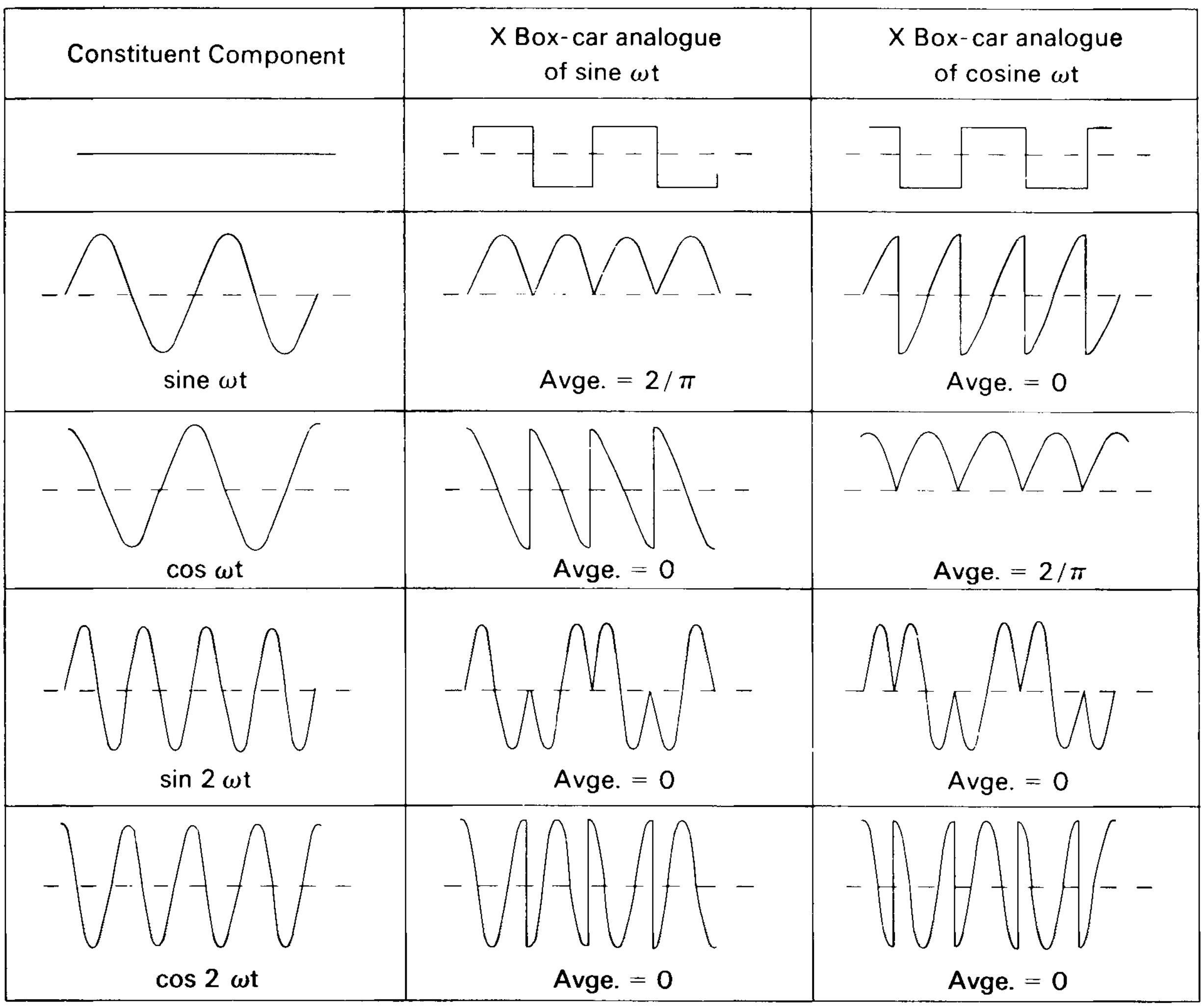


FIG. 23. Numerical filtering of tidal constituents by multiplication of signal by box-car functions.

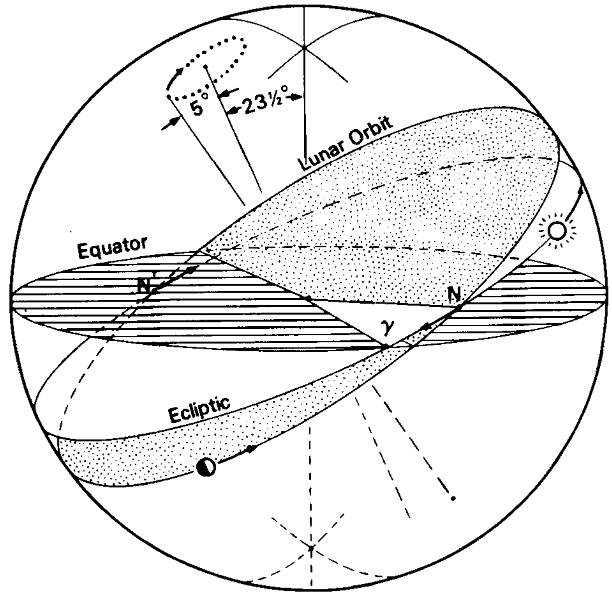


FIG. 24. Celestial sphere, showing equator, ecliptic, lunar orbit and regression of moon's ascending node, N.

3.4 Nineteen-year Modulation of Lunar Constituents

The moon's orbit about the earth is in a plane always inclined at approximately 5° to the plane of the earth's orbit about the sun (the ecliptic). but the line of intersection of the two planes rotates once every 18.6 years about the pole of the ecliptic. Figure 24 shows a celestial sphere (a sphere of infinite radius, with the earth's centre as its centre) onto which are projected the earth's equator and its polar axis, the plane of the ecliptic and its axis. and the plane of the moon's orbit and its axis. The spherical screen of a planetarium is a model of a portion of a celestial sphere. The line NN' is the intersection of the moon's orbital plane with the ecliptic, and N and N' are referred to as "nodes." N is the "ascending node," since the moon is moving from south to north of the ecliptic as it passes N. As the axis of the moon's orbit rotates about the axis of the ecliptic, tracing out the 5° cone once every 18.6 yr, the point N moves around the ecliptic from east to west in the same period. Since the moon travels its orbit in the opposite sense (west to east), this is referred as the "regression of the moon's ascending node," and it has a major influence on the moon's declination over the approximate l9-year period. The inclination of the ecliptic to the equator is 23 1/2°, so over the course of a year the sun changes declination between 231/2° north in summer and 231/2° south in winter. Since the moon's orbit is inclined at 5° to the ecliptic, the moon's declination may change over the course of a month between 281/2° (i.e. 231/2° + 5°) north and south during one part of the l9-year cycle, and between only 181/2° (i.e. 231/2°—5°) 91/2 years later in the cycle. The moon's maximum monthly swing in declination occurs when its ascending node, N, coincides with the vernal equinox, and its minimum swing occurs when its ascending node coincides with the autumnal equinox. The vernal equinox (γ) is the point at which the sun crosses the equator on its way north, and the autumnal equinox is the point at which it crosses it on its way south.

The regression of the moon's ascending node has the effect of modulating both the amplitude and the phase of the lunar tidal constituents in a l9-year period. Because the period is so long, it is assumed that the modulation of the constituents in the real tide is the same as that of the equilibrium constituents. The amplitude modulation is represented by a nodal factor, f, which varies about a mean value of unity over the period of 18.6 years. The phase modulation is represented by a nodal correction, u, which varies about a mean value of zero over the same period. There is no nodal modulation of the solar constituents, and the f and u values are different for each lunar constituent. Values of the nodal parameters are tabulated, and may also be computed from formulae involving the astronomical variables. f of K_2 varies between about 0.75 and 1.30, while f of M varies only between 0.96 and 1.04. u of K_2 varies between plus and minus 17°, while u of M_2 varies only between plus and minus 2°.

To conform with the above, the equilibrium phase, E_i', used in the harmonic analysis must be the phase of the mean equilibrium constituent, E_i, plus the nodal correction, U_i, for that constituent at that time. The amplitude, H_i', that results from the analysis will be the amplitude of the mean constituent,
H_i, times the nodal factor, f_i, for that constituent at that time. Thus, in section 3.3

$$E_i' = E_i + u_i, \text{ and } H_i' = f_i H_i$$

The tidal constants that are retained from the analysis are the amplitudes of the mean constituents ($H_i = H_i'/f_i$) and the phaselags of the mean constituents (g_i).

3.5 Shallow-water Constituents

Section 1.12 discusses, and Fig. 12 illustrates, the distortion of a tide wave as it travels in shallow water. The Fourier theorem suggests that this distortion could be represented by adding harmonics of the fundamental tidal frequencies, a procedure that is attractive because it is compatible with the methods of harmonic analysis and prediction. This, then, is the origin of the shallow-water constituents (sometimes called "over-tides"). They are introduced as a mathematical convenience to represent distortion of the tide wave, and do not arise directly from the tidal forces. As an example, M_6, is the second harmonic of the fundamental tidal constituent M_2, with three times its angular speed. The most common shallow-water constituents are the quarter-diurnals M_4 and MS_4, with frequencies twice that of M_2 and the sum of those of M_2, and S_2, respectively. Their combination produces a quarter-diurnal tide whose amplitude is modulated at their difference frequency, which is the same as the modulation frequency of the semidiurnal tide produced by the combination of M_2 and S_2. This part of the quarter-diurnal tide is thus able to increase and decrease in the same spring-neap cycle as the part of the semidiurnal tide produced by the combination of M_2 and S_2. Figure 25 shows graphically the distortion produced in a fundamental constituent by the addition of a first harmonic. This may be compared to the distortion of the semidiurnal tide in the St. Lawrence River, shown in Fig. 12. The shallow-water constituents can be included in the harmonic analysis procedure described in section 3.3.

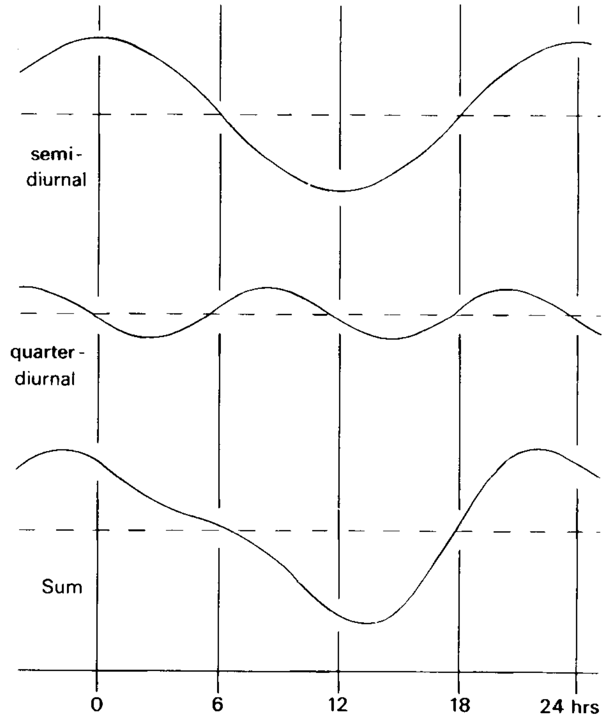


FIG. 25. Representation of shallow water distortion by addition of a first harmonic to the fundamental frequency.

3.6 Record Length and Sampling Interval

Theoretically, if a water level record contained nothing but a pure tidal signal consisting of the contributions fromthe amplitudes and phaselags of the n constituents could be determined from almost any set of 2n observation points. Of course, nothing in this world is so pure, and tidal records are contaminated with "noise," both of meteorological and observational origin. This is why it is necessary to rely on statistical averaging and filtering of long tidal records, as described in section 3.3, to resolve the tidal signal from the background noise and to distinguish the individual harmonic constituents. In practice, the lowest frequency constituent that could possibly be distinguished in a tidal record is one whose period equals the length of the record, and the highest frequency constituent is one whose period is twice the sampling interval. Whether two neighbouring constituents can be separated from each other in an analysis depends both on the difference in their frequency and the length of the record. The "Rayleigh criterion" for the separation of two constituents requires that they should change phase with respect to each other by at least 360° during the record period. If n_1 and n_2 are the constituents' angular speeds in degrees per hour and T is the record length in hours, the Rayleigh criterion for separation is

(3.6.1) $(n_1 - n_2)T$ is greater than, or = 360°

Sometimes, if a record seems relatively free of noise, this criterion might be relaxed, and the right side of 3.6.1 made 180°. Diurnal constituents could be separated from semidiurnal constituents on the basis of a single day's record, but to separate constituents of the same species requires a much longer record. As an example, consider the length of record required to permit the separation of constituents M_2 and S_2. From Appendix A we see that their difference in speed is 1.016°/h, so that 3.6.1 gives the length of record necessary to separate them as T = 360/1.016 = 354 h, or 14.8 days. But to separate N_2 from M_2 would, by the same reasoning, require a record length of T = 360/0.544 = 662 h, or 27.6 days; and to separate K_2 from S_2, would require T = 360/0.082 = 4390 h, or 183 days. It should not be surprising that the separation periods turn out to be the basic astronomical periods or tractions of them, since it was from these that the constituents inherited their frequencies.

When a record is too short to allow separation of all the constituents that are known to contribute significantly to the tide in that region, relationships must be assumed between the inseparable constituents. These are called regional relations, because the justification for their adoption is that waves of such nearly equal frequencies must behave very similarly, and so maintain very nearly the same relation to each other over a large region. The ratio of amplitudes and the difference of phaselags of the two constituents are therefore assumed to be the same as those observed at the nearest comparable location for which a more complete analysis is available. In any tidal analysis, every harmonic constituent must be (a) included directly in the analysis, (b) allowed for through regional relations with other constituents, or (c) omitted because it is known to be negligible in the region of observation. The longer the tidal record, the greater is the number of constituents that can be analysed, and the higher is the accuracy of their determination. For temporary water level gauges installed in tidal waters, a minimum recommended length of record is I month.

Tidal analyses are usually carried out on data that has been read at hourly intervals from the original record. A sampling interval of I h is short enough to detect the highest frequency constituents of interest in most tidal work. However, if higher frequencies are seen to be present, even though they may not be of direct interest, they should be smoothed out of the record before the hourly samples are taken. This is to avoid the process known as aliasing, by which frequencies higher than the sampling frequency may masquerade as lower frequencies. Figure 26 illustrates the principle of aliasing. It is usually more important to smooth the samples from a short record than from a long one, because the aliased signal is likely random with respect to the tidal signal, and so can be adequately eliminated in the averaging and filtering of the long analysis. Today's computer programs for tidal analysis are sufficiently flexible to accept data that has been sampled at irregular intervals, and even data that has gaps of several days in it. In fact, two records taken at different times may be more valuable than a single continuous record of their combined length. For example, 1 day of record at spring tide and I day of record at neap tide could permit separation of M_2 and S_2, whereas 2 days of continuous record could not. A single continuous record covering the whole period from spring to neap tide would, of course, be more valuable still.

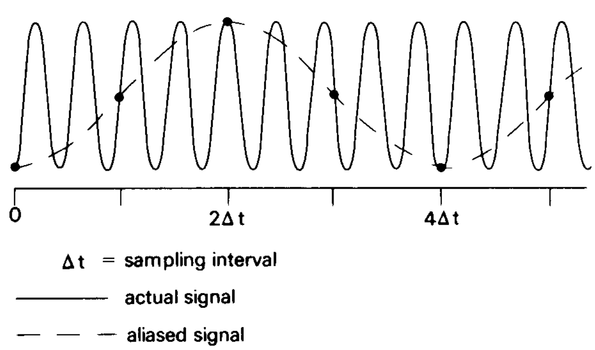


FIG. 26. "Aliasing" to a lower frequency by an oscillation whose period is less than the sampling interval.

3.7 Harmonic Analysis of Tidal Streams

The only difference between the analysis of a water level record and the analysis of a current meter record is that the current speed and direction data must first be resolved into two mutually perpendicular horizontal vector components. Two separate harmonic analyses are then performed, one on each time series of component speeds. The choice of the two component directions is arbitrary, as long as it is well recorded what they are. A common choice, and one that is recommended, is that of the true azimuths North and East. If the current observations were taken in a well defined channel or passage, the directions along and across the channel might be chosen for the components. The result of the analysis of a current record is two sets of harmonic constants (amplitudes and phaselags), one for each component direction. Even though the two directions are known to be perpendicular to each other, it is im portant to quote both of them, to avoid a possible 180° uncertainty. Sometimes the constants for each harmonic constituent are used to calculate that constituent' s tidal ellipse (see section 1.11). The ellipse may be described by the direction of its major axis, the amplitude and phaselag of the tidal stream component in that direction, the tidal stream amplitude in the minor axis direction, and a statement that the sense of rotation of the stream is clockwise or counterclockwise. In tidal ellipse form, the phaselag on the minor axis is always 90° different from that on the major axis. The sense of rotation of the stream tells how the difference should be applied. There is no objection, of course, to stating explicitly the direction of the minor axis and the phaselag as well as the amplitude for that direction. The tidal ellipse form for the results of a tidal stream analysis has some advantage if it is wished to display the individual harmonic constituents graphically, but it has the significant disadvantage that the stream components are resolved in different directions for each harmonic constituent.

Tidal streams reflect the presence of internal tides as well as of surface tides, and. because of the properties of internal waves discussed in section 1.7, tidal streams may vary in depth, may vary seasonally with changes in stratification, and may vary spatially in a pattern different trom that of the surface tide. For these reasons, the coherence between tidal stream records taken on different moorings at different times is usually poor, and the records should not be combined into a single analysis. Even if a long current record is obtained from a single mooring, it is usually wise to break the record up into one month lengths for separate analysis. Care should also be used in applying regional relations tor the constituents in tidal stream analysis, because of the greater variability in tidal streams than in surface tides in the same region .

3.8 Harmonic Method of Tidal Prediction

Prediction of the tidal height at any desired time, t, involves summing the contributions from all the important harmonic constituents in their proper phase for that time, and adding their sum to the mean water level, Z_O. Expressed mathematically, this is

$$(3.8.1)\quad h(t) = Z_o + \sum_{i=1}^{n} f_i H_i \cos(E_i + u_i - g_i)$$

where the symbols have the same meanings as in sections 3.3 and 3.4. The nodal parameters, f and u, and the Greenwich phase of each equilibrium constituent, E, may be obtained for time t either from tables or from formulae involving known astronomical parameters. According to the convention explained in section 3.3, the values of E must be obtained for GMT numerically equal to t, for t in the same time zone to which the phaselags, g, refer. Values for the constituent amplitudes and phaselags, H and g, come from a harmonic analysis of tidal data previously recorded at that location, or, perhaps, from interpolation between known values at nearby locations. There is no model of world tides that is accurate or detailed enough to provide local tidal predictions from first principles. Previous observation of the tide in a region is a prerequisite to its prediction. The word "prediction" as used here includes calculation of tidal heights for past times (hindcasting) as well as for future times (forecasting).

The prediction of tidal streams is basically the same as the prediction of tides, except that predictions must be made separately in the two component directions, and then combined vectorially to give speed and direction. In 3.8.1, Z_O would be the component of the steady current in that direction. Many requirements for current and tidal stream predictions are for channels in which the flow is nearly rectilinear along the axis of the channel. In these cases it is possible to limit the predictions to one component direction, that along the axis of the channel .

3.9 Prediction of Tidal and Current Extrema

The prediction of the times and heights of high and low water (HW and LW) is usually done by generating a series of predicted heights at hourly intervals and scanning it for changes in trend. When a change in trend (e.g. from increasing to decreasing height) occurs, intermediate predictions are inserted until the extreme height at which the trend reverses is located within a narrow interval. The time and height of the extremum (HW or LW) are then interpolated in the interval. Ingenious mechanical analogue apparatus was used in the past to generate the time series, and the scanning was done visually. Today, the task is accomplished almost exclusively by electronic digital computers programmed to perform all the steps.

Most locations for which current predictions are made are in channels or passages where the current is nearly rectilinear along the axis of the channel. Predictions are usually published only for the times and speeds of maximum flood and ebb and for the times of slack water. Using the set of harmonic constants for the component direction along the channel, a time series of current speeds is generated, with the positive sign for the flood direction and the negative sign for ebb. This series can be scanned to identify maxima and minima in the absolute value of the speed (i.e. without regard to sign). When the maxima are found, the sign of the speed at those times identifies them as floods (+) or ebbs (—). The minima usually occur at zero speed, and thus give the times of slack water. However, when there is a large residual current, the tidal streams may not be large enough to cause a reversal in the flow, and the minima may not be zero. In these cases the minima are identified as minimum flood or ebb according to the sign of the speed, and there is no slack water and no "time of turn."

For locations in which the currents are not rectilinear, a time series of current vectors may be produced from the two component sets of harmonic constants and scanned for maxima and minima in the magnitude (speed) of the vector. Since the concept of flood and ebb is inexact when the current is not rectilinear, the extrema should be identified by time, speed, and direction. In quoting the direction of a current the convention used is opposite to that used for wind direction: the direction of a current is the direction toward which it is flowing. Figure 11 illustrates some of the many patterns that may result from the combination of a steady current with a rectilinear or rotary tidal stream. If there is a large diurnal inequality in the tidal stream, the patterns could be even more variable.

3.10 Cotidal Charts

Cotidal charts of the major harmonic constituents of the tide are frequently constructed to illustrate their different propagation patterns and to locations where no observations exist. A cotidal chart consists of a set of co-phase lines and a set of co-range lines drawn on a suitable chart. Each co-phase line traces out the locus of points at which the constituent has a particular phaselag, and each co-range line traces out the locus of points at which it has a particular amplitude. Provided sufficient information is available, there is no limit to how large an area may be covered by a cotidal chart for a single harmonic constituent, and some have been constructed covering the whole world ocean. Figure 27 and Figure 28 show cotidal charts of Hudson and James bays for the constituents M_2 and K_1 respectively. The marked difference in the two reflects the fact that the basin responds differently to waves of different frequencies. Charts of constituents within the same species of tide usually resemble each other quite closely over fairly large regions, and may be combined into composite cotidal charts for the diurnal and semidiurnal species separately. Figure 29 and Figure 30, respectively, show cotidal charts for the semidiurnal and diurnal tides of the East Coast and Gulf of St. Lawrence. Figure 31 and Figure 32 show the same thing for the West Coast. Large differences are apparent between the semidiurnal and diurnal charts, with respect to location of amphidromes and other characteristics. Clearly, where the tide is of the mixed type (MSD or MD), a cotidal chart that attempted to combine the two species to represent the total tide could cover only a very small region.

As will be discussed more fully in Part II,

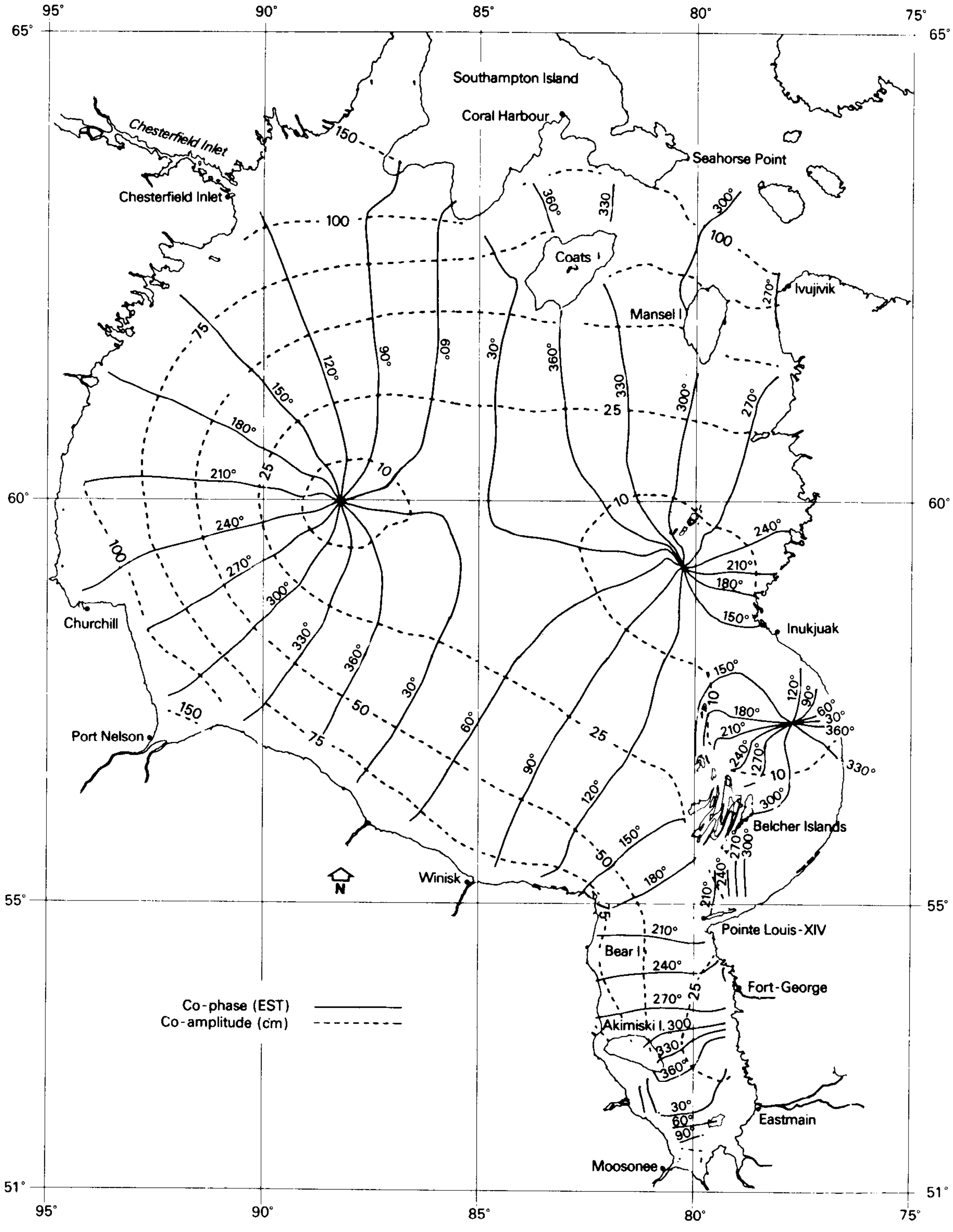


FIG. 27. Cotidal chart of M_2 constituent in Hudson and James Bays by numerical modelling. (from fig. 7 of Freeman and Murty, J. Fish. Res. Board Can. 33(10), 1976).

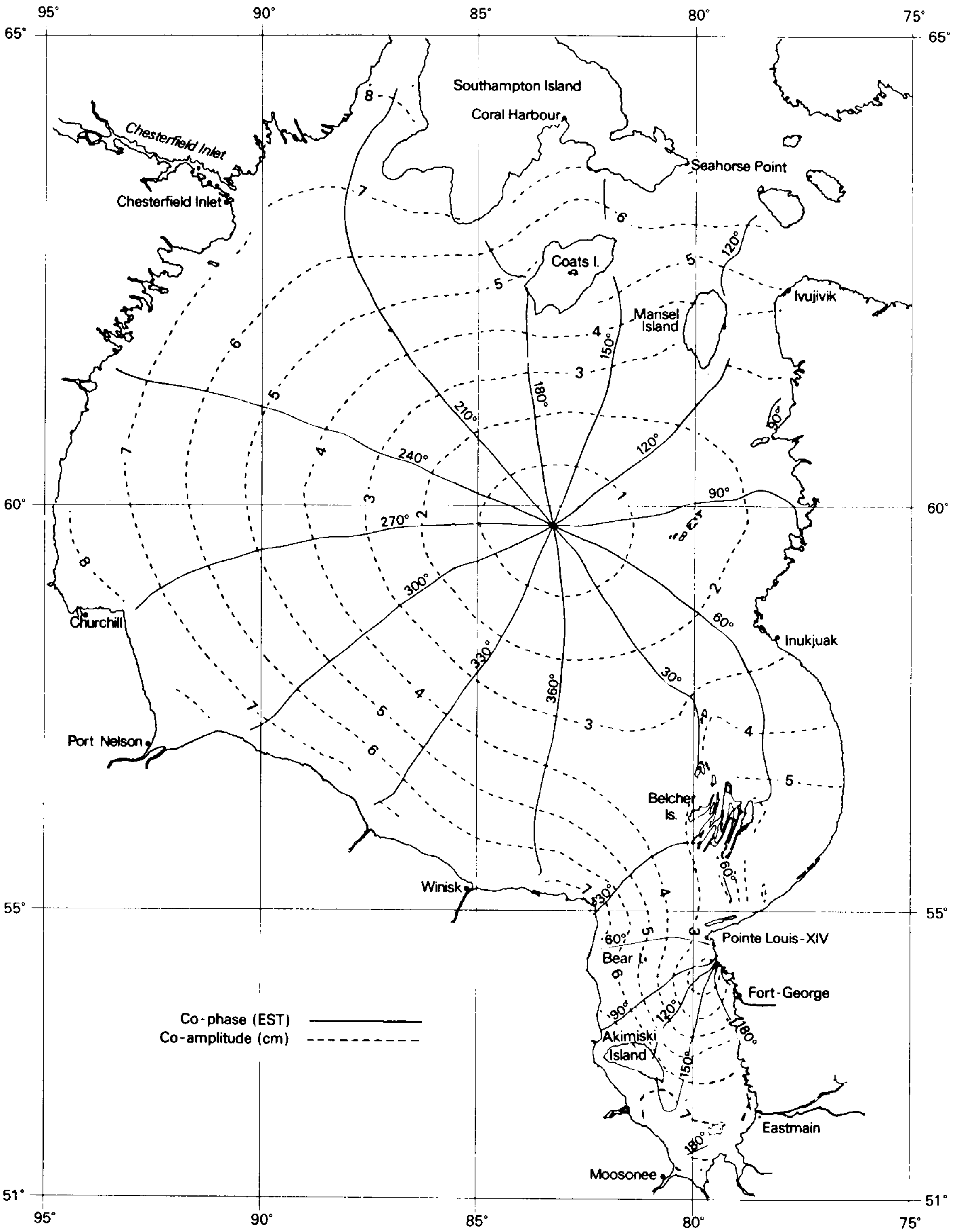


FIG. 28. Cotidal chart of K_1 constituent in Hudson and James Bays by numerical modelling. (from fig. 10 of Freeman and Murty, J. Fish. Res. Board Can. 33(10), 1976).

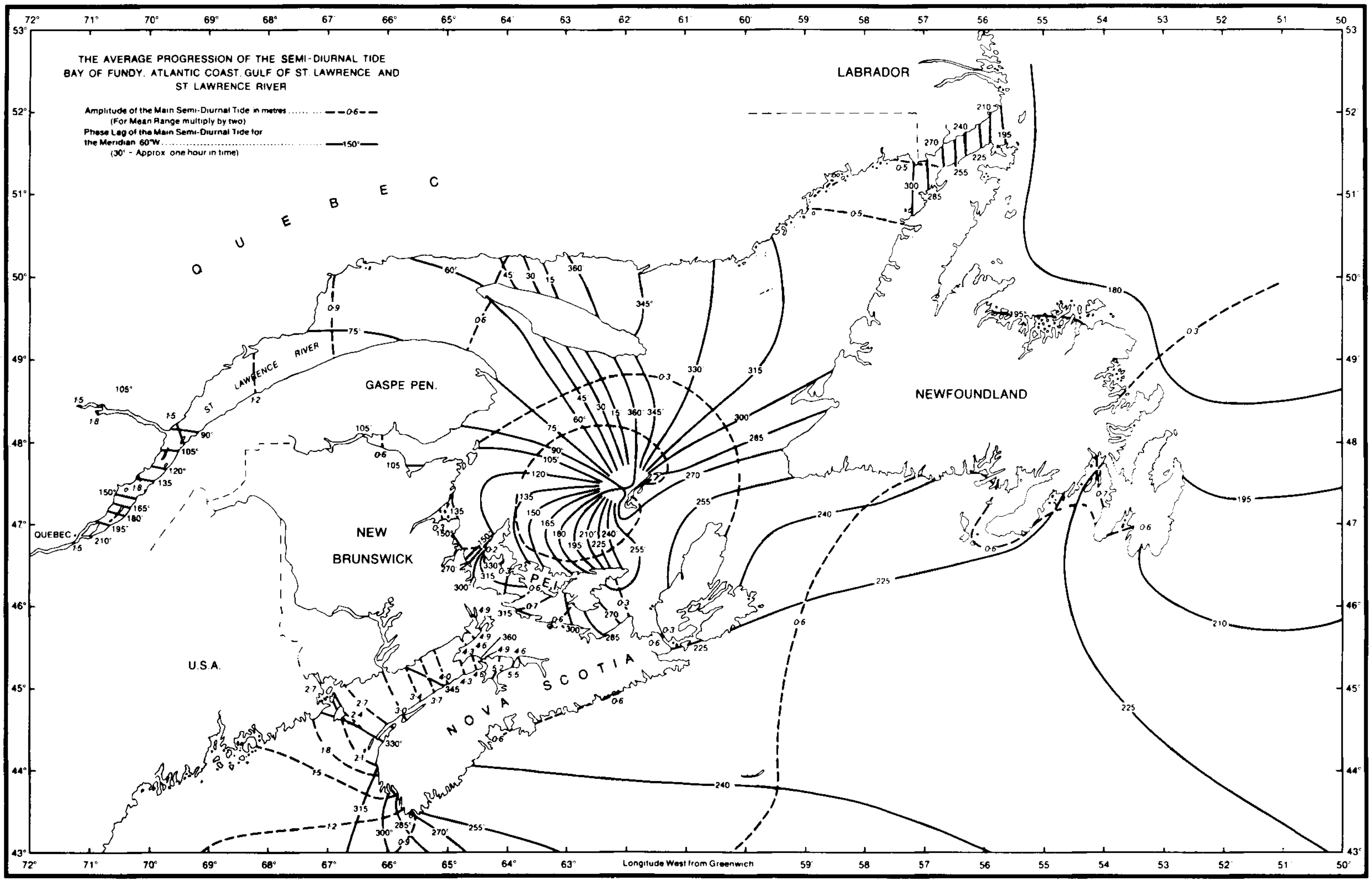


FIG. 29. Cotidal chart of semidiurnal tide on East Coast of Canada. (from fig. 10 of *Tides in Canadian Waters*, by G. Dohler).

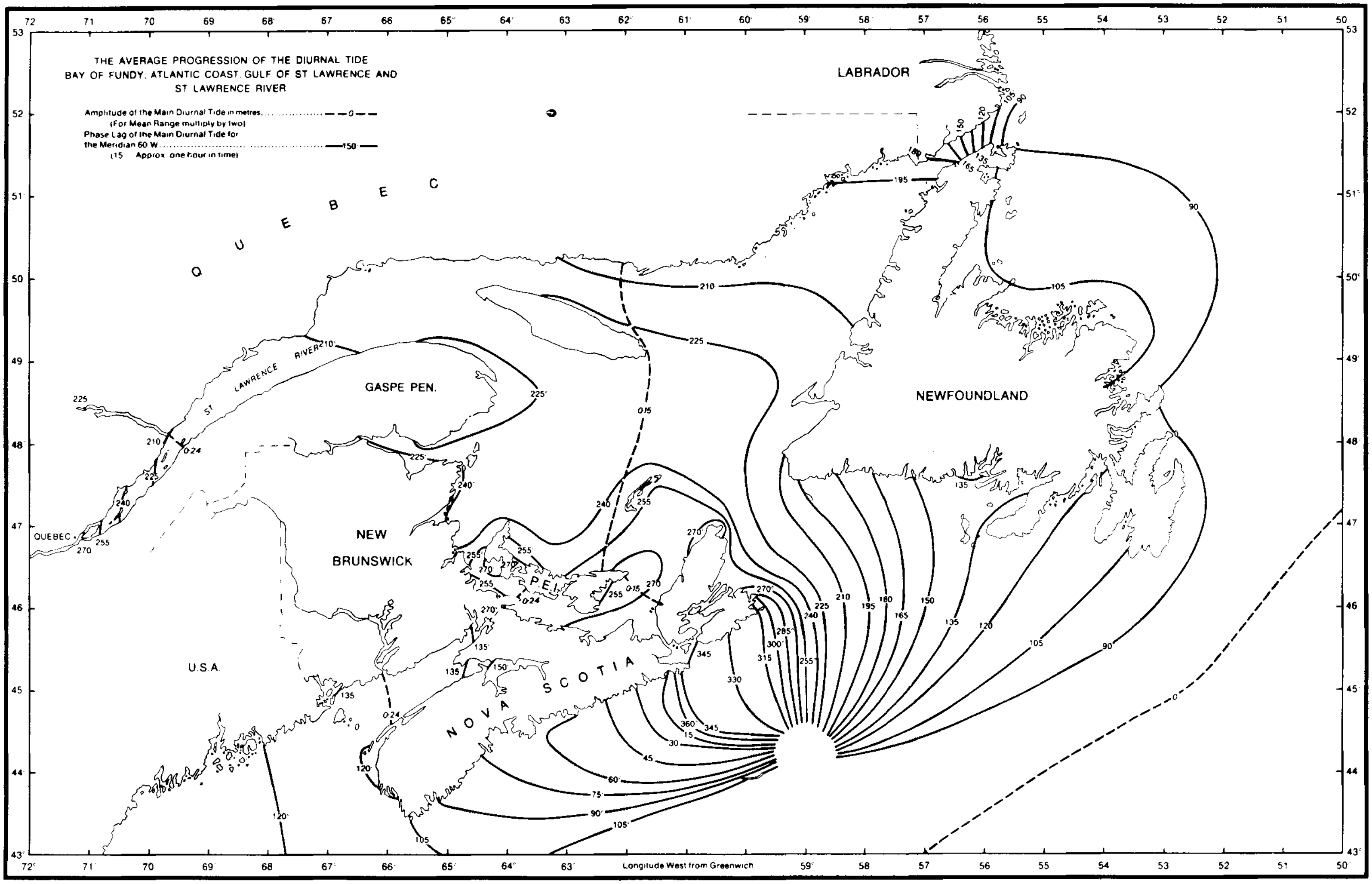


FIG. 30. Cotidal chart of diurnal tide on East Coast of Canada. (from fig. 11 of *Tides in Canadian Waters*, by G. Dohler).

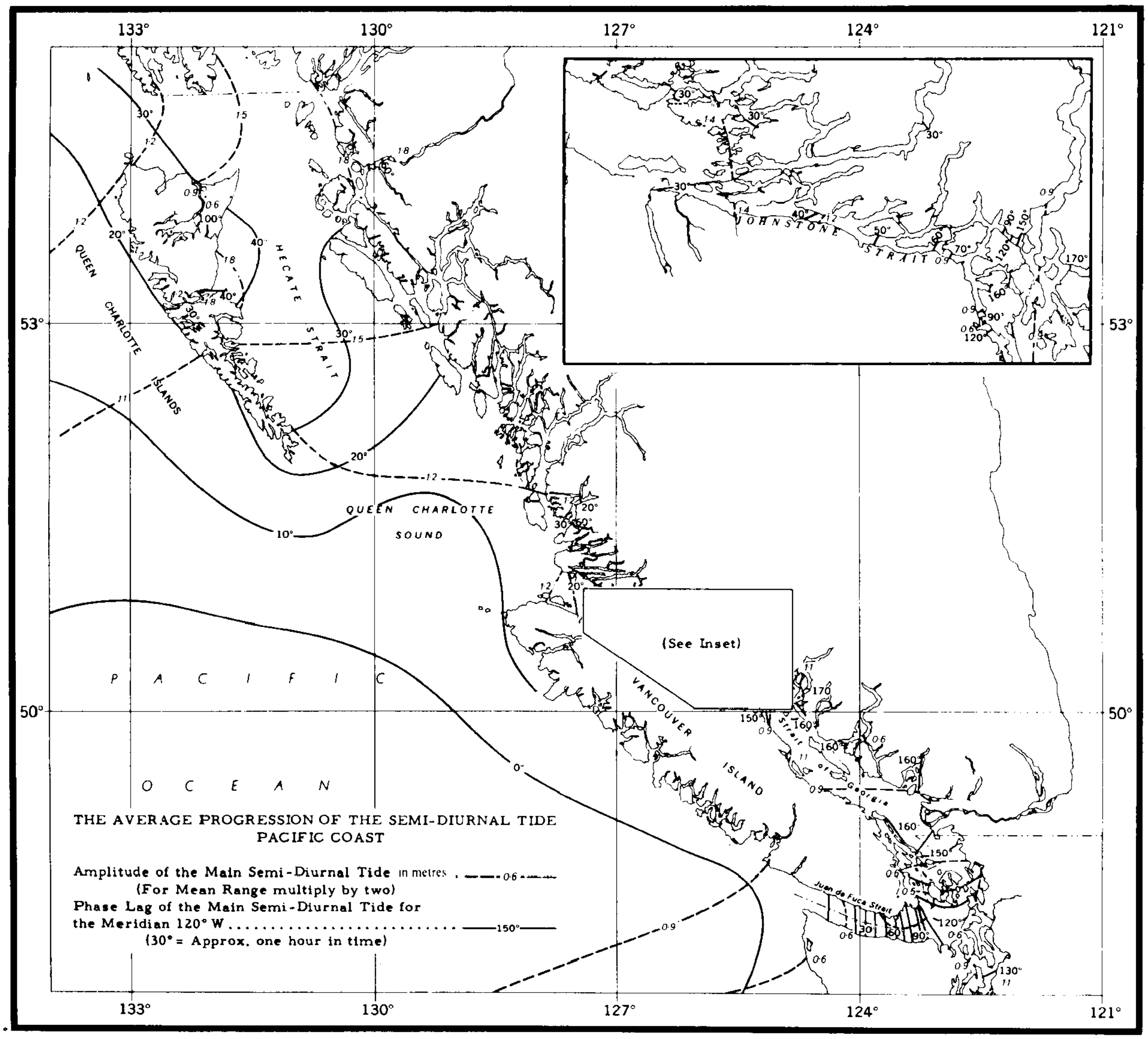


FIG. 31. Cotidal chart of semidiurnal tide on West Coast of Canada. (from fig. 18 of *Tides in Canadian Waters*, by G. Dohler).

cotidal charts must often be prepared for use in reduction of soundings to datum on offshore surveys. Since they usually attempt to represent the total tide, the extent of the region they may cover depends upon the extent to which the tide is of the mixed type. In cotidal charts for sounding reduction, times and heights are most often referred to the tide at a nearby reference port. The region is commonly divided into two sets of overlapping zones, one set being zones in which the range is considered to bear a constant ratio to that at the reference port. and the other being zones in which the arrival time of the tide is considered to differ by a constant from that at the reference port.

3.11 Numerical Modelling of Tides

Numerical modelling is becoming more and more common in modern tidal studies, being encouraged by the ever-increasing capabilities of the electronic digital computer. Models incorporate the physical principles of the equations of motion and of continuity, a description of the shape and the bathymetry of the basin, and a set of boundary conditions that must be preserved. The boundary conditions consist of the harmonic constants at all gauge and current meter sites for which analyses exist, and a statement of the character of the tide along any open boundaries.

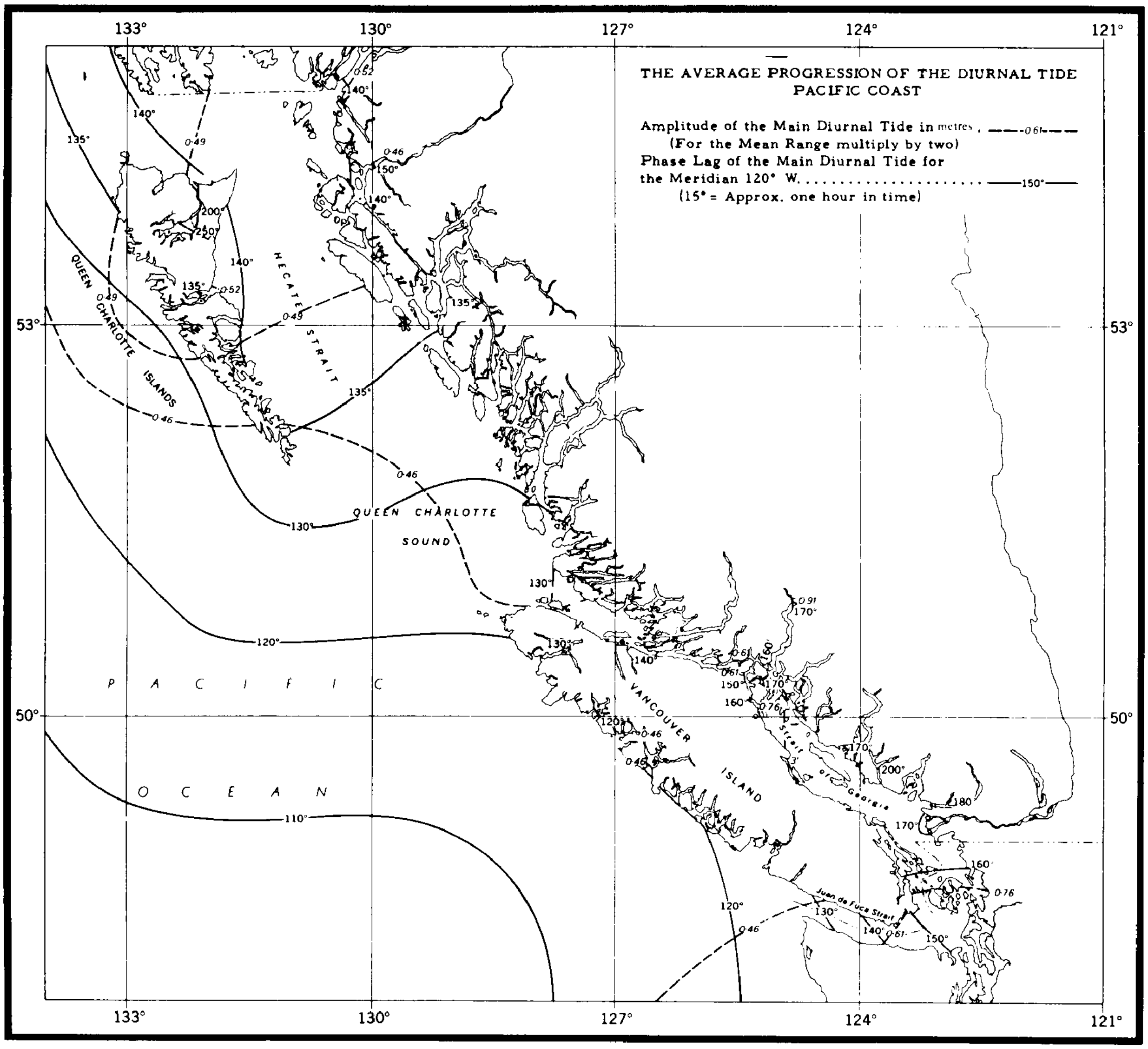


FIG. 32. Cotidal chart of diurnal tide on West Coast of Canada. (from fig. 19 of *Tides in Canadian Waters*, by G. Dohler).

The region is divided into a set of grid points, closely enough spaced to define the characteristics being investigated. The computer is programmed to commence with an initial set of elevations at all grid points and to change the elevations progressively in accordance with the physical principles and the boundary conditions. In this way, the progression of the tide can be modelled over as many cycles as desired. The tidal streams are also modelled in the same operation, through the application of the principle of continuity. The cotidal charts of Figure 27 and Figure 28 are the result of numerical modelling of the tides in Hudson Bay. Figure 33 shows current vectors for one stage of the tide in Chignecto Bay and Minas Basin, as deduced by numerical modelling, and Figure 34 shows a similar result from numerical modelling in the Strait of Georgia and Juan de Fuca Strait.

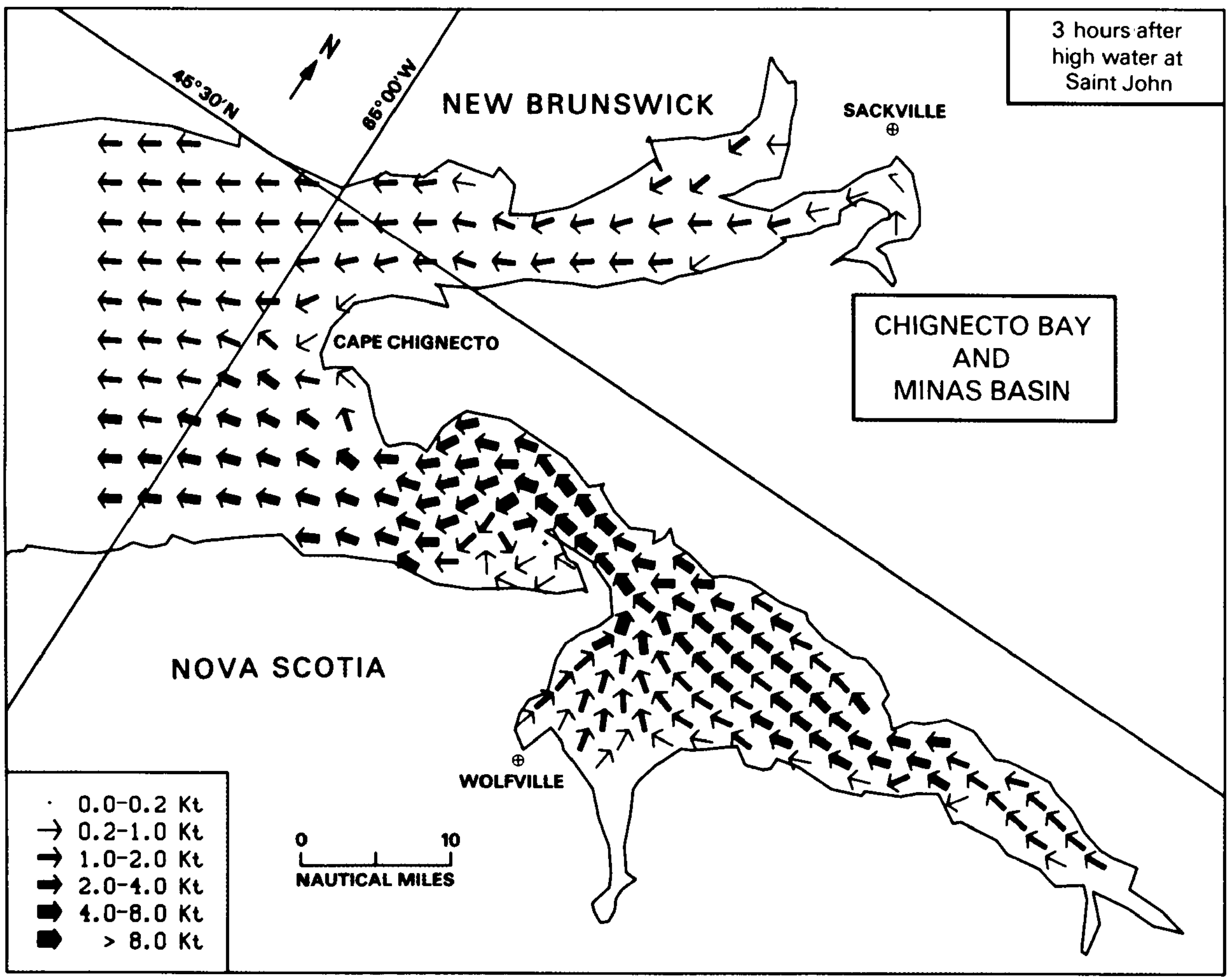


FIG. 33. Sample of tidal current information from numerical modelling in Chignecto Bay and Minas Basin. (from page 28 of CHS *Atlas of Tidal Currents — Bay of Fundy and Gulf of Maine*).

CHAPTER 4

Meteorological and Other Non-Tidal Influences

4.1 Introduction

Because the tide usually dominates the spectrum of water level and current fluctuations along the ocean coasts, it is common to think of non-tidal fluctuations mostly in connection with inland waters. The tide in the deep ocean, however, can be quite insignificant, and, as shown in section 1.5 and Table 2, the tidal streams completely negligible from the standpoint of navigation. Wind-driven surface currents in the deep ocean, on the other hand, are of major importance to navigation. Water levels along ocean coasts are just as surely affected by atmospheric pressure and wind as are water levels along the shores of inland bodies of water. The range, however, is generally small compared to that of the tide on the coast, and the importance may not be fully realized until an extreme of the non-tidal fluctuations coincides with a corresponding extreme (high or low) of the tidal fluctuation. In using tidal predictions, such as those in the Canadian Tide and Current Tables, it should be borne in mind that they contain no allowance for non-tidal effects, other than for the average seasonal change in mean water level. The non-tidal influences are discussed below with reference both to ocean and inland waters.

4.2 Wind-driven Currents

The major current systems of the ocean are driven by the wind stress acting on the surface. The direct effect of the wind stress is transmitted only to a limited depth by viscosity and turbulence, but the pressure gradients resulting from the induced surface slopes can set up deep flows in directions different from those of the surface flows. The main surface current systems of the Atlantic and Pacific oceans are in the form of large gyres that occupy most of the width of the ocean and are clockwise in the Northern Hemisphere and counter-clockwise in the Southern Hemisphere. The Coriolis acceleration is responsible for these circular patterns, deflecting both the winds and the currents driven by the winds. It may seem surprising that these "permanent" large-scale features of the ocean circulation could be the result of something as capricious as the wind. But, while the wind does vary from day to day with the passage of weather systems, it has a fairly consistent average pattern over much of the ocean, as witnessed by the Doldrums near the equator, the Trades in the tropics, and the Westerlies at mid-latitudes. Changes in the ocean current systems associated with seasonal changes in the average wind field are not well documented, except in certain areas such as the northern Indian ocean, where there is a marked difference between the current pattern during the southwest Monsoons of northern summer and that during the northeast Monsoons of northern winter.

Ekman, a Swedish mathematician and oceanographer, demonstrated that, in the absence of constraining boundaries, the surface current should flow in a direction 45° to the right of the wind stress in the Northern Hemisphere, and that over the whole water column there should be a net transport of water 90° to the right of the wind stress . Observations indicate that the wind-driven surface current flows at about 20° to 25° to the right of the wind as measured ten metres above the surface, and with a speed about two per cent of that of the wind. If, in the Northern Hemisphere, the wind blows parallel to a coastline on its right, the Ekman transport piles water against the coast until a surface slope is created to balance the Coriolis force; the current then flows parallel to the coast in the direction of the wind. If the coastline is to the left of the wind, the surface water is displaced away from the coast and deeper water rises to replace it. This "upwelling'' is of biological importance in that it brings chemical nutrients back up into the euphotic zone (depth penetrated by sunlight), where they can be utilized in the growth of marine vegetation.

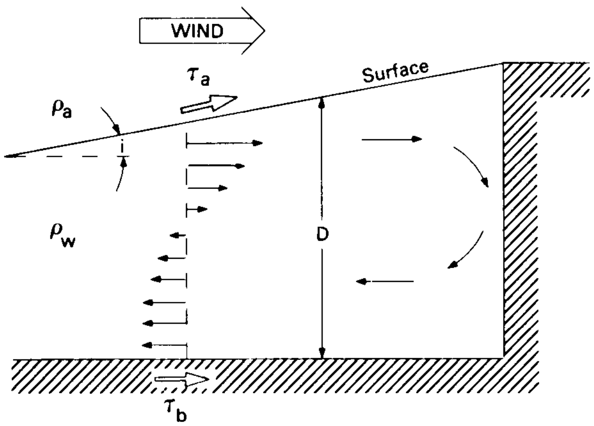


FIG. 35. Circulation of water and balance of forces associated with wind set-up.

4.3 Wind Set-up

The term *wind set-up* refers to the slope of the water surface in the direction of the wind stress. The slope perpendicular to a wind blowing along a coast, mentioned in the previous section, balances the Coriolis force on the along-shore current driven by the wind. It is an indirect effect of the wind, but is not usually thought of as wind set-up. When a wind commences to blow across the water surface, the wind stress is initially occupied in accelerating the water. When a steady state has been achieved, and the water is no longer accelerating, the balance of forces must be between the pressure gradient force due to the surface slope and the surface and bottom stress on the water (due to wind and bottom drag). Figure 35 illustrates the balance of forces in the direction of the wind stress for a wind blowing toward shore, or along the axis of a lake or coastal embayment. The wind is assumed to have been blowing long enough for a steady state to have been reached, so the currents and the surface slope are not changing with time. The wind stress is just sufficient to maintain the surface current against the surface slope and the drag of the slower moving water beneath it. The pressure gradient caused by the surface slope is the same at all depths, and below a certain depth it drives a return flow. This flow is opposed by the bottom stress, or drag. The total drag force on a column of water of unit cross section and spanning the entire depth, D, is τ_a, + τ_b, where τ_a is the surface wind stress and τ_b is the bottom stress. The horizontal pressure gradient is $\rho_w gi$, where ρ_w is the water density, g is gravity, and i is the inclination of the surface in the wind direction. The pressure gradient force on the total volume of the same column of water is therefore $\rho_w giD$. The bottom stress is usually considered small and proportional to the wind stress, and the balance of forces is approximately

(4.3.1) $\tau_a = \rho_w giD$

The wind stress is equal to the drag coefficient for air on water, C, times the density of air, ρ_a, times the square of the wind speed, W. We may thus write 4.3.1 as

(4.3.2) $i = (C\rho_a W^2)/(\rho_w gD)$

The drag coefficient is not a precise constant. but has a value of approximately 2×10^{-3}, ρ_a/ρ_w, is 1.3 x 10^{-3} , and g is 10 m/s^2. Thus 4.3.2 becomes

(4.3.3) $i = 2.6 \times 10^{-7} (W^2/D)$

for W in metres per second and D in metres.

Observations on lakes Ontario, Erie, and Huron, the Gulf of Bothnia and elsewhere indicate that the constant in 4.3.3 is too low, and should be between 4×10^{-7} and 5×10^{-7}. This is probably because of the neglect of the bottom stress in 4.3.1 and because of some funneling effects toward the ends of the lakes. Taking the larger experimental value for the constant, and expressing i as $\Delta h/L$, where Δh is the difference in water level over the length L,
4.3.3 gives

(4.3.4) $\Delta h = 4.5 \times 10^{-7} (W^2L/D)$

with all dimensions in metres and seconds.

We see from 4.3.4 that the difference in elevation between two ends of a lake, caused by a wind blowing along its length, is proportional to the square of the wind speed and the length, but inversely proportional to the depth. Wind set-up is thus of particular importance in shallow bodies of

TABLE 3. Samples of wind set-up on Lakes Erie and Ontario derived from equation 4.3.4

Wind speed	Δh(Erie)	Δh(Ontario)
2 m/s	0.04 m	0.00 m
5 m/s	0.26 m	0.02 m
10 m/s	1.03 m	0.07 m
15 m/s	2.32 m	0.16 m
20 m/s	4.13 m	0.29 m
25 m/s	6.45 m	0.45 m

Erie: length 390 km, mean of 1/depth = 1/17 m
Ontario: length 275 km, mean of 1/depth = 1/170 m

water with large horizontal extent. This is demonstrated by comparison of the values of Δh for Lakes Erie and Ontario given in Table 3 for various wind speeds along the lengths of the lakes. While the values in Table 3 have been derived from substitution in 4.3.4. examination of wind and water level data on these lakes confirms the relationship very closely. Because of the action of the Coriolis force, the axis of greatest slope would be slightly to the right (NH) of the wind direction. The line of flow of the surface current and the return deep current (Fig. 35) would also be oriented to the right of the wind.

4.4 Atmospheric Pressure Effect

The depression of the water surface under high atmospheric pressure, and its elevation under low atmospheric pressure, is frequently referred to as the "inverted barometer" effect. In a standard mercury-in-glass barometer, one atmosphere of pressure supports 0.76 m of mercury; if water were used instead of mercury in the barometer, the height of the column supported would be 10 metres. Since one atmosphere is approximately 100 kilopascals (kPa), we have the barometric equivalence of 10 cm of water and I kPa (or 1millibar of pressure and I centimetre of water). Of course, if the water level is to rise in one place, it must fall in another; clearly, the level in a glass of water does not drop by 10 cm when the atmospheric pressure rises by 1 kPa. It is the slope of the water surface that adjusts to the atmospheric pressure gradient along the surface, so that in the absence of other forces, if the atmospheric pressure at A exceeds that at B by I kPa, the water level at B will exceed that at A by 10 cm.

Stated another way, other forces again being absent, the water level at any location on a body of water differs from the mean surface level by an amount equivalent (but in the opposite sense) to the difference between the local and the average atmospheric pressure over the same body of water. The ocean is sufficiently large that it is fairly safe to assume that the average atmospheric pressure over its surface is constant, and that the inverted barometer effect is therefore fully experienced at each location. On lakes, however, a constant average pressure cannot be assumed, and water level differences from place to place must be treated instead of changes at one location only.

The change in water level caused by pressure change cannot easily be separated from that caused by wind set-up, because the winds are driven by the pressure gradients, and the two are closely correlated. It is usually best to assume that the pressure compensation is complete, and to credit the wind with the remaining surface slope. The justification for the assumption that the pressure compensation is complete is that the surface disturbance travels at the speed of a long wave, $(gD)^{1/2}$, which is usually fast enough to keep pace with moving weather systems. An interesting and useful result of the inverted barometer effect is observed in records from self-contained pressure gauges moored on the ocean bottom (section 6.7). Since these gauges are not compensated for atmospheric pressure, they record the sum of atmospheric and hydrostatic pressure. The compensation for changes in atmospheric pressure by changes in hydrostatic pressure, provided by the inverted barometer effect, is so nearly complete that most of the ''noise" usually found in a tidal record disappears.

Because there is no significant tidal signal in the water level fluctuations imposed by variations in atmospheric pressure, the loss of this part of the water level record simply leaves a cleaner record for tidal analysis. This is not, of course, a desirable feature if it is wished to record actual water levels for navigation or charting.

4.5 Storm Surges

As the name suggests, storm surges are pronounced increases in water level associated with the passage of storms. Much of the increase is the direct result of wind set-up and the inverted barometer effect under the low pressure area near the centre of the storm. There is. however, another process by which the surge may become more exaggerated than would be anticipated from these two effects alone. As the storm depression travels over the water surface, a long surface wave travels along with it. If the storm path is such as to direct this wave up on shore, the wave may steepen and grow as a result of shoaling and funneling, as discussed for long waves in general in section 1.12. The term ''negative surge" is sometimes used to describe a pronounced non-tidal decrease in water level. These could be associated with offshore winds and travelling high pressure systems, and are not usually as extreme as storm surges. Negative surges may, however, be of considerable concern to mariners, since they can create unusually shallow water if they occur near the low tide stage.

4.6 Seiches

A seiche is the free oscillation of the water in a closed or semi-enclosed basin at its natural period. They were discussed in section 1.6, and the formulae for the natural period of closed and open basins were given in equations 1.6.1 and 2. Seiches are frequently observed in harbours, lakes, bays. and in almost any distinct basin of moderate size. They may be caused by the passage of a pressure system over the basin or by the build-up and subsequent relaxation of a wind set-up in the basin. Following initiation of the seiche, the water sloshes back and forth until the oscillation is damped out by friction. Seiches are not apparent in the main ocean basins, probably because there is no force sufficiently co-ordinated over the ocean to set a seiche in motion. The tides are not seiches, being forced oscillations at tidal frequencies. If the natural period, or seiche period, is close to the period of one of the tidal species, the constituents of that species (diurnal or semidiurnal) will be amplified by resonance more than those of other species. The constituent closest to the seiche period will be amplified most of all, but the response is still a forced oscillation whereas a seiche is a free oscillation.

A variety of seiche periods may appear in the same water level record because the main

TABLE 4. Sample seiche periods and typical large ranges.

Basin	Seiche type	Large range(m)	Period(h)
Lake Erie	closed	2	14
Lake Ontario	closed	0.2	5
Lake Huron	closed	—	6
North Channel of	closed	0.1	5
Lake Huron	open	0.1	10
Gore Bay (off North Channel)	open	0.1	0.2
Sydney Harbour, N.S.	open	0.3	2
Bay of Fundy	open	—	12
Bay of Fundy plus Gulf of Maine	open	—	13

body of water may oscillate longitudinally or laterally at different periods, it may also oscillate both in the open and closed mode if the open end is somewhat restricted, and bays and harbours off the main body of water may oscillate locally at their particular seiche periods. Seiches generally have half-lives of only a few periods, but may be frequently regenerated. The largest amplitude seiches are usually found in shallow bodies of water of large horizontal extent, probably because the initiating wind set-up can be greater under these conditions. Table 4 lists a few seiche periods that have been observed and/or calculated for some Canadian waters. The list is by no means exhaustive, since seiches can be identified on almost any water level record; the entries in the table have been chosen simply to illustrate some of the principles mentioned above. The 'typical large" ranges listed are not extremes, but are typical of perhaps the largest 10 or 20% of observed seiches. The last two entries in Table 4 are included not because of actual seiche activity in the Bay of Fundy, but because of interest in the part played by resonance in the large Bay of Fundy semidiurnal tides. For a long time study was concentrated on the resonant (or seiche) period of the Bay of Fundy alone, whereas it is now believed the important resonance is that of the tide with the oscillation of the combined Bay of Fundy - Gulf of Maine system.

4.7 Precipitation, evaporation, and runoff

The precipitation and evaporation considered here are those that occur at the water surface, not those that occur elsewhere in the drainage basin. The runoff is all the water that flows into the water system in question, and thus is the net result of precipitation, evaporation, and absorption of water over the land portion of the drainage basin. In the water budget of a system, precipitation is a positive term, evaporation a negative term, and runoff usually a positive term. In very arid regions, runoff could be negative by virtue of absorption of water into the parched soil along the shores; we are fortunate that in Canada this would be a rarity indeed. If the rate of input to the system from the sum of the three terms exceeds the rate of outflow at its mouth, the water levels within the system must rise, and, conversely, if the rate of outflow exceeds the rate of input, the water levels must fall. If there is no control on the outflow of a system, such as might come from dams, log jams or ice jams, the outflow would increase or decrease steadily with the rise or fall of the water level until an equilibrium was achieved between input and outflow. This is the basis for establishing "stage-discharge relations" from which the flow can be judged from the water level; they are valid only at locations below which there are no control structures or barriers.

There are seasonal variations in precipitation and evaporation that reflect in seasonal variations in water level and outflow of inland water systems, but the most dramatic changes are those associated with changes in runoff. Runoff reflects precipitation over the whole drainage basin, which may cover many times the area of the actual water surface. During a heavy sustained rainfall, only a portion of the water can be absorbed into the ground, and the runoff from a large drainage area can cause "flash flooding" of a water system. The Great Lakes system is unusual in that it has a small drainage area in relation to its large water surface, so no dramatic changes in level or outflow occur in that system. The water storage capacity of the land area of a drainage basin is greatly increased in winter, when much of the surface and ground water is locked up as ice, and the precipitation accumulates as snow cover. As the ice and snow melt in the spring, the runoff can increase rapidly, resulting in the spring "freshet" in streams and rivers, and in raised water levels throughout the system. Sea level along open coasts is not noticeably affected by precipitation, evaporation, and runoff because their net average for the whole ocean is close enough to zero not to affect the elevation of such a large surface area. Water level records from a harbour at the mouth of a river may, however, reflect fluctuations in runoff. While the average water budget for the ocean is essentially zero, the local budgets are not, and water must be moved from place to place to minimize the occurrence of bumps and hollows on the surface. Significant currents are thus set up in the ocean to disperse water away from regions of high precipitation and/or runoff, and to divert water toward regions of excess evaporation (the ocean "deserts"). These currents do not flow directly from regions of budget surplus to regions of budget deficit, but are deflected, as are all currents, by the Coriolis force.

4.8 Effect of Coriolis Force on Currents

The Coriolis force was introduced and discussed in section 1.8. Its effect on tide propagation was considered in sections 1.8 and I.10, and its effect on wind and wind-driven currents was mentioned in section 4.2. The fresh water runoff from land provides another example of the Coriolis force in action. Instead of continuing to flow directly away from the coast, this water is deflected to the right (NH) and forms a coastal current flowing along the coast to the right, for an observer facing seaward in the Northern Hemisphere (left in SH). In large lakes, bays and gulfs the runoff and the Coriolis force contribute to a cyclonic circulation, and around a large island they contribute to an anticyclonic circulation. The term "cyclonic" refers to rotation in the same sense as the earth's rotation on its axis. It is counter-clockwise when viewed from the Northern Hemisphere, and clockwise when viewed from the Southern Hemisphere. The term "anti-cyclonic" has just the opposite meaning. Use of these terms avoids continual reference to the reader's chosen hemisphere. The main wind driven ocean current gyres are, in this terminology, anti-cyclonic. The Coriolis effect on currents is illustrated in surface current charts of the north Atlantic and Pacific (Fig. 36), the Gulf of St. Lawrence and East Coast (Fig. 37), and the Strait of Georgia (Fig. 38) between Vancouver Island and the B.C. mainland.

It must be remembered that the Coriolis force does not generate currents, nor does it speed them up: its action is always perpendicular to the motion, and so can only change the direction of flow. The main anti-cyclonic gyres in both oceans south of latitude 45° N are driven by the anti-cyclonic wind stress and are reinforced in their anti-cyclonic pattern by the Coriolis force. The smaller cyclonic gyres north of latitude 45° N have this pattern partly because of the shape of the bathymetry and the coastline, and partly because the winds have a more cyclonic pattern at higher latitudes. The cyclonic circulations in the Gulf of St. Lawrence and Hudson Bay result from runoff

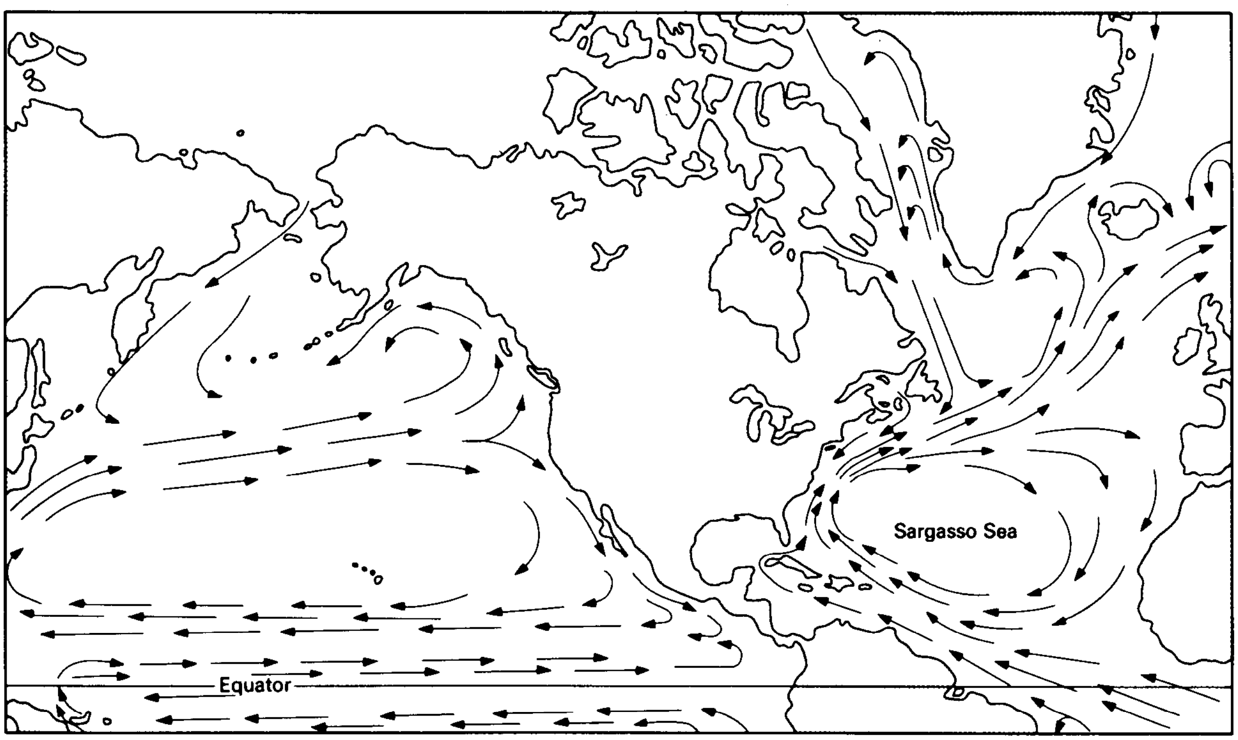


FIG. 36. Surface circulation in north Atlantic and Pacific Oceans.

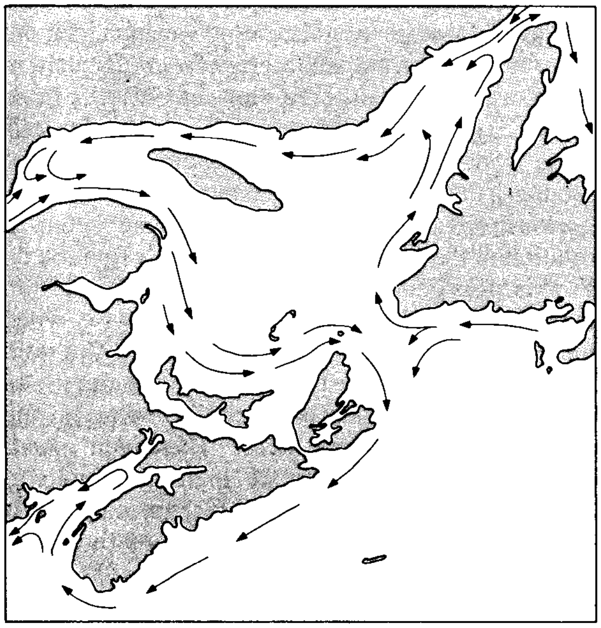


FIG. 37. Surface circulation in Gulf of St. Lawrence and off East Coast.

and a slightly cyclonic average wind stress, assisted by Coriolis force, which likes to pile water up on the right-hand shore in the North. The circulation pattern in the Strait of Georgia fits the pattern of coastal runoff deflected to the right by the Coriolis force. In all of the above cases, the bathymetry of the basins also plays an important role in shaping the circulation patterns, but we shall not pursue that aspect here.

Inertial currents were the subject of section 1.9, but are mentioned here again because of their frequent appearance in current records from ocean moorings. They may be recognized in a record by their characteristic frequency, (12/sinj) hours, where j is the latitude. Inertial oscillations are not continuous in most records, but may appear and reappear several times, somewhat in the manner of a seiche in a water level record. The inertial signature is almost always present in current records from the deep ocean, but is rarely seen in records from shallow coastal regions. The sense of rotation is always anti-cyclonic around the inertial circle.

4.9 Estuarine Circulation

An estuary is, for our purposes, any semi-enclosed body of water that has free access to the .sea, a significant intrusion of sea water, and an inflow of fresh water. The mouths of rivers that flow into the sea are estuarine as far upstream as the limit of salt penetration, and most coastal harbours receive enough fresh water runoff to qualify as estuaries. The St. Lawrence system is estuarine from the limit of salt penetration at Quebec City through the Gulf to Cabot Strait and the Strait of Belle Isle. Estuarine circulation is a system of oppositely directed surface and deep currents driven by the mixing of the outflowing fresh water with the underlying salt water. Figure 39a illustrates in a simplified manner the principles of estuarine circulation; the fresh water is shown as if it entered only at the head of the estuary. The surface of the estuary slopes down toward the sea, and the fresher surface water flows seaward down this pressure gradient, becoming saltier as it mixes with the underlying water along its way. How much mixing takes place between the fresh and the salt water depends to a large extent upon the wind and the tidal action, but, qualitatively at least, the result is to tilt the isohalines (surfaces of equal salinity) down toward the head of the estuary as shown. The average density, which is roughly proportional to the salinity, is seen to be less for a column of water at the head of the estuary than for a column at the mouth. Because of this, the hydrostatic pressure increases more rapidly with depth at the mouth than at the head of the estuary, and the seaward-sloping pressure gradient near the surface may be replaced by a pressure gradient in

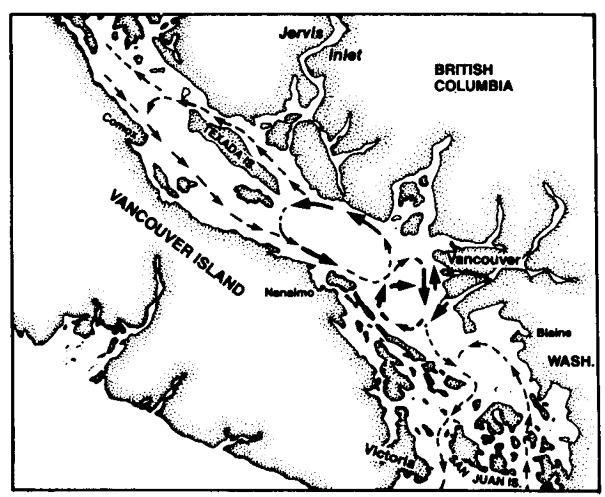


FIG. 38. Surface circulation in Strait of Georgia. (from fig. 10.23 of *Oceanography of the British Columbia Coast*, by R.E. Thomson).

the opposite direction at greater depths. This deep horizontal pressure gradient drives a return flow of saltier water into the estuary beneath the fresher outflow.

If conditions remain unchanged for a sufficiently long time, a steady state would be reached in which as much salt is being transported into the estuary in the deep layer is being transported out in the surface layer, and the volume outflow of water exceeds the volume inflow by exactly the amount of the fresh water input. Without further consideration of the dynamics (i.e. the forces involved). we may find an interesting relation simply from the principles of continuity (i.e. the conservation of matter and the continuous nature of a liquid).

Consider a vertical cross-section of the estuary across the flow, and let S_o, and S_i be the average salinities of the outflowing and inflowing water respectively, V_o and V_i be their volume transports, and R be the rate of volume input of fresh water. To conserve a steady state for the volume of water inside the estuary,

(4.9.1) $V_o - V_i = R$

and to conserve a steady state for the amount of salt inside,

(4.9.2) $S_oV_o = S_iV_i$

Solution of 4.9.1 and 2 for V_o and V_i gives

(4.9.3) $V_o = R\ [S_i / (S_i - S_o)]$

and $V_i = R\ [S_o / (S_i - S_o)]$

The volume transports in and out of the estuary depend critically on their salinity difference as well as on the fresh water input rate. The amount of mixing between the fresh and salt water along the estuary is therefore very important since it determines the salinity difference. If we imagine the unrealistic situation in which there is no mixing, and the fresh water simply flows out over the undisturbed salt water beneath, S_o would be zero and 4.9.3 would give $V_o = R$ and $V_i = 0$. A more realistic example is the St. Lawrence estuary near Rimouski, Que., for which S_o is approximately 30 parts per thousand, S_i is 34 parts per thousand, and R is about 10 000 m^3/s. From these values, 4.9.3 gives V_o as 85 000 m^3/s and V_i as 75 000 m^3/s. Taking the width of the estuary at Rimouski as 45 km, the depth of the upper layer as 50 m, and that of the lower layer as 250 m, we may calculate the cross-sectional areas through which these volume transports are flowing, and so convert the transports into mean velocities in the two layers. This gives the mean outflow velocity in the upper layer as 0.04 m/s, and the mean inflow velocity in the lower layer as 0.01 m/s. These values agree well with observation, but they must be recognized as average values only. There is usually a strong vertical shear in the velocity, with the largest values near the surface, and both the outflow and inflow are usually stronger on their respective right-hand sides because of the Coriolis force.

The effect of earth rotation (i.e. Coriolis force) on a simple estuarine circulation is shown in Fig. 39b, a cross-sectional view of the estuary. The surface slopes up across the channel toward the right side of the outflow, and the isohalines slope down toward the same side. The strongest outflow is at the surface on the right side (NH) facing out. The strongest inflow is on its right side facing in, but is not always strongest at the bottom. The outflowing Gaspe Current (Fig. 37) is at least partly the result of intensification of the surface estuarine outflow along the Gaspe shore because of earth rotation.

The Mediterranean Sea is an example of a body of water in which the evaporation exceeds the sum of the precipitation and the runoff. Such a body of water does not qualify by our definition as an estuary, but the term “negative estuary” is sometimes applied to it, and the formulae in 4.9.3 may be used with the negative value of R to calculate what is now a surface volume transport inward and a deep volume transport outward. What happens physicalIy in this case is that evaporation lowers the level of the surface, causing surface water from the ocean outside to flow inward down the slope. The evaporation also raises the salinity of the inside water, making the average density of a column of water inside greater than that of a corresponding column of water outside. Below a certain depth this reverses the direction of the pressure gradient, and drives a

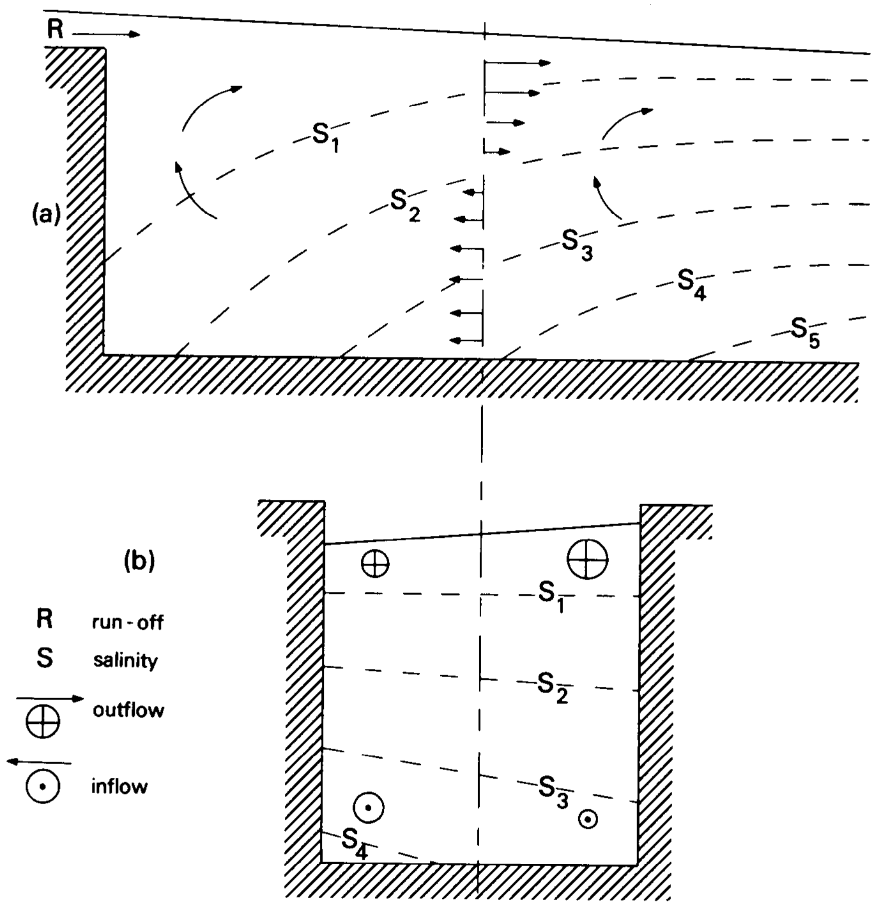


FIG. 39. Circulation and salinity patterns of estuarine circulation, (a) in a vertical section along the estuary and (b) in a vertical section across the estuary.

deep flow of high salinity water from the inland sea to the ocean. The effect of earth rotation is to tilt the surface up on the right side of the inflow and to tilt the isohalines in the opposite sense.

Figure 40a and b illustrates the situation when R is negative. The picture is in every way similar to that in Fig. 39a and b, but with the directions of flow reversed. The warm and salty water that flows out from the Mediterranean through the Strait of Gibraltar can be detected at intermediate depths far out in the mid-Atlantic.

In the above discussion, no mention has been made of temperature as a factor in determining the density of seawater. In the open ocean, where salinity differences are small, temperature is in fact the controlling factor for density, and colder water almost invariably underlies warmer water. In estuaries and other coastal regions, however, large salinity variations are common, and it is they that usually determine the density, with temperature inversions frequently occurring.

4.10 Melting and Freezing

When seawater freezes, it is only the water that forms into ice crystals. The salt becomes trapped between the crystals in a concentrated brine that eventually leaches out, leaving mostly pure ice floating on the surface, surrounded by sea water of increased salinity and density. Since the ice displaces its own weight in this denser water, it does not displace as much volume as it occupied before freezing. Because of this, freezing has an effect similar to that of evaporation - it lowers the water level and increases the surface salinity and density. Surface water must therefore flow toward a region of freezing, while the cold salty water that is formed must sink and flow away

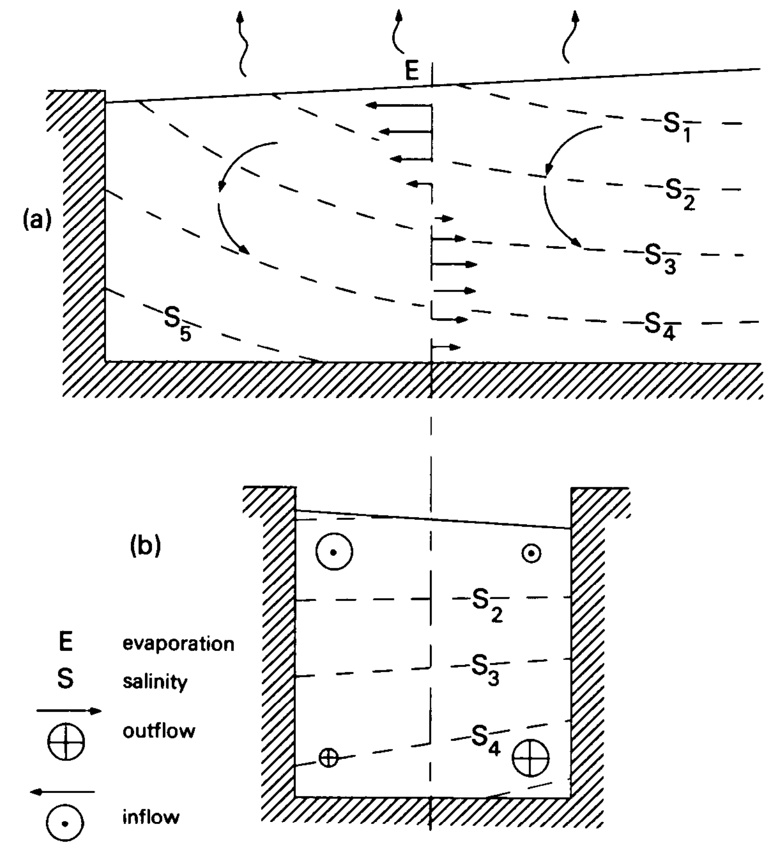


FIG. 40. Circulation and salinity patterns for a sea (e.g. Mediterranean) in which evaporation exceeds runoff plus precipitation, (a) in a vertical section along the axis and (b) in a vertical section across the axis.

from the region. In the polar regions, particularly in the Antarctic. freezing produces cold salty water that sinks and flows along the ocean bottom for thousands of kilometres. When sea ice melts, mostly fresh water is released, and this decreases the salinity and the density of the surrounding water. Melting thus has an effect similar to that of precipitation - it raises the water level and decreases the surface salinity and density. Surface water must therefore flow away from a region of melting ice. The speed of currents associated with freezing and melting in the ocean are never great.

4.11 Tsunamis

A tsunami is a disturbance of the water surface caused by a displacement of the sea-bed or an underwater landslide, usually triggered by an earthquake or an underwater volcanic eruption. The surface disturbance travels out from the centre of origin in much the same pattern as do the ripples from the spot where a pebble lands in a pond. In some directions the waves may almost immediately dissipate their energy against a nearby shore, while in other directions they may be free to travel for thousands of kilometres across the ocean as a train of several tens of long wave crests. Being long waves, they travel at the speed $(gD)^{1/2}$, giving them a speed of over 700 km/h (almost 400 knots) when travelling in a depth of 4,000 m. The period between crests may vary from a few minutes to the order of 1 h, so that in a depth of 4,000 m the distance between crests might range from less than a hundred to several hundred kilometres. The wave heights at sea are only the order of a metre, and over a wavelength of several hundred kilometres this does not constitute a significant distortion of the sea surface. When these waves arrive in shallow water, however, their energy is concentrated by shoaling and possibly funneling (section I.12) causing them to steepen and rise to many metres in height. Not only are the tsunami waves high, but they are also massive when they arrive on shore, and are capable of tremendous destruction in populated areas. Because of the relative gentleness of tsunamis in deeper water, ships should always leave harbour and head for deep offshore safety when warned of an approaching tsunami. The origin of the word is, in fact, from the Japanese expression for “harbour wave.” This name has been adopted to replace the popular expression “tidal wave,” whose use is to be discouraged since there is nothing tidal in the origin of a tsunami. Another expression sometimes used for these waves is “seismic sea wave,” suggesting the seismic, or earthquake, origin of most tsunamis.

A tsunami warning system for the Pacific has been established by the United States, with its headquarters in Honolulu, Hawaii. Other countries, including Canada, that border on the Pacific have since been recruited into the system. Canada’s direct contribution consists of two automatic water level gauges programmed to recognize unusual water level changes that could indicate the passage of a tsunami, and to transmit this advice to Honolulu. The gauges are at Tofino on the west coast of Vancouver Island, and at Langara Island off the northwest tip of the Queen Charlotte Islands group. The tsunami warning centre at Honolulu receives immediate information from seismic recording stations around the Pacific of any earthquake that could possibly generate a tsunami; it calculates the epicentre and intensity of the quake and the arrival time of the as yet hypothetical tsunami at the water level sensing stations in the network; it initiates a “tsunami watch” at all water level stations in the path, for a generous time interval around the ETA of the hypothetical tsunami; and it issues tsunami warnings to the appropriate authorities in threatened locations if the water level interpretation indicates that a tsunami has indeed been generated.

CHAPTER 5

Datums and Vertical Control

5.1 Vertical Datums

It is apparent that the elevation of something can only be expressed relative the elevation of something else, whether the reference elevation be that of the centre of the earth, the mean surface of the sea, the orbit of a satellite, or simply a bolt set in bedrock. The chosen zero to which other elevations are referred is called a datum of vertical reference or simply a vertical datum. The latter term is most commonly used. but it must not be misinterpreted to mean that the datum is vertical. The plural of “datum” is “datums” in this context, to distinguish it from the word “data” that signifies any set of observed values of a parameter. If the datum is defined over an area, it is called a datum surface.

A distinction must usually be made between the concept chosen to define a vertical datum and the realization of the concept in practice. For example, two agencies may both choose mean sea level as their reference surface, and they may independently determine a value for the elevation of the same benchmark. Because of differences in technique and errors of observation, the two values would almost certainly differ. The two datums would then be said to differ by that amount at the location of the benchmark, even though they profess to have the same ideal reference surface. It is not unusual to have elevations assigned to the same benchmark by several survey organizations.

5.2 Equi-geopotential or Level Surfaces

These are surfaces along which no work is done by or against gravity in moving from one point to another. The concept of a force potential was introduced in section 2.4, and equi-geopotential or level surfaces are simply surfaces of constant potential in the earth’s gravity field. Gravity acts everywhere perpendicularly to level surfaces, and they are the surfaces to which all water levels would eventually conform in the absence of all forces other than gravity. The geoid is the level surface that most closely fits the mean surface of the world’s oceans. The term “Mean Sea Level (MSL)” is frequently loosely used, without clear definition of its intended meaning; for our purposes, the surface of MSL will be defined as identical to the geoid. By this definition it is clear that the mean elevation of the sea surface at a particular location need not be the same as the elevation of MSL, since the elevation of MSL (the geoid) could be determined only by fitting a level surface to observations of the mean level of the sea surface over the whole of the ocean. The local mean water level (MWL) departs from MSL in the ocean because of surface slopes caused by prevailing wind stress patterns, persistent anomalies in the distribution of precipitation, evaporation, freezing, melting, heating and cooling, and by the deflection of ocean currents by the Coriolis force.

5.3 Geopotential, Dynamic, and Orthometric Elevations

The difference in geopotential between two level surfaces equals the work done in raising a unit mass from the lower to the higher surface. Since this amount of work equals the vertical distance between the two surfaces times the average gravity along the vertical path, division by a standard value of gravity (e.g. the average value of gravity at sea level for a specified latitude) gives a number equal to the linear vertical separation of the two surfaces at a location where gravity equals the standard gravity. This number is the difference in geopotential elevation, and is quoted in units such as geopotential metres. All points with the same geopotential elevation above the geoid (MSL) must, by definition, lie on the same level surface, since the geoid is itself a level surface. To perform geopotential levelling in the field requires a knowledge of the value of gravity along the path of the levelling. To obtain differences in geopotential elevation, all instrumentally observed differences in elevation along the line must be multiplied by the ratio of the local gravity to the standard gravity.

The concept of dynamic elevation is precisely the same as that of geopotential elevation, the only difference being that geopotential levelling uses an observed value of local gravity in correcting the instrumental differences in elevation, whereas dynamic levelling uses a value calculated from a gravity formula involving latitude and altitude only. Local gravity anomalies can thus introduce local errors into dynamic levelling, in addition to any instrumental errors committed. The errors introduced by the anomalies tend to cancel out as the anomalies are passed, and do not accumulate over a long line of levelling.

The name given to the concept of vertical linear distance above the geoid is orthometric elevation. Although this may at first seem to be the most straightforward definition of elevation, it will be seen to have some drawbacks. The average shape of a level surface on the earth is that of an oblate spheroid, with its centre at the earth's centre and its axis of revolution along the earth's axis. The family of level surfaces are concentric, but they are not all the same shape; their spheroidal shape becomes progressively flatter with distance from the centre of the earth. This progressive flattening is illustrated in Fig. 41, and simply follows from the definition of geopotential and the fact that gravity is greatest at the poles and decreases with latitude, being least at the equator. The decrease of gravity with latitude results from the increasing outward centrifugal force near the equator and from the flattened shape of the earth itself, which is probably also attributable to centrifugal force during the earth's formative period. The oblateness of all surfaces in Fig. 41 is greatly exaggerated to illustrate the principle, since if drawn to scale at this reduction, all level surfaces would be indistinguishable from spheres. Local gravity anomalies cause local increases or decreases in the separation between neighbouring level surfaces, but no attempt has been made to illustrate this in Fig. 41.

To demonstrate the correction to instrumental differences in elevation that is required to obtain orthometric differences, consider a line of levelling run from south to north along a level surface from A to B in Fig. 41. Since the levelling is along a level surface, there would be no difference in elevation detected instrumentally; however, because the level surfaces converge toward the north, B is at a lower orthometric elevation than is A. The orthometric correction to the instrumental difference observed in levelling from A to B is the height BB', where AB' lies along a surface that is parallel to the geoid. The amount of the correction is calculated from formulae involving latitude, altitude, and the north - south extent of the line. The formulae for the dynamic and the orthometric corrections are based on the same model of earth gravity, making the two systems mutually convertible. Local gravity anomalies introduce local errors into orthometric levelling, but, as in dynamic levelling, the errors tend to cancel out as the anomalies are passed, and do not accumulate over a long line of levelling.

The greatest objection to an orthometric system of elevations is that points on the same level surface are not given the same elevation if they are at different latitudes. This is particularly disturbing for hydraulic and hydrodynamic studies on lakes and rivers. As an example, the orthometric elevation of the level surface of Lake Winnipeg (no wind, etc.) would be about 0.08 m less at the north end than at the south end. When working at sea level there is no orthometric or dynamic correction to apply, because both corrections are approximately proportional to the altitude. In small local surveys, such as between the control benchmarks and a water level gauge, there would probably never by any need to correct the instrumental differences to either the dynamic or orthometric system, because the ranges of elevation and latitude would be too small to generate a significant correction.

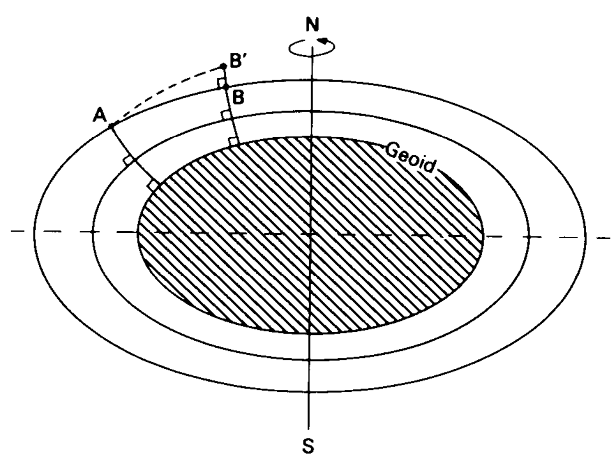


FIG. 41. The geoid and higher level surfaces, illustrating correction required to obtain orthometric elevation.

5.4 Geodetic Datum

Geodetic Datum (GD) is the reference surface to which the Geodetic Survey of Canada refers elevations. It is referred to as a sea-level datum because it professes in concept to be the geoid, which is also called Mean Sea Level. In practice, of course, it can only be an approximation to the geoid, and its physical location is precisely defined only with reference to Geodetic benchmarks in a region. Geodetic Datum exists only as a concept in regions not yet included in the Geodetic network of vertical control. Elevations above the Geodetic Datum are always quoted in the orthometric system. Geodetic Datum (i.e. the elevations of the benchmarks in the network) is based on a 1928 adjustment of the Canadian levelling network, in which the mean water levels at the gauging stations of Halifax, Yarmouth, Pointe-au-Pere, Vancouver, and Prince Rupert were all held fixed at zero. Since the mean sea surface is known not to be a level surface (section 5.2), Geodetic Datum is seen to have departed immediately from the precise concept of the geoid. It was realistically reasoned, however, that the errors introduced by equating MWL to MSL were less than those incurred in long lines of land levelling.

5.5 International Great Lakes Datum (1955)

International Great Lakes Datum (1955), or IGLD, is a datum established by the Canada - U.S. Coordinating Committee on Great Lakes Basic Hydraulic and Hydrological Data, to provide a unified datum for use in hydraulic and hydrological studies on both sides of the border along the Great Lakes and St. Lawrence River. The establishment of IGLD also met the need for a revision of datums in the Great Lakes region caused by the cumulative effect of crustal movement over the years. Crustal tilting in the Great Lakes basin appears capable of raising one end of a lake with respect to the other end by as much as one metre in three hundred years. IGLD may be referred to as a sea-level datum, but, in recognition of the fact that sea level varies from place to place, it was defined as the level surface passing through the mean water level at the outlet of the system. In practice, this was determined as the mean level at Pointe-au-Père over the period from 1941 to 1956, and extended throughout the system by a network of over-land levelling and water level transfers. Dynamic elevations are used in the IGLD system because it was wished to have the same elevation quoted for all points on the same level surface, since water surfaces seek level surfaces, not surfaces equidistant above the geoid. The geopotential system would have been more desirable still, but insufficient gravity information was available at the time. The standard gravity used to convert geopotential numbers into dynamic elevations is the average sea-level value of gravity at 45° latitude. The length of a dynamic metre at a particular location therefore equals the length of a linear (or orthometric) metre times the ratio of the standard to the local gravity, a ratio that is very close to unity in the region served by IGLD. The distinction in units can usually be ignored when dealing with the small instrumental differences in elevation encountered in small local surveys.

Because the physical location of a datum is defined locally by the elevations of benchmarks that move with the earth's crust, crustal movement has continued to distort IGLD with respect to level surfaces since its establishment (as it has all datums in the region). In the interest of consistency within the system, the elevation of a new benchmark should be established only by transfer from a nearby original benchmark, on the assumption that the difference in crustal movement between two nearby locations is small. Eventually, the distortion within the system may become intolerable, and a complete re-levelling and readjustment of the network undertaken. New elevations would then be assigned to all benchmarks, and the new datum would be identified by its date of adjustment.

5.6 Hydrographic Charting Datums

Depths and elevations shown on hydrographic charts must be below and above specified datum surfaces. For purposes of navigational safety, depths are referenced to a low water datum and elevations to a high water datum, so only rarely could there be less depth or less clearance than that charted. Water level gauge measurements and tide height predictions must also refer to specified datums. It is universal practice to reference the water levels and the tide predictions to the same datum as that used for charted depths, so addition of the observed or predicted water height to the charted depth will give the appropriate total depth.

Chart datum (CD) is the datum to which depths on a published chart. all tide height predictions, and most water level measurements are referred. It was agreed in 1926 by member states of the International Hydrographic Organization that chart datum "should be a plane so low that the tide will but seldom fall below it." The wording indicates that the resolution was formulated with only tidal waters in mind, and, since the word "seldom" was left undefined, it provides but a qualitative instruction for the choice of chart datum. The following three criteria place somewhat more restriction on its choice: chart datum should be:

I) so low that the water level will but seldom fall below it,
2) not so low as to cause the charted depths to be unrealistically shallow, and
3) it should vary only gradually from area to area and from chart to adjoining chart, to avoid significant discontinuities.

On most Canadian coastal charts the surface of lower low water, large tide, or LLWLT (see section 5.7), has been adopted as chart datum, but the term "lowest normal tide," or "LNT," has been retained on the charts since it encompasses a variety of other choices for chart datum on some older charts. On United States charts, chart datum is taken as mean lower low water (MLLW), a surface somewhat higher than LLWLT. It has been agreed by the two countries that on charts covering both Canadian and U.S. waters Canadian chart datum is to be used on the Canadian side of the boundary, and U.S. datum on the U.S. side, regardless of which country publishes the chart. This policy causes a discontinuity in chart datum along the international boundary, but preserves the principle that charts of the same waters should all have the same chart datum, and that it should be the same datum as used for tidal predictions in those waters.

The choice of a chart datum is usually more difficult on inland waters than on coastal waters because inland waters lack the stabilizing influence the huge ocean reservoir exerts on the mean water level. Whereas a 2-month water level record at a coastal location provides sufficient tidal information to determine a reasonably accurate chart datum, many years of record may be necessary to provide the information on seasonal and secular fluctuations in mean water level required to determine chart datum on lakes and rivers. Shorter term fluctuations, such as those due to seiches and wind set-up may also be considered in setting chart datum, but information on these can be obtained over a fairly short record period. Dry and wet periods in many drainage basins (e.g. the Great Lakes basin) seem to occur in almost regular ' cycles" of several years, causing corresponding periods of low and high water levels in the drainage systems. The chart datums must be set with low stage years in mind, and may appear to be pessimistically low over several years of high stage. There are a fortunate few inland waters for which chart datum is easily chosen - those in which the minimum water level is controlled during the navigation season. A guideline sometimes used in setting inland chart datums is that the water level may fall below the datum 5% of the time, but this may not be severe enough if the water level undergoes large fluctuations. A preferred guideline is that the daily mean water level should never fall more than 0.2 m below the chart datum during the navigation season.

It should by now be apparent that chart datum need not be a level surface even over the extent of a single chart. Along a river, chart datum must slope with approximately the slope of the water surface of the river at low stage. Even along the coast, where there is no appreciable slope of the mean water level, the surface of chart datum must slope down from regions of small tidal range toward regions of larger tidal range to accommodate the lower low waters. On most lakes, however, it is common to adopt a single level surface as chart datum over the whole lake.

Sounding datum is simply the datum to which soundings are reduced when compiling a "field sheet" during a hydrographic survey. It may or may not remain as the chart datum. While a sounding datum may be chosen rather arbitrarily to facilitate an immediate start for a sounding survey, it is imperative that its elevation and the elevation of the zero for any water level records be referenced to permanent benchmarks on shore. This is required to permit adjustment of the soundings to the final chart datum, and to permit recovery of the chart datum in future surveys of the same region.

The *datum for elevations* on a chart is the surface to which the charted elevations of prominent targets (lights, beacons, steeples, chimneys, etc.) and clearances under obstacles (bridges, power lines, etc.) are referred. It is usually the same high water datum used to define the shoreline on a chart. On most Canadian coastal charts the surface adopted as datum for elevations is higher high water, large tide, or HHWLT (see section 5.7). On Canadian charts of non-tidal inland waters, however, for reasons that are no longer apparent, the low water chart datum is also used as the datum for elevations, while a high water surface is used to define the shoreline.

5.7 Special Tidal Surfaces

It is found useful to define and name several average tidal elevations that can be used in comparing tidal characteristics from place to place. Some of these have already appeared in the preceding text. From a single gauge site only one elevation can be determined for each definition, but it is proper to think of each elevation as only one spot on a continuous tidal surface over the whole ocean. The tidal surfaces presently in vogue in Canada are listed and discussed below.

MWL - mean water level - average of all hourly water levels over the available period of record.
HHWLT - higher high water, large tide - average of the highest high waters, one from each of 19 years of predictions.
HHWMT - higher high water, mean tide - average of all the higher high waters from 19 years of predictions.
LLWMT - lower low water, mean tide - average of all the lower low waters from 19 years of predictions
LLWLT - lower low water, large tide - average of the lowest low waters, one from each of 19 years of predictions.
LNT - lowest normal tide - in present usage it is synonymous with LLWLT, but on older charts it may refer to a variety of low water chart datums.

Of the above tidal surfaces, MWL is the only one whose elevation is determined in practice by straightforward application of the definition. The others are at present calculated from semi-empirical formulae involving the harmonic constants of the major tidal constituents. Today's high speed computers, however, possess the capability of generating nineteen years of prediction and applying the definitions directly with no great difficulty. Another possibility is to generate only one year of predictions, that being a year in which the moon experiences its average excursions in declination (nodal factors, f, near unity), and to take the appropriate averages and extremes from that year of predictions only. These options are under consideration, but in the meantime the semi-empirical method of calculation gives values that have been shown to simulate the definitions very well. Figure 42 illustrates these tidal surfaces and their relation to the charting datums and other charted features.

A variety of other tidal surfaces are defined and used by hydrographic agencies in different countries. Chart datum for United States charts on both the Atlantic and Pacific coasts is now determined as mean lower low water (MLLW), which is defined as the average of all the lower low waters over a specified 19-year period. Previous to 1980 the chart datum for U.S. Atlantic coast charts was defined as mean low water (MLW), the average of all the low waters over an earlier specified l9-year period. Because of the small diurnal inequality on the east coast and because of a difference in sea level between the two l9-year periods, the change from MLW to MLLW has made only a minor change in east coast chart datums, one that is not reflected in the

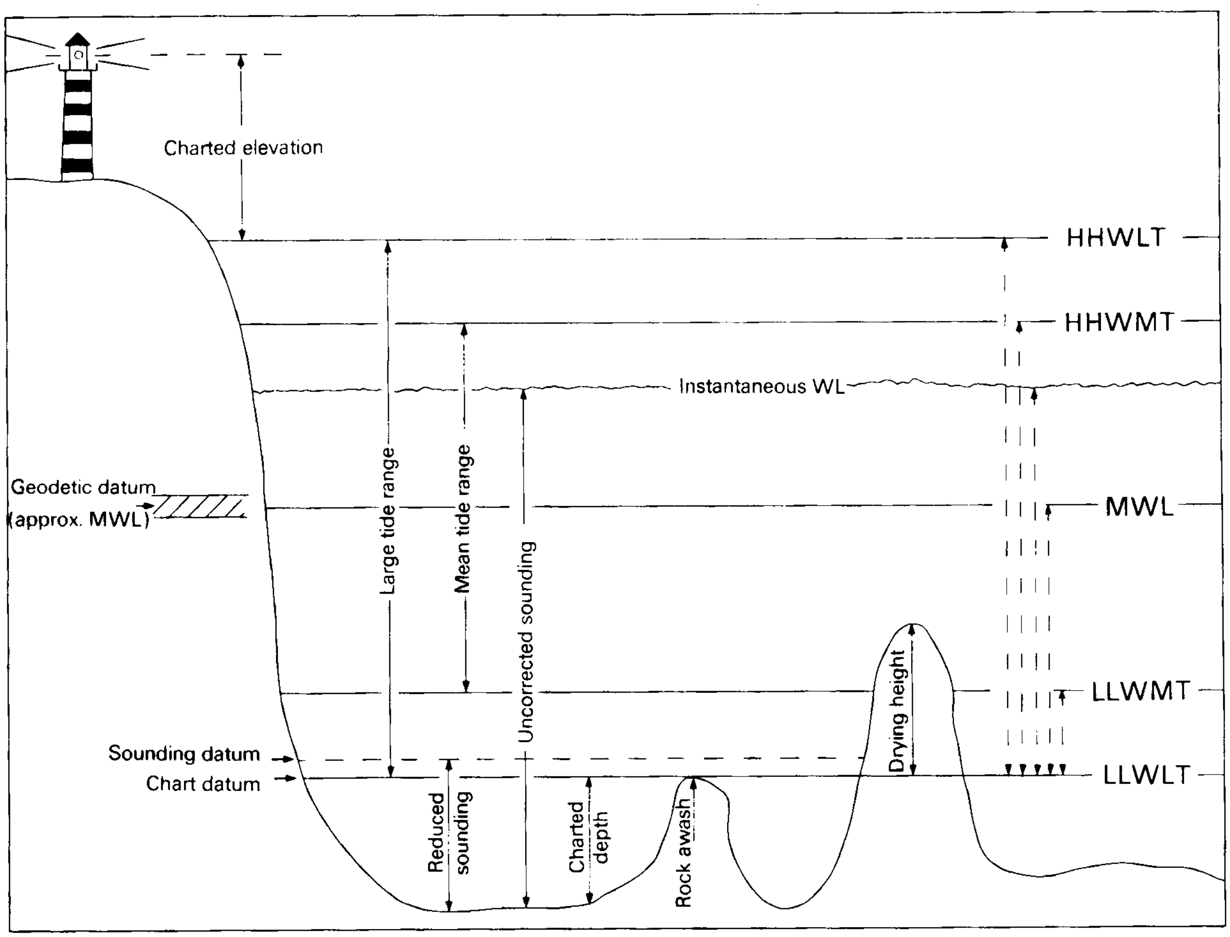


FIG. 42. Relation between tidal surfaces, charting datums and physical features.

charted depths. The discontinuity in datums between adjoining U.S. and Canadian charts remains. Chart datum for British charts is now defined by the tidal surface of lowest astronomical tide (LAT), which is the lowest water level predicted in a l9-year period. Partly because this is difficult to determine, and partly to accommodate older charts, the definition of chart datum is relaxed to permit it to be 0.1 m above a rigorous LAT. A tidal surface used to define chart datum on many older British Admiralty charts, including some in Canadian waters, is mean low water springs (MLWS). It is the average of all available low water observations at the time of spring tide, and applies only where the diurnal inequality is small. While it is no longer in general use, MLWS provides a simple example of how the harmonic constants may be used to approximate the elevations of tidal surfaces. MLWS is approximated as the height of MWL above chart datum minus the sum of the amplitudes of the semi-diurnal lunar and solar constituents, or, symbolically,

$$MLWS = Z_0 - (M_2 + S_2)$$

The semi-empirical formulae used to approximate LLWLT, etc. are much more complicated, and will not be given here.

5.8 Land Levelling and Water Transfers

Anyone who installs or operates a water level gauge must ensure that the zero of the gauge is always accurately referenced to local benchmarks. This is done by standard spirit levelling around the closed network of benchmarks and gauge. It is convenient if one of the local benchmarks is a Geodetic benchmark, but if no Geodetic BM is accessible within a kilometre of the gauge, the task of tying the gauge and its BMs into Geodetic Datum is left to the Geodetic Survey of Canada, the experts in long-distance overland levelling.

Water transfers were mentioned in section 5.5 in connection with the establishment of IGLD. They provide a means of transferring elevations across large expanses of water, on the assumption that the slope of the water surface can be estimated from the hydraulic and meteorological factors. Installation of water level gauges is usually essential for accurate water transfer because reasonably long records are required to average out seiche activity and to span a variety of meteorological conditions, whose effects may then be evaluated. Water transfers are used most frequently over large lakes, because on the average their surfaces approximate very closely to level surfaces. Water transfers of chart datum along the sloping surface of a river may also be carried out, but since the transfer is not along a level surface, elevations may be determined only with respect to the sloping chart datum, not with respect to sea level. In performing a water transfer along a river, interpolation should be made between two reference gauges, because the slope of the river may not be the same with respect to the slope of the chart datum at all stages. Water transfer of sounding datum from gauge to gauge along the sea coast is also a common practice, as described in Chapter 6. This is also the transfer of a sloping rather than a level datum, and is based on the assumption that the tide curves at the neighbouring station have the same shape, but that one may lag the other in time and have a different vertical scale (i.e. different range). Only where the tide has a small diurnal inequality are the assumptions likely to be valid.

It is partly for this reason that it is referred to here as a transfer of sounding datum, rather than of chart datum, because the final chart datum would almost certainly be based on an analysis of the full tidal record available at the end of the survey, rather than on the preliminary water transfer.

5.9 Purpose and Importance of Benchmarks

The purpose of permanent Hydrographic benchmarks is to identify locally the elevation of the physical surface that is chart datum. Since all other charting datums and tidal surfaces are referred to chart datum, the Hydrographic BMs are the fundamental references for vertical control in charting and water level gauging on navigable waters. While other agencies, such as the Geodetic Survey, frequently tie the Hydrographic BMs into their networks and provide elevations for them on their own datums, it remains the elevation of the BM above chart datum that is basic to charting and gauging procedures. Only the responsible Hydrographic agency may assign or alter the elevation quoted for a BM above chart datum. Although it is not necessary for charting purposes, it is desirable that chart datum be referenced to Geodetic Datum, so that the Geodetic elevation of chart datum can be supplied to engineers and surveyors and documented on the charts. Subsequent readjustment of the Geodetic network could provide new Geodetic elevations for the Hydrographic BMs and chart datums, but would not affect the quoted elevations of the BMs above chart datum. On the Great Lakes, where chart datum is defined on each lake as a fixed elevation above IGLD, it is necessary that the Hydrographic BMs be tied in to IGLD as part of the procedure for establishing datum for charting. Ultimately an adjustment will need to be made to all BMs in the IGLD network, to correct for the crustal movement since 1955 and to incorporate new levelling. This will provide new IGLD elevations for Hydrographic BMs and chart datums, but will not change the BM elevations relative to chart datum. These can be changed only if a new chart datum is defined, and the charts revised accordingly.

As part of the installation procedure of any water level gauge, a minimum of three BMs are established in the immediate vicinity (1/2 km) of the gauge, with no two in the same feature or structure. The elevation difference between the preliminary gauge zero and each of the BMs is then determined by accurate spirit levelling. When the elevation of chart datum is finally chosen with respect to the preliminary gauge zero, the BM elevations are converted and recorded in the BM descriptions as elevations above chart datum. If the water level gauge is to continue in operation, its permanent zero would be set to chart datum. The BMs provide for the recovery of chart datum in future surveys and for consistency in the setting of gauge zero for all water level measurements at the same site.

One BM at a site is insufficient because there would be no comparison by which to test its stability over the time since its installation. Two BMs are insufficient because if one is found to have moved with respect to the other, there would be no way to know whether one, the other, or both were unstable. Three BMs provide the possibility of identifying one unstable member of the group. This is why three is the minimum required number of control BMs at each gauge site. More than three BMs is, of course, desirable because there is no guarantee that two BMs may not be found to have been unstable. When a BM is found to be unstable, it must be destroyed and replaced by a new one in a different location. The elevation of the new BM above chart datum is determined by levelling from the remaining stable BMs. The elevation and description of the new BM are recorded, along with notice of the destruction of the unstable BM.

It is worth noting that chart datum may be precisely (a few millimetres) related to other datums, such as GD and IGLD, only at gauge sites where those datums have been tied in to the Hydrographic BMs. This is true for the following reasons: firstly, because away from the gauge sites the chart datum is determined only to the accuracy to which the soundings are observed and corrected, and secondly, because chart datum at each sounding site is determined in effect by water transfer from the gauge site along the water surface, and the shape of the water surface with respect to the geoid is not determined as part of the sounding survey. It is thus not possible to define the continuous surface of chart datum in terms of its accurate elevation above Geodetic or other survey datums, the relation being accurately known only at gauge sites.

PART II
Instruments and Procedures

CHAPTER 6

Establishment of Temporary Water Level Guage

6.1 Introduction

From the viewpoint of a field hydrographer, the immediate function of a temporary water level gauge is to provide the information necessary for the transfer or establishment of sounding datum (and, eventually chart datum) and for the reduction of soundings to this datum. If properly recorded and documented, however, the information from such gauges may serve other functions as well, such as provision of harmonic constants for tidal prediction and of information on short-term water level fluctuations. Since the hydrographer may not know to what use the water level and benchmark information may be put in the future, the care expended in installation and operation of a gauge should not be limited to that necessary to achieve the required accuracy of sounding reduction. This is why some of the accuracy standards recommended below may at first appear to be unnecessarily severe: they should, however, be readily achievable with the exercise of moderate care. As discussed in section 3.6, every effort should be made on tidal waters to obtain at least one month of water level record, to permit proper tidal analysis of the data. One of the fringe benefits that may accrue to the field hydrographer as a result of added care in the installation and operation of gauges is the improvement of cotidal charts for use in future surveys of the same or nearby areas.

6.2 Stilling Wells

A stilling well is a vertical enclosure with only limited access to the outside water; its purpose is to damp out most of the rapid vertical oscillation of the water surface whose elevation is being measured. A stilling well is always required for use with a float-type water level gauge because rapid rise and fall of the float may cause its suspension cable to slip over, or even jump off, the pulley wheel. Other problems that may be cured by installation of a stilling well are excessive "chatter" in a pen-on-paper record, and excessive scatter among readings taken at fixed intervals by a digital recorder. A small portable stilling well is useful when it is wished to level to the water surface, as is required in checking the zero setting of a submerged pressure gauge. Such a portable well could be simply a length of metal or plastic pipe sealed at one end except for a small intake hole far enough above the sealed end to avoid obstruction. With the well set vertically in the shallow water near shore (secured by rocks or other temporary supports), a levelling rod may more easily and accurately be held on the water level inside the well than on that outside.

A much more substantial stilling well than that described above is required for use with an automatic recording gauge. It may be constructed from wooden planks, metal or plastic pipe, sections of culvert, etc. It must be vertical and have sufficient cross-sectional area to accommodate the float and counter-weight clear of the sides of the well at all water level stages; it must extend from below the lowest to above the highest water levels anticipated (including wave action outside the well); except for the small intake hole, it must be water-tight over the portion of its length that may be submerged; and it must be sturdily constructed and mounted to withstand wave action without significant motion of the well. Particular care should be given to strengthening the bottom of the well, since a sudden surge (up or down) in the water level outside the well creates a pressure (in or out) against the bottom of the well of 0.1 atmospheres per metre of surge.

The intake opening should be so placed as to be submerged at all times, but should not be so close to the bottom of the well that it could become blocked by the accumulation of silt inside or outside. It is sometimes difficult to find a location that is both convenient and suitable for construction of a stilling well. For example, the vertical side of a pier provides a convenient surface to which to attach a stilling well, and the pier can provide easy access to the gauge; but in regions with large tidal range (e.g. Bay of Fundy), the area around the pier may dry out at some low water stages. In such a case, it may be feasible to dig the bottom of the well down below the low water stage and feed it through the siphon action in a hose running from deeper water off shore to the inside of the well; the hose or pipe may pass through the wall of the well at any convenient spot near the bottom of the well, but it must be assured that the end of the hose inside as well as that outside the well remain submerged at all times. A similar arrangement may be used when silting is a problem near the bottom of the well, feeding the well through a hose or pipe whose outside end is secured in deeper water where silting is not a problem .

The damping action of a stilling well is a function of the ratio of the cross-sectional area of the inside of the well to that of the intake. the larger the ratio the greater the damping. Figure 43, shows for different intake ratios, the rate of adjustment of the water level inside a well to a sudden and sustained surge in the outside level. While the decay rate of the level difference is not strictly exponential, it is nearly enough so over much of the curves to make the concept of a response time meaningful. Taking the response time of a well as the time required for the inside water level to adjust half way to a sudden and sustained surge in outside level, Fig. 43 provides the following response times for wells of various intake ratio, R:

6 seconds for R = 50

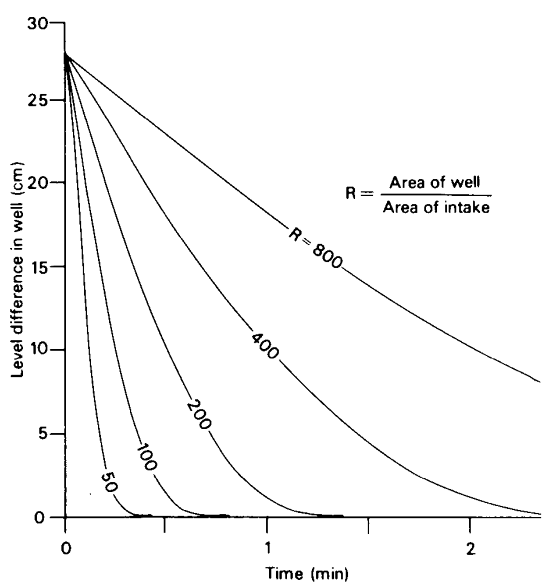


Fig. 43 Response of water level in stilling well to sudden and sustained surge of outside water level, for various intake ratios.

11 seconds for R = 100
22 seconds for R = 200
45 seconds for R = 400
90 seconds for R = 800

The data for Fig. 43 was obtained by measurement in a section of plastic pipe whose cross-sectional area was 900 cm^2 and whose wall thickness was 3 mm. The wall thickness and the roughness of the intake surface may influence the response times somewhat. It is recommended that the area of the intake be 1/1 00th that of the well (R = 100), or that the diameter of the intake be 1/10th that of the well . If a long intake pipe or hose is employed, the pertinent intake area is the smallest cross-sectional area along its length. Since friction in a long intake pipe increases the response time of the well, the cross section of a very long pipe or hose may need to be greater than that indicated by R = 100. Figure 44 illustrates the damping effect of a well with R= 100 on waves of 1/2 metre amplitude and periods of 12 hours, 6 minutes, and 6 seconds, respectively. These three periods were chosen to represent a possible tide wave, harbour seiche, and

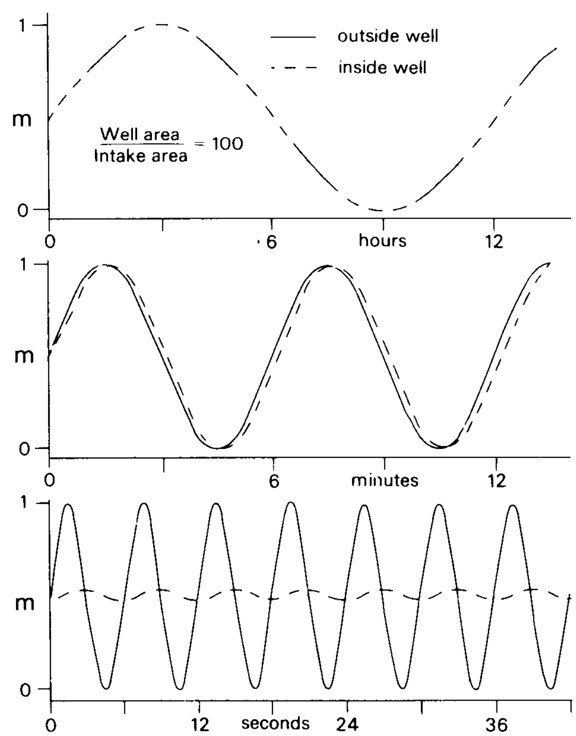


FIG. 44. Damping effect of stilling well with intake ratio of 100, for oscillations with period (a) 12 hours, (b) 6 minutes and (c) 6 seconds.

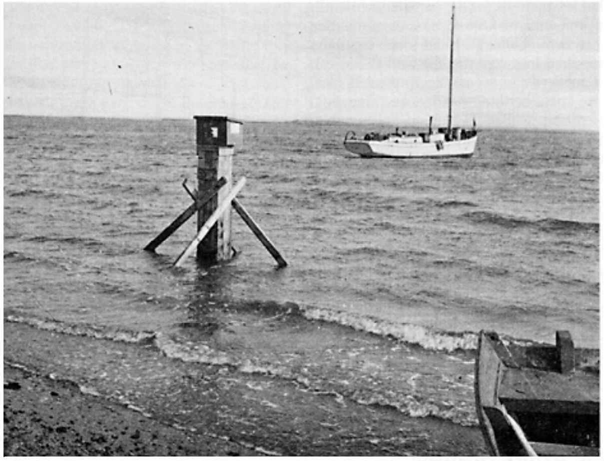
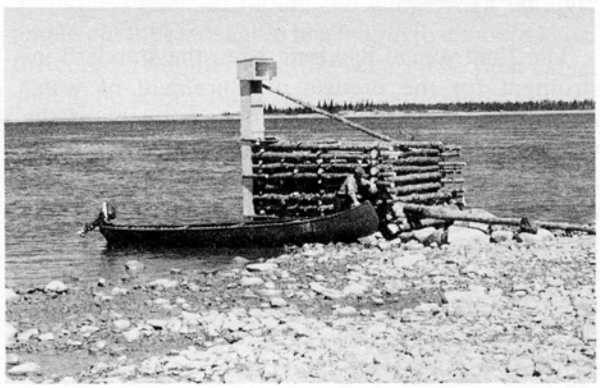
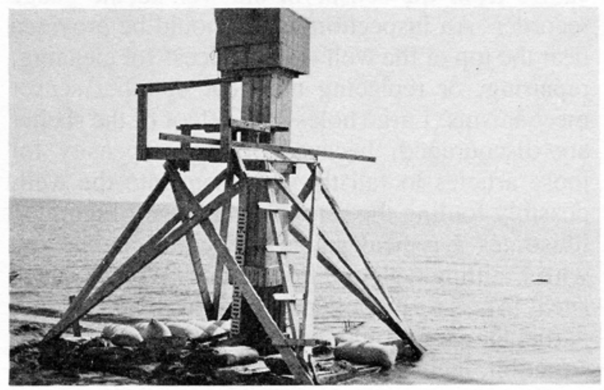


Plate 8. Various makeshift structures to support temporary water level gauges. The gauge shelters shown house float gauges mounted over stilling wells constructed of wooden planks. (Photos by Canadian Hydrographic Service.)





surface swell. The well is seen to damp out the high frequency waves effectively, while passing most of the intermediate frequency seiche, and all of the low frequency tide signal.

Detailed instructions for the installation of stilling wells are not given here because each situation presents its own challenges. and some ingenuity may be required to assure that the well is vertical, rigid and motionless, accessible, protected from damage by boats berthing nearby, and free from excessive silting, while still being deep enough to have its intake below the lowest water level. Figure 45 illustrates schematically the use of a stilling well in conjunction with a float-actuated water level gauge.

6.3 Gauge Shelters

Most installations will requireconstruction of some sort of protective shelter for the gauge against the weather and interference from curious passers-by. At permanent gauge sites, a small walk-in gauge house is usually provided, but at temporary sites, an enclosure large enough to accommodate the gauge itself is sufficient. It is conveniently constructed from plywood, with a door hinged at the bottom so it can be dropped down out of the way or secured horizontally by hooks and chains to form a working surface. If there is a stilling well, the gauge shelter may be fastened securely to the top of the well for support, with a weather-tight connection between the well and the shelter. Holes drilled in the floor of the shelter should be no larger than necessary to accommodate passage of the leads or cables from the sensor in the well to the gauge recorder. An inspection hatch should be provided near the top of the well to give access for cleaning, repairing, or replacing the float or other sensor mechanisms. Large holes in the floor of the shelter are discouraged, because of the propensity for loose articles to fall through them into the well, possibly fouling the sensor mechanism. Figure 45 illustrates a typical gauge shelter in conjunction with a stilling well and float gauge. When a gauge site is in an area inhabited or travelled by the public, particular care should be given to neatness of construction, including the painting of the gauge shelter and any supporting framework. When unattended, the shelter should be securely padlocked. A notice attached to the shelter identifying the installation as a water level gauge and briefly describing its function will satisfy most people's curiosity, and, it is believed. decrease the likelihood of meddling.

6.4 Float gauges

The float gauge has long been the standard instrument for the precise measurement of water levels. It provides a direct measurement of the water level, and so does not require calibration over its range of operation, although its zero adjustment must be regularly monitored. Its major drawback is the rigid requirement for a stilling well, provision of which may be a problem at many locations. Figure 45 illustrates in principle the operation of a float-actuated water level gauge (float gauge). The illustration does not pretend to represent any particular make or model of instrument, and the drives, linkages, etc. encountered on actual equipment may differ considerably from the simple ones shown.

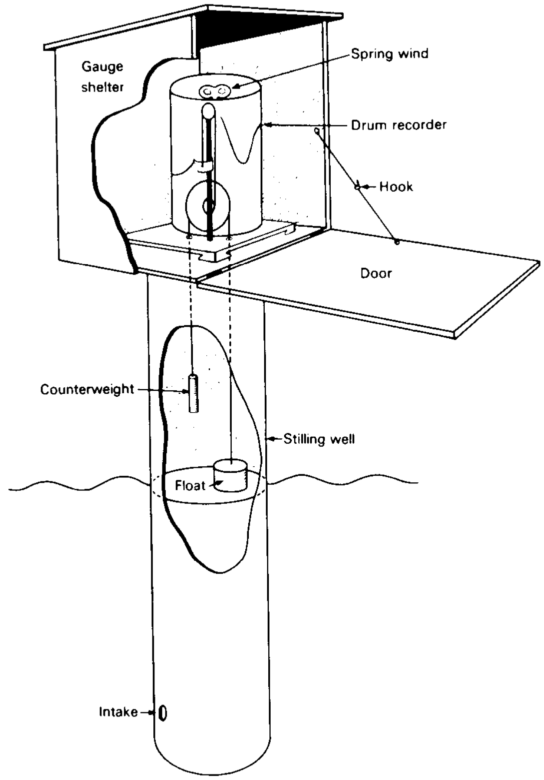


FIG. 45. Float gauge, stilling well, gauge shelter and drum recorder.

The water level information is transmitted from the float to the recorder by the thin cable which is attached at one end to the float, passes over the pulley on the recorder, and is attached at the other end to the counterweight. The float is usually cylindrical in the centre and spherical at the ends; it is hollow, and the level at which it floats may be adjusted by the addition or removal of lead shot. When deployed, the water line on the float should come about half way up the cylindrical section, to assure a linear change in buoyancy with change in depth. Increasing the cross-sectional area of the float increases the sensitivity of the gauge to changes in water level, but a practical limit is set by the size of the well and the inspection hatch. The counterweight must be heavy enough to keep sufficient tension in the float cable to prevent if from slipping on the pulley, but not so heavy as to lift the float too high up in the water. It should be solid and made all or mostly of lead, to minimize its loss of weight to buoyancy if it is submerged over part of the range. Since the float will ride slightly lower in the water when the counterweight is submerged than when it is in air, a small but systematic error can thus be introduced into the readings near high water with respect to those near low water. At permanent gauge installations an effort is made to avoid this source of error, either by mounting the gauge high enough above the highest water level that the counterweight need never reach the water, or by providing a separate water-tight dry well to accommodate the counterweight. These refinements are not required for temporary installations, but if the range of water levels is small, it should be a simple matter to mount the gauge high enough and to cut the float cable to a length that would keep the counterweight out of the water. The float cable should be strong but light. It is particularly important that it be light when the range of the tide is large, because the effective weight of the counterweight is increased by the weight of cable on its side of the pulley, and decreased by the weight of cable on the float side. Further details concerning the installation of a particular model of float gauge should be obtained from the instrument manual accompanying it or from the agency issuing the equipment. A variety of types of recorder may be used in conjunction with a float gauge, and some of these are discussed below; in Fig. 45 a drum-type paper chart recorder is shown.

6.5 Pressure gauges—diaphragm type

The hydrostatic pressure at depth h in a column of water is pgh, where p is the mean density of the water in the column above the depth h, and g is the acceleration due to gravity. Gravity may be considered constant for the purpose of water level measurement, but differences in water density from place to place may be important. particularly if large differences in salinity occur. The difference in density of ocean water at 0°C and of fresh water at 25°C is about 3%, 2½ % being due to the salinity difference, and ½ % due to the temperature difference. Clearly, if water levels are to be interpreted from hydrostatic pressure measurements, different calibration scales would be required for fresh and salt water. Use of pressure sensors instead of float gauges to measure water levels at temporary locations for the control of hydrographic surveys has become almost standard practice. This is because installation of the pressure gauge is much simpler, especially if no wharf is available. A stilling well is not normally required with a pressure sensor, any damping that is required usually being supplied by the design of the sensor head itself and by the natural damping of the pressure signal of short waves with depth (see section 1.4). Figure 46 illustrates the damping of the pressure signal from a 6 second and a 12-second sinusoidal wave at depths of 10, 20, and 30 metres. The vertical scale in Fig. 46 is shown in metres, after conversion from pressure units. There is, of course, no damping of the pressure signal from long waves (tides, seiches, etc.).

Figure 47 shows schematically a diaphragm type pressure gauge assembly. The pressure sensor is a flexible rubber diaphragm that forms one face of a hollow air chamber; the outside of the diaphragm is exposed to the water pressure through holes in a protective housing. Adjusting the size of the holes controls the damping of the response much as in a stilling well. The air chamber behind the diaphragm has an air-tight connection through a small (1-2 mm inside diameter) capillary tube to the inside of a Bourdon tube, bellows or aneroid chamber at the recorder site on shore. These devices translate changes in differential pressure (inside vs. outside) into a motion which can be conveyed by various linkages to a recorder. A Bourdon tube uncoils slightly as its internal pressure increases, a bellows extends lengthwise, and an aneroid lid becomes more convex. The gauge depicted in Fig. 47 has a bellows linked to the pen arm of a drum type recorder, but other arrangements are possible. The diaphragm housing must be securely mounted face down on some supporting structure below the lowest water level, such that it may not move (especially not vertically) during the recording period. If a wharf is available, the attachment may be to one of its pilings, but since the sensor may be 200 m or more from the recorder, a small rock crib or other support can usually be constructed in a reasonably protected location on firm bottom off shore. The principle of operation of this type of pressure gauge is that the static air pressure is uniform within any closed system (except for the

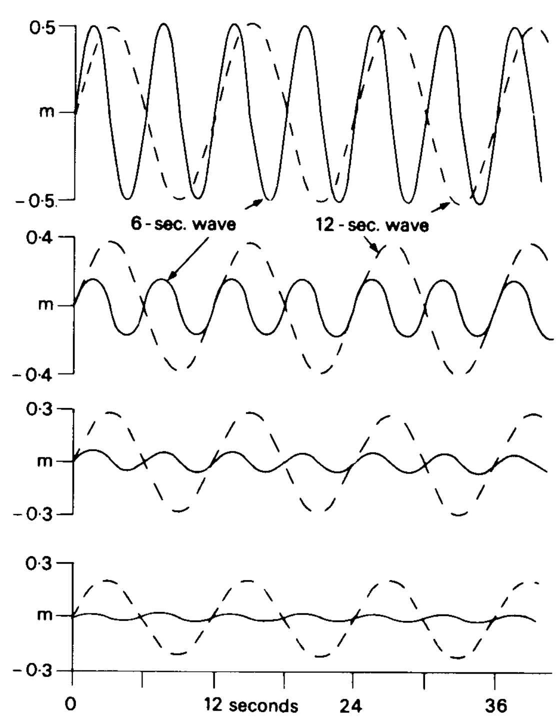


FIG. 46. Damping of pressure signal with depth, for 6-second and 12-second sinusoidal waves.

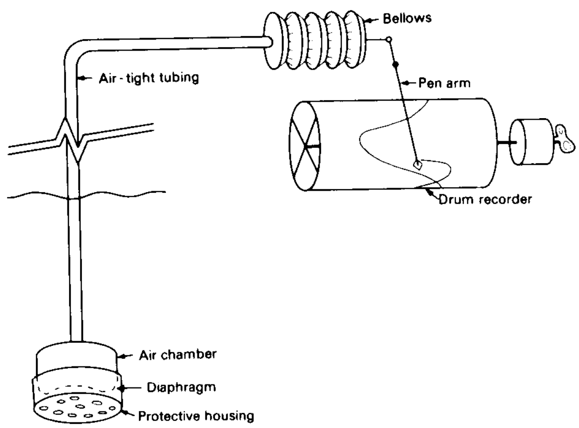


FIG. 47. Diaphragm-type pressure gauge, bellows pressure transducer and drum recorder.

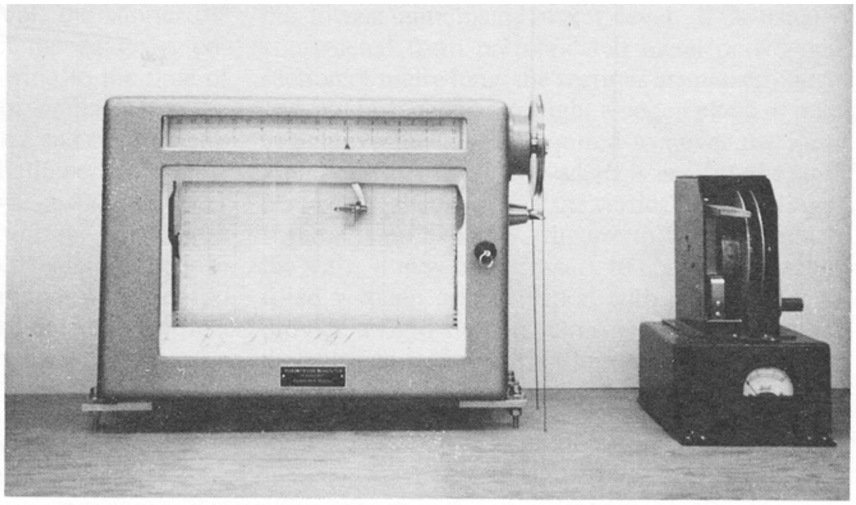
negligible weight of the vertical column of air). Thus, as long as all seals are tight, the pressure inside the bellows equals the pressure at the diaphragm, which consists of the hydrostatic pressure due to the column of water plus the atmospheric pressure at the water surface. Since the pressure outside the bellows is the atmospheric pressure, the pressure difference to which the bellows responds is the hydrostatic pressure due to the column of water above the diaphragm.

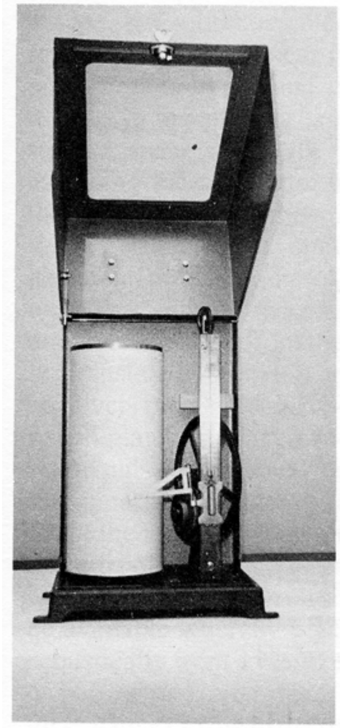
6.6 Pressure gauges - bubbler type

Bubbler gauges (also called gas purge gauges) are not as frequently used as diaphragm gauges at temporary gauge sites in Canada, but they have many of the same advantages, e.g. they do not require a stilling well and the pressure sensor can be installed a considerable horizontal distance from the recorder. Figure 48 shows schematically a bubbler-type pressure gauge assembly. The pressure sensor is simply the orifice at the underwater end of a long flexible air-tight tube. The tube may be larger than that of a diaphragm gauge because the volume of gas contained in the system is not a limiting factor; an inside diameter of 5 mm is recommended. At the recorder site on shore compressed air or nitrogen is continuously introduced into the system from a cylinder (A) through a reduction valve (B) that lowers the gas pressure to the working range of the recorder and other equipment. The pressure at the high and low side of the reducing valve is displayed on the needle gauges (C and D). The pressure at the low side (gauge D) must always exceed the greatest hydrostatic pressure that could be experienced at the orifice. The gas then passes through a flow control valve (E), which is a valve like that used in underwater breathing apparatus to maintain a steady flow of air regardless of changes in pressure at the downstream end. At this point the tube branches, one branch going to the recorder and the other continuing on to feed the orifice (F). At a convenient spot in the system below the flow control valve a bubble chamber (G) containing oil or water is inserted so the rate of gas purging can be monitored and controlled to about one bubble per second. As long as gas is issuing from the orifice at about this rate, the pressure throughout the system below the flow control valve will be sensibly uniform, and equal to the pressure in the water at the orifice. Again, various recorders could be used, but Fig. 48 depicts a Bourdon tube linked to the pen arm of a drum-type recorder. Since the outside pressure on the Bourdon tube is atmospheric, the pressure difference to which it responds is equal to the hydrostatic pressure due to the column of water above the orifice. If the flow of air is allowed to stop, water will flow through the orifice up the tube, giving a faulty reading; and if the flow of air is too rapid, there will be a slight pressure drop along the tube toward the orifice, giving slightly too high a reading. One cylinder of gas should operate the gauge for four or five weeks.

6.7 Pressure gauges - deep sea

These are gauges that are self-contained in their own protective case, which can be anchored on the sea bottom to record changes in pressure for periods up to a year and in depths up to several kilometres. The type most commonly used at present senses pressure by means of a quartz crystal which forms part of an electrical oscillator circuit. The resonant frequency of the crystal, and hence of the oscillator, depends upon the pressure applied to the crystal (and to a lesser extent upon the temperature of the crystal). By exposing the crystal to the external pressure through a pressure port in the case, the frequency of the oscillation is made to depend upon the pressure outside the





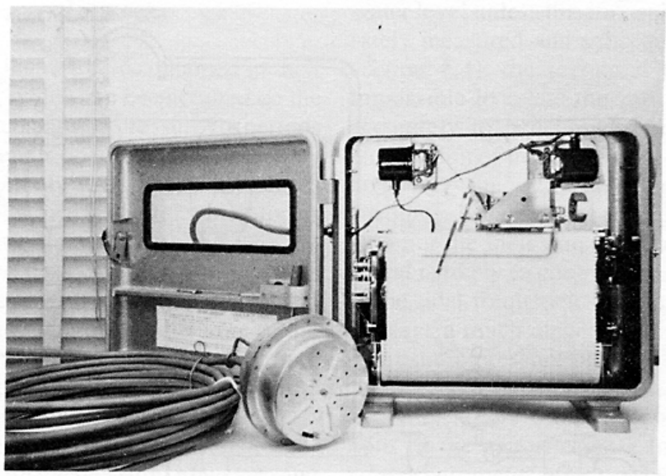


Plate 9. (Lower) Diaphragm-type pressure gauge, showing diaphragm housing, coiled-up capillary tubing, and strip-chart recorder. (Top right) Drum-type recorder for use with float gauge. (Top left) Strip-chart recorder for use with float gauge with sight (electrical tape) gauge. (Photos by Canadian Hydrographic Service.)

case. The frequency response is not linear with the pressure, however, and the instruments must be calibrated to relate frequency to pressure. The oscillator frequency is recorded regularly at a pre-selected sampling interval. Most present models record data on magnet- ic tape, but solid-state memory banks may largely replace tapes in the near future, thus reducing the power requirement and eliminating a source of trouble in the moving tape drive assembly. Data storage space is usually the limiting factor determining the maximum record length attainable, the shorter the sampling interval, the shorter the record. In deep sea operation the gauge is un- attended from the time of mooring to the time of recovery, and the record is not available for use until it is removed from the gauge and electronically translated. For use at shallower locations, however, most models have an acoustic transducer which may be engaged to transmit the readings as sound pulses that can be received by hydrophone in real time. Alternatively, if the mooring is not too deep, the readings can be transmitted by electrical cable from the gauge to an auxiliary recorder in a moored buoy, or even on shore. A system may soon be available whereby a gauge can be interrogated acoustically from a vessel and made to play back all or part of its stored data. Such features may give these gauges a role in the reduction of hydrographic soundings, but at present they are mostly used for the study of tides at offshore locations.

From the standpoint of water level measurement, there are two major difficulties with the self-contained gauges moored far from shore: the first is that they sense total pressure (hydrostatic plus atmospheric), and hence do not reflect the true water level unless the atmospheric pressure is separately measured and subsequently subtracted (see section 4.4); the second is that it is difficult or impossible to relate the zero setting of the gauge accurately to benchmarks on shore. For the de- termination of tidal constants at offshore

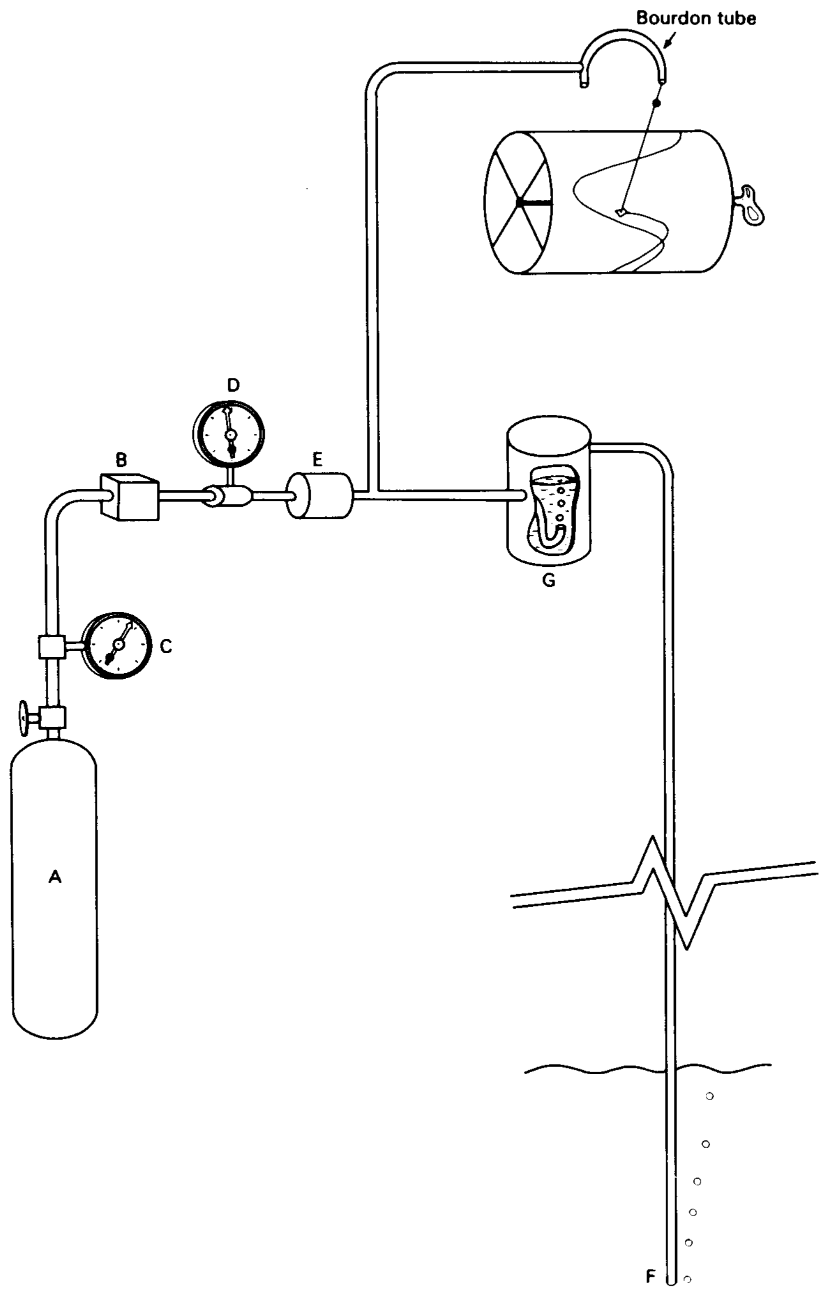


FIG. 48. Bubbler-type pressure gauge, Bourdon tube pressure transducer and drum recorder.

locations, however, the self-contained gauges are most adequate, because neither a slight drift in the gauge zero nor the inclusion of the atmospheric pressure signal is likely to contribute any significant energy at the tidal frequencies. In fact, the total pressure displays a much cleaner tidal signal than does the hydrostatic pressure alone. This is because in the ocean the fluctuations in local atmospheric pres- sure are largely offset by corresponding fluctuations of the opposite sense in the water level (in- verted barometer effect; section 4.4). In spite of their shortcomings in water level measurement, the deep-sea gauges may be useful in improving the accuracy of sounding reductions over shallow banks far from shore (e.g. the Grand Banks).

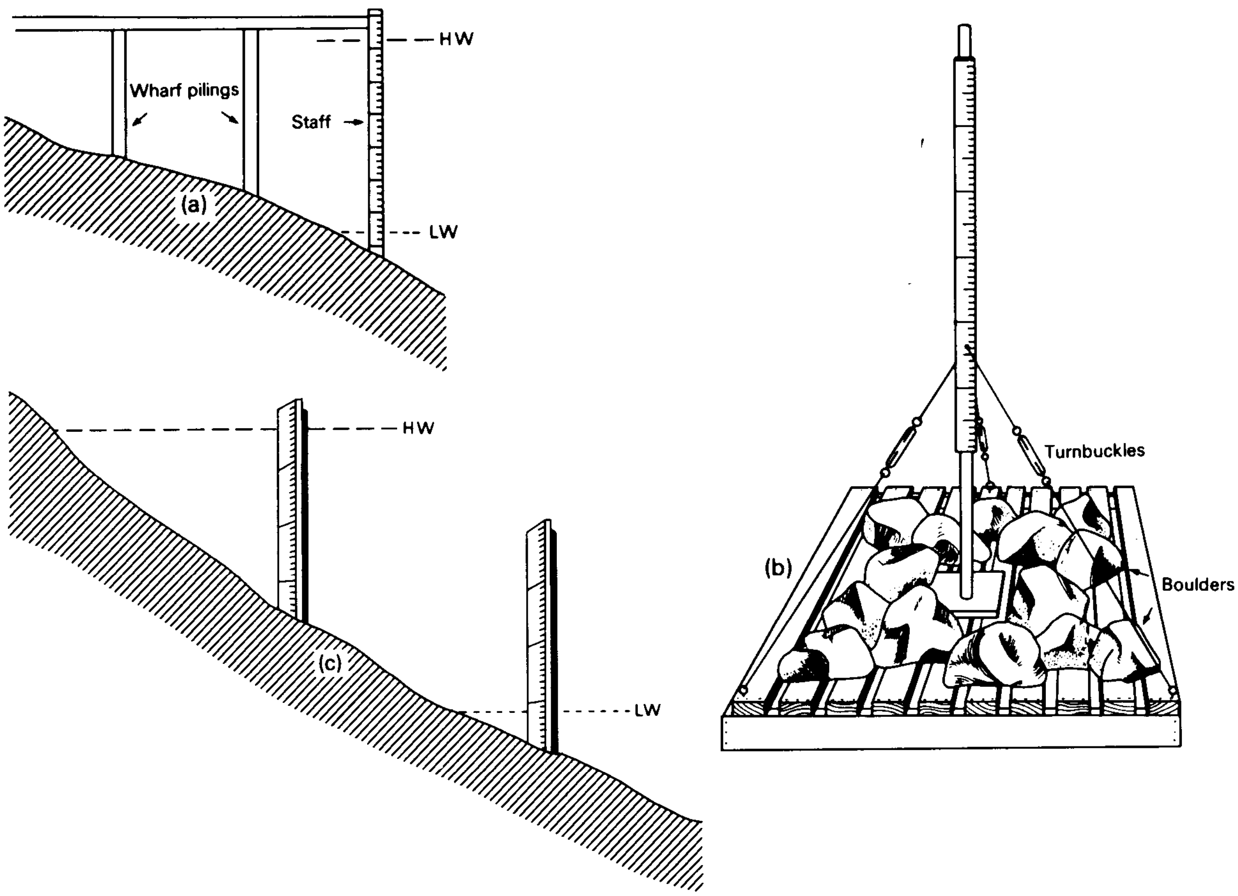


FIG. 49. Staff gauge installations (a) on wharf piling, (b) on submerged platform and (c) on long sloping beach.

6.8 Staff gauges

A staff gauge is simply a graduated staff (usually marked in metres, decimetres, and centimetres) mounted vertically in the water with its zero below' the lowest anticipated water level. It is usually constructed from prefabricated metal or wooden sections I metre long; the sections are placed end to end and fastened to a straight wooden plank or pole to achieve the length required to cover the local range of water level. If a wharf is nearby, the staff may be attached to one of the vertical wharf pilings. If there is no existing structure to which to attach the staff, it may be necessary to construct a stone-filled wooden crib or tripod with a wide base to provide a rigid support. On some types of bottom the pole supporting the staff may be driven into the bottom until it is firm, and secured vertically by at least three guy wires fastened to anchors. Sometimes, when the tidal range is large and the bottom slope is small, the intertidal zone is so broad that two staff gauges may be required, one near shore to be read during the upper part of the range, and one farther off shore to be read during the lower part of the range. It is even possible that more than two staff gauges could be required under rare circumstances (e.g. the tidal flats of the upper Bay of Fundy or Ungava Bay). When more than one staff gauge is used to cover the range of water levels, they should be related to each other so that there is a slight overlap in the part of the range that they cover, and so that they give the same reading in the region of overlap. Figure 49 sketches several possible staff gauge installations, but the hydrographer's ingenuity may produce others; all are satisfactory as long as the staff is rigid and steady and is convenient to read.

A staff gauge is required at every gauging site. It may serve as the only gauge for a brief local survey at a location where the tidal constants are already known, or on non-tidal waters, although

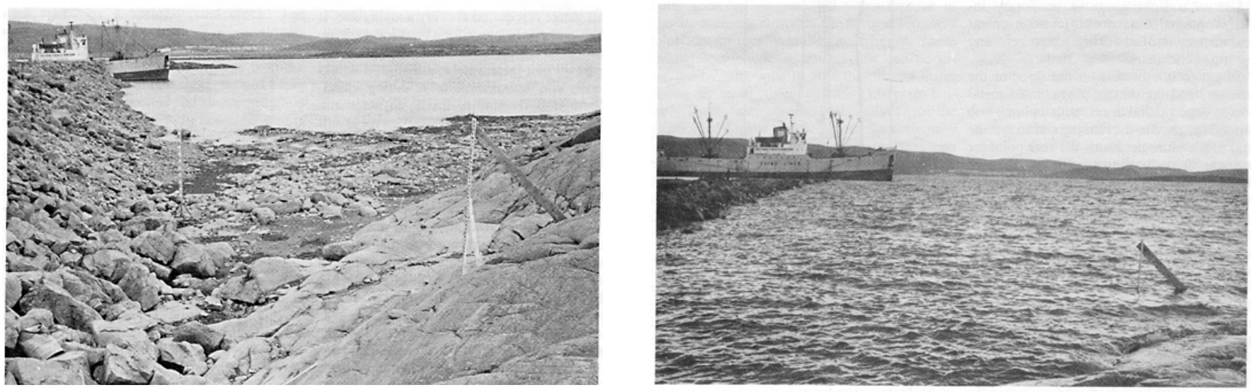


Plate 10. Use of multiple staff gauges at Frobisher Bay, Northwest Territories, to span the large range of tide between low and high waters. (Photos by Canadian Hydrographic Service.)

it would then require continuous monitoring during sounding. Where it is feasible to install an automatic gauge (float, diaphragm, etc.), a staff gauge is still required, against which to make checks of the accurate operation of the automatic gauge.

6.9 Sight gauges (electrical tape gauges)

A sight gauge is used to make spot readings of the water level inside a stilling well, usually as a check on the operation of the automatic gauge. It provides a more accurate check than can be obtained from a staff gauge, but may not be considered as a replacement for the staff gauge. This is because both the sight gauge and the automatic gauge read the level inside the well, and a comparison of their readings tells nothing about possible blockage of the well intake. At permanent gauge installations, comparisons are always made of the automatic gauge with both the staff and sight gauge. Use of a sight gauge at temporary installations is optional, but it does offer a convenient and accurate means of checking gauge zero and referring it to benchmarks.

A sight gauge is mounted on the floor of the gauge shelter, and consists of a graduated metal tape spooled onto a metal drum, with a plumb bob (or plummet) fastened to the running end so that the lower end of the plummet forms the zero point for the tape graduations. As shown in Fig. 50, the core of the metal drum is electrically connected in series with a low-voltage battery, a needle galvanometer, and with the water in the well (either through the wall of a metal well or through a wire on the inside of a non-metallic well). A flat-topped peg, called a "gnomon," is set alongside the

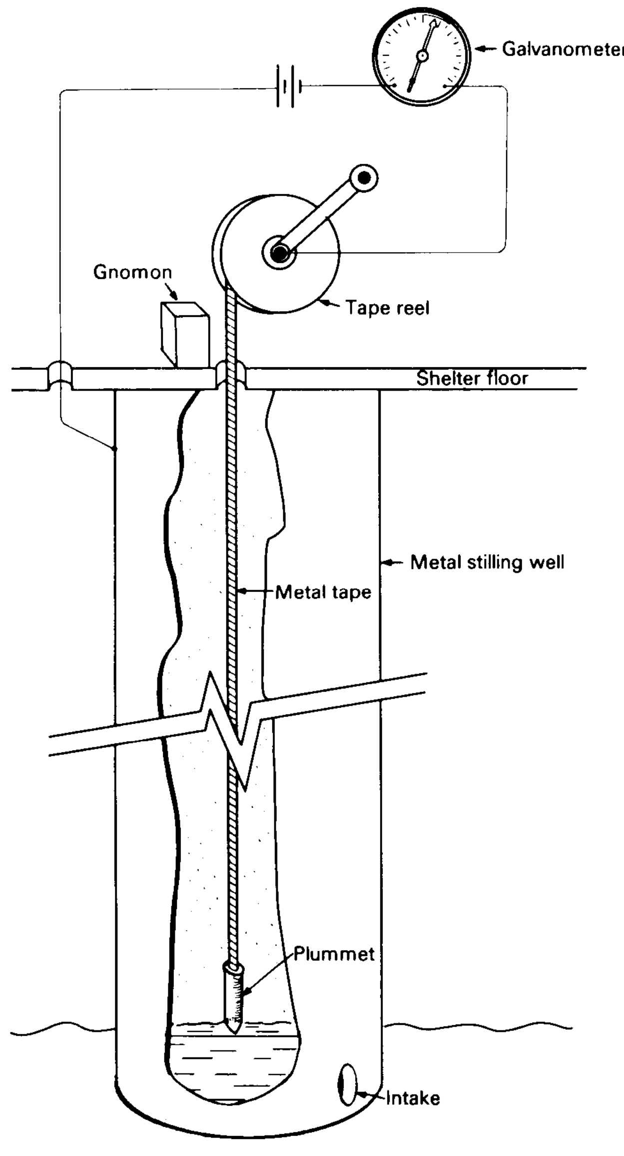


FIG. 50. Electric sight gauge, with metal stilling well.

tape on the shelter floor. To read the vertical distance of the water level below the gnomon, the tape is unrolled from the drum through a small hole in the shelter floor until the tip of the plummet touches the water surface, completing the electrical circuit through the water and causing the galvanometer needle to jump. At this instant the length of tape out at the level of the top of the gnomon is read. The tape is then slowly raised until the galvanometer needle drops back again, and another reading of the tape against the gnomon taken. The mean of the two readings should be taken as the distance of the water level in the well below the top of the gnomon at the central time of the operation. The gnomon must be mounted so as to be clearly in view through the open shelter door, to permit its elevation, and hence that of the gauge zero, to be referenced to benchmarks.

6.10 Data recorders

The type of recorder most commonly used at present with automatic water level gauges at temporary locations provides a continuous pen-on-paper trace of the water level on a chart driven at a constant speed by a spring-wound clock. The movement of the pen is mechanically controlled by the movement of the float pulley or the pressure element (bellows, etc.). The drum-type recorder (Fig. 45, 47,48) uses a single sheet of chart paper that fits exactly once around the drum, which is driven to rotate once per day. In non-tidal waters the chart on the drum recorder must be changed each day, to avoid confusing overlaps of the traces. In tidal waters, however, the chart may be used over several days because the daily advance of the tide (50 minutes per day) distinguishes one day's record from another. The strip-chart recorder uses a long strip of chart paper that feeds from a supply spool over a recording plate and onto a take-up spool. The chart is long enough to contain a full month of record, but segments may be documented for identification and cut off for use during the sounding survey, provided that the gauge is the responsibility of the sounding party. Segments of record may not be removed from gauges at permanent stations, whose records serve purposes other than sounding reduction.

At many permanent gauging stations data are digitally recorded on punched paper tape or in solid-state memory core that can be read remotely by telephone or locally in the gauge house. There are also tele-announcing gauges that can be interrogated by telephone to give the present water level and the trend (rising or falling) in plain language. There are obvious advantages to the application of similar technology to the temporary gauges; for example, to telemeter water level data by radio link to the survey vessel, either in real time or in blocks of specified length. Equipment of this type will probably soon replace the traditional equipment, and the hydrographer should keep abreast of such developments.

6.11 Selection of gauge site

The first consideration in choosing gauge sites for vertical control of hydrographic sounding surveys should be given to how well the water level fluctuations at the gauge sites reflect those in the survey area. This will depend not only upon the distance between the gauge and the survey area, but also upon the rate at which the tidal character may change in the region, the change in slope of a river along its length, the response of a lake to wind set-up and seiches, etc. If a survey is to cover a long stretch of coast-line, it may be desirable to have two gauges in simultaneous operation and to leap-frog them along the coast as the survey progresses. Two gauges may also be required in the survey of a long tidal inlet, since the tidal character can change significantly between the entrance and the head. More than one gauge is often required along a strait that joins two bodies of water of different tidal character because the tide must change character rapidly along the strait. The surface slope along a river may be different at different stages of flow, and so may not always be parallel to the slope of the low-water stage chosen as chart datum: for this reason, two gauges may again be necessary, one at the upstream end and one at the downstream end of the survey area, with no rapids, waterfalls, locks, or other datum discontinuities between them. The approximate location of temporary gauges should be planned before entering the field, and the Regional Tidal Officer should be consulted when advice or assistance is required.

The detailed local selection of a gauge site should be made with the following considerations in mind:

(1) Ease of installation: the existence of ready-made structures to which the automatic gauge and the staff gauge may be attached (wharf, fish stakes, bridge pilings, etc.); presence of firm bottom on which to construct support for

gauges if no ready-made structure is available; presence of sufficiently deep water near shore to assure that the gauge does not "dry out" or the surrounding water become impounded in a tidal pond at low water; availability of materials from which to construct support structures; accessibility of the site by water and/or land; and suitability of nearby ter- rain or structures for establishment of benchmarks.

(2) Ease of maintenance and operation: natural protection provided against full impact of waves and current; likelihood of silting around gauge intake or sensor; possibility of damage from or obstruction to marine traffic; and accessibility of the gauge and recorder both by launch and by foot.

6.12 Benchmarks - general

Benchmarks are the fixed elevation markers against which the zero setting of the gauge is checked during its operation, from which hydrographers may recover chart datum for future surveys, and through which surveyors and engineers may relate their surveys and structures to chart datum. The function and importance of benchmarks, and the reason that a minimum of three is required at each gauge site, has been discussed in section 5.9. The benchmarks (BMs) should be in place by the time the gauge is to be set in operation. To minimize the length of the levelling lines, an attempt should be made to keep all three BMs within a radius of half a kilometre of the gauge. The primary consideration, however, is that they be solidly set in stable structures, bedrock, or firm ground. No two BMs should be in the same structure or within 70 m of each other in horizontal distance, to minimize the likelihood of two of them experiencing the same instability.

Outcrops of bedrock provide the most stable setting for BMs, but structures with substantial foundations that extend below the frost line (public buildings, water towers, bridges, etc.) are usually also excellent. Permission should, of course, be obtained before placing a BM in a private structure. A BM should never be placed in a hollow or depression in which water might collect and freeze, and care should be given to the final appearance of the BM, since this reflects indirectly on the credibility of the other aspects of the survey. As an aid to future recovery, BMs set in bedrock should, whenever possible, be set close to a distinctive feature in the rock or to some easily describable landmark. Existing BMs that may have been established by other agencies in the vicinity of the gauge may serve as reference BMs for the gauge if they meet the stability and accessibility standards. Indeed. the use of such BMs is encouraged, since it determines the local relation between chart datum and the other survey datum. Enquiries about existing BMs should be made to pertinent survey agencies before entering the field.

6.13 Benchmarks - standard type

The standard Canadian Hydrographic Service BM tablet is illustrated in Fig. 51. It is made of bronze, and has a cap 5.6 cm in diameter with a shank 6.4 cm long and 1.5 cm in diameter. In the lower end of the shank there is a slot about 1.5 cm deep to receive a bronze wedge, which spreads the prongs of the shank against the sides of the hole when the tablet is set in rock or concrete. On the face of the cap there is a groove about 3.0 cm in length. When the tablet is set into a vertical face (i.e. with the shank horizontal), the groove marks the BM elevation, and the tablet must be set so that the groove is horizontal. During levelling, a benchmark chisel inserted into the groove provides the horizontal surface on which to rest the levelling rod. When the tablet is set in a horizontal face (i.e. with the shank vertical), the BM elevation is the highest point on the slightly convex surface of the cap, and the flat bottom of the levelling rod may be rested directly on the face of the tablet.

The standard method for setting the tablet into a horizontal or vertical face of rock or concrete is to drill a hole with a rock chisel, slightly deeper than the length of the shank of the tablet and slightly wider than its diameter. If the hole has the proper dimensions, the underside of the cap will come flush with the surface of the rock or concrete and the wedge will spread the prongs on the shank to grip the sides of the hole when the tablet is driven in. If the hole has been made slightly too deep, a small pebble may be placed in it

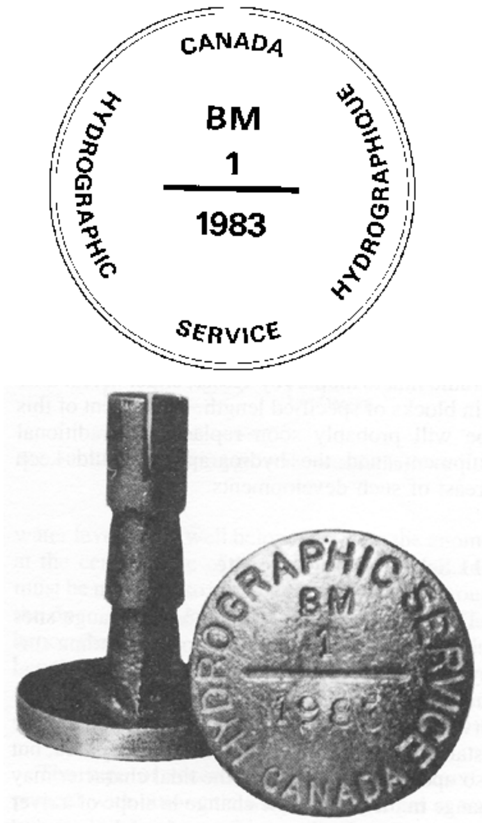


Fig. 51 Standard Canadian Hydrographic Service benchmark tablet.

to bear on the wedge. Under no circumstances may the shank be shortened to fit into a hole that is too shallow. Before finally setting the tablet, the hole should be cleaned out and filled with sufficient cement mix to squeeze into all the spaces around the shank and under the cap where it does not seat perfectly against the rock or concrete when it is wedged into the hole. Excess cement should be cleaned off, leaving the tablet neatly sealed against the penetration of water and frost. It is stressed that the purpose of the cement is to seal the small hollows left when the underside of the cap of the tablet is as nearly as possible flush with the original surface: cement is not to be used to fill in a space left because the hole was not drilled deep enough.

6.14 Benchmarks - special types

Benchmarks may be placed in suitable soil by fastening them to an iron pipe (called a "soil post"), when no rock or concrete structure is available. Figure 52 illustrates the setting of a soil post BM. A standard bronze tablet is welded to the top of a smooth iron pipe that is long enough to reach below frost level (2 or 3 metres). Holes are drilled through both sides of the pipe at its base, through which are fitted steel rods about 20 cm long to help anchor the pipe. The pipe must have an inside diameter large enough to accommodate the shank of the tablet, and an outside diameter no greater than that of the cap of the tablet. A hole of about 20 cm in diameter must be dug deep enough that only about 15 cm of the pipe protrudes. A post hole auger should be of assistance in this. Sufficient cement is placed in the bottom of the hole to encase the anchoring rods and seal the bottom of the pipe against the intrusion of ground water. Soil is then tamped in firmly around the pipe to fill the hole, with the surface soil mounded

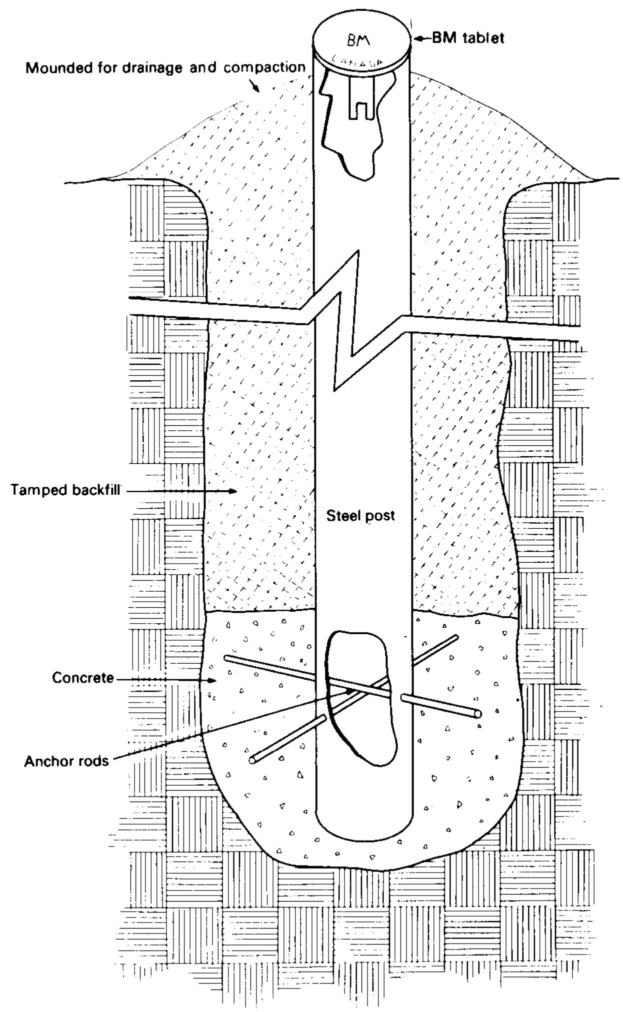


FIG. 5′. Soil post benchmark installation.

up around the pipe to allow for consolidation and shrinkage of the back-filled soil. The outside surface of the pipe should be smooth, so that frost heaving of the surface soil is less likely to affect the pipe. Soil posts should be placed on high ground with good drainage. Sandy soil is usually excellent for the installation of soil posts, but clay soil should be avoided, being too subject to frost heaving. If it is anticipated that soil post BMs may be required, a supply of pipes should be prepared before leaving for the field, having the tablets welded to the tops and the anchor-pin holes drilled near the bottoms. Rather than preparing a variety of lengths of pipe, one could prefabricate the pipes in short sections of three types - top sections, threaded on the inside at one end, and with the tablet welded to the other end; bottom sections, threaded inside at one end, and with anchor holes drilled at the other end; and insert sections, threaded inside at both ends. In the field, the sections could be fastened together with threaded plugs to form the desired length. The sections should be joined on the inside by threaded plugs, rather than on the outside by threaded collars, to preserve a smooth exterior.

Special techniques for setting BMs in areas of perma-frost, muskeg, and spongy soil do exist, but they are often laborious, and sometimes require special motori7.ed equipment. If at all possible, gauge sites should be chosen to avoid the need for such difficult BM installations. The Geodetic Survey of Canada has a great deal of equipment and experience related to the installation of special types of BM. Sometimes, if a request is made before the final planning of their field season, a Geodetic benchmark party may be able to visit problem gauge sites to install special BMs. The Regional Tidal Officer should be consulted if BM installation problems are anticipated, and requests for assistance from the Geodetic Survey or other agencies should be made through him.

6.15 Benchmarks - descriptions

The face of each BM tablet must be stamped with the BM number and the year of its establishment (see Fig. 51). The stamping is done with metal dies before the tablet is set. Because Hydrographic BMs do not form a continuous network, but exist in discrete clusters around gauge sites, it is acceptable to repeat the same set of consecutive numbers (I, 2, 3, etc.) for the BMs at separate sites. If a BM has been lost or destroyed, its number must be retired, and the BM that is installed to replace it should receive the next number that has not been previously used from the numbering sequence at that gauge site. To enable a BM to be recovered and used at a future date, a description of its appearance and location must be recorded on the Temporary Gauge Data form (Appendix B), copies of which will be retained and updated by the Regional Tidal Officer. In all records, the BM name should appear exactly as stamped on the tablet, e.g. BM 1, 1983. The BM description consists of three elements:

(a) a verbal description,
(b) a sketch of the immediate vicinity, and
(c) photographs of the BM and surrounding area

The verbal description should tell the type of BM, its number, how and in what it is set, its distance and direction from any easily identifiable marks (e.g. the corner of a building), and any other information that might assist in its recovery. The sketch should be kept as simple as possible, but must include at least the following basic information: true north direction, distance scale, high water line, drying areas and their type (e.g. rock, sand, shingle), prominent structures or features and their names, and distances between BMs and from BMs to structures or features. Two photographs should be taken, one a close-up to show the tablet and the surface or structure in which it is set, and the other a more distant view of the BM in relation to its surroundings, particularly identifiable features such as buildings, boulders, the shoreline, or even trees. In the second photograph, someone should point to or hold a levelling rod on the BM so there is no doubt as to its location.

If a previously occupied gauge site is revisited and the original BMs are located, their catalogued descriptions should be checked for accuracy and for possible changes that may have occurred at the site. Necessary revisions to the descriptions are to be noted on the Temporary Gauge Data form. If an old BM is found to have been destroyed, it is to be replaced with a new one, bearing a different number from the one that was destroyed. If an old BM is determined to be unstable, as demonstrated by a shift in its elevation with respect to the surrounding BMs. it is to be destroyed and replaced by a new BM with a new number. Notice of the destruction of BMs and the descriptions of BMs planted to replace them must be reported on the Temporary Gauge Data form, so that this information can be kept up to date in the benchmark catalogues.

6.16 Levelling - General

Levelling is an essential part of the gauge installation procedure, whose function is to establish the elevations of the BMs with respect to each other and with respect to the staff gauge and the automatic gauge zeros. When sounding datum is decided upon relative to the gauge zero, its elevation can thus be referenced to the BMs. When chart datum is ultimately confirmed, the catalogued elevations of the BMs above chart datum will depend for their accuracy upon the levelling done during the gauge installation, and from time to time during its operation. Because of the importance of levelling, the basic principles and procedures are discussed here. but a novice may wish to refer to a manual of surveying or civil engineering for a more detailed treatment. The three systems for calculating elevations above sea level that were discussed in section 5.2 (geopotential, dynamic, and orthometric) should be understood by the hydrographer, but need not concern him in connection with the local gauge site levelling. This is because there would be no significant difference in the three systems over the short distances and the small elevation differences

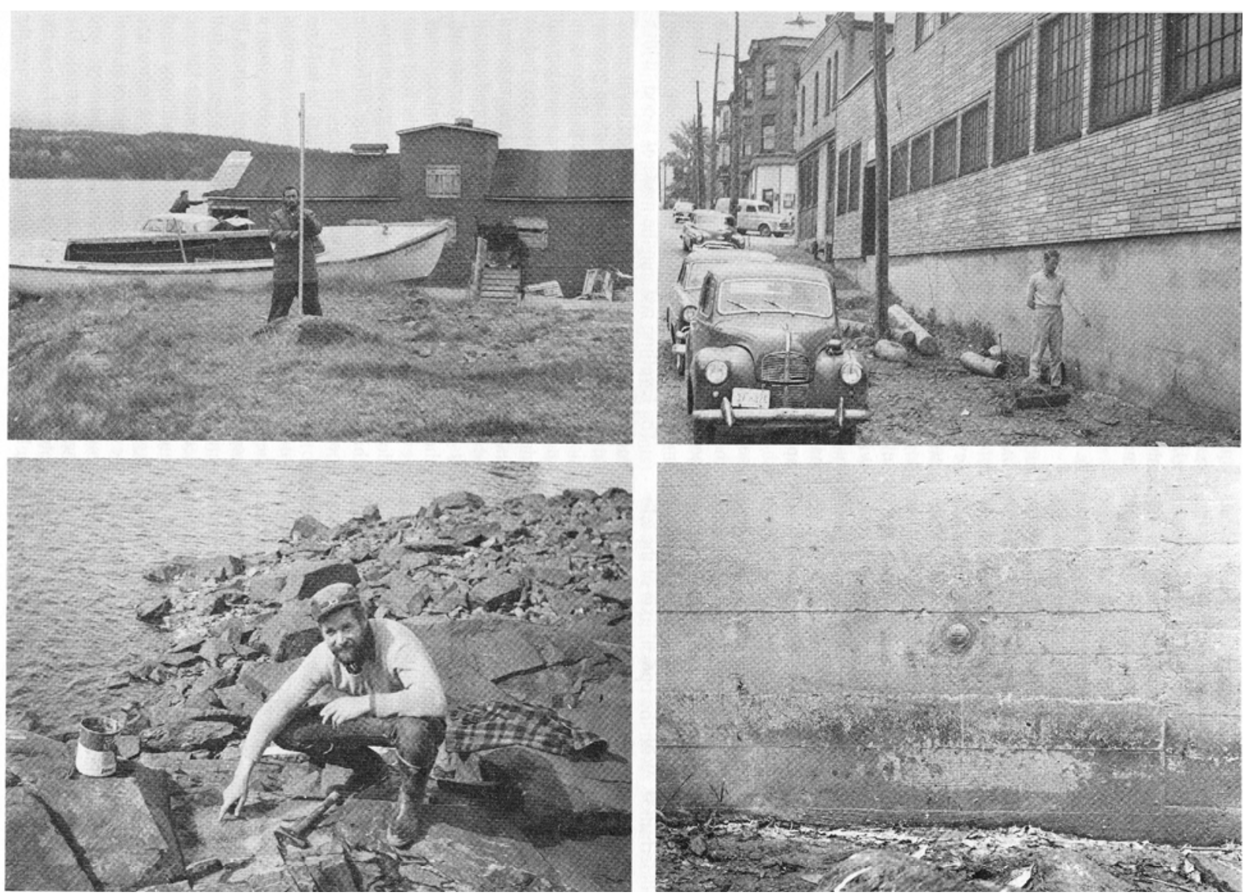


Plate 11. Benchmark photographs, used to aid in recovery of the benchmarks on future surveys. Location of the benchmark may be indicated by someone pointing to it or holding a levelling rod on it when photographed. Photo in lower right is a close-up of the benchmark whose location relative to its surroundings is shown in the upper right photo. (Photos by Canadian Hydrographic Service.)

involved, and it is acceptable to use the observed values directly. If the Hydrographic BMs are later tied by another agency into a more extensive levelling network, their elevations above that agency's datum will be measured in one of the three systems of section 5.2, but this will not change their quoted elevations above chart datum, for which the Canadian Hydrographic Service alone is responsible .

6.17 Levelling - method and terminology

The traditional method of levelling employs a levelling instrument (or simply a "level") with a viewing telescope, which is set up on a tripod between two points whose difference in elevation is to be determined. With the optical axis of the telescope horizontal, a reading is made on a graduated rod resting vertically on one of the points; the rod is then placed vertically on the second point, the telescope is swung around in the same horizontal plane, and another reading is taken on the rod. The difference in rod readings gives the difference in elevation between the points, the larger rod reading corresponding to the lower point. If the distance between the two points is large, if the difference in their elevations is greater than the length of the rod, or if they are not both visible from a single instrument set-up, the process must be repeated in steps, using intermediate points called turning points between the ends of the line. This is illustrated in Fig. 56. If the levelling is proceeding from BM1 to BM2, the sightings on the rod when it is closer to 1 are called backlights, and those when it is closer to 2 are called foresights. The difference in elevation from 1 to 2 should always be measured twice, once by running the line from 1 to 2, and again by running it from 2 to 1 . The discrepancy between the values obtained is called the closing error, or the closure. This method of levelling has long been referred to as "spirit levelling" because the horizontality of the telescope was set with reference to the bubble in a vial containing alcohol, called a spirit level. Many of today's levelling instruments have a system of optics suspended on fibres to eliminate the need for the literal spirit level, but the name persists. A more apt descriptive name for the method is *differential levelling.*

6.18 Levelling - equipment

In addition to the levelling instrument itself, the following items are required for the performance of proper differential levelling: instrument tripod, levelling rod, rod level, benchmark chisel, and portable turning point. The tripod usually is mated to the particular levelling instrument. Most have telescoping legs, and care is required to see that all wing nuts are tight after the tripod is set up. The base plate on which the instrument sits sometimes has a small spherical level, whose bubble should be approximately centred when the tripod is set up. If there is no spherical level, the rough levelling can be accomplished by sighting along the surface of the base plate at the horizon. On uneven terrain it is permissible to clamp the legs of the tripod at different lengths to obtain a more stable set-up.

The levelling rod may be made of wood, with a metal foot plate and with a graduated invar metal scale mounted on its face, the zero of the graduations being at the base of the foot plate. There is on most rods a sliding section that can be extended and clamped, to double the usable length of the rod; in using this, one should be certain that the extended section is fully seated and securely clamped against its stops. While it may be wise to carry a spare rod, the same rod should be used over a complete line of levelling. This is so that if the base of the foot plate does not coincide with the zero of the graduations, this zero error will cancel out in the difference between foresights and backsights.

It is important that the levelling rod be held no more than a few degrees off the vertical when it is being read. To aid the rodman in this endeavour, a rod level is supplied. It is a piece of metal about 10 cm long, with a right-angle groove along its full length, and a small spherical level mounted at one end. The rodman holds the rod level with one hand against the corner edge of the rod, just below eye level, and swings the rod until the spherical level bubble is centred; the groove in the rod level, and hence the rod itself, should then be vertical. The observer can tell from the vertical cross-hair in the telescope if the rod is off vertical to one side or the other, but he can not tell if it is off vertical fore-andaft: this is why rodmen are sometimes asked to sway the rod slightly back and forth so the observer can take the lowest reading, which should be the reading obtained when the rod is vertical. Use of the rod level provides a much easier solution, especially when three-wire levelling is called for, as described below

A benchmark chisel is a flat piece of metal about 3 cm wide and 30 cm long, with a knife-edge at one end to fit into the horizontal groove of a BM tablet that has been set in a vertical face. The other end of the chisel is bent in a right-angle to form a handle, and a small level vial is mounted on the face of the chisel to indicate when it is horizontal. When the chisel is set into the groove of the tablet and is horizontal, it forms the base on which the levelling rod is rested while being read. If the rod is not extended and if it is not windy, the rodman alone can usually perform this operation; otherwise, two men may be required, one to hold the rod and one to hold the chisel.

A portable turning point is a metal plate that is placed on the ground to provide a rest for the levelling rod at intermediate points (turning points) in the levelling between terminal points. It is usually an aluminum plate about 20 cm in diameter, with three pointed feet on the bottom and a small raised pedestal at the centre on the top, on which to rest the rod. The purpose of the feet is not to raise the plate off the ground, but to prevent if from slipping sideways, and on soft ground the feet should be pressed down so that the underside of the plate is bearing on the ground over most of its area.

6.19 Levelling - instruments

Two types of levelling instrument are in general use today, the "spirit level" and the "automatic (or self-levelling) level." We will examine first the characteristics they have in common. Both have three toot-screws and a small spherical level on the base of the instrument, by which to level it on the tripod. Both have a reticle with one vertical crosshair and three equally-spaced horizontal crosshairs (or fine lines etched on glass). Their telescopes have an objective lens to focus the image of the rod onto the plane of the reticle, and an eyepiece through which to view the reticle and rod image. Some instruments have azimuth locking screws, which must be loosened when the instrument is being swung through a large horizontal angle, and tightened when the azimuth tangent screw is to be used for precise aiming or when a reading is being taken. Other instruments may have only the azimuth tangent screw, the horizontal rotation being tight enough not to require a locking screw.

The spirit level has a long sensitive level vial mounted to the telescope tube, with adjusting screws by which the horizontal axis of the vial may be set parallel to the optical axis of the telescope. The departure of the two axes from parallel is called the collimation error. On some spirit levels the telescope and level assembly can be flipped through 180° about its longitudinal axis; this feature is used only in the procedure for removing the collimation error, as described below. On older instruments the bubble was viewed in a tilting mirror, and was centred between graduations etched on the surface of the level vial. On newer instruments the bubble is seen through a viewer with a split optical path, so that half of one end of the bubble is juxtaposed alongside half of the other end of the bubble in the field of view. The telescope is leveled before each reading by turning the telescope tilting screw until the two halves of the bubble ends match up alongside each other in the field of view, as shown in Fig. 53. Illumination of the level vial may be by a built-in battery light, or by natural light directed onto the vial by a movable mirror.

The automatic level has no precise level vial. Instead, it has as part of its optical train an ingenious combination of prisms and/or mirrors, by which every ray of light that enters the system horizontally is deflected until it is parallel to the axis of the telescope. It might be said that with the spirit level the telescope axis is set parallel to the horizontal light rays, whereas with the automatic level the horizontal light rays are set parallel to the telescope axis. In both systems the horizontal rays are focused at the centre of the reticule. The principle of the optical compensator is illustrated in Fig. 54. All components are shown here as mirrors, but in practice, prisms with one face silvered may be used. The two upper mirrors are fixed to the telescope tube, while the lower mirror

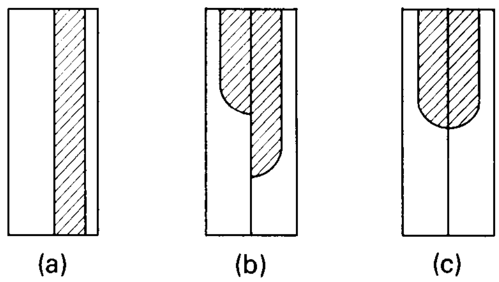


Fig. 53. Level bubble as seen through split optical viewer when instrument is (a) far off level, (b) nearly level and (c) level.

is suspended by tour filaments from the top of the tube. The geometry of the suspension is such that when the telescope axis is tilted up or down by an angle θ from the horizontal, the suspended mirror will be tilted up or down by exactly the angle 3θ/2. In Fig. 54 the path of a horizontal light ray through the system is shown (a) with the telescope axis horizontal, (b) with the telescope axis tilted up, and (c) with the telescope axis tilted down. The angles of tilt in Fig. 54 b and c are greatly exaggerated for purposes of demonstration, the actual operative range of the compensator being only about plus or minus 10 minutes of arc. This operative range is sufficient as long as the spherical level on the instrument base has been centred by use of the foot-screws. Note, however, that levelling with the foot-screws may not be done between the reading of a backsight and a foresight, because it can change the height of the instrument.

6.20 Levelling - instrument adjustments

The adjustment of the spherical level on the instrument base should be checked first. To accomplish this, the telescope is positioned over one of the foot-screws. and the bubble is centred in the spherical level by means of the foot-screws. The telescope is then rotated 180° in azimuth, and. if the bubble has not remained in the centre of the scribed circle, it is brought half way back to the centre by means of the foot-screws. The instrument should now be level, and the spherical level can be adjusted by centering the bubble the rest of the way by means of the spherical level adjusting screws. The adjustment may be checked by returning the telescope to its original azimuth; the bubble should remain centred. The reasons for positioning the telescope over a foot-screw at the start are to provide a reference for its reversal in azimuth, and to facilitate the centering of the bubble with the footscrews.

The collimation of the instrument should next be tested, and adjusted if necessary. For a spirit level this involves setting the axis of the main level vial parallel to the optical axis of the telescope; for an automatic level it involves matching the cross-hairs to the axis of the optical compensator. The first step is to establish a truly horizontal line of sight. Some spirit levels are designed so the

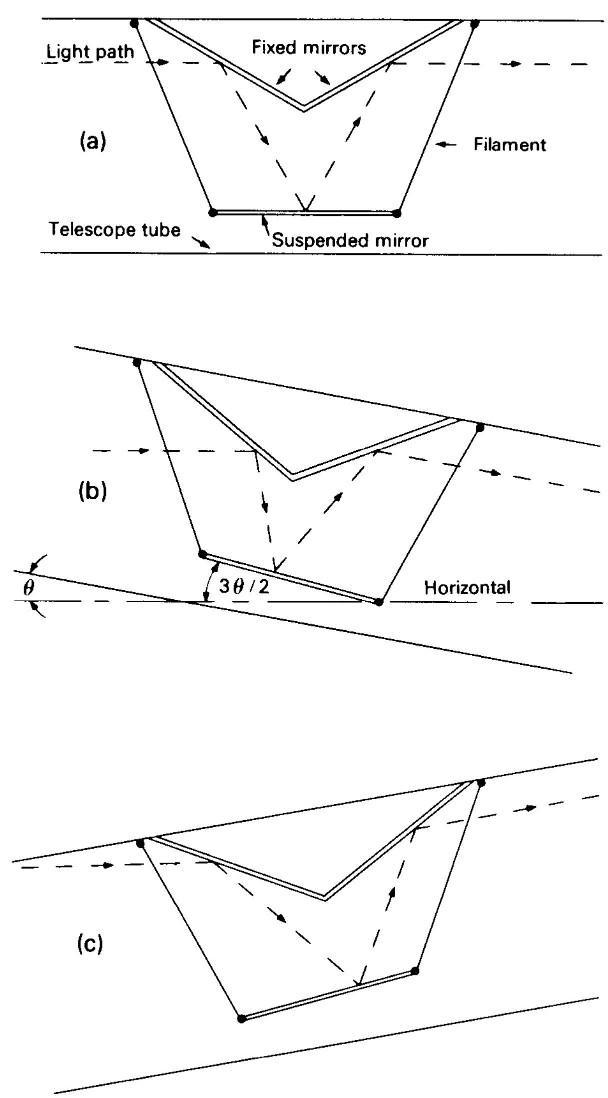


Fig. 54. Illustration of optical compensation in self-levelling instruments for (a) telescope level, (b) telescope tilted up by an angle θ and (c) telescope tilted down by an angle θ.

telescope can be rotated 180° about its longitudinal axis, permitting rod readings to be taken in both the upright and the inverted position. Since the collimation error appears with the opposite sign in the two readings, their average is the true instrument height above the reference point on which the rod is resting. With the telescope back in the upright position. it is moved with the tilting screw to read the average of the two previous readings on the rod. The telescope should now have its optical axis horizontal, and it remains only to adjust the axis of the precise level vial by moving its adjusting screw until the bubble is centred. The distance from the instrument to the reference point should be about 40 m. Since only one point is required, and since a peg driven into the ground is frequently used as the reference point, this procedure is often called the "one-peg test."

Instruments that do not have the reversing feature for the telescope require what is called a "two peg test" for adjustment of the collimation; some spirit levels and all automatic levels fall into this class. The true difference in elevation between two reference points (pegs) is first established by setting the instrument up exactly equidistant from the two points and reading the rod on each of them. Since the collimation error will appear in both readings with the same sign and magnitude, the difference between the readings should give the true difference in elevation of the points. The instrument is then set up as close to one point as is possible while still retaining the ability to focus on the rod, and a reading is taken on the rod at that point. Applying the known difference in elevation of the two points to this reading gives the reading that should be obtained on the rod at the distant point if there is no error. The instrument is pointed at the distant rod and, if it is an automatic level, the reticule is adjusted in position until the centre horizontal cross-hair is at the required rod reading. If the instrument is a spirit level, the telescope is moved with the tilting screw to the required rod reading, and the precise level vial is adjusted until the bubble is centred. The distance between the two points in the test should be about 50 m. When taking the reading very close to the rod, only a few divisions on the rod may be visible, and it is wise to have the rodman verify the approximate reading by pointing to it with a pencil when the observer calls it out.

An instrument should receive a collimation test at the start of a season, after it has been transported, and whenever it is suspected that it may have received a jolt or been exposed to rough handling. The effect of any collimation error that may not have been detected can be greatly reduced by keeping the total distance of the foresights as close as possible to that of the backsights in each line of levelling. Detailed descriptions of the locations and methods of manipulation of the various parts of a particular instrument will be found in the instruction manual accompanying the instrument.

6.21 Levelling - observation and recording procedures

Suppose levelling is to be run from BM 1 to BM 2. The instrument is set up on its tripod at a point not more than about 30 m from BM 1 and in the general direction of BM 2. The eyepiece is focused to give a sharp image of the cross-hairs, if possible against a blank field of view such as the sky. The rod is then held to rest vertically on BM 1, and the telescope is directed at and focused on the rod by means of the objective lens focusing screw. The test for proper focus of the objective lens is that there should be no parallax between the cross-hairs and the image of the rod; i.e. when the observer moves the viewing position of his eye up and down slightly, there should be no relative motion of the image and the cross-hairs. When this is achieved, no further adjustment of the objective lens should be made during the reading, but the image and the cross-hairs may be sharpened up by further adjusting the eyepiece if necessary. Readings are made on the rod at the positions of the three horizontal cross-hairs (upper, middle, and lower) and are recorded as the A, B, and C entries in the 'Backsights" column of the standard levelling torm, a sample of which is shown in Fig. 55. If a spirit level is being used, the bubble must be centered at the time of each reading. using the telescope tilting screw to do so. If the image of the rod as viewed through the eyepiece is inverted. the "upper" crosshair is the

LOCATION ______________ **LEVELLING** YEAR ________

METRIC

LINE		DIFFERENCE OF ELEVATION		
FROM	TO	FORWARD	BACKWARD	MEAN

	FORWARD	BACKWARD
SUM OF BACKSIGHTS	+	+
SUM OF FORESIGHTS	−	−
DIFFERENCE OF ELEVATION	+	

REMARKS

Date   LEVELLING   AT   Page   of

Inst. No.   Observer

Weather   Recorder

Visibility   Rodman

Line   Forward/Backward

METRIC

Backsights			Station	Elevations	Set-up	Foresights		
A						A		
B	D	F	B.S. +			B	D	F
C	E	G	H.I.			C	E	G
H	L	J	F.S. −			H	L	J
K						K		
A						A		
B	D	F	B.S. +			B	D	F
C	E	G	H.I.			C	E	G
H	L	J	F.S. −			H	L	J
K						K		
A						A		
B	D	F	H.I. +			B	D	F
C	E	G	B.S.			C	E	G
H	L	J	F.S. −			H	L	J
K						K		
Sums						Sums		

A – Rod reading upper hair

B – Rod reading middle hair

C – Rod reading lower hair

Sums – Sums of L and Sums of J

$D = A - B$ $F = \frac{D-E}{3}$ $H = A+B+C$ $K = A - C$

$E = B - C$ $G = B + F$ $J = \frac{H}{3}$ $L = D + E$

Checks $K = L$ $G = J$ $D = E \pm \cdot 002$ m. (max.)

Form L1

FIG. 55. Standard Canadian Hydrographic Service levelling form.

one that gives the highest rod reading, even though it appears in the lower part of the field of view.

As soon as the recorder determines that the differences in the pairs of readings, A-B and B-C. agree to within 2 mm. the rodman is directed to a suitable turning point, TP 1, in the general direction of BM 2, and no more than about 30 m from the instrument. Readings are taken on the rod at TP 1, in the same manner as before, and recorded in the "Foresights" column of the levelling form, to complete the readings for set-up # 1 . It is essential that the height of the instrument not be changed between reading of backsight and toresight, which means that neither the tripod nor the foot-screws may be adjusted during this interval. After assuring thatA-B and B-C agree. the instrument is moved to a new position in the general direction of BM 2, up to 30 m from TP 1 . The process of leapfrogging the rod and instrument along is continued until the final foresight readings are taken on BM 2. The series of set-ups is illustrated in Fig. 56. Calculation of the remaining quantities (F to L) on the levelling form, and completion of the summary on the opposite side of the form finishes the forward running of the line. The line must, however, be run again in the opposite direction (i.e. from BM 2 to BM 1). If satisfactory agreement is obtained between the forward and backward running, the mean value is accepted as the difference in elevation between BM 1 and BM 2.

The levelling form shown in Fig. 55 may appear at first glance to be unnecessarily complicated, but the safeguards against error that are provided by three-wire levelling and the check calculations in the levelling torm more than pay for themselves by reducing the need tor re-levelling. Although the form is laid out so it can be completed mechanically step-by-step, a little time spent in understanding the logic of the steps can make the task more satisfying, and perhaps more efficient. The check on the mean of three readings (G = J) comes from the simple algebraic identity

$$(A + B + C)/3 = B + ((A-B) - (B-C))/3 .$$

The difference between the upper and lower reading (A-C) is the "stadia interval," and is proportional to the distance of the rod from the instrument. Multiplication of this interval by the "stadia factor" for the instrument would give the actual distance, but since our interest is simply to equalize the total distance of the foresights to that of the backsights, the stadia interval serves just as well. The length of the backsight and foresight at each set up should as nearly as practicable equal each other, but the recorder should keep an eye on the running sums of the stadia intervals and advise the observer, so that any imbalance can be corrected by the end of the line. The check on the stadia interval (K=L) is simply the identity

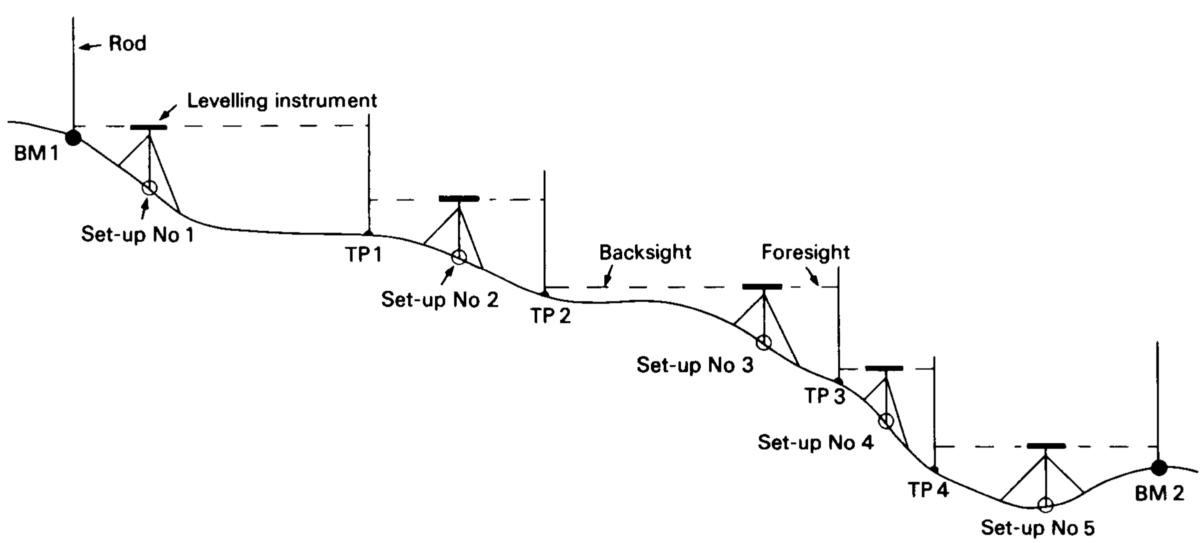


FIG. 56. Illustration of differential levelling procedure.

A-C = (A-B) + (B-C).

Use of a pocket-size electronic calculator can be of great assistance in completing the levelling torm. There are available programmable pocket calculators which, once the three readings (A, B, and C) are entered, will carry out the remaining calculations successively at the press of a button. All values must, of course, still be entered on the torm. Levelling form sheets without the summary section on the reverse are also provided, to supplement that shown in Fig. 55 when more than three instrument set-ups are required between primary points (BMs, gauge gnomon, staff gauge zero). One summary page must, however, be completed for each section of levelling between primary points. These summaries provide the input for the benchmark and datum information that must also be shown on the Temporary Gauge Data form.

6.22 Levelling—Accuracy

The accuracy demanded of hydrographic levelling exceeds that necessary strictly for sounding reduction because the information may be used as well for other purposes, because the increased accuracy may help to identify an unstable mark more quickly, and because the small extra investment in time and effort required does not add significantly to that expended at lower standards.

The collimation error may never be completely removed from an instrument. If the error is found to be no more than 20 seconds of arc, which corresponds to a reading error of 3 mm over a distance of 30 m, no further adjustment need be made to the collimation, provided that care is taken to balance the sums of foresight and backsight distances to within 10 m over each segment of line between primary points.

The precision of the rod readings is judged by the agreement between the two halves of the stadia interval, i.e. the upper stadia reading minus the middle wire reading. and the middle wire reading minus the lower stadia reading. If the difference is greater than 2 mm, the readings should be repeated.

The most revealing test of levelling precision is the closing error, the disagreement between the elevation differences determined on the forward and backward running of the line. When BMs are so close together that only one instrument set-up is required, the backward running of the segment may amount simply to moving the instrument to a different location and repeating the measurements. Some errors are constant or systematic, and may be made to cancel out of the calculations; examples are the zero error on a levelling rod, which cancels out in subtracting foresight from backsight, and a small collimation error, which can be made to cancel out by equalizing foresights and backsights. There are also systematic errors that may not cancel out, such as might be caused by a rod with an expanded or contracted scale. Fortunately, most systematic errors would not accumulate a large error in the short distances and small elevation differences usually involved in hydrographic levelling. The criterion on which the agreement between forward and backward runnings is judged assumes that the errors involved are random errors, so that their effect would be expected to accumulate as the square root of the distance covered in the levelling. It is, of course, reasonable to hope that if the random errors have been kept small, so also have the systematic ones. The criterion is that the difference between forward and backward values must not exceed the greater of

3 mm or $8(K)^{1/2}$ mm,

where K is the length of the levelling line (one way) in kilometres. Following are the values of $8(K)^{1/2}$ mm up to about I km:

190 m (or less)	3 mm
I 91 m to 316 m	4 mm
317 m to 472 m	5 mm
473 m to 660 m	6 mm
661 m to 878 m	7 mm
879 m to 1129 m	8 mm

6.23 Setting gauge zeros

There are usually two gauge zeros to set, that of the staff gauge and that of the automatic gauge. The zero of the staff gauge is normally set first, and the zero of the automatic gauge then set to agree with it. If, however, it is felt that the staff gauge zero has been set too high, the zero of the automatic gauge may be set lower to avoid negative readings; if this is done, the difference in zeros should be some simple amount such as 20 or 30 cm. The gauge zeros ought not to be changed once they have been referenced to the BMs and water level recording commenced.

If the gauge site has been previously occupied, it should be possible to recover the old BMs, whose elevations above chart datum are known. By standard levelling from one of the BMs, the height of the levelling instrument above datum is found for a set-up from which the staff gauge is visible. The staff gauge is then juggled and secured in position when the instrument reading on the staff equals the instrument height above datum. Since this operation cannot be performed perfectly, the actual zero of the staff must still be referenced to the BMs by repeat levelling, and the information recorded on the Temporary Gauge Data form as well as on the levelling sheets. If an electric sight gauge is installed in conjunction with an automatic gauge and stilling well, the elevation of the gnomon on the floor of the gauge shelter is found by standard levelling from the BMs. (Note: a short length of level rod or metre stick is required to fit in the door of the shelter for the reading on the gnomon, and the zero of its scale must be matched to that of the main level rod .) The elevation of the gnomon above datum should then be marked on a card and prominently displayed inside the gauge shelter. The length of tape required to reach from the top of the gnomon to the water surface in the well is subtracted from the gnomon elevation to give the height of the water above chart datum at that instant. The automatic gauge is then made to read that water level, thus setting its zero to chart datum. Setting the water level on a float gauge is done by holding the float wire and slipping the pulley past it until the desired reading is obtained. Pressure recorders usually have an adjusting screw with which to make the setting, but the appropriate instrument manual should be consulted for particulars. Use of a sight gauge wherever a stilling well is available is encouraged, but is not mandatory. If there is no sight gauge, the setting of the automatic gauge must be done against readings taken on the staff gauge. In either case, the setting should be done in two stages, a coarse setting against the first water level reading, followed by a fine setting against a second water level reading. It should then be checked against a third reading.

In some regions the relation between chart datum and Geodetic Datum or IGLD may be well enough established that BMs in one of those nets can be used to set gauge zeros, even when there has been no gauge at the site before.

When it is not possible to set the gauge zeros from known BM elevations, the zero of the staff gauge may be set rather arbitrarily, provided only that it be safely (about half a metre) below the lowest water level anticipated. A few observations on a temporary staff may be compared with observed or predicted water levels at nearby locations to help in making this judgement. The staff gauge zero must then be related to the BMs by standard levelling. If a sight gauge is being used, the elevation of its gnomon above the zero of the staff gauge is determined by levelling, and the value is displayed in the gauge shelter. The zero of the automatic gauge may be set to agree with that of the staff gauge, and may be checked from time to time, by using the sight gauge as described above. If there is no sight gauge, the setting of the automatic gauge must be done against readings taken on the staff gauge.

In all of the above cases, the elevations of the gauge zeros with respect to each other and with respect to the BMs must be entered onto the Temporary Gauge Data form in the appropriate slots.

CHAPTER 7

Gauge Operation and Sounding Reduction

7.1 Introduction

This chapter treats the general procedures that should be followed in operating a temporary water level gauge, and in applying the information from that gauge to the accurate reduction of soundings taken in the vicinity. Detailed operating instructions for a particular model of gauge (paper loading, pen filling, etc.) may be obtained from the instruction manual accompanying the instrument. Although several methods by which to determine a satisfactory sounding datum are described, situations will almost certainly arise for which no single method is exactly suitable. In such a situation, it is hoped that the principles explained here may be combined with a little common sense to suggest a solution. It is highly desirable that the sounding datum chosen be as close as possible to the final chart datum, but it is recognized that there will be occasions when sounding must be started before sufficient water level information is available to permit an elegant choice of sounding datum. The sounding datum should not be altered during the survey, since this would be more likely to give rise to error in the final reduction to chart datum than would consistent use of a poorly chosen sounding datum. Cotidal charts for the reduction of soundings in tidal waters not in the immediate vicinity of a gauge are treated here from the standpoint of the user in the field, not from that of the tidal officer who must prepare them. It is important, however, that the field hydrographer advise the tidal officer of his requirements for cotidal charts well in advance of the survey, and discuss them with him, to obtain the most suitable presentation of the cotidal information .

7.2 Sounding datum from existing BMs

Usually when an area is being re-surveyed, it is possible to recover BMs from the previous survey, whose elevations have been determined with respect to chart datum. When this is the case, the gauge zeros are set to chart datum as described in section 6.23, and sounding datum at the gauge site is gauge zero. Presumably sounding datum and chart datum would be identical in such a case, but until confirmed by the Regional Tidal Officer, sounding datum remains just that. If there are Geodetic or IGLD BMs in thc immediate vicinity of the gauge, and if the local relation between chart datum and these datums is known approximately, then the gauge zeros and the sounding datums may be set by reference to these BMs, as in section 6.23. Sounding datum and gauge zero should then be close, but not identical, to chart datum.

7.3 Sounding datum by water transfer—tidal waters

When sounding datum cannot be determined from existing BMs, it may be obtained by water transfer from a site nearby where a gauge has been or is in operation, and for which chart datum has been established. The method described here for datum transfer in tidal waters requires three or four days of high and low water heights measured with respect to gauge zero at the new gauge site, and a corresponding set of high and low water heights (observed or predicted) with respect to chart datum at the reference gauge site. The method is more accurate if the sets of heights can be obtained when the range of tide is large, usually around the spring tide. The assumptions are made that the mean water levels at the two sites over this short period are the same as they would be over the long-term average, and that the tide curves at the two places have the same shape, although they may differ in range and arrival time. Since the distance of chart datum below mean water level is determined by the range at large tide, it follows from the assumptions that the ratio of these distances at the two sites should be the same as the ratio of the ranges of the tide at the two places. If the tide at the two sites is semidiurnal or mixed-semidiurnal, all of the HWs and LWs are averaged to give a mean HW and a mean LW for each site. The mean water level (MWL) is taken to be the same as the mean tide level (MTL), i.e. half way between MHW and MLW. The average range over the short interval being treated is the mean HW minus the mean LW. The distance of sounding datum below the observed mean level at the new site is then taken as the distance of chart datum below the mean level at the reference site times the range ratio (new/reference). Subtracting the observed height of the mean level from this value gives the distance of the sounding datum below the gauge zero at the new site. These calculations and entries are all to be entered on the Temporary Gauge Data forlll. The gauge zero should not be changed to agree with sounding datum, since there is less chance of error if all water levels are recorded on the same gauge zero throughout the survey .

The method is less accurate, and the assumptions less easy to justify, when the character of the tide is mixed-diurnal or diurnal. The calculation is nevertheless carried out in much the same way as described above for the semidiurnal case, with the major exception that only the HHWs and LLWs in each set are included in the averages. It is important that there be a one-to-one correspondence between the HHWs at the new site and those at the reference site (and for the LLWs as well). Recognition of matching pairs at the two sites is not always easy, particularly if the reference site is far away. A little care in making the selection and carrying out the calculations should, however, produce a satisfactory sounding datum, although a further adjustment to chart datum will probably be required later when the complete water level record is available. When there is a choice of acceptable reference sites, it is best to choose the one with the greater tidal range.

Tables 5 and 6 show examples of the determination of sounding datum by water transfer in tidal regimes with small and large diurnal inequalities, respectively. R and r are the ranges at the reference and new sites, respectively; M is the MWL at the reference site, above its chart datum; m is the MWL at the new site, above its gauge zero; and m' = Mr/R is the calculated distance of sounding datum at the new site below its MWL. The height of the gauge zero above the sounding datum at the new site is thus d = m'—m = (Mr/R) - m. Provision is made on the Temporary Gauge Data form for the entries and calculations shown in the tables to be made directly on the form. There is also provision for the calculation of the high water datum to which elevations are referred. This datum at the reference site is HHWLT, and its height above chart datum, which we will call H, should be available either from the Tide Tables or from the Regional Tidal Officer. The height of the datum for elevations at the new site is calculated as h = Hr/R, above sounding datum, and therefore is h - d above the gauge zero. Just as sounding datum is provisional, pending final determination of chart datum, the datum for elevations is also provisional, pending a final determination when all data is available.

7.4 Sounding datum by water transfer—lakes

Chart datum on lakes is usually chosen as a level surface, whose elevation above one of the survey datums (e.g. Geodetic or IGLD) is known. Only when sounding datum cannot be found by levelling from one of these BMs or from a previously established Hydrographic BM would the water transfer method be used. To transfer datum by water transfer on a lake requires that there be a reference gauge in actual operation at a location on the lake where chart datum has been established: there is no equivalent to the predicted values that may be used in tidal waters. The data input required is a set of hourly (or more frequent) water levels above chart datum for a period of two or three days at the reference gauge, and a simultaneous set of water levels above gauge zero at the new gauge. The data should be gathered on days when the wind is light and the seiche action is small. The wind is usually the more important factor, since averaging over 2 or 3 days will filter out most seiche effect, whereas it will not remove the effect of a steady wind set-up. The assumption is made that the mean surface of the lake over the sampled period is a level surface, and so is the same distance above chart datum at all locations. If M is the mean water level at the reterence gauge above chart datum, and m is the mean water level at the new gauge above gauge zero, then the distance of sounding datum below the gauge zero at the new site is d = M - m. If a high water datum for elevations has been established at the

TABLE 5. Datum transfer in tidal waters with small diurnal inequality.

	New gauge site (metres, rel. to gauge zero)			Reference gauge site (metres, rel. to chart datum)	
	HW	LW		HW	LW
	1.40	0.46		2.19	0.70
	1.58	0.70		2.07	0.43
	1.40	0.46		2.23	0.70
	1.58	0.70		2.10	0.40
	1.40	0.43		2.23	0.70
	1.58	0.67		2.10	0.43
Sum	8.94	3.42	Sum	12.92	3.36
Mean	1.49	0.57	Mean	2.15	0.56

$m = (1.49 + 0.57)/2 = 1.03$ $\quad$ $M = (2.15 + 0.56)/2 = 1.36$

$r = (1.49 - 0.57) = 0.92$ $\quad$ $R = (2.15 - 0.56) = 1.59$

$m' = Mr/R = 0.79$; $d = (m' - m) = -0.24$

TABLE 6. Datum transfer in tidal waters with large diurnal inequality.

	New gauge site (metres, rel. to gauge zero)			Reference gauge site (metres, rel to chart datum)	
	HW	LW		HW	LW
	2.59*	0.30*		2.87*	0.43*
	2.44	2.23		2.35	2.13
	2.47*	0.49*		2.71*	0.58*
	2.26	2.10		2.44	2.04
	2.50*	0.73*		2.53*	0.82*
	2.26	1.89		2.53	1.92
Sum	7.56*	1.52*	Sum	8.11*	1.83*
Mean	2.52*	0.51*	Mean	2.70*	0.61*

$m = (2.52 + 0.51)/2 = 1.51$ $\quad$ $M = (2.70 + 0.61)/2 = 1.66$

$r = (2.52 - 0.51) = 2.01$ $\quad$ $R = (2.70 - 0.61) = 2.09$

$m' = Mr/R = 1.60$; $d = (m' - m) = +0.09$

NOTE: Only values marked with (*) used in calculation.

reference site, then the datum for elevations at the new site should be set the same distance above sounding datum as that at the reference site is above chart datum.

7.5 Sounding datum by water transfer - rivers

Once again, if chart datum is known precisely or even approximately with respect to local BMs, sounding datum should be established by levelling from a BM. If this cannot be done, datum may be established by water transfer from an operating reference gauge, much as described for lakes in section 7.4. The reference gauge must not only be on the same river, but must be on the same stretch of the river, with no locks, dams, or major changes in cross-section between it and the new gauge. This is because the assumption will be made that the water level is the same distance above chart datum at both gauges, for any given river discharge (i.e. that the “stage-discharge relation” of the river is the same at both locations). Because of this assumption, the mechanics of the determination of sounding datum on a river by transfer from a single reference gauge are the same as on a lake, and, in the terminology of section 7.4, d = M-m. Unfortunately, the surface slope of a river is unlikely to be the same at all high and low stages, and so cannot legitimately be expected to be exactly the same as the slope of chart datum, except when the river is at the low water stage chosen to define chart datum. For this reason it is desirable to have two reference gauges, one above and one below the new gauge, and to interpolate the datum transfer between them. Let L_u and L_d be the distances of the upstream and downstream reference sites from the new site, M_u and M_d be the mean water levels above their respective chart datums for the period of interest, and m be the mean water level above gauge zero for the same period at the new site. If m' is the distance of sounding datum below the mean water level at the new gauge site, we have

$$m' = (L_u M_d + L_d M_u) / (L_u + L_d)$$

This follows simply from weighting the contributions from the two reference gauges in inverse proportion to their distances from the new gauge. The distance of sounding datum below the gauge zero is therefore d = m'-m.

Figure 57 attempts to illustrate the principles and implied assumptions involved in the transfer of sounding datum on tidal waters, lakes and rivers. CD denotes chart datum at the reference gauge, and SD denotes sounding datum at the new gauge. Situations will undoubtedly arise where there are no existing BMs and also no suitable reference gauges. The hydrographer must then examine the limited water level data and whatever other information he has been able to gather before soundings are to be commenced, and choose what appears to be a reasonable sounding datum. Conscientious operation of the gauge for as long as possible before, during, and after the sounding survey will greatly assist in the eventual selection of chart datum.

7.6 Daily gauge inspection

Every water level gauge should be inspected at least once each day. In tidal waters an attempt should be made to inspect the gauge near high water and near low water on alternate days. The first aspect of the inspection consists of a superficial visual check to see if any of the installation has been disturbed; e.g. the staff gauge, stilling well, gauge shelter, or pressure sensor mounting shifted, weakened, or damaged in any way. If it appears that the staff gauge may have been shifted, its zero must be checked again by levelling to the BMs. If an electric sight gauge (tape gauge) is in use, the elevation of its gnomon must again be checked against the BMs if it appears the gauge shelter may have been shifted. When any alterations are required, two sets of comparison readings should be made and recorded as described in the tollowing paragraph, one set before and one set atter the alteration .

Figure 58 shows a sample of the water level gauge Comparison Form in common use by the Canadian Hydrographic Service for recording daily gauge inspection information. The form is self explanatory, but the comparison procedure requires some elaboration. Rather than completing the entries in order from column 1 to 15, the sequence below is recommended.

1) Enter the general information requested in columns 1 to 3 and 12 to 15, and ensure that the header on the sheet has been filled in.
2) Read the water level on the staff gauge and enter it in column 9; enter the true time of the reading in column 7. This reading is taken first because the height readings should be as nearly as possible simultaneous, and reading the staff may require a little time to mentally filter out the wave and swell fluctuations to obtain a proper reading .
3) Make a short mark with the recorder pen on the diagram, parallel to the height axis. On a pressure-activated recorder this is done by lightly moving the pen arm; on a float-operated recorder it is done by rotating the float pulley back and forth, taking care not to let the float wire slip on the pulley. The true time of this operation is entered in column 5.
4) If a sight gauge (tape gauge) has been

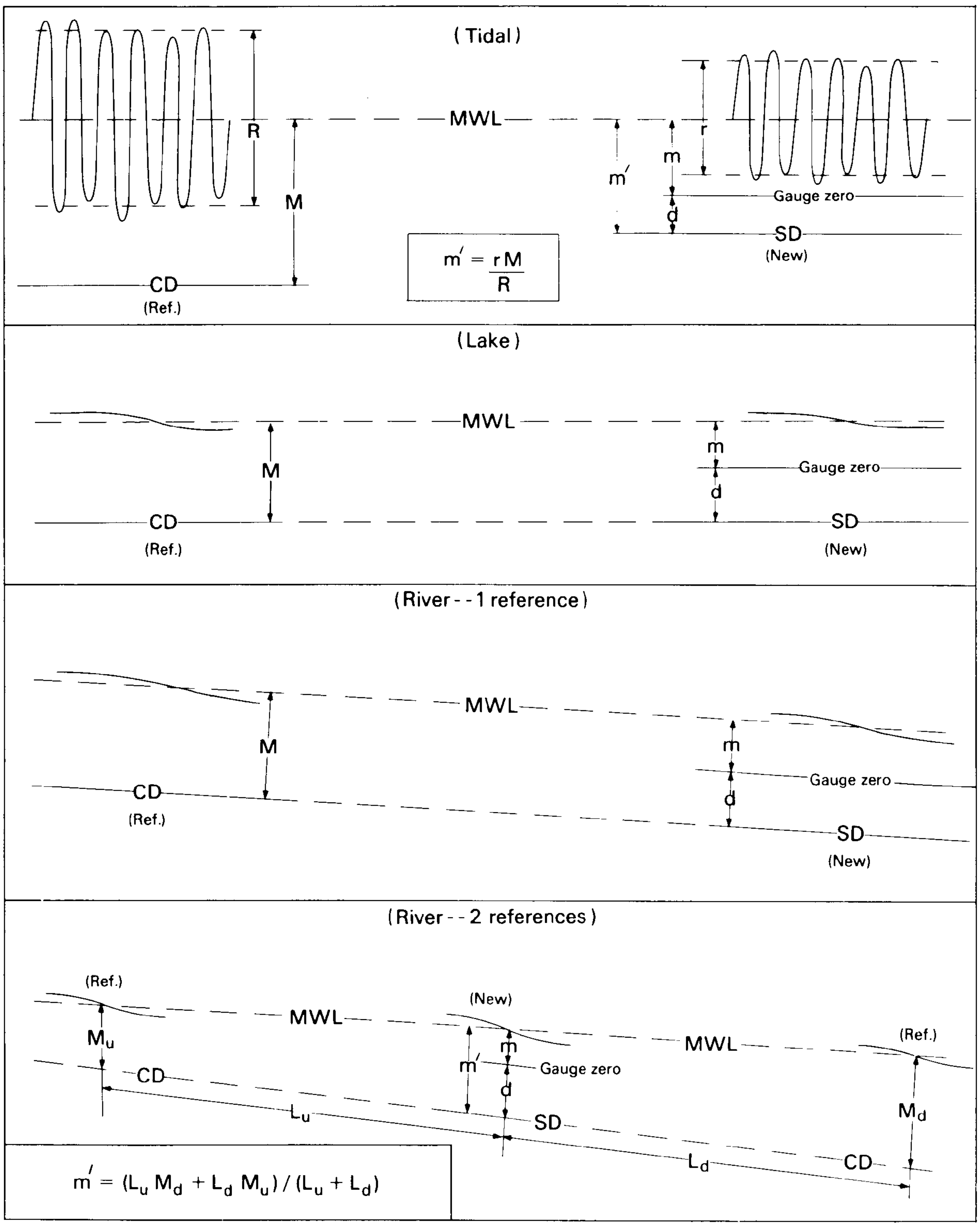


FIG. 57. Transferring sounding datum by water transfer in (a) tidal waters, (b) lakes, (c) rivers with only one reference gauge and (d) rivers with two reference gauges.

METRIC

COMPARISON FORM

FORM NO. 105A/83.

LOCALITY .... GAUGE TYPE .... YEAR 19 ....

CHIEF OF PARTY .... GAUGE NO .... TIME USED ....

MONTH			TIME COMPARISON			HEIGHT COMPARISON					WIND AND SEA			
1 DIAGRAM NO.	2 WEEK DAY	3 DATE	4 DIAGRAM TIME	5 TRUE TIME	6 TIME ERROR (4-5)	7 TRUE TIME	8 TAPE GAUGE	9 STAFF GAUGE	10 DIAGRAM	11 DIFF. * (9-10)	12 WIND DIRECTION	13 WIND FORCE	14 STATE OF SEA	15 READ BY

I HEREBY CERTIFY that the information given on this sheet is correct.

Signed .... PARTY CHIEF

* NOTE TO HYDROGRAPHERS. Column (11) is the height of automatic gauge zero above staff gauge zero (column 9 minus column 10). The average difference in column (11) is entered directly in section 1A of form TWL-502 and in section 1B with opposite sign.

FIG. 58. Standard Canadian Hydrographic Service Gauge Comparison form.

installed, read it and enter the value in column 8. The value may be either the distance of the gnomon above the water or the height of the water above gauge zero, as long as specified by a note on the form.

5) Look back at the mark made with the recorder pen in step (3); read the time from the diagram at the location of the mark, and enter it in column 4; read the water level height from the diagram at the mark, and enter it in column 10. Step (4) is inserted before step (5) to make the height readings on staff, automatic, and tape gauges as nearly simultaneous as possible.
6) Enter the time error in column 6; its value is column 4 minus column 5 . Enter the height difterence in column 11; since the tape gauges are not commonly installed at temporary gauge sites, this is taken as column 9 minus column 10, and so is a measure of the height of the zero of the automatic gauge above that of the staff gauge.
7) If it is convenient to mark on the recorder diagram without disturbing anything, mark the true time (from column 5) and the date on the diagram opposite the mark made with the pen in step (3). A soft-tipped pen should be used to mark lightly.

The inspection is not complete until indications of possible trouble are investigated. If the record trace seems unusually smooth, if the tidal range seems unduly small on the record, or if the automatic gauge and the tape gauge agree with each other, but not with the staff gauge. clogging of the intake by silting or marine growth is indicated. Flat spots in the record may be caused by the stilling well being too short, the float wire being too short, the intake or pressure sensor being exposed at low water, the water around the intake or sensor being impounded near low water, a pressure connection leaking, or by the movement of the diaphragm in a pressure sensor being restricted by silt. Problems peculiar to particular recorders may also arise (clogged pen, paper jammed or off sprockets, etc .), and these, along with the routine recorder maintenance (fill pen, wind clock, etc.), must be attended to as detailed in the particular instruction manual. When any adjustment or alteration is made that could change the time or height readings, a gauge comparison (steps (1) to (7)) must be made immediately before, a note written on the comparison form describing the alteration, and another comparison made and recorded immediately afterwards.

Small discrepancies in height or time that can be tolerated within the accuracy of the sounding reductions should not be removed by adjustment of the gauge or recorder. This is because a record with a small continuous error is easier to treat in the final analysis than is one with frequent adjustments. How large an error should be allowed to become before it is adjusted is left to the discretion of the field hydrographer, within limits that may be suggested by the Regional Tidal Officer.

7.7 Documentation of gauge records

The Temporary Gauge Data torm, the Comparison Form, and the levelling sheets and summaries are sufficiently documented and identified when all the pertinent entries have been made in the spaces provided. The first step when starting a fresh form should always be to enter the header information (station name, location, date, etc.). If it is not done at first, it may be overlooked altogether, and the information could become orphaned later on. It i.s even more important to identify each piece of water level record (sheet or strip-chart) with the station name, date and starting time immediately recording is commenced. The word “Start” should be marked at the beginning of a strip-chart record, and the word “End” marked at the finish; station name, and the end time and date should also be marked at the end of the record, to avoid having to unspool a record to identity it. If pieces of record are cut ot-f tor use during the survey, each piece of the record must be identified at each end as described above. When recording is completed, all pieces of record should either be taped or pasted back together consecutively, or each piece should be marked on the outside with its consecutive number and the total number (e.g. #l of 5), and all of them bundled together. This applies only to temporary gauge records, since records and parts of records may not be removed from permanent gauges by the field hydrographer .

The above advice has been written with stripchart recorders mainly in mind, since they are in most common use with temporary gauges at the time of writing. The principles, however, remain the same regardless of the method of recording, whether it be punched paper tape, magnetic tape, solid-state memory banks, or whatever: each piece of record and each supporting document must be ed as to station, date, time (including time zone), and any other parameter that seems appropriate.

7.8 Datum notes on field sheets

Every field sheet on which soundings are marked must have a datum note defining the elevation of the sounding datum used in the reduction of soundings shown on the sheet. On surveys of deep offshore regions it may be that no sounding reduction is considered necessary or practicable, or that a constant amount is subtracted from each sounding just to be on the safe side. In such cases there is no standard format for the datum note, but the note must still be supplied: it might read

"Reduced to a sounding datum which is the surface (or x metres below the surface) of the water at the time of the sounding."

There is a standard format tor the sounding note to be used when the sounding reductions have been made with reference to the water level at a particular location. The basic note in this case is to read

"Reduced to a sounding datum which at (name of gauge location) is x metres below BM (name of benchmark). "

A supplement to the basic note should be added, however, stating how the reduction at the sounding site was obtained from the water level at the gauge site (e.g. applied directly or calculated from cotidal chart).

When sounding datum has been determined at the gauge site by levelling from a BM on International Great Lakes Datum (IGLD) or Geodetic Survey of Canada Datum (GD), it is desirable to add the following sentence to the basic sounding note:

"The elevation of BM (name of benchmark) was determined in (year) to be y metres above IGLD (or GD)."

This wording respects the fact that the Canadian Hydrographic Service is not authorized to assign benchmark elevations except with respect to its own chart datums.

7.9 Submission of records and documents.

All records, documents, and explanatory information pertaining to the operation of water level gauges, the establishment of benchmarks, and the method of sounding reduction are to be submitted to the Regional Tidal Officer at the first opportunity following completion of that phase of the survey. This is not, of course, an invitation to terminate operation of a gauge before sutficient record is obtained to permit useful tidal analysis (absolute minimum 15 days; desirable minimum of 29 days). The material to be submitted includes the originals of

1) Temporary Gauge Data form, showing sounding datum determination and relation to gauge zeros and BMs, BM descriptions and sketches. and general descriptive information;
2) BM photographs;
3) Water level gauge Comparison Forms;
4) Complete water level gauge records (pen-on-paper traces, punched paper tapes, magnetic tapes, memory bank print-out, or whatever);
5) Levelling notes and summaries; and a copy of
6) The datum notes from the field sheets.

While it is desirable to submit as much of the above information as possible before the end of the field season, no original records should be sent by mail or other third party carrier unless a usable copy has been retained. It is always wise, in l:act, to make copies of records whenever possible, and to store the originals and copies separately.

7.10 Sounding reduction - general

Sounding reduction based on surtace elevations recorded at a gauge site is very similar in principle to the determination of sounding datum by water transfer from one gauge to another, as described in sections 7.3, 4 and 5. Sounding datum is, in effect, carried by water transter from the gauge site to the sounding location. The water transter in this case, however, is based on only one reading at each end of the line, and does not have the benefit of averaging over several days of record as in the previous case. Because of this, when accurate sounding reductions are required, the region of sounding should not be far trom the control gauge. It is not possible to specity a fixed distance from a gauge, beyond which sounding should not be carried. That decision must take into account such factors as the required accuracy of the survey, and the local surmay be generated by wind, seiche, river discharge, etc.

7.11 Sounding reduction - cotidal charts

Cotidal charts have already been discussed in section 3.10. Their application to sounding reduction is mostly in offshore regions, where it is not feasible to place gauges close enough to the sounding area to justify taking the correction directly trom the gauge, and where a lesser accuracy in the reduction can be tolerated. The cotidal charts most commonly used for sounding reduction implicitly assume that the shape of the tidal curve at any point on the chart has exactly the same shape as that at the control gauge, but that it may be shifted in time and magnified or reduced in vertical scale. This assumption may be valid over an extensive region when the tide is mostly semidiurnal, but over only a restricted region when the tide is strongly diurnal. A cotidal chart is prepared and provided by the Regional Tidal Officer. atter discussion with the concerned field hydrographer about required accuracy, extent of the survey, proposed gauges, etc. The area covered by the chart is divided into two sets of zones, one set defined according to the time differences, and the other set defined according to the amplification factors (ratios of ranges), relative to the tide at the control gauge.

Figure 59 shows a cotidal chart for a fictitious region. Boundaries between time difterence zones are shown as solid lines, and those between amplification zones (range zones) as broken lines. Each time zone is labelled with the average number of minutes by which the occurrence of a tidal event (e.g. HW, LW, etc.) within the zone lags behind the occurrence of the corresponding event at the control gauge. Each range zone is labelled with the average ratio of tidal ranges within the zone to the tidal range at the control gauge. The size of the step between time zones and between range zones must be chosen to accord with the required accuracy of the soundings and the average range of the tide. If, for example, the tide is semidiurnal with a 5-m range, a time shift of 10 minutes produces an average height change of 0.13 m, and a maximum change of 0.21 m; a shift 0.1 in range factor produces tor the same tide an average height change of 0.32 m, and a maximum change of 0.50 m. Improved accuracy in the use of cotidal charts may be obtained if values are interpolated, rather than taken as constant over a zone. At the very least, when a sounding location is closer to the boundary than it is to the centre-line of a zone, the value chosen should be the average of those on either side of the boundary. To help illustrate the reduction method, Table 7 contains a hypothetical set of water level data supposed to have been read from the control gauge for the cotidal chart of Fig. 59. The times shown in the table are the times taken directly off the gauge record (or diagram), corrected only for

TABLE 7. Control gauge readings for sounding reduction

Gauge diagram time (AST)	Water level above sounding datum (metres)
10:40	4:05
11:00	3.70
11:20	3.35
11:40	2.90
12:00	2.50
12:20	2.10
12.40	1.65
13:00	1.30
13:20	0.95

gauge clock error, if any.

The following two examples demonstrate how to calculate the sounding reduction from the cotidal chart information and the water level data.

Example (a):

At 11:05 AST a sounding of 9.6 m is taken at point A on the cotidal chart. From Fig. 59. the time lag of the tide at A relative to the tide at the control gauge is interpolated approximately as T = 12 min. and the range factor as R = 1.03. Since the time lag is positive, it must be subtracted from the sounding time to obtain the diagram time at which the corresponding tidal phase occurred at the gauge. Thus, the gauge reading should be taken at 11:05 - 00:12 = 10:53. By interpolation between the first two readings in Table 7, the gauge reading at 10:53 is 4.05 - 0.35 x (13/20) = 3.82 m. This times the range factor gives the height of the water level at A as 3.82 x 1.03 = 3.93 m above sounding datum. The sounding reduction, to the nearest decimetre, is thus 3.9 m. and the corrected sounding is 9.6- 3.9 = 5.7 m below sounding datum at point A.

Example (b):

At 12:45 AST a sounding of 12.3 m is taken at point B on the cotidal chart. From Fig. 59, T = -7 min, and R = 0.95. Since the time lag is negative, tidal events arrive at B before they arrive at the control gauge, and so T must be added to the sounding time to obtain the appropriate diagram time. The gauge reading is therefore taken at 12:45 + 00:07 = 12:52. By interpolation between the seventh and eight readings in Table 7, the gauge reading at 12:52 is 1.65 - 0.35 x (12/20) = 1.44 m. This times R gives the water level at B as 1.44 x

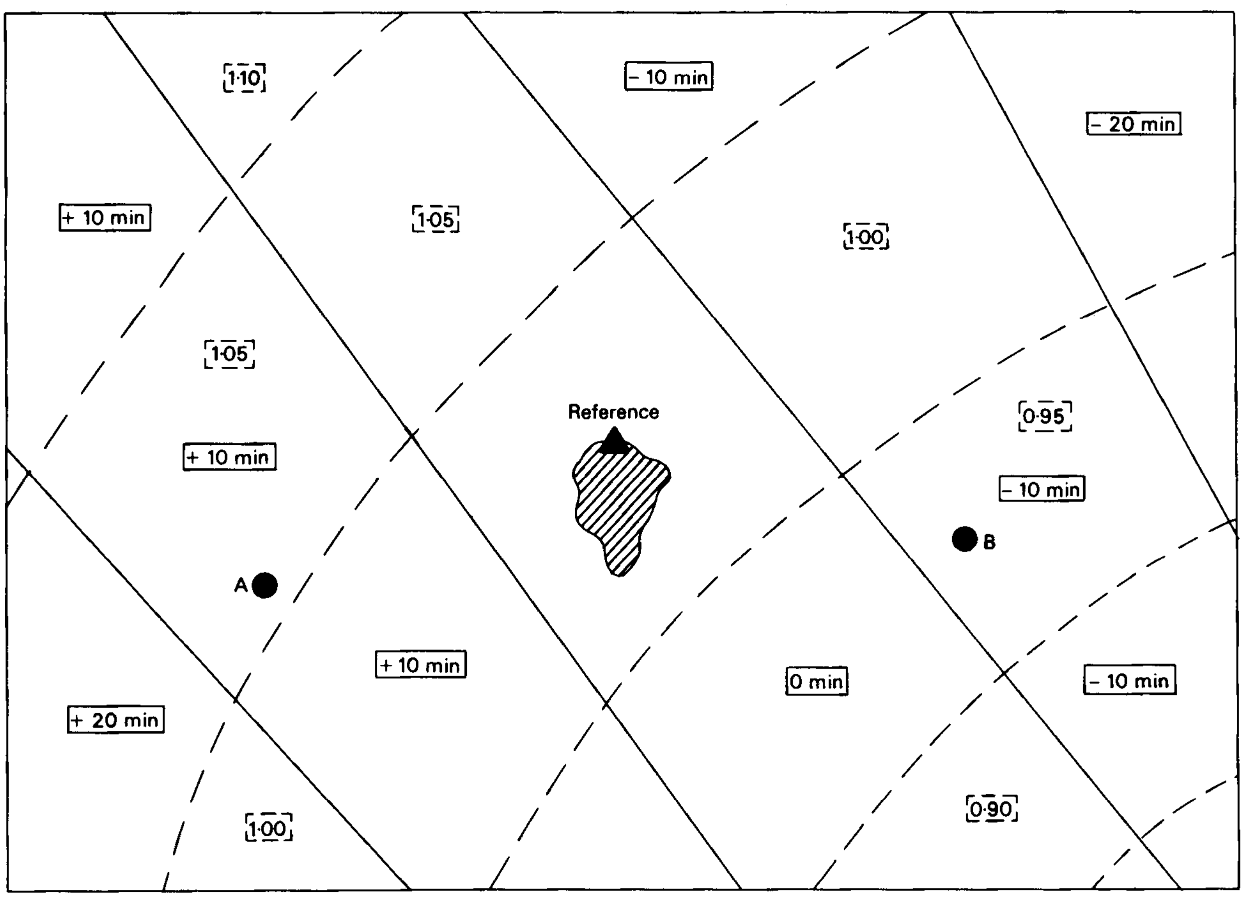


FIG. 59. Specimen cotidal chart of a fictitious region.

0.95 = 1.37 m above sounding datum. The sounding reduction, to the nearest decimetre, is thus 1.4 m, and the corrected sounding is 12.3 - 1.4 = 10.9 m below sounding datum at point B.

As an alternative to a cotidal chart drawn out as in Fig. 59, a program might be supplied for use in a mini-computer, to calculate R and T as functions of grid coordinates, and pertorm the sounding reduction from the sounding times and tabulated gauge readings. Coupling such a system to a telemetering water level gauge can provide real-time automated sounding reduction. When it is not feasible to have an operating gauge to control the sounding reduction, tidal predictions may be used in conjunction with the cotidal chart or computer program. Predicted water levels for the control site admittedly do not include the non-tidal fluctuations that would be detected by an operating gauge, but then, neitherdo the non-tidal oscillations behave in the manner prescribed by the cotidal chart.

Sounding surveys covering an extensive area in which the tide displays a large diurnal inequality may not be well served by the type of cotidal chart (or computer program) described above. because the assumptions of constant time lags and amplification factors are not justified. In these cases it may be necessary to supply a separate cotidal chart for each of the major tidal harmonic constituents (usually M_2, S_2, 0_1, and K_1). From the cotidal charts, the constituent amplitudes and phaselags for the immediate sounding area may be read, and tidal predictions made in the manner described in section 3.8. No control gauge site is directly invoved in this field procedure, but information from several neighbouring gauge sites would be used in preparation of the cotidal charts. In preference to graphic cotidal charts for the constituents, computer programs might be provided to generate the tidal constants as functions of grid coordinates, calculate sounding datum relative to MWL, predict the water level relative to sounding datum, and so automatically provide corrections for soundings. In the above procedure, the distance of sounding datum below MWL would usually be taken as the simple sum of the major tidal constituent amplitudes.

7.12 Sounding reduction—non-tidal waters

Sounding datum on a lake is chosen as a level surface, whose elevation is defined relative to BMs at the control gauge site. The sounding correction is usually taken directly as the water elevation above sounding datum at the gauge. The correction is thus accurate only insofar as the water surface between the gauge and the sounding is level at the time. Improved accuracy may be obtained by interpolating between values from two control gauges, weighting each value in inverse proportion to the distance of the sounding from the particular gauge. Even this procedure provides no compensation for surface slope in the offshore direction, unless one of the control gauges is itself off shore. Because of this, sounding should not be carried out far from a control gauge on a lake when large wind set-up or seiche activity is suspected; this is particularly true for lakes that are shallow, and of large horizontal extent (see sections 4.3 and 4.6).

Sounding datum on a river should be a surface that approximates closely to the actual water surface when the river is at its lowest stage of the navigation season. If a sounding correction is transferred directly from a single gauge on the river, it is accurate only insofar as the river slope at the time of sounding is parallel to the river slope at low stage. It may be necessary, particularly on shallow rivers, to establish two control gauges and to interpolate sounding corrections between them, except when soundings can be done at or near low stage. The method of interpolation and its justification are the same as those given for the determination of sounding datum by water transfer on rivers (section 7.5).

7.13 Sounding reduction—offshore gauging

In general, offshore sounding does not require as high an accuracy as that near shore, because of the greater depths offshore. There are occasions, however, when sounding must be done in shallow water far from shore, over offshore reefs, shoals, or banks. in such a case, sounding reductions taken from a distant shore gauge, whether by direct transfer or by cotidal chart, may not be sufficiently accurate. Some sort of offshore water level measurement is then necessary to control the soundings. If the water is very shallow, it may be possible to drive a long pole into the bottom and attach a staff gauge to it: it may not be feasible to station a vessel to read the staff every hour, but it could be visited twice or four times a day near high and low waters to observe the maxima and minima, from which an adequate tidal curve could be constructed. If it is not possible to set an offshore staff gauge, a vessel or launch equipped with a sounding device may be moored over a level bottom, to take a series of soundings at the fixed location. The vessel acts as an inverted float gauge, and the series of soundings is the water level record. Alternatively, a bottom-mounted pressure gauge of the type described in section 6.7 may be employed. Records from these gauges require subtraction of a corresponding set of atmospheric pressure measurements before being interpreted as water levels (section 4.4). The pressure record is stored in the gauge, but may also be acoustically telemetered to a surface float or vessel for more immediate use in sounding reduction. While it is not possible to reference the datums from offshore water level measurements to BMs on shore, the records may be very useful in tidal studies and the preparation of future cotidal charts. They must, therefore, be submitted to the Regional Tidal Officer, just as are the other gauging records and documents.

CHAPTER 8

Current Measurement

8.1 Introduction

The responsibility of a Hydrographic Service to provide current information, mainly as an aid to navigation, was mentioned in the Preface to this Manual . The responsibility for gathering the necessary current information lies mainly with the field hydrographer, and this aspect must be recognized as an integral part of any hydrographic survey. Current information that is obtained on a survey will be analysed, compared with previous information, interpreted, and eventually incorporated into the charts, *Tide and Current Tables, Sailing Directions*, or possibly tidal atlases. Since aiding safe and efficient navigation is the main goal of a hydrographic survey, current measurement will be called for mostly in narrow or shallow channels, harbour entrances, congested shipping lanes, or other areas where the margin for navigational error is small. Current information from deeper and less restricted offshore areas is also greatly appreciated by mariners, but its measurement may often require the assignment of dedicated current survey teams with specialized mooring equipment. Even in the offshore areas, however, bits of current information can be gathered during the course of a sounding survey, from analysis of the drift of the ships and launches.

Various aspects of currents and have already been described in Part I of the Manual. Some of the terminology and characteristics associated with tidal streams were discussed in sections 1.5 and 1.11; harmonic analysis and prediction of tidal streams were dealt with in sections 3.7 and 3.8; and some of the causes and characteristics of non-tidal currents were treated in sections 4.2, 4.3, and 4.7 to 4.10. This chapter will deal with practical aspects of gathering current information as part of a hydrographic survey - preliminary investigation, methods of measurement, types of current meters, etc.

8.2 Preparatory investigation

The effort expended on current measurement can be more effectively directed when something of local conditions and navigational problems is understood. Before leaving for the field, the hydrographer should study the current information given for the survey area in the *Sailing Directions*, navigational charts, *Tide and Current Tables*, and current atlases, if they cover that area. The Regional Tidal Officer should be consulted for advice and for information that may not yet have been published, and also for copies of correspondence that may have been received complaining of errors or omissions in the publications, or requesting additional current information. When in the field, the opinions of experienced local mariners and fishermen should be sought. They should particularly be asked to comment on the information concerning currents in the area as shown in the Hydrographic charts and publications. On the basis of evidence gathered as above, and by first-hand reconnaissance, it must be decided if and where current observations are required to verify or supplement the existing information.

8.3 Location and depth of current measurement

Current measurements will normally be required only where appreciable velocities (0.2 knots or more) are encountered in shallow, narrow, or congested shipping routes, in harbour entrances, in berthing areas, etc. Where possible, the observations should be taken where the velocity is greatest, but if this is in the centre of heavy traffic, it may be necessary to move the observation site to one side of the channel. When this is the case, a few spot readings should be taken in the centre of the channel, to relate the current there to that at the site of the more complete observations. Indeed, it is always wise to take spot readings at various locations and at various times (e.g. at maximum flood and ebb) during the main observations: this can help to define the spatial variability of the currents with minimal effort. The exact position of each main observation site will, of course, be influenced by the availability of a suitable bottom for mooring current meters or anchoring a vessel, or of an existing platform such as a bridge footing or drilling barge.

Since the current information is intended mainly for the use of mariners the depths of observation should span the depth of the deepest draught vessels frequenting the area; if only one depth is occupied, it should be about one-half the deepest draught; if two depths are occupied, they should be about one-third and two-thirds the deepest draught, etc. A moored current meter, however, may not be placed so near the surface that the subsurface float from which it is suspended could break the surface in any sea conditions or at any stage of the tide.

8.4 Time and duration of measurements

The length of record required to permit separation of the major tidal constituents by harmonic analysis is the same for currents as for water levels, a minimum of 29 days. This length of record should, therefore, be the target for current measurement in tidal waters. If, for some reason, 29 days of record cannot be obtained, an honest attempt should be made to get at least 15 days. The above is intended to encourage longer series of current observations, but is in no way intended to discourage short series when circumstances absolutely preclude the possibility of longer ones. Almost any carefully observed data are better than no data at all. In waters whose tide shows a large diurnal inequality, observations should be obtained over at least 25 hours, whereas if the tide shows only a small diurnal inequality, this may be relaxed to 13 hours, if necessary. Short series are more valuable if they can be observed when the range of the tide is large (usually at spring tide), since the tidal streams should then be large as well. Two short series taken one at spring tide and one at neap tide are much more informative than a single series of their combined length. In general, the more frequently the current is sampled, the more reliable is the data, because irregularities can more easily be smoothed out of the record. Frequent sampling is most important in short records, a sampling interval of 15 minutes being desirable, but one of 30 minutes being acceptable. The sampling interval might be extended to one hour if so doing permitted a longer record to be obtained through conservation of limited battery power or data storage space. As mentioned in section 8.3, spot readings taken at various locations and times during recording at the main sites are very valuable. While there can be no set sampling interval prescribed for these, they are most useful if taken near the times of maximum flood and ebb, in tidal waters. In fact, series of measurements of the time. rate, and direction of the current at maximum flood and ebb (or at the times of maximum and minimum ebb in the case of some tidal rivers) are very worthwhile even when it may not be feasible to observe a proper time series.

In non-tidal waters, the continuity of current observations is less important than in tidal waters. What is more important is to obtain measurements over the range of conditions that influence the current, these being mainly river discharge, runoff, and wind. Long continuous periods of record are certainly satisfactory if they span a sufficient range of conditions, but it is frequently more convenient to schedule several shorter periods to coincide with such events as the spring freshet, the dry season, the stormy season, etc. The sampling interval for observations in non-tidal waters may, in general, be longer than that recommended for tidal waters, one hour being a reasonable value unless seiche activity is thought to contribute significantly to the current (see section 4.6). The seiche contribution may be studied from a record of duration equal to several seiche periods, observed with a sampling interval about one-tenth of the seiche period: seiches are not, of course, always present, and care should be taken that seiche activity is included in the record if it is to be studied.

8.5 Observation methods - general

Current observations may be categorized as “direct” or “indirect” on the basis of whether the velocities are measured as such or are deduced from their relation to other measured parameters. Five examples of indirect current observation are described below—the continuity, the hydraulic, the long wave, the electromagnetic, and the geostrophic methods. Of these, only the first three are likely to find even limited application on hydrographic surveys, but they may all contribute to the information in Hydrographic publications through the courtesy of other investigators. Direct current measurement is sub-classified as either"Eulerian" or"Lagrangian." Eulerian measurements are taken at a fixed location over a period of time; Lagrangian measurements are taken by tracking the path of an object that drifts with the water over a period of time. Results from the two methods could be comparable only if the drifter remained within the same current regime throughout its path, which would usually require the path to be very short. In general, it is easier to interpret Eulerian than Lagrangian measurements. The methods described below that involve drifters of one sort or another may be considered to be Eulerian, because it is envisaged that the drifters would be recovered and re-set, to traverse the same small re the series of observations.

8.6 Self-contained moored current meters

These instruments consist of sensors to detect current rate and direction, data recorder (usually magnetic tape), clock, power supply, and a watertight pressure case to house the vulnerable components. In some instruments. additional sensors may be supplied for such things as pressure, temperature, and electrical conductivity (from which salinity is determined): while these other parameters may not be of direct concern to a hydrographer, they may be observed with no extra effort by using an instrument that is so equipped. To date, direction sensors still rely on the magnetic compass, gyro-compasses not yet having been successfully engineered into small current meters. This is a serious drawback only in the far North, where the horizontal component of the earth's magnetic field is weak near the magnetic pole. The rate sensor is usually either a propeller mounted on a horizontal shaft, or a drum-shaped Savonius rotor mounted on a vertical shaft. Some instruments have a tail fin and are suspended so that the whole body of the meter turns to trace the current, in the manner of a weather vane; others, particularly among those using Savonius rotors, have a direction vane that rotates independently of the instrument case. The common recording procedure is to store a pair of rate and direction readings at a fixed sampling interval, typically about 15 minutes. More sophisticated instruments may be programmed to accumulate the vector average of readings taken every few seconds over a fixed interval, and to store only the average rate and direction after each recording interval. In this way, data storage is conserved, but wave "noise" is filtered out of the record by high frequency sampling. Solid-state sensors are available on some models to replace the propeller or rotor. There are two acoustic types, one of which detects the change in travel time of a sound pulse between two probes, and the other of which detects the Doppler shift in frequency of a sound pulse reflected from particles in the moving water. There is also the EMF (electro-magnetic force) type sensor, which senses the voltage generated as the moving water (a conductor) cuts the lines of force of a magnetic field in the sensor.

Figure 60 shows a typical configuration in which to moor a string of current meters in moderately shallow coastal waters. The subsurface float must be deep enough not to break the surface at any wave or tide condition; the cable supporting the meters is non-rotating stainless steel wire, with a swivel above the anchor, below the float, and above and below each meter; the ground line and the line to the surface marker are buoyant polypropylene rope; and the anchor release is triggered by a coded acoustic signal. The ground line should be a least as long as twice the water depth, and be laid out in a known direction. Normal recovery procedure is to trigger the release, pick up the subsurface float, and retrieve the meters and the rest of the mooring in order. If the release should fail, the mooring may be recovered in reverse order, starting with the surface marker buoy. If the surface buoy is lost, or was not considered necessary when the mooring was placed, the string may be recovered by grappling for the ground line normal to its length. In very shallow water, a single instrument might be mounted on a tripod or frame and weighted to the bottom. Further details and alternative suggestions for mooring current meters may be obtained from the Regional Tidal or Current Officer or from oceanographic field personnel.

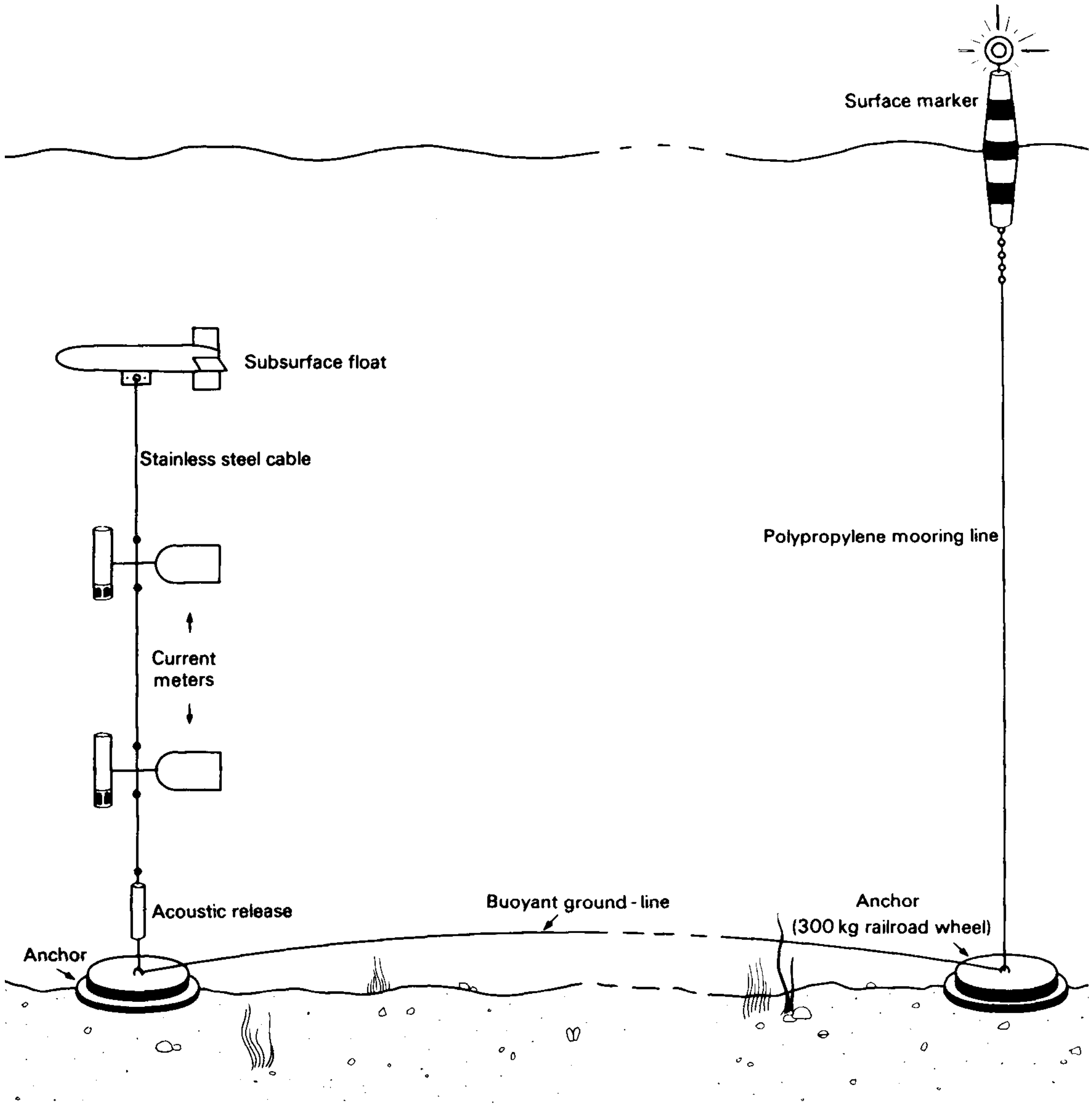


FIG. 60. Typical current meter mooring configuration for coastal waters.

8.7 Over-the-side current meters

These instruments are basically the same as those described above, but the internal recorder is usually replaced by a visual display and/or recorder on deck, to which the information is fed through electrical conductors in the suspension cable. With the vessel at anchor, the meter is lowered by winch and readings taken at selected depths. The procedure should be repeated at intervals of about 30 minutes for as long a series as is feasible. If the system has an automatic recorder, or if personnel are available to monitor the visual display continuously, the meter should be left to record at a selected control depth rather than being brought inboard between lowerings. A pressure indicator that can be set to zero at the surface is a desirable feature in over-the-side current meters, to provide an automatic record of the various pressures (hence depths) at which readings were taken. If this is not provided, the depth must be determined from the length of cable out and its angle from the vertical.

One of the self-contained meters described in section 8.6 may be used for over-the-side operation instead of for mooring, if so desired. It must then, however, have a pressure sensor as well as rate and direction sensors and a clock. A record, independent of that in the meter, should be kept of the times at which the instrument was recording at particular depths. This, coupled with the pressure record, provides additional time checks on the record; or, if the pressure sensor fails, helps to identify the depths of the readings. Since the record is not usually accessible during operation, the initial estimate of the depth of the self-contained meter is determined from the length of wire out and its angle from the vertical. Some meters may be monitored acoustically by hydrophone during operation, but the added equipment, including a decoder, perhaps unduly complicates the procedure. Again, between lowerings to selected depths, the instrument should be left to record at a chosen control depth.

A current meter suspended from a movable platform records the movement of the water relative to the platform, so it is necessary to remove the platform motion from the record or, at least, to identify parts of the record that should be ignored because of excessive platform motion. To achieve this when the platform is a vessel positioned by a single anchor, a record should be kept of the direction of the ship's head and the scope of the anchor.

8.8 Suspended current cross

This is a piece of equipment that can be simply constructed to measure currents from a reasonably stationary platform . Its design and use are illustrated in Fig. 61. A rigid cross, about half a metre in each of its three dimensions, is weighted at the bottom and suspended from the top by a thin wire. The drag force (D) of the water on the cross swings it in the direction of the current until the suspension wire is hanging at an angle θ off the vertical. At this point, the horizontal component of the tension in the wire must balance the drag force on the cross, and the vertical component must balance the weight minus the buoyancy of the weighted cross (i.e. the weight in water, W), so

(8.8.1) $\tan\theta = D / W$

The drag force of a moving fluid on a bluff object

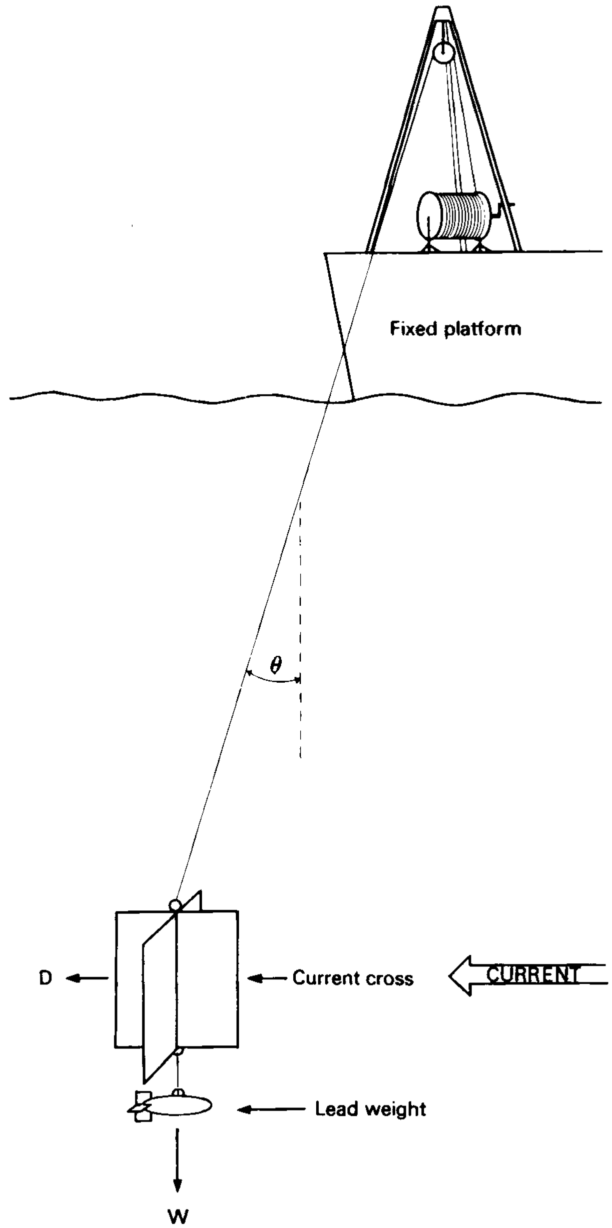


FIG. 61. Current measurement by suspended current cross.

(not streamlined) is

(8.8.2) $D = ½C\ p\ Av^2$

where A is the cross-sectional area of the object normal to the flow, v is the speed of the fluid relative to the object, p is the density of the fluid, and C is the drag coefficient, a dimensionless constant approximately equal to unity. The procedure for measuring current is as follows:

(1) lower the current cross over a fixed pulley to

the desired depth of water (allow for wire angle),

(2) measure the angle (θ) between the downward vertical and the wire (wire angle indicators are available for this purpose),

(3) estimate the azimuth toward which the suspension wire is heading in the horizontal plane, to get current direction,

(4) record the time and date, the inclination angle, and the azimuth of the suspension wire,

(5) assure that the weight of the loaded cross in water and the cross-sectional area of the cross plus weights are recorded for each measurement, because different combinations may be required to give reasonable inclinations at different speeds,

(6) to confirm the validity of the measurements, at least some observed sets of θ, W and A should be converted to current speed: if the apparatus was supplied with calibration tables, this may be done by table look up; if it is an *ad hoc* piece of equipment assembled in the field, speeds calculated from the formula

$$(8.8.3)\quad v = [0.019(\frac{W}{A})\tan\theta]^{1/2}\,\mathrm{m\ s^{-1}}$$

where W is the weight in water in kg, and A is the area in m^2.

The constant (0.019) in 8.8.3 was derived by substitution of 8.8.2 into 8.8.1, with C = 1.0, p = 1020 kg m^{-3}, g = 9.8 m s^{-2}. Multiplication of W by the acceleration of gravity, g, in 8.8.1 was necessary to convert weight units (kg weight) to fundamental force units (kg m s^{-2}).

A current cross may be employed down to depths of 15 m if the suspension wire is thin enough (about 2 mm) to keep the drag on the wire small relative to that on the cross. If a cross is constructed in the field to meet an unexpected requirement, it should be retained for later calibration, because the expression in 8.8.3 is only approximate, and because an error could be made in determining W/A. The area and/or the weight of the cross must be chosen so that a reasonable inclination angle (θ) is obtained in the current speeds anticipated. If the angle is too small, it cannot be read accurately; and if it is too large, the cross may plane on its side. The maximum

TABLE 8. Current speeds associated with 40° deflection angle, for current crosses with various weight/area ratios.

W/A (kg wt.)/(m^2)	*v* (m s^{-1})	*W/A* (kg wt.)/(m^2)	*v* (m s^{-1})
1	0.1	100	1.3
5	0.3	140	1.5
10	0.4	200	1.8
20	0.6	300	2.2
40	0.8	500	2.8
60	1.0	1 000	4.0

inclination that should be encountered during operation is about 40°. To assist in the design of a current cross for a particular application, Table 8 gives the current speeds that would produce the maximum deflection of 40° for various values of W/A, according to expression 8.8.3. If it is wished to work in knots, a sufficiently accurate conversion is 1 m s^{-1} = 2 knots.

While a wooden cross is usually employed in this application, other material and forms may be used as well. If a large value of W/A is required, it may be convenient to use a concrete cube. For solid cubes, WM is proportional to the length of the side, and for a concrete cube of 30-cm side, W/A = 500 (kg wt.)/(m^2). In theory, a sphere is the best shape to use, because it presents the same cross-sectional area to the current regardless of its attitude. A cross, however, is easy to as and it serves well.

8.9 Drift poles - general

A drift pole is a long buoyant spar of uniform cross-section, weighted at its lower end so that it floats vertically in the water column, with only a small portion of its upper end above the surface. A small flag, radar reflector, light, or even a radio transponder may be fastened to the upper end to help locate it. The area exposed to the wind, however, must be kept small in relation to that exposed to the current, to prevent the wind from unduly affecting the drift of the pole. For hydrographic studies, the length of the pole under the water should approximate the deepest draught of vessels regularly operating in the area, so that it will experience an average current similar to that experienced by a ship. Drift poles may be assembled to the desired length from prefabricated sections, or may be made up from local lumber for temporary use. In some areas it may be difficult to acquire and to manipulate drift poles as long as the draught of the largest vessels. How long a pole can be accommodated will depend a great deal on the survey vessels, equipment, and manpower, but a l 0-m pole would probably be the maximum for any operation.

8.10 Drift poles - tethered

This procedure is designed for use from an anchored vessel. A drift pole is tethered to the stern of the vessel (anchored at the bow and riding the current) at the end of a measured line. The line should be made of buoyant polypropylene rope, about 100 m long, and marked off in 10-m sections . Starting with the line coiled on deck, the drift pole is released and allowed to drift away from the stern of the vessel as the line is payed out fast enough to provide slack, but slowly enough to prevent fouling. The time taken for the pole to drift the full length of the line (or a measured fraction of it. if the current is slow) is recorded, along with the relative bearing of the pole at the end of its run and the direction of the ship's head. From this, the current speed and direction may be calculated, if it is assumed that the vessel has not moved during the exercise. The ship movement should be monitored as well as is possible, so the observations may be adjusted accordingly. The procedure should not be undertaken when the tide is turning and the vessel is coming about on its anchor, but the time of turn should be recorded as such. The drift pole releases may be repeated at 30-minute intervals to provide a meaningful series of current measurements.

8.11 Drift poles – free-floating

This procedure is useful when it is wished to study current patterns over a fairly extensive area. It requires several drift poles, at least one launch capable of placing and recovering the poles, and a fairly accurate method of position fixing (visual or electronic). Drift poles are released one at a time in representative sections of the region, and their positions, along with the times, are recorded as soon as they are floating free from the launch. As the poles continue to drift freely, their positions are fixed about every half hour, or more frequently if the region is small and/or the currents are strong. A drift pole should be picked up and repositioned if:

(a) it is in danger of going aground,
(b) it is about to leave the region of interest,
(c) the distribution of the poles no longer provides representative coverage of the region, or
(d) two poles are too close together to independent information.

Any one of a variety of methods may be used to fix the drift pole positions: if a launch has the capability of fixing its own position, it may come gently alongside the pole and take a fix; if the launch also has radar, it may fix the position of a drift pole from a fair distance, possibly fixing several poles from the same location; a centrally stationed vessel with proper radar may be able to monitor all the poles, using a launch only to recover and reset them; positions may be determined by theodolite observations from shore; or the poles may carry radar transponders or radio beacons and interact with an electronic positioning system. The number of drift poles that can be monitored at one time will depend on the size of the region, the speed of the current, the number and speed of the launches. the quality of the radar or other positioning system, etc . In a simple region, such as a short and narrow channel. one or two drift poles frequently positioned and frequently reset may provide better coverage than a larger number of poles less frequently positioned and reset.

8.12 Current drogues

Current drogues are deployed and monitored in the same manner as drift poles, but are designed to drift with the current at specific depths, instead of with the average current over an interval of depth. They consist of a surface marker with floatation, from which is suspended an object with a large surface area to intercept the current at the desired depth. They must be designed so that the area exposed to the wind and to the current at other depths is small compared to the area of the drogue at the selected depth. Square crosses, similar to those discussed in section 8.8, may be constructed from plywood or sheet metal and weighted at the bottom to serve as drogues. A drogue that has a large effective surface area. but is not bulky to transport, is the ‘ parachute drogue,” which is in appearance just what its name suggests, except that the chute is deployed horizontally instead of vertically. Care must be taken in launching a parachute drogue to assure that the shrouds do not get tangled or fouled. There is a common misconception that the chute must be fully open at all times to operate properly. The chute, however, opens only to oppose the water movement past it, i.e. to oppose being dragged by the surface float and marker. If the drogue is moving freely with the current at that depth, the chute may properly appear collapsed, as long as it is free to open if needed. Figure 62 illustrates current drogues of the square cross and the parachute type, and also a drift pole, for comparison .

8.13 Continuity method

It follows from the continuity principle, which is basically the principle of the conservation of mass, that the rate at which the total mass of liquid inside a container increases must equal the net rate of transport through the entrance to the container. Neglecting the small changes in density that may occur, we may equally well say that the rate at which the volume of water inside an embayment increases must equal the net inward volume transport through its entrances. The simplest application of this principle to current determination is to estimate the average tidal streams flowing through the single entrance to an embayment. It is required to know the surface area of the embayment (A), the crossectional area of the entrance (a), and the harmonic constants for the main tidal constituents in the mean vertical tide over the surface of the embayment. Suppose H and g are the average amplitude and phaselag of a constituent of the vertical tide in the embayment. Let h(t) be the contribution of the constituent to the tidal height in the embayment at time t, and V(t) be its contribution to the volume of water in the embayment. Assuming that the surface area is roughly the same at all stages of the tide,

(8.13.1) $$V(t) = Ah(t) = AH \cos(\omega t - g)$$

where ω is the angular frequency of the constituent. Differentiation of 8.13.1 with respect to time, and use of some simple trigonometric relations, gives the rate of change of V(t) as

(8.13.2) $$\frac{d}{dt} V(t) = -\omega AH \sin(\omega t - g) = -\omega AH \cos(\omega t - g + 90)$$

By continuity, the rate of change of volume must equal the rate at which water is being transported in through the entrance, and this, divided by the crossectional area of the entrance, is the mean tidal stream speed through the entrance, u(t), whence, from 8.13.2, divided by a,

(8.13 .3) $$u(t) = (\omega AH/a) \cos(\omega t - g + 90)$$

The amplitude and phaselag of the tidal stream constituent are thus $\omega H(A/a)$ and $(g - 90)$ in the appropriate units. Consider the following numerical example, using the tidal constituent M_2, for which $\omega = 0.00014$ radians per second. Let A = 20 km^2, a = 0.007 km^2, H = 1.0 m, and g = 025°. By 8.13.3, the amplitude of the average M_2 tidal stream in the entrance is 0.00014(1.0)(20/ 0.007) = 0.4 m s^{-1}, and its phaselag is 025°-90° = -065°, or, adding 360° to conform to the convention that phaselags are positive angles, 295°.

The field hydrographer would not be expected to perform the continuity calculations necessary to convert water level observations into current information, but could be expected to provide sufficient water level data from inside an embayment to support the calculations. Observations from one gauge site are sufficient for many small embayments, but observations from several gauge sites may be required to represent the average tidal surface of a long and shallow inlet. The usefulness of the method has been demonstrated by the calculation of tidal stream

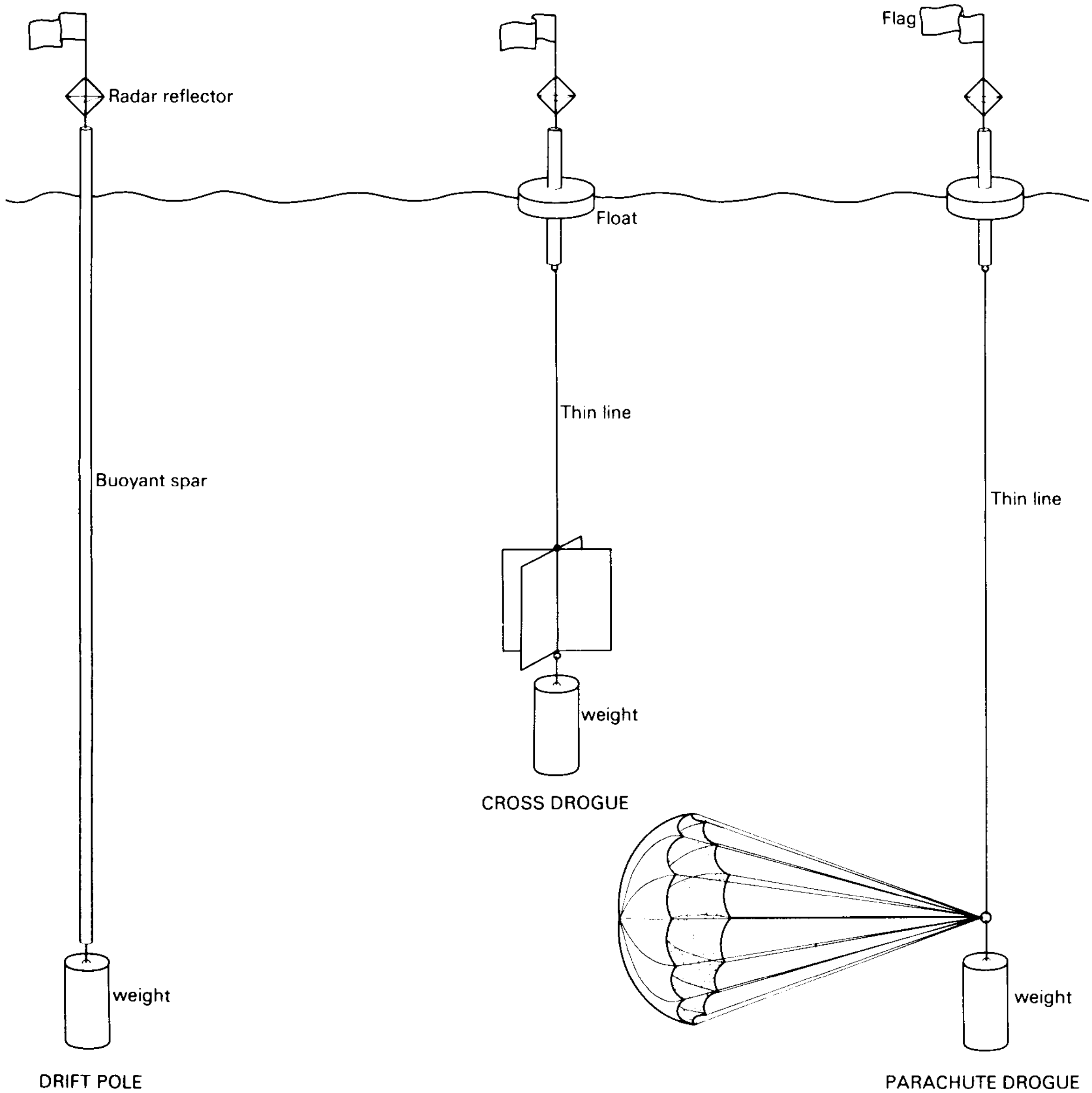


FIG. 62. Drift pole, standard current drogue and parachute drogue.

constants for various crossections of the St. Lawrence estuary and river between Pte. des Monts and Lake St. Peter. A steady river discharge through the system is not reflected in the water levels, and its effect on the current at the outlet must be added as a constant current, with speed equal to the volume discharge rate divided by the cross-sectional area of the outlet.

8.14 Hydraulic method

The cross-sections of some narrow and shallow passages are too small to accommodate the large volume transports of water associated with the propagation of very long waves (tides, seiches, etc.). The result is that the water level rises or falls at one end of the passage, creating a hydraulic head between the two ends. The flow in the passage is said to be "hydraulic" if water enters the passage at very nearly zero velocity and is accelerated down the pressure gradient created by the hydraulic head. Neglecting frictional losses, the law of the conservation of energy tells us that the gain in kinetic energy per unit mass ($v^2/2$) along a streamline must equal the loss in potential energy (gh), where v is the current speed, g is the acceleration of gravity, and h is the hydraulic head.

Thus, we might expect to find the relation

(8.14.1) $v^2 = 2gh$

In practice, however the gauges at the ends of a passage might not be far enough apart to detect the full hydraulic head, and their zeros might not be set to exactly the same datum. The practical form of 8.14.1 is, therefore

(8.14.2) $v^2 = ah + b$

where a and b are constants to be determined by calibration against direct current measurements. The constant b may also include allowance for the initial kinetic energy possessed by water entering the system at a non-zero velocity.

Self-contained pressure gauges of the type described in section 6.7 are convenient to place at each end of the passage to measure the fluctuating hydraulic head, since they detect the sum of hydrostatic and atmospheric pressure, and it is to this combined pressure gradient that the water responds. These gauges can be left to operate unattended for several months, but during part of the period direct current measurements should be taken in the passage to permit determination of appropriate values for the constants a and b. Whether or not the method is applicable in a particular passage is revealed by how well the results of the calibration conform to equation 8.14.2, with a and b constant. It may be reasonable to permit choice of two sets of constants, one for flow in one direction and one for flow in the other, but if the results cannot even then be fitted satisfactorily to expression 8.14.2, it must be assumed that the flow is not sufficiently hydraulic for the method to apply.

8.15 Long wave method

The relation between the particle motion and the wave form was discussed for long waves in section 1.5, both for standing and progressive waves. If water level measurements in a region are available from enough locations, it may be possible to identify the propagation characteristics of long waves (e.g. tides and seiches), and so to deduce a great deal about the streams associated with them. This is another reason why water level measurements during a hydrographic survey should not be limited to the absolute minimum needed for sounding reduction.

8.16 Electromagnetic method

This method works on the same principle as the electric dynamo: if an electrical conductor is moved through a magnetic field, a voltage is developed along the conductor in proportion to the rate at which the conductor cuts through lines of magnetic force. Although pure water is a very poor conductor of electricity, most naturally occurring water (especially seawater) is a reasonably good conductor, because of the dissolved salts in it. The vertical component of the earth's magnetic field therefore causes an electric voltage to be generated in water that flows through it, and the voltage is proportional to the speed of flow as well as to the strength of the magnetic field. In theory, therefore the flow through a channel may be measured by placing the probes of a sensitive voltmeter in the water, one at each side of the channel, to detect the voltage generated by the flow. The voltage that is measured can be shown to be proportional to the total transport through the channel, rather than to the flow at a particular depth. This is because higher voltages generated at depths where the flow is greater are partially short-circuited by the water at depths where the flow (and hence the voltage) is less, so that an average voltage is detected. A further complication arises from the fact that the material on the bed of the channel, and beneath it, is not a perfect insulator, so that part of the signal is also short-circuited by this path. Because the conductivity of the material in and below the bed is never well enough known to permit calculation of its effect on the measurements, direct observations of the flow must be made during part of the installation period, to permit calibration of the system.

There are many practical difficulties to face in implementing such a measuring system. An insulated electrical cable must be led from the voltage recorder to the electrode at the far side of the channel; this is usually done by laying it along the bottom, unless there is a bridge along which it can be strung. The electrodes at the two sides of the channel must be carefully matched, since the voltage generated by their "battery effect" may otherwise be as great as the signal being measured. ln channels through which the flow is small, it is difficult to separate the signal from extraneous "noise" generated in the system. ln channels through which the flow is large enough to generate a strong signal, it is often difficult to lay and maintain the electrical cable intact to the far side. This is not a method of current measurement that is recommended for routine use on hydrographic surveys.

8.17 Geostrophic method

The Coriolis force, that results from the earth's rotation and deflects currents to the right in the Northern Hemisphere, was discussed in section 1.8. The forces acting on a current that experiences no acceleration must be in balance; the balance in the direction of flow being between the pressure gradient and the frictional forces, and the balance in the direction normal to the flow being between the pressure gradient and the Coriolis forces. If, in addition to zero acceleration, the assumption is made that friction is negligible, there would be nothing to balance a component of the pressure gradient in the direction of flow. The current would then have to flow in a direction normal to the pressure gradient, and at a speed just sufficient to produce a Coriolis force equal and opposite to the horizontal pressure gradient force. Such a current is said to be in "geostrophic" equilibrium. The hypothetical current whose Coriolis force just balances the horizontal pressure gradient force is called the "geostrophic current," and if the pressure field in the ocean is known, the rate and direction of the geostrophic current may easily be calculated. The geostrophic current will, however, resemble the actual current only insofar as the assumptions of zero acceleration and zero friction are true, and as the horizontal pressure gradient is accurately determined.

The horizontal pressure gradient in the ocean depends upon the horizontal atmospheric pressure gradient, the slope of the sea surface, and the distribution of water density within the body of the ocean. Oceanographers can calculate the density distribution from measurements of the water temperature and salinity, and so determine how the horizontal pressure gradient changes with depth. To convert these relative values to absolute values of the pressure gradient requires a knowledge of the actual pressure gradient for at least one depth. Since it is rarely possible to know the slope of the sea surface, the assumption is usually made that the horizontal pressure gradient is zero at some large depth (e.g. 2000 m). This depth is called the "depth of no motion," because the horizontal pressure gradient can be zero only if the Coriolis force, and therefore the current, is also zero. The relative geostrophic currents calculated from the density distribution may then be referred to zero at the depth of no motion, to obtain estimates of the absolute values of geostrophic current. In the open ocean, where friction and accelerations are small, the geostrophic currents resemble the actual currents reasonably well, and much has been learned about ocean circulation by this method.

In coastal waters, where friction and acceleration may not properly be neglected, great care must be taken if geostrophic currents are to be interpreted in terms of actual currents. If the current is known to have a fairly uniform direction of flow, we may consider only the balance of forces normal to the flow, and so avoid friction, which acts parallel to the flow. Also, if observations are taken over a long enough period, the effects of acceleration must average to near zero. Therefore, if the density distribution is determined as an average over a period sufficiently long to remove acceleration effects, and if only the component of the horizontal pressure gradient perpendicular to the main flow is used, the profile of the geostrophic currents so calculated should resemble reasonably well the profile of the actual average current over the same period. Selection of a reference for a geostrophic current profile in coastal water is difficult, since a depth of no motion cannot be assumed reasonably to exist in shallow water. This difficulty may be partly overcome by adjusting the current profile to satisfy the estimated transports of water and salt through the channel, or by measuring the average current directly at a particular depth, and fitting the profile to that value at that depth.

This discussion of geostrophic currents is given here not with the expectation that field hydrographers would be required to execute the associated observations or calculations, but with the realization that a great deal of the existing body of ocean current information has come from this source, so that the method should be understood and appreciated.

8.18 Current surveys - general remarks

Shipping and marine interests should be notified in a Notice to Mariners of any planned current survey that might in any way interfere with their operations. The information should appear in a Notice about a month before the commencement of the survey, early enough to be acted upon, but not so early as to be forgotten. It should tell the purpose of the survey, the general area and time period involved, the nature of the operation (e.g. current meter moorings, anchored vessels, tracking of drift poles, etc.), and should describe the appearance of any surface markers and surface drifters that are to be used. The locations of moorings and anchor stations that are planned should be given as closely as possible. An attempt should also be made to have the same basic information broadcast on the marine radio during the survey, particularly if the operation is in or near shipping lanes or fishing grounds. Another effective outlet for the information is sometimes the fishing and marine broadcast over the local commercial radio station. Such notices not only reduce the risk of lost or damaged equipment and of lost data, but they foster better public relations, by satisfying natural local curiosity. The issuing of notices, however, does not relieve the surveyor of the need to choose mooring locations and conduct operations in a manner that will cause the least disruption of other interests, while still providing the desired data. From the standpoint of equipment safety, the mooring of current meters in an area frequented by fishing draggers is particularly hazardous.

The convention for quoting the direction of currents is the exact opposite of that for winds: the current direction is the direction toward which it is flowing, whereas the wind direction is that from which it is blowing. The compasses in current meters are designed to record in agreement with this convention, and all manual records must accord with it as well.

As it is for water level observations, accurate time keeping is also important for current observations, particularly in tidal waters, where it is wished to relate the phases of the tidal streams to those of the constituents in the equilibrium tide. In the mooring and recovery logs, the time of every event that could be reflected in the current record should be recorded and described; e.g. rotor spinning in the wind, meter in water, anchor on bottom, anchor release tripped, etc. This information provides supplementary time checks on the records, which can frequently resolve uncertainties caused by an error in either the initial or final time check. In all records involving time, the zone time being used must be clearly indicated on every sheet by the appropriate abbreviation (GMT, AST, PDST, etc.).

It may be that the same current meter is to be moored more than once during the season, or even during a single survey. Even though the data storage capacity of the meter may be large enough to accommodate the combined records from several moorings, the data record should always be removed, and replaced by fresh magnetic tape (or film, etc.), before the meter is moored again . Keeping separate records for each installation reduces the possibility of confusion later on, but the main reason for removing the record before resetting is to protect if from loss or damage. The time, effort, and expense invested in a successful mooring are worth a great deal more than the material on which the data are recorded, and there is no economy in risking the data simply to conserve magnetic tape. There is, of course, no objection to cutting off the used portion and continuing with the unused portion of a tape or film, if this can be done with no risk to the record and if sufficient storage space remains on the unused portion. Magnetic tape records should be stored in ferrous metal containers and kept out of strong magnetic fields and excessive heat until the data has been extracted from them. All current records and supporting documentation should be submitted to the Regional Tidal Officer at the end of the season, or earlier if an opportunity is afforded. Where possible, copies should be made of records and documents, and stored separately from the originals.

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Vol. 4: Arctic and Hudson Bay
Vol. 5: Juan de Fuca and Georgia Straits
Vol. 6: Barkley Sound and Discovery Passage to Dixon Entrance

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British Columbia (two volumes)
Great Slave Lake and Mackenzie River
Labrador and Hudson Bay
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APPENDIX A

MAJOR TIDAL HARMONIC CONSTITUENTS

This is by no means a complete list of all the possible tidal harmonic constituents, but it does contain all the larger ones.

The "ratio" in column 2 is the amplitude of the constituent in the equilibrium tide divided by the amplitude of the M_2 constituent in the equilibrium tide. The "speed" in column 3 is the angular speed of the constituent in degrees per solar hour.

Description	Symbol	Ratio	Speed °/h
Mean value	Z_o	-	0.0000
Annual constituent (see section 2.6)	S_a	0.013	0.0411
Semi-annual constituent (see section 2.6)	S_{sa}	0.080	0.0821
Monthly constituent (see section 2.6)	M_m	0.091	0.5444
Fortnightly constituents	M_f	0.172	1.0980
(see section 2.6)	MS_f	0.009	1.0159
Diurnal constituents	K_1	0.584	15.0411
(section 2.5)	0_1	0.415	13.9430
	P_1	0.193	14.9589
Semidiurnal constituents	M_2	1.000	28.9841
(section 2.5)	S_2	0.465	30.0000
	N_2	0.194	28.4397
	K_2	0.127	30.0821
	L_2	0.028	29.5285
	T_2	0.027	29.9589
Quarter-diumal shallow-	M_4	-	57.9682
water constituents	MS_4		58.9841
(section 3.5)			

APPENDIX B

CANADIAN HYDROGRAPHIC SERVICE **METRIC**

FORM TWL-502/83

Temporary Gauge Data

NAME OF STATION

YEAR

SECTION 1. DATUM COMPUTATIONS AND LEVELLING SUMMARY

SECTION 1A. BY RECOVERY OF PREVIOUSLY ESTABLISHED CHART DATUM.

Reference bench mark .... is .... m. above chart datum.

BM(a) .... above chart datum	....	BM(a) .... above chart datum	....
BM(b) .... above/below BM(a) ..... :		BM(b) .... above/below BM(a) ..... :	
BM(b) .... above chart datum		BM(b) .... above chart datum	
BM(a) .... above chart datum	....	BM(a) .... above chart datum	....
BM(b) .... above/below BM(a) ..... :		BM(b) .... above/below BM(a) ..... :	
BM(b) .... above chart datum		BM(b) .... above chart datum	

BM .... above chart datum .... + ...

Zero of staff gauge below BM .... —

∴ Zero of staff gauge + above/— below chart datum ....*

Zero of automatic gauge + above/—below zero of staff gauge ....**

∴ Zero of automatic gauge + above/—below chart datum ....

SECTION 1B. BY TRANSFER OF A PREVIOUSLY ESTABLISHED CHART DATUM AT ______________ (Z)

USING OBSERVED HEIGHTS ☐, OR PREDICTED HEIGHTS ☐. (INDICATE).

MONTH	OBSERVED HEIGHTS AT TEMPORARY GAUGE		OBSERVED OR PREDICTED HEIGHTS AT (Z)	
DAY	H.W.	L.W.	H.W.	L.W.
SUM				
MEAN				
	(1)	(2)	(3)	(4)

(1) − (2) = r = ... (2) + ½ r = m = ...

(3) − (4) = R = ... (4) + ½ R = M = ...

ZERO OF AUTOMATIC GAUGE
REFERRED TO SOUNDING DATUM

= (M .... xr .... /R ....) − m ....

= .... − m .... = d = ______

DATUM FOR HEIGHTS IS HIGHER HIGH WATER LARGE TIDES

HHWLT at (Z) .... xr .... /R ....

Datum for heights ______________ above Sounding Datum

Zero of automatic gauge + above/− below sounding datum (d) ....*

Zero of staff gauge + above/− below zero of automatic gauge ....**

Zero of staff gauge + above/− below sounding datum (e) ....

BM .... above zero of staff gauge .... + ...

Zero of staff gauge + above/−below sounding datum (e) ....

BM .... above sounding datum .... +

BM(a) .... above sounding datum	....	BM(a) .... above sounding datum	....
BM(b) .... above/below BM(a) ...... :		BM(b) .... above/below BM(a) ...... :	
BM(b) .... above sounding datum		BM(b) .... above sounding datum	
BM(a) .... above sounding datum	....	BM(a) .... above sounding datum	....
BM(b) .... above/below BM(a) ...... :		BM(b) .... above/below BM(a) ...... :	
BM(b) .... above sounding datum		BM(b) .... above sounding datum	

*use appropriate signs. **From comparison Form 105-A.

SECTION 2. GENERAL INFORMATION

NAME OF STATION . . . . LOCATION . . . .

PROVINCE . . . . LATITUDE . . . . N. LONGITUDE . . . . W.

FIELD SHEETS AFFECTED . . . .

TIME ZONE OF OBSERVATIONS + ... OR . . . . . . PERIOD GAUGE IN OPERATION: FROM . . . . TO . . . .

O.I.C. . . . . AGENCY . . . .

MAKE AND MODEL OF GAUGE: . . . . SERIAL NO. . . . . TYPE OF SENSOR: . . . .

TYPE OF RECORD: . . . . RANGE OF GAUGE: . . . . .m.

SECTION 3. TYPE OF RECORD OBTAINED AND LOCATION OF GAUGING SITE.

Year . . . . TYPE OF RECORD: Continuous ☐ Or Staff gauge readings: hourly ☐ or . . . . .times daily ☐

The gauge was located . . . .

. . . .

Year . . . . TYPE OF RECORD: Continuous ☐ Or Staff gauge readings: hourly ☐ or . . . . .times daily ☐

The gauge was located . . . .

. . . .

Year . . . . TYPE OF RECORD: Continuous ☐ Or Staff gauge readings: hourly ☐ or . . . . .times daily ☐

The gauge was located . . . .

. . . .

Year . . . . TYPE OF RECORD: Continuous ☐ Or Staff gauge readings: hourly ☐ or . . . . .times daily ☐

The gauge was located . . . .

. . . .

SECTION 4. METHOD BY WHICH BENCH MARK ELEVATIONS WERE ORIGINALLY ESTABLISHED AND WERE MAINTAINED DURING THE PERIODS OF OPERATION OF THE GAUGE SITE.

Year

Year

Year

Year

SECTION 5. TABULATION OF BENCH MARK ELEVATIONS

Date	O.I.C.	Establishment	B.M. No.	B.M. No.	B.M. No.	B.M. No.	B.M. No.	Datum Used

SECTION 6. DATUM FOR HEIGHTS . . . . **above Sounding Datum** . . . . **above Chart Datum**

SECTION 7. BENCH MARK DESCRIPTIONS

B.M. NO .... A .... (agency) bronze tablet stamped .... set

Sounding datum .... m. below.

Chart Datum .... m. below.

G.S.C. elevation ....

IGLD (1955) elev. ....

BM condition ....

correct ☐ amended ☐

B.M. NO .... A .... (agency) bronze tablet stamped .... set

Sounding datum .... m. below.

Chart Datum .... m. below.

G.S.C. elevation ....

IGLD (1955) elev. ....

BM condition ....

correct ☐ amended ☐

B.M. NO .... A .... (agency) bronze tablet stamped .... set

Sounding datum .... m. below.

Chart Datum .... m. below.

G.S.C. elevation ....

IGLD (1955) elev. ....

BM condition ....

correct ☐ amended ☐

SECTION 8. SKETCH correct ☐ amended ☐

PLACE

FILE NO.

F.TWL-502/83

Temporary Gauge Data

This form combines a computation sheet for datums and Bench Mark elevations, a gauge history for past and present data and a Bench Mark sketch with descriptions along with detailed instructions for standardizing its use. The gauge history will be supplied along with Bench Mark data when required.

Section 1. This section is for computing the elevation of all Bench Marks and the zero of the automatic gauge relative to chart or sounding datum. The Bench Marks should follow the pattern indicated by the letters a and b, and the convention + above and - below should be used throughout having careful regard to the exact context.

Section 1A. is used by successive parties when the elevation of Chart Datum below a Bench Mark is known. The elevation of all Bench Marks in the net and the zero of the staff gauge are related to Chart Datum. The zero of the automatic gauge relative to the zero of the staff gauge is found by comparing simultaneous readings recorded on the comparison form and hence the zero of the automatic gauge relative to Chart Datum.

Section 1B. This section is used by the initial party to transfer Chart Datum from a place (Z) where it has already been established. The table should be completed using the days with maximum tidal range available. The mean values are used as indicated to compute r, R, m, M, and hence "d" the amount the zero of the automatic gauge is above (+) or below (-) Sounding Datum. The zero of the staff gauge relative to the zero of the automatic gauge is found by comparing simultaneous readings recorded on the comparison form and hence the zero of the staff gauge is found relative to Sounding Datum. The elevation of all Bench Marks in the net are then computed above Sounding Datum. The Higher High Water Large Tides (HHWLT) datum for heights can be calculated using the HHWLT at (Z) and r/R as indicated. The HHWLT at (Z) is found by applying the HHWLT height difference at Z to the HHWLT height at the appropriate reference port. If this data is not published, it will be supplied by the Tides and Water Levels Section.

Section 2. All parties insert appropriate data (In sections 3, 4 and 5 all parties complete one section of each).

Section 3. Indicate type of record and insert a brief description of the site.

Section 4. Insert a brief description of the method used for establishing the Bench Mark elevations e.g. water level transfer from "Z" or levelling run from Geodetic Bench Mark No. "CCX". Similarly for successive years e.g., levelling run from controlling Bench Mark No. 1.

Section 5. The elevations of Bench Marks are accepted as those computed by the party which establishes them. Successive parties tabulate the results of levelling lines and identify the controlling Bench Mark with an asterisk e.g. 12.23*.

Section 6. The initial party inserts Datum for Heights. The calculation of this Datum in section 1 B is a water level transfer and it is therefore referred to Sounding Datum. Its elevation above Chart Datum will be entered by the Tides and Water Levels Section.

Section 7. The initial party inserts Bench Mark data and concise descriptions using the printed format. The elevations of Bench Marks are always above Sounding Datum unless a Bench Mark elevation above Chart Datum has been used in the levelling run. Otherwise, elevations above Chart Datum will be entered by the Tides and Water Levels Section. Photographs of each Bench Mark with their number and location marked, should be submitted along with other gauge data. Successive parties insert the condition of each Bench Mark e.g., good, unreliable, destroyed or not located, and check accuracy of descriptions making amendments if necessary.

Section 8. The initial party draws a sketch showing location of Bench Mark, automatic gauge and staff gauge, showing distances from conspicuous fixed points. Successive parties check for amendments if necessary.

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