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NOTE ON ITERATIVE METHODS FOR CALCULATING DIRECTIONAL SPECTRA

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Abstract

There have appeared in the literature iteration schemes for improving the resolution of Maximum Likelihood estimates of directional spectra from heave/pitch/roll wave buoy data. In this note we provide illustrations showing,that when iteration schemes with a firmer foundation are used, the iteration converges to spectra very closely resembling Maximum Entropy spectra. Comparisons between Maximum Entropy and iterated Eigenvector spectra are also made.

Résumé

Il a été publié dans la littérature des schémas d'itération pour améliorer la resolution des estimations du maximum de vraisemblance des spectres directionnels à partir de données de bouées mesurant le pilonnement, le tangage et le roulis. Le présent article contient des illustrations montrant que lorsque des schémas d'itération à fondement plus solide sont utilisés, l'itération converge vers des spectres ressemblant étroitement à des spectres d'entropie maximale. Des comparaisons entre les spectres d'entropie maximale et de vecteur propre itéré sont également effectués.

In his 1983 paper, Pawka showed that the resolving power of the Maximum Likelihood Estimate (MLE) of a directional-spectrum may be improved by a certain iteration process (type "P" iteration). In 1984 Oltman-Shay and Guza gave a simple formula for calculating the MLE spectrum from heave/pitch/roll data. They used type "P" iteration to improve the spectrum and suggested a variation (type "O" iteration) which seemed to produce a somewhat better spectrum. In 1986 Lygre and Krogstad described a simple formula for calculating the Maximum Entropy (MEM) directional spectrum from the data of a heave/pitch/roll buoy.

A variety of directional spectrum methods have been tested out in our organization to find those suitable for use in data products. In looking at spectrum plots from Wavec data it became evident that the iterated MLE and the MEM spectra were generally very much alike, the main difference being that the MEM peaks were more pronounced. The authors of the iteration techniques have not claimed any rigorous basis for their schemes, so the question arises: Is there another variation having a more rational foundation, and which leads to spectra very close to or identical with MEM spectra? If so, then perhaps wherever iteration is used, it could be replaced by the simpler and more direct MEM method.

In general terms, as applied to heave/pitch/roll data, the MLE iteration schemes work as follows: The MLE directional spectrum $E0(\theta)$ at a particular frequency is calculated from the six given cross-spectral coefficients at that frequency. Here θ is direction, in $(-\pi,\pi)$. The MLE spectrum is known to be a smoothing of the true spectrum. Substituting it in the six integral formulas defining the cross-spectral coefficients (Oltman-Shay and Guza,1984) yields a new set of coefficients, and a "twice-smoothed" MLE spectrum called $T(\theta)$ is gotten from these. By applying the difference between E0 and T to E0 in a backward direction a first estimate of the unsmoothed spectrum $E(\theta)$ is obtained. The process may be iterated, and will reach convergence when E yields the original cross-spectral coefficients, and the T spectrum is identical to E0. The "O" version uses straight backward differencing, and the "P" version uses scaled backward differencing. The iteration is evidently a de-convolution, or de-smoothing process. There are an unlimited number of solutions all of which are possible true spectra.

Smoothing is customarily done either by convolution, or by application of the response function to the fourier transform of the data. In the present case, the response function can be estimated by decomposing the E0 and T spectra into their fourier coefficients (auto-covariances) and taking the ratio of corresponding coefficients. The estimated unsmoothed spectrum E can then be obtained by applying the inverse of the response to the coefficients of the E0 spectrum and then performing a fourier synthesis. This appears to be a more logical approach to de-smoothing than the backward differencing schemes. The iterative algorithm may be described as follows:

Let $C0_k$ be the fourier coefficients of the MLE spectrum $E0(\theta)$, where k= ...,-1,0,1,... and $C0_{-k}$ is the complex conjugate of $C0_k$. Similarly CT_k^i and C_k^i are the coefficients of the ith iteration of the $T(\theta)$ and $E(\theta)$ spectra respectively. The iteration formula is:

$$C_k^i = C_k^{i-1} (C0_k / CT_k^{i-1})^{1/\gamma} ,$$

where γ is some number 1.0 or greater,which slows down and stabilizes the iteration. $E^i(\theta)$ is obtained by fourier synthesis of the C_k^i . $E0(\theta)$ is used as the original estimate of $E(\theta)$, and $T^{i-1}(\theta)$ is the MLE spectrum from cross-spectral coefficients yielded by $E^{i-1}(\theta)$.

We tried out this "Response" type iteration using both real and artificially created sets of cross-spectral coefficient data. In some cases, a spectrum very close to the MEM spectrum was obtained, but very often, especially with low noise data, severe convergence problems were encountered. Instability problems are to be expected in de-smoothing, so we tried iterating on the logarithm of the spectrum rather than the spectrum itself, in order to lessen the scale of reduction of sharp peaks in the response and to ensure a positive spectrum. The only additions needed in the algorithm are to take the log of the spectrum before each fourier analysis, and the exponential after each synthesis. There were occasional convergence problems, but this "Log-Response" scheme generally converged to something very close to MEM when applied to both real and artificial data. Figure 1 shows a varied sampling of spectra from real data. Type ”O” iteration is shown by the dotted line, type ”P” by short dashes, ”Log–Response” by long dashes, and MEM by the solid line. The small circle and arrows show the mean direction and angular width calculated by standard formulas. There is evidently a distinct sequence in the resolution of peaks by the three iterative methods, and the ”Log–Response” method generally matches the MEM very closely. In some of the spectra it is hard to distinguish the solid line from the long dashes. With artificial data the match was usually even closer, but exact convergence to MEM was never obtained.

Convergence of the ”Log–Response” iteration (and of the ”Response” iteration when it did converge) was usually quick. In the tests we used 16 iterations with γ=4 for the first eight iterations, 2 for the next four, and 1 for the remaining four. For integration and fourier analysis the spectrum was sampled at 128 points over $-\pi$ to π . For types ”P” and ”O” iteration we used γ=10, with 200 iterations.

The functional form of a MLE spectrum is the reciprocal of a second degree fourier sum. This sum is evidently a simpler and generally more gently varying function of direction than the MLE spectrum itself. Because of this one might expect the iteration schemes to work better when applied to the inverse spectrum rather than the spectrum itself. This was tried on the four iteration schemes that have been discussed so far, with the following results:

The ”Log–Response” method gave exactly the same result as before. (Taking the reciprocal merely changes the sign of the log and does not otherwise affect the iteration process.) The ”P” spectrum was noticeably closer to the MEM than before. The ”O” and ”Response” methods converged exactly to the MEM. These two methods preserve the original functional form of the MLE spectrum throughout the iteration. This form plus the convergence condition that the final spectrum $E(\theta)$ yield the original cross–spectral coefficients are equivalent to the defining relations for the MEM spectrum. (Smylie et al. 1973) So when convergence occurs, it would be expected to be to MEM exactly.

All of the above does not prove that the MEM directional spectrum is the one and only ”proper” end result of iterating on a MLE spectrum. But it does seem fair to say that the original iterated ”P” and ”O” spectra do resemble MEM spectra, and when a more logical basis for the iteration is introduced the resemblance becomes very close, and in some cases identity.

Another method for calculating directional spectra that has been put forward recently is the Eigenvector (EV) method of

Marsden and Juszko, 1987. They applied type ”P” iteration to EV spectra and found improved resolution in many cases. The uniterated EV method generally produces sharp peaks to start with, however out of general interest we have repeated for EV the iteration tests just done for MLE. The results are shown in Figure 2. As before the short dashes represent the type ”P” spectrum, the long dashes the ”Log Response”, and the solid line MEM. The type ”O” spectrum is not plotted as it was never used by Marsden and Juszko. The ”Log–Response” spectra show a distinct resemblance to MEM, but not as close as with iteration on MLE. It is interesting to note that the type ”P” spectra are considerably closer to the MEM here than with iteration on MLE.

As with MLE we also tried iterating on the reciprocal of the EV spectrum, and exactly the same comments apply here as for MLE. (The EV spectrum also has the functional form of an inverse second degree fourier sum.)

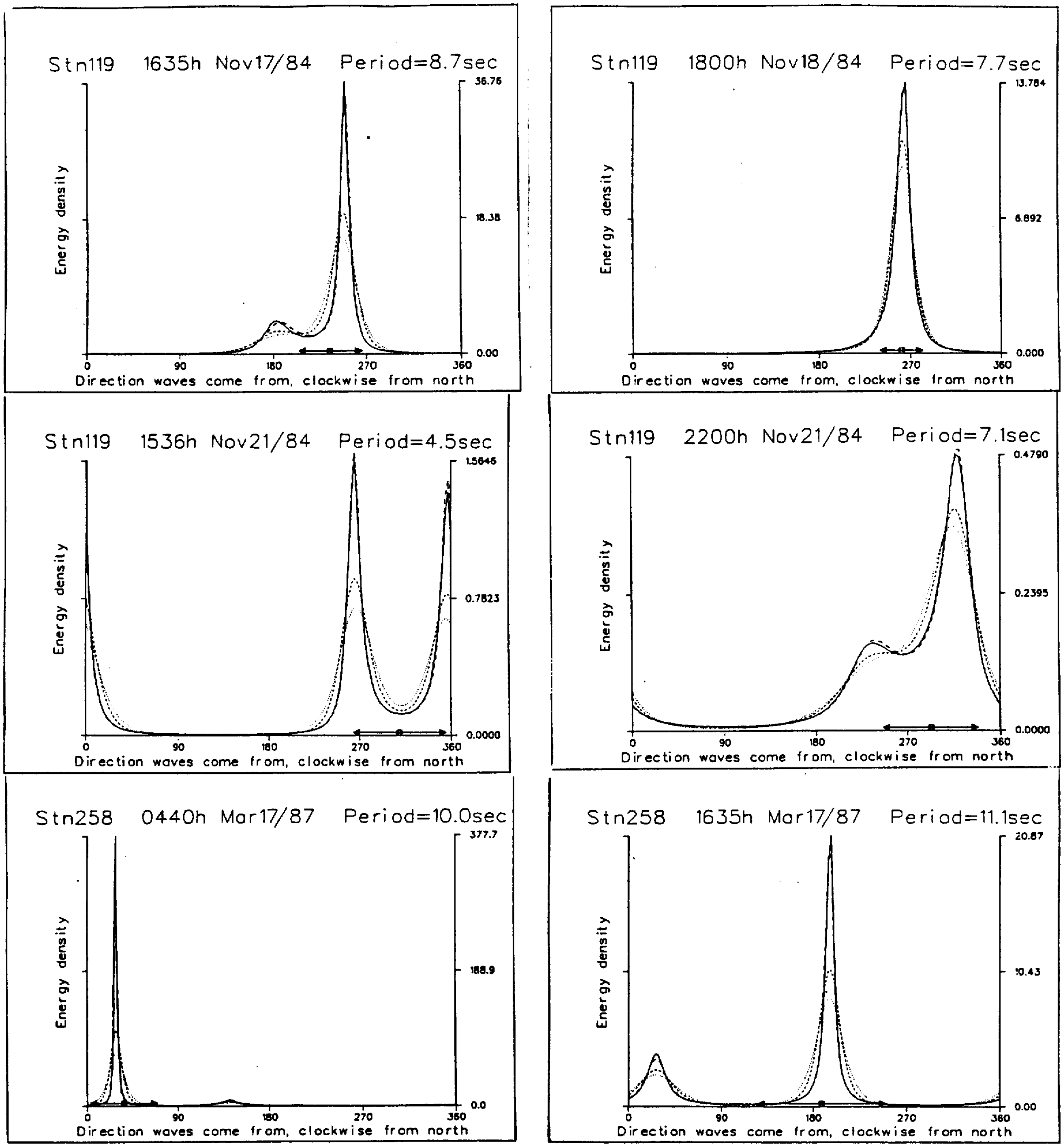

Figure 1. Comparison of MEM and Iterated MLE Directional Spectra

Solid line: MEM spectrum

Long dashes: Log-Response type iteration

Short dashes: Type "P" iteration

Dotted line: Type "O" iteration

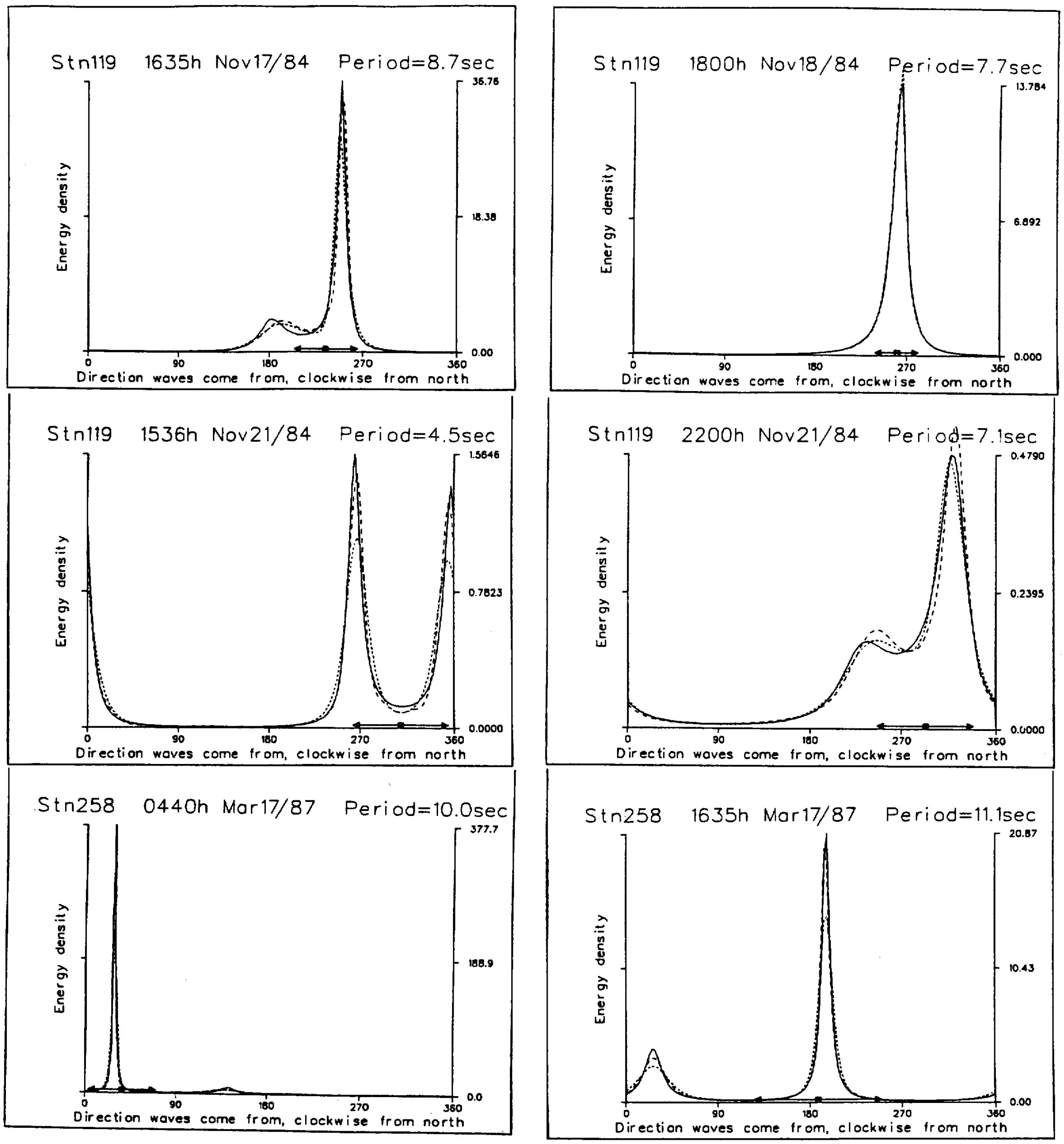

Figure 2. Comparison of MEM and Iterated EV Directional Spectra

Solid line: MEM spectrum

Long dashes: Log-Response type iteration

Short dashes: Type "P" iteration

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