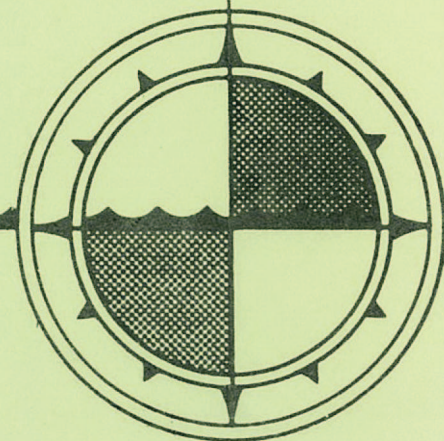


KITIMAT PHYSICAL OCEANOGRAPHIC STUDY 1977-1978
PART 6
TEMPORAL VARIATIONS OF THE
BAROCLINIC TIDE IN DOUGLAS CHANNEL

by
Ian Webster
Dobrocky SEATECH Ltd.

For
INSTITUTE OF OCEAN SCIENCES
Sidney, B.C.



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ABSTRACT

This report is one of a series describing the implementation and results of a study of the circulation in the channels which form the seaward approaches to the port of Kitimat, British Columbia. It was evident from the analyses of current meter data collected throughout the study area that the tidal currents in these channels had a pronounced degree of baroclinicity. In the following report, the nature of the baroclinicity in Douglas Channel is examined.

The variations in this baroclinicity as they occur on a seasonal basis and over shorter time scales are described. On the assumption that the observed baroclinicity is due to internal Kelvin waves the observed currents are fitted by a sum of such waves whose forms are calculated from the observed vertical density structure in Douglas Channel. Finally, the possible origin of the baroclinicity and the implications of its presence to the practical tidal current prediction problem in Douglas Channel are discussed.

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PREFACE

The purpose of the present study was to gain an understanding of features of the tidal, estuarine, and wind driven circulations in the seaward approaches to the port of Kitimat. To collect sufficient data to describe these circulations in such a complex system of channels as the one studied was recognized at the outset as being beyond the available resources of the study. Nevertheless, much information on these circulations in the region was learned which is of great value in itself, but perhaps is even more valuable as a framework from which future studies can be designed and interpreted. Major deficiencies in our understanding of the circulations remain which could be resolved by further work both in the field and on data which have already been collected. Time constraints dictated that most of the data collected during the study could not be examined in detail. The data analysis procedures were chosen to outline those features of the data necessary for the application of a tidal circulation model and for descriptions of the estuarine and wind driven circulations which were mostly non-dynamical. It was hoped that by providing a generalized description of the circulations in the channel approaches to Kitimat rather than concentrating on several smaller scale phenomena, which may be more scientifically interesting, the utility of the reporting of the study results would be maximized.

The implementation and results of the study are presented in the six volumes:

Kitimat Physical Oceanographic Study 1977 - 1978
Data Collection and Analyses

Kitimat Physical Oceanographic Study 1977 - 1978
A Manual for General Data Access

Kitimat Physical Oceanographic Study 1977 - 1978
Estuarine Circulation

Kitimat Physical Oceanographic Study 1977 - 1978
Tidal Circulation Model

Kitimat Physical Oceanographic Study 1977 - 1978
The Response of Douglas Channel to Meteorological Forces

Kitimat Physical Oceanographic Study 1977 - 1978
Temporal Variations of the Baroclinic Tide in Douglas Channel

1. INTRODUCTION

In the period July 1977 to June 1978, Dobrocky SEATECH undertook a study of the circulation in the channels which form the seaward approaches to the port of Kitimat in northern British Columbia (see Figure 1). As part of this study a one-dimensional, barotropic tidal model was developed for the channel complex. This model is described in "Kitimat Physical Oceanographic Study 1977-1978 - Tidal Circulation Model". The barotropic tidal currents are caused by pressure forces arising from long channel variations in the tidal height. Outside the bottom boundary layer the velocity field associated with a travelling barotropic wave is dependent upon the ratio of the wavelength to the water depth. Since the wavelength of the barotropic tide (typically > 2,000 km in the Kitimat channels) is much greater than the water depths (up to 600 m) the barotropic tidal velocity field is expected to be virtually independent of depth outside the bottom boundary layer.

During the Kitimat study, tide gauge and current meter data were collected throughout the region for use as input into the barotropic tidal model. It became apparent in analyzing the data from the current meters that the tidal currents in the Kitimat channel system showed a marked degree of baroclinicity, that is they varied in amplitude and in phase with depth. This baroclinicity makes the determination of the barotropic component of current from current meter data more difficult. Also, since the baroclinic components of current appeared to be comparable in size to the barotropic current components they are an important consideration in the practical tidal current prediction problem for the region.

In this report the nature of the baroclinic currents are examined for a location near the southern end of Douglas Channel which forms part of the Kitimat channel system. Analyses of current meter data from this location showed major temporal changes in the degree of baroclinicity in the currents. This variability and the possible origin of the baroclinicity are discussed.

2. DATA COLLECTION IN DOUGLAS CHANNEL

2.1 Area Description

Douglas Channel provides direct oceanic access to the port of Kitimat. (see Figure 2). It is defined on its western side by the mainland of British Columbia and on its eastern side by Hawkesbury Island. At its northern end it adjoins Kitimat Arm and Devastation Channel; at its southern end it opens into Wright Sound. The length of Douglas Channel (about 55 km) can be divided into two halves. The southern half running almost directly in a north/south direction has an average width of about 4 km; the northern half which runs in a direction of about 45° true is somewhat narrower having a width of 2 to 3 km. Depths within Douglas Channel are mostly in excess of 350 m. A sill of depth 240 m which crossed the channel between its northern and southern halves is the only prominent bathymetric feature (see Figure 3).

2.2 Data Collection

Current meter and thermistor chain data, which are referred to in this report, were collected from locations in the southern half of Douglas Channel. Five current meters were deployed on the two moorings designated CM 2 and CM 3. Three current meters were supported at depths of 30 m, 165 m and 305 m by the subsurface mooring CM 2; the surface mooring CM 3 supported current meters at depths of 5 m and 17 m. Mooring CM 2 was located in mid-channel approximately 10 km south of the sill in Douglas Channel whereas CM 3 was located 1.3 km further south (see Figure 3 and 4). A thermistor chain designated TMS 2 was located 0.9 km due north of CM 2. This thermistor chain which was supported by a Geodyne buoy made temperature measurements at depth increments of 5 m from the surface to a depth of 50 m.

Data were collected from the thermistor chain TMS 2 and the current meters on moorings CM 2 and CM 3 over a five month period between the middle of July 1977 and the middle of December 1977. These data records were broken in the middle when the instruments were recovered, serviced and redeployed near the end of September. After the current meters on mooring CM 2 were recovered in December 1977 they were serviced and reinstalled. Apart from an additional servicing in March 1978 these instruments collected data until June 1978. Details of the times of instrument operation as well as instrument descriptions and deployment procedures are provided in "Kitimat Physical Oceanographic Study 1977-1978 - Data Collection and Analyses".

In addition to the measurements made in Douglas Channel reference in this report is made to tidal heights measured by tide gauge TG 4 located in Otter Channel (see Figure 4). This tide gauge was operational for the time period July 1977 to December 1977. A comparison of the tidal heights and phases for the M_2 constituent between TG 4 and a tide gauge located in Kitimat Arm (TG 1) over the $2\frac{1}{2}$ month period September 1977, to December 1977, showed the amplitude at TG 1 to be 6% higher than that at TG 4 and the phase at TG 4 to lead that at TG 1 by about 3° . Thus the tidal heights and phases observed at TG 4 should be fairly representative of those within Douglas Channel.

3. TIDAL CURRENTS IN DOUGLAS CHANNEL

3.1 Seasonal Variations

Harmonic analyses were applied to each of the two 2½ month sections of velocity record from the five current meters. These analyses showed that at all depths the tidal currents were dominated by the semi-diurnal constituents M_2 , S_2 , and N_2 in decreasing order of importance. The M_2 constituent has a period of 12.42 hours, the S_2 a period of 12.00 hours and the N_2 a period of 12.66 hours. Next in order of importance were the K_1 and O_1 diurnal constituents having periods of 23.93 hours and 25.82 hours, respectively.

The semi-diurnal currents in Douglas Channel at the current meters ran primarily back and forth in a north/south direction. The cross channel tidal flows were generally less than 20% of the long channel tidal flows. Such flows are expected in a long, fairly straight section of channel such as the southern half of Douglas Channel. The amplitudes and phases of velocities in the report will refer to the amplitudes and phases of the north/south components of the current velocities. Note that if $u(t)$ is the contribution to the total north/south component of velocity at time, t , from the tidal constituent having a frequency, ω , then the amplitude, A , and the phase, θ , of this contribution are defined such that:

$$u(t) = A \cos(\omega t - \theta)$$

The comparison of the tidal velocities from different depths showed these to vary markedly in amplitude and phase, that is a high degree of baroclinicity was indicated. Figure 5 compares the velocity amplitudes of the M_2 , S_2 and N_2 constituents for the first 2½ month period, July to September. The M_2 amplitude decreases from almost 20 cm/sec at 5 m depth to less than 10 cm/sec at the bottom two meters. Although the S_2 and N_2 amplitudes are less than half the size of the M_2 amplitude these show similar decreases in amplitude with depth. The corresponding changes of phase of the three semi-diurnal velocity constituents are compared in Figure 6. For each constituent the phases are relative to the phase of tidal height obtained from the harmonic analysis of data from tide gauge TG 4. The phases at 17 m and 30 m depths for the three constituents lead those at 5 m, 165 m and 305 m depths. The similarity of the depth dependencies of amplitudes and phases for the three largest semi-diurnal constituents suggests that the nature of their baroclinicity is likewise similar.

The velocity amplitudes and phases of the M_2 constituent for the July to September period are compared to those from the successive periods September to December, December to March, and March to July in Table 1. The indicated phases are relative to the phase of the M_2 constituent of tidal height which varied by less than 5° over the four periods. Whereas in the period July to September the M_2 velocity amplitudes decreased with depth, in the period September to December the largest amplitude occurred at 17 m depth. At 30 m depth the largest M_2 velocity amplitude (14 cm/sec) occurred in the September to December period. In the March to June period this amplitude had dropped to 9 cm/sec. Less pronounced changes occurred in the M_2 velocity amplitudes at 165 m and 305 m depths. Dramatic changes occurred in the depth dependence of the M_2 velocity phases over the four periods. As in the July to September periods, in the September to December periods the phases achieved a maximum at 17 m and 30 m depths. The phases at the five depths fell within a 27° range in the first period and within a 20° range in the second. However, in the December to March period which followed the difference in phase between 30 m depth and 305 m depth was 70° .

3.2 Weekly Harmonic Analyses

Evidently, significant changes in the depth dependence of the M_2 tidal velocities occurred over the four time periods examined. To examine the temporal variations of the baroclinicity over shorter time scales harmonic analyses were carried out on the velocity time series from the current meters on a weekly basis. A weekly analysis is not able to differentiate the contributions of the individual semi-diurnal constituents (M_2 , S_2 and N_2); the resultant semi-diurnal constituent which was calculated for the M_2 frequency was strongly modulated by the presence of the other constituents both in amplitude and in phase. In effect, the weekly harmonic analysis procedure has a transfer function which considerably modifies the calculated amplitudes and phases of the M_2 constituent.

One of the results of the barotropic tidal model was that the semi-diurnal barotropic tidal velocities in Douglas Channel were almost directly proportional to the semi-diurnal constituents of tidal height. Weekly harmonic analyses were performed on the tidal height records from the tide gauge TG 4. The resulting semi-diurnal tidal height amplitudes, h , were used to calculate 'barotropic' semi-diurnal velocity amplitudes, A_b , for each week according to:

$$A_b = 4.9 \times 10^{-2} \text{sec}^{-1} h$$

The proportionality factor was obtained from the results of the tidal model. The velocity amplitudes, A , and the velocity phases, θ , obtained from the weekly analyses of the data from the five current meters were normalized as:

$$A_N = A/A_b$$

$$\theta_N = \theta - \theta_h$$

TABLE 1 Seasonal Variation of the M₂ Velocities in Douglas Channel

<u>M₂ VELOCITY AMPLITUDE (cm/sec)</u>					
<u>Time Period</u>	<u>5 m Depth</u>	<u>17 m Depth</u>	<u>30 m Depth</u>	<u>165 m Depth</u>	<u>305 m Depth</u>
July - Sept.	20	18	13	8	8
Sept. - Dec.	15	17	14	6	8
Dec. - Mar.			10	8	9
Mar. - June			9	8	8

<u>M₂ VELOCITY PHASE</u> (Relative to phase of M ₂ tidal height constituents)					
<u>Time Period</u>	<u>5 m Depth</u>	<u>17 m Depth</u>	<u>30 m Depth</u>	<u>165 m Depth</u>	<u>305 m Depth</u>
July - Sept.	-77	-62	-50	-75	-72
Sept. - Dec.	-82	-64	-64	-69	-84
Dec. - Mar.			-45	-79	-115
Mar. - June			-122	-110	-128

Where A_N and 0_N are the normalized semi-diurnal velocity amplitudes and phases and 0_h is the phase of the semi-diurnal tidal height constituent. The legitimacy of this normalization procedure depends upon the velocity amplitudes of the M_2 , S_2 and N_2 constituents at each depth being in similar ratios to the tidal height amplitudes for the three constituents. Also, the relative phases of these constituents of tidal velocity should be similar to the relative tidal height phases. Based upon the results of the harmonic analyses of the $2\frac{1}{2}$ month records, both these criteria appear to be fairly well satisfied.

Figure 7 shows the normalized weekly semi-diurnal velocity amplitudes for the five current meter depths. These data are presented for twenty-one consecutive weeks (except for week #11) starting on July 17, 1977 and ending on December 12, 1977. These normalized amplitudes undergo significant temporal variations. At 5 m depth the amplitudes on weeks #5 and #6 exceed three times the 'barotropic' amplitude; on week #15 the amplitude at 5 m depth is less than the 'barotropic' amplitude. At greater depths the magnitudes of the fluctuations are less, but these still represent a large proportion of the average amplitude at each depth. Figure 8 shows the normalized velocity phases of the semi-diurnal constituents at depths of 5 m, 30 m and 305 m. The phases at 17 m depth tend to be similar to those at 30 m depth and the phases at 165 m depth are mostly similar to those at 305 m depth. Over the period of the record the phases undergo large variations sometimes between one week and the next. Between week #1 and week #21 the normalized phases range from -108° to -59° at 5 m depth; from -76° to -44° at 17 m depth; from -81° to -29° at 30 m depth; from -85° to -63° at 165 m depth and from -103° to -65° at 305 m depth.

It is presumed that changes in the baroclinicity of the semi-diurnal currents are associated with changes in the density structure and in the mean horizontal velocity shear in Douglas Channel. In common with most inlets along the British Columbian coast with appreciable freshwater inputs Douglas Channel has a relatively low salinity, low density, surface layer of the order of 10 m in thickness. Figure 9 shows the average weekly water densities (plotted as σ_t) obtained from the five current meters for weeks #1 to #21. Marked changes occur in the water density at 5 m depth over this period; smaller changes occur at greater depths. The densities at 5 m depth indicate two minima one probably prior to week #1 and the second in weeks #15 and #16. These minima are associated with periods of high freshwater discharge into Douglas Channel (see "Kitimat Physical Oceanographic Study 1977-1978 - Estuarine Circulation").

The north/south components of velocity low pass filtered to remove the diurnal and semi-diurnal tidal fluctuations are shown

plotted in Figure 10 for 5 m, 17 m 30 m and 165 m depths. The behaviour of these currents up to week #11 is different from that in week #12 on. Up to week #12 the currents at 5 m and 17 m depths tended to be southwards; those at 30 m and 165 m depth were weaker and changed direction over this period. From weeks #12 to #21 the currents at 5 m, and 17 m and 30 m depths were stronger and underwent a number of reversals in direction apparently caused by the wind.

If changes in the baroclinicity of the semi-diurnal tidal velocities in Douglas Channel are viewed with respect to changes in the density and mean velocity structure, three characteristic periods are evident. In the first period which includes the weeks up until about week #12, the baroclinicity as indicated by the difference in the semi-diurnal velocity amplitude between the top two and the bottom two meters increases to about double the 'barotropic' velocity. During this time the density at the 5 m depth current meter shows an overall increase of about $3 \sigma_t$ units and the near surface non tidal flows in the channel are predominantly southwards.

In the second period which includes weeks #14 to #18 roughly, the differences in amplitudes between the semi-diurnal velocities at the five depths decreases somewhat. The differences in phase between these velocities remains large so that the level of baroclinicity could still be considered high. This period is characterized by the appearance of relatively low water densities at the top three current meters and by the occurrence of strong wind driven currents in Douglas Channel.

The nature of the baroclinicity changes again in the third period which includes weeks #19, #20, and #21. In this period the phase differences in the semi-diurnal velocities at the five depths are smaller than they are prior to week #19. Whereas in the weeks up to week #12 the amplitude of the semi-diurnal velocity at 30 m depth is approximately half way between the amplitudes at the top two and the bottom two depths, in weeks #19, #20 and #21 the 30 m amplitude is about the same as those at the top two depths. During this time the density differences between the top three depths are less than $0.5 \sigma_t$ units.

4. DESCRIPTION OF VELOCITY STRUCTURE OF DOUGLAS CHANNEL AS BAROCLINIC WAVES

4.1 Theoretical Considerations

It is conjectured that the observed baroclinicity in Douglas Channel is due to propagating internal Kelvin waves. The governing equation for the propagation of an internal Kelvin wave along a straight vertical boundary can be shown to be:

$$\frac{\partial^2 \phi(z)}{\partial z^2} + \frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial z} \frac{\partial \phi}{\partial z} + \frac{1}{c^2} \left(g \frac{\partial \bar{\rho}}{\partial z} - \omega^2 \right) = 0 \quad 1)$$

This equation is solved subject to the boundary conditions:

$$\phi(0) = \phi(h) = 0$$

where:

- $\phi(z)$ is the vertical displacement amplitude of the internal wave as a function of depth, z .
- c is the horizontal component of the phase speed of the internal wave parallel to the direction of the boundary (the 'x' direction).
- ω is the angular frequency of the internal wave.
- h is the water depth.
- $\bar{\rho}(z)$ is the mean water density as a function of depth.

Equation 1) can be derived from the fluid momentum and continuity equations subject to the conditions that:

1. The fluid is inviscid and incompressible.
2. The waves are of small amplitude so the equations can be linearized.
3. The water depth, h , is constant.
4. The mean velocities in the fluid are all zero.
5. The horizontal derivatives of density are zero.
6. The vertical displacement of water particles as a function of distance, x , and time, t , takes the wave-like form:

$$\phi(x, z, t) = \phi(z) e^{i \left(\frac{\omega x}{c} + \omega t - \theta \right)}$$

where θ is a phase constant.

The solutions of equation 1) for specified vertical profiles of density were obtained numerically using computer software developed by P. Fryer (unpublished manuscript). The wave frequency, ω , was taken to be the M_2 frequency, that is $\omega = 1.41 \times 10^{-4} \text{sec}^{-1}$. For the solution to exist as a propagating wave a necessary condition is that:

$$N = \sqrt{\frac{-g}{\rho} \frac{\partial \rho}{\partial z}} > \omega$$

Even in the deep water of Douglas Channel where the stratification was less strong, values of N always exceeded 10^{-3}sec^{-1} over the period July to December, 1977. The solution of equation 1) consists of a series of eigenfunctions which will be termed the first, second, etc., baroclinic modes. Let $\phi_n(z)$ be the n 'th mode eigenfunction for a wave propagating in the positive x direction and let c_n and θ_n be the horizontal phase speed and phase corresponding to this solution. Through application of the continuity equation it can be shown that the horizontal velocity field associated with this wave, $\psi_n(x, z, t)$, is given by:

$$\psi_n(x, z, t) = c_n \frac{\partial \phi_n(x, z, t)}{\partial z}$$

The amplitude of the horizontal velocity field is thus given by:

$$\psi_n(z) = c_n \frac{\partial \phi_n(z)}{\partial z} \quad 2)$$

The Kelvin wave solution of equation 1) propagates in a direction parallel to and to the left of the boundary. Although the influence of the Earth's rotation does not appear in equation 1) and so does not affect the calculation of the eigenfunctions it does result in the amplitude of the Kelvin wave being dependent on the distance, y , away from the boundary. If the boundary is defined as the line $y=0$ then this dependence can be shown to be:

$$\phi_n(y, z) = \phi_n(z) e^{-\frac{fy}{c_n}}$$

where f is the Coriolis parameter.

Thus the wave amplitude is highest against the boundary and decreases with increasing distance away from the boundary. The horizontal velocity field associated with the wave will be modified by the same exponential decay factor.

Since Douglas Channel has two fairly straight parallel sides it is possible for internal Kelvin waves to propagate in both directions along it. Values of c_n for first baroclinic mode were computed to be of the order of 0.8 m/sec. Near the current meters the channel is about 4 km wide. For these parameters the amplitudes of fluctuations

due to a passing internal Kelvin wave will differ by a factor of 1.8 on the two sides of the channel. For an internal Kelvin wave propagating northwards along Douglas Channel the amplitudes will be largest on its eastern side; for a southward propagating wave the amplitudes on the channels' western side will be largest.

4.2 Determination of Theoretical Baroclinic Modes for Douglas Channel

4.2.1 Estimation of Vertical Profiles of Density

The solution of equation 1) requires the specification of the vertical mean density profile, $\rho(z)$. Data from the current meters and from the thermistor chain TMS 2 were used to estimate these profiles for weeks #1 to #10.

Time series of water temperatures and densities from the current meters at 5 m, 17 m and 30 m depths were used to determine a relationship between temperature and density appropriate to each week. During weeks #1 to #10 the temperature in the top 50 m or so of Douglas Channel was a monotonically decreasing function of density. For this time period the temperature data from the thermistor chain were processed to provide weekly averaged temperatures for each of the eleven thermistor depths over the 50 m length of the chain. Application of the derived temperature-density relationships to these averages provided weekly estimates of density corresponding to each thermistor depth. An appropriate density profile for the entire water column for each week was obtained by fitting polynomials to the density data from the thermistor depths and from the current meters at 165 m and 305 m depths.

4.2.2 Horizontal Velocity Eigenfunctions in Douglas Channel

Amplitude eigenfunctions, $\phi_n(z)$, for the first three baroclinic modes ($n=1,2,3$) were calculated for weeks #1 to #10 using the appropriate density profiles. Application of equation 2) provided the eigenfunctions of horizontal velocity for each of the first three modes. Figure 11 shows the density profile and the horizontal velocity eigenfunctions calculated for week #3. Note that these eigenfunctions have been normalized with respect to their values at the surface. All three eigenfunctions have their maximum amplitudes at the surface and decrease rapidly with depth. The first mode eigenfunction goes through zero and changes sign at a depth of 130 m; the second mode eigenfunction changes sign twice, once at 18 m depth and again at 182 m depth. The third mode eigenfunction has a more rapid decline away from the surface and changes sign three times over the water column. The horizontal phase speeds for modes one, two and three are 0.8 m/sec, 0.5 m/sec and 0.3/sec, respectively. These speeds are much less than the 60 m/sec phase speed of the barotropic tide in Douglas Channel.

Although the intensity of the stratification varies somewhat over weeks #1 to #10, all the density profiles display similar features to that shown in Figure 11. The density increases fairly rapidly

from the surface, but by 50 m depth the rate of increase slows markedly.. Although the eigenfunctions of horizontal velocity computed from the density profiles have similar appearances, significant changes occur over the ten week period. These changes are exemplified in Figure 12 where the first mode eigenfunctions from weeks #1, #6 and #10 are compared. The primary difference in the density structure between the same three weeks is the reduction in the near surface density gradients. (see Figure 9).

4.3 Fitting of Theoretical Baroclinic Modes to Observed Currents

4.3.1 Fitting Procedure

Theoretical eigenfunctions of horizontal velocity computed for the first and second mode baroclinic tides and a horizontal velocity field associated with a barotropic tide were fitted to the observed semi-diurnal velocities in Douglas Channel for weeks #1 to #10. The locations of the moorings CM 2 and CM 3 were considered to be close enough together (1.3 km) to have the same position as far as the analysis was concerned. The difference in phase at the two moorings for a second mode baroclinic wave propagating down channel at a phase speed of 0.5 m/sec (an average value) would be 20° .

A least squares fitting procedure was used to obtain the amplitudes and phases of the barotropic contribution and the first and second modes of the baroclinic contributions to the fitted velocities. This procedure was completed as follows:

Let the observed horizontal component of semi-diurnal velocity at each current meter depth, $u(z_i, t)$, ($z_1 = 5\text{m}$, $z_2 = 17\text{m}$, $z_3 = 30\text{m}$, $z_4 = 165\text{m}$, $z_5 = 305\text{m}$) be represented as :

$$u(z_i, t) = A(z_i) \cos(\omega t - \theta(z_i))$$

The amplitudes, $A(z_i)$, and phases, $\theta(z_i)$, are determined from the harmonic analyses of the current meter data.

Let the normalized horizontal velocity eigenfunction associated with the n'th baroclinic mode be $\psi_n(z)$. If the phase of the n'th mode is δ_n and its surface amplitude is B_n , then the horizontal velocity field associated with the n'th mode would be:

$$U_n(z, t) = B_n \psi_n(z) \cos(\omega t - \delta_n)$$

A fitted velocity at the depth of each current meter, $V(z_i, t)$, is the sum of contributions from the barotropic mode ($n=0$), the first baroclinic mode ($n=1$) and the second baroclinic mode ($n=2$). (For the barotropic mode $\psi_0(z) = 1.0$). Thus:

$$V(z_i, t) = \sum_{n=0}^2 B_n \psi_n(z_i) \cos(\omega t - \delta_n)$$

The fitting procedure involves determining the six unknowns B_0 , B_1 , B_2 , δ_0 , δ_1 , and δ_2 . The velocities $u(z_i, t)$ can be decomposed into contributions from $\cos(\omega t)$ and $\sin(\omega t)$ as follows:

$$u(z_i, t) = A(z_i)\cos(\theta(z_i))\cos(\omega t) + A(z_i)\sin(\theta(z_i))\sin(\omega t)$$

$$U_n(z, t) = B_n\psi_n(z)\cos(\delta_n)\cos(\omega t) + B_n\psi_n(z)\sin(\delta_n)\sin(\omega t)$$

A standard least squares fitting is carried out on the $\cos(\omega t)$ and $\sin(\omega t)$ components of velocity separately. This results in the six equations:

$$\text{for } n=0,1,2 \quad \frac{\partial}{\partial(B_n\cos(\delta_n))} \left[\sum_{i=1}^5 (A(z_i)\cos(\theta(z_i)) - \sum_{n=0}^2 B_n\psi_n(z_i)\cos(\delta_n))^2 \right] = 0$$

$$\text{for } n=0,1,2 \quad \frac{\partial}{\partial(B_n\sin(\delta_n))} \left[\sum_{i=1}^5 (A(z_i)\sin(\theta(z_i)) - \sum_{n=0}^2 B_n\psi_n(z_i)\sin(\delta_n))^2 \right] = 0$$

from which the six unknowns can be determined.

4.3.2 Results of Fitting Procedure

A barotropic mode and the two baroclinic modes were fitted to the velocity data from each of weeks #1 and #10. The results of the fitting procedure are presented in Figure 13 and 14. Figure 13 shows the fitted amplitudes for the barotropic mode and for the first and second baroclinic modes; the phases of these modes are compared in Figure 14. Note that the amplitudes are normalized with respect to the 'barotropic' velocity and the phases are relative to the phases of the tidal height for each week.

4.3.2.1 Barotropic Mode

The normalized amplitudes for the fitted barotropic mode undergo relatively little variation over the ten week period. These vary from 1.16 to 1.45 averaging 1.26. The phases vary over a 60° range over the ten week period the average phase being -61° . The tidal model predicts that the phase of the barotropic velocity with respect to tidal height in the southern end of Douglas Channel should be around -85° , that is the character of the tide in Douglas Channel is that it is predominantly a standing wave. For a pure standing wave the horizontal tidal velocity would lag the tidal height by 90° and the net energy transmission into Douglas Channel over a tidal cycle would be zero. For a pure travelling wave propagating northwards through Douglas Channel the tidal height and horizontal tidal velocity would be in phase. The actual relative phase between the tidal height and horizontal velocity of around -60° indicates that the tide is somewhere between a standing and a travelling wave and that there is a considerable net northwards transmission of tidal energy through Douglas Channel over each tidal cycle.

4.3.2.2 Baroclinic Modes

The modal fitting technique yields values of the normalized first and second baroclinic amplitudes mostly between 1.0 and 2.0. The phases of the first mode vary from -74° to -13° ; the phases of the second mode fall in the range -15° to 80° .

A measure of the goodness of the fit was taken to be the degree to which the sum of the three modes could account for the observed variance at each current meter depth. Let $\vec{V}(z_i)$ be a vector representing the amplitude and phase of the observed semi-diurnal velocity at current meter i and let $\vec{U}(z_i)$ be a vector representing the fitted velocity.

Thus if the vector representing the n 'th fitted baroclinic mode is $\vec{U}_n(z_i)$ then:

$$\vec{U}(z_i) = \vec{U}_0(z_i) + \vec{U}_1(z_i) + \vec{U}_2(z_i)$$

At each depth a residual, r_i , can be defined as the proportion of the actual velocity not accounted for by the fitting procedure. Accordingly:

$$r_i = \frac{|\vec{V}(z_i) - \vec{U}(z_i)|}{|\vec{V}(z_i)|}$$

Let the average residual for each week, $\bar{r}_i$, be the average of the r_i from the five current meter depths. Over the ten weeks the values of r_i range up to 0.2; their average value is 0.15.

Figure 15 shows a composite diagram of the normalized vector residuals $\vec{R}_i$, for the ten weeks. Here, $\vec{R}_i$ is defined as:

$$\vec{R}_i = \vec{V}(z_i) - \vec{U}(z_i)$$

The amplitude of $\vec{R}_i$ is normalized with respect to the amplitude of the 'barotropic' tidal current and the phase of $\vec{R}_i$ is relative to the phase of the tidal height. It is evident that the $\vec{R}_i$ at each depth are clustered implying that the differences between the observed and fitted semi-diurnal velocities are consistent from week to week.

5. SOURCE OF BAROCLINICITY

The major part of the observed baroclinicity in Douglas Channel is almost certainly due to the interaction of the barotropic tide with the topography of the channels forming the Kitimat channel system. The analyses of the 2½ month long records of current meter data show that the nature of the baroclinicity is similar for the three tidal constituents. If the baroclinicity were generated by a semi-diurnal wind stress acting on the surface of Douglas Channel one would expect that the baroclinicity would be pronounced at the S_2 frequency, but not at the M_2 or N_2 frequencies. The most obvious source for the generation of baroclinic waves is the sill which crosses Douglas Channel about 10 km north of the current meter moorings (see Figure 3 and 4).

The energy transported northwards past the current meters by the barotropic tide over one tidal cycle can be shown to be given approximately by:

$$E = \frac{\rho g v a B H T}{2} \cos(\theta)$$

where:

- v is the amplitude of the barotropic tidal velocity
- a is the tidal height amplitude
- B is the channel width
- H is the water depth
- T is the tidal period
- θ is the phase difference between the tidal height and the barotropic tidal velocity

Considering the semi-diurnal frequency for week #6 which has $v=0.11$ m/sec, $a=1.33$ m and $\theta=62^\circ$, E is calculated as 2×10^{13} joules. The northward energy transmission over a tidal cycle in the other weeks was within a factor of two of this value.

Once the amplitude of a baroclinic mode is determined, its kinetic energy can be readily calculated. The kinetic energy, E_k , contained in a length of channel equal to the wavelength of a particular baroclinic mode $\left(\frac{2\pi c_n}{\omega}\right)$ is given by:

$$E_k = \frac{\rho}{2} \int_0^H \int_0^B \int_0^{\frac{2\pi c_n}{\omega}} u_n^2(x,y,z,t) dx dy dz$$

For an order of magnitude calculation of this energy the channel width, B, and the channel depth, H, are both assumed constant along the channel axis. Taking the velocity eigenfunctions for week #6 the kinetic energy in one wavelength of the first mode is calculated to be 4×10^{10} joules and in one wavelength of the second mode it is calculated to be 2×10^{10} joules. The ratios of these two energies to the energy transported into Douglas Channel in one tidal cycle are 2×10^{-3} and 10^{-3} , respectively. Evidently, there is plenty of energy available from the barotropic tide for generation of baroclinic waves.

The phase speeds of the first baroclinic mode are plotted against the phases of the fitted first mode in Figure 16. It appears that the phases are little dependent on the phase speeds. If the baroclinic waves were generated on the sill and propagated downstream a negative relationship between the phases and phase speeds would be expected. The dotted line in Figure 16 shows the relationship expected if the relative phase of the first internal mode were to be -140° at the sill. From Figure 16 it is evident that the phase changes of the fitted first mode are not due only to changes in the phase speed of this mode.

The propagation of energy in baroclinic waves takes place along their characteristics. The equations for these characteristics in a zero mean flow can be shown to be:

$$\frac{dz}{dx} = \pm \left(\frac{\omega^2 - f^2}{N^2 - \omega^2} \right)^{\frac{1}{2}}$$

Assuming the baroclinic waves are generated at the top of the sill then this equation can be used to estimate the distance away from the sill at which the wave energy reaches the surface. If N is assumed constant between the current meter depths then the energy characteristic will decrease depth by Δz over a distance Δx where:

$$\Delta x = \pm \left(\frac{N^2 - \omega^2}{\omega^2 - f^2} \right)^{\frac{1}{2}} \Delta z \quad 3)$$

Using values of N calculated from the current meter density data and taking the Δz 's as the changes in depth between current meters then equation 3) can be evaluated in a stepwise fashion to determine the total horizontal distance travelled before a characteristic originating at 240 m depth (sill depth) reaches the surface. This technique is schematicised in Figure 17. Table 2 details the calculation for week #1 for which the characteristic reached a depth of 5 m a distance of 37 km away from the sill. For the other nine weeks this distance was not less than 31 km.

Since this distance is at least three times the distance of the current meters to the sill (10 km) baroclinic energy propagated directly from the sill does not appear to be responsible for the observed baroclinicity at the current meters.

TABLE 2 - Calculation of characteristic depths for week #1.

Depth Range	$N(\text{sec}^{-1})$	$\Delta z(\text{m})$	$\Delta x(\text{km})$
240 m to 165 m	0.006	75	5
165 m to 30 m	0.01	135	18
30 m to 17 m	0.03	13	5
17 m to 5 m	0.06	12	9
		Total	37

6. SUMMARY AND CONCLUSIONS

A high degree of baroclinicity was indicated in the semi-diurnal tidal currents measured in the southern part of Douglas Channel over the period July 1977, to June 1978. Both the amplitudes and the phases of these currents showed marked changes with depth. The amplitudes of the semi-diurnal current at 5 m depth often exceeded twice the amplitude of the estimated barotropic tidal currents in this section of the channel; at mid-depth and near the bottom of the channel the tidal current amplitudes were similar to the expected barotropic current. The semi-diurnal current phases differed by as much as 70° for current measurements made at different depths. A comparison of the tidal currents between four seasons of the year indicated that significant changes in the nature of the baroclinicity occurred. Furthermore, when the tidal current analyses were completed on a weekly basis the results showed that major changes were also taking place over time scales at least as short as a week.

Based on the assumption that the observed baroclinicity was due to internal Kelvin waves propagating past the site of current measurement the observed currents were fitted by a sum of these waves having mode numbers of one and two and a barotropic wave. The theoretical velocity structure of each mode of the internal waves was determined numerically using measured density profiles from Douglas Channel. The mean horizontal velocity in the channel was assumed to be zero. This analysis was completed on a weekly basis over ten weeks between July 1977 and September 1977.

The differences between the observed velocities and the fitted velocities for each current meter depth have a consistent nature from week to week. This consistency suggests that the differences are not primarily due to a noise-like phenomenon in the current meter data. Perhaps the fitting procedure should include baroclinic waves of mode higher than two. Also, the baroclinic wave structures were calculated from vertical profiles of density which had to be interpolated from the available data. As a result these calculated wave structures used in the fitting procedure are subject to considerable uncertainty.

Perhaps a more serious shortcoming of the analysis used is the assumption of zero mean horizontal velocity in the derivation of the baroclinic eigenfunctions. Douglas Channel has significant horizontal water movements evidently associated with its estuarine and wind driven circulations. Near the surface (5 m depth) where these mean horizontal currents tended to be largest their magnitudes usually exceeded 10 cm/sec and were often over 20 cm/sec in the ten week period over which the fitting was accomplished. Since these currents are not insignificant compared to the horizontal phase speeds of the first and second baroclinic modes one might expect that their presence would have a major influence on the nature of the baroclinic waves in the channel.

It appears that the baroclinicity results from the interaction of the barotropic tide with the topography of the area. The energy transmitted northwards into Douglas Channel by the barotropic tide is many times larger than the energy evident in the baroclinic tide so that there is plenty of energy available from the barotropic tide for the generation of baroclinicity. The most obvious source of the baroclinicity in Douglas Channel is the sill which crosses the channel at a depth of 240 m about 10 km north of the site of the current measurements. However, the method of characteristics suggests that the sill is too close to the current measurement site for baroclinicity generated there to be observed by the current meters near the surface.

The determination of the source and the generation mechanism of the baroclinicity is essential to the prediction of near surface tidal currents in Douglas Channel. Probably the amount of energy which appears in the baroclinicity is dependent upon the seasonally variable density and mean flow structure of the channel. If the energy is distributed among a series of baroclinic waves having different mode numbers then the baroclinic contribution to the total current will change in amplitude and phase with increasing distance away from the source of the baroclinicity due to the differing horizontal phase speeds of the modes. A semi-diurnal first mode baroclinic wave having a horizontal phase speed of 0.8 m/sec changes phase by 555° along the 55 km length of Douglas Channel; a semi-diurnal second mode baroclinic wave having a horizontal phase speed of 0.5 m/sec changes phase by 885° in the same distance. By comparison, the phase change along the channel of a propagating barotropic wave of phase speed 60 m/sec is only 7° . The first and second mode baroclinic waves will therefore each make additive and subtractive contributions to the barotropic current simultaneously in different locations in the channel. If a superposition of one or more baroclinic waves is the appropriate representation of the baroclinicity in Douglas Channel then predictive schemes for near surface tidal currents should consider potentially significant spatial variability in the currents which arises as a consequence.

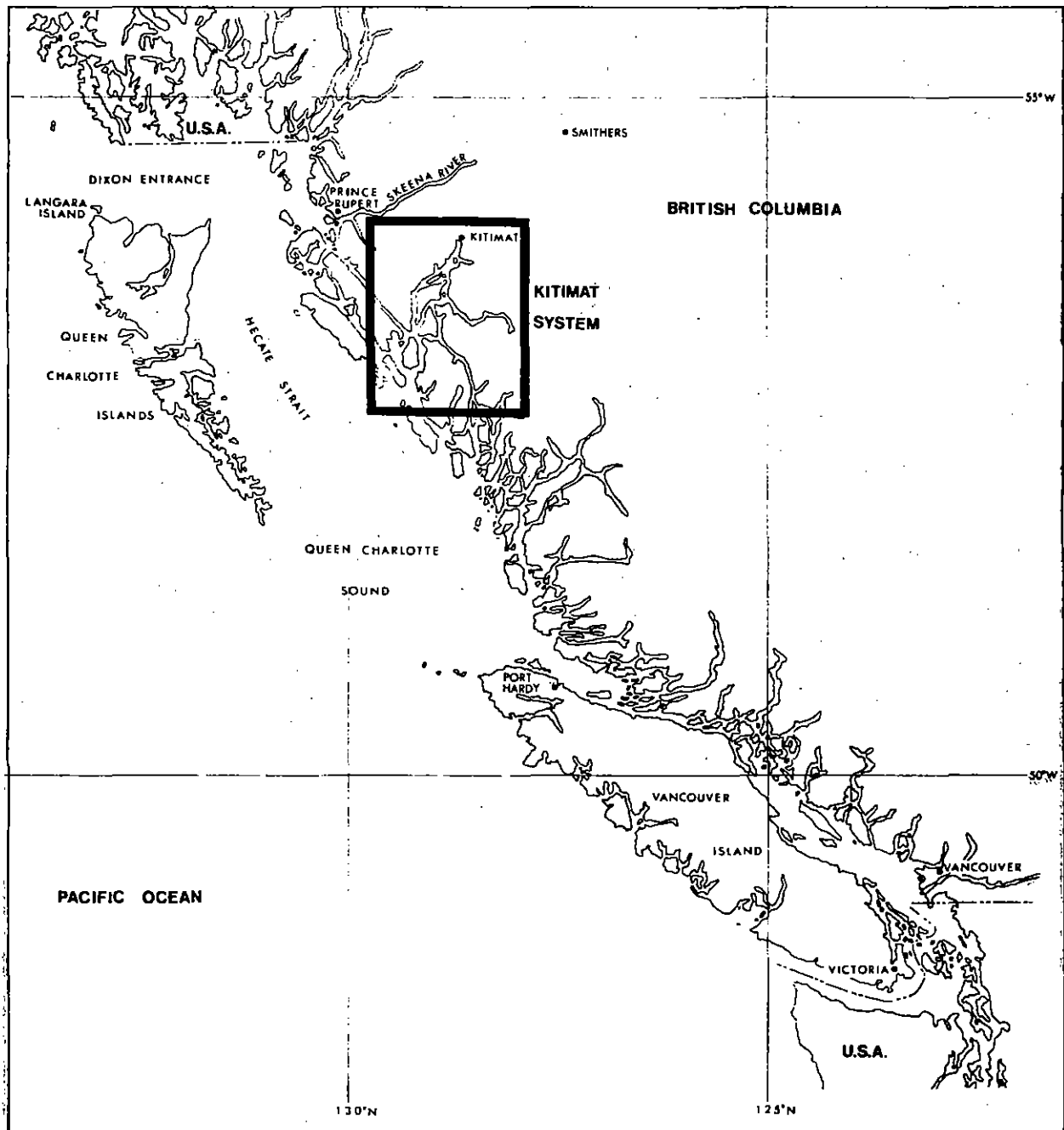


Figure 1. Location of Kitimat system.

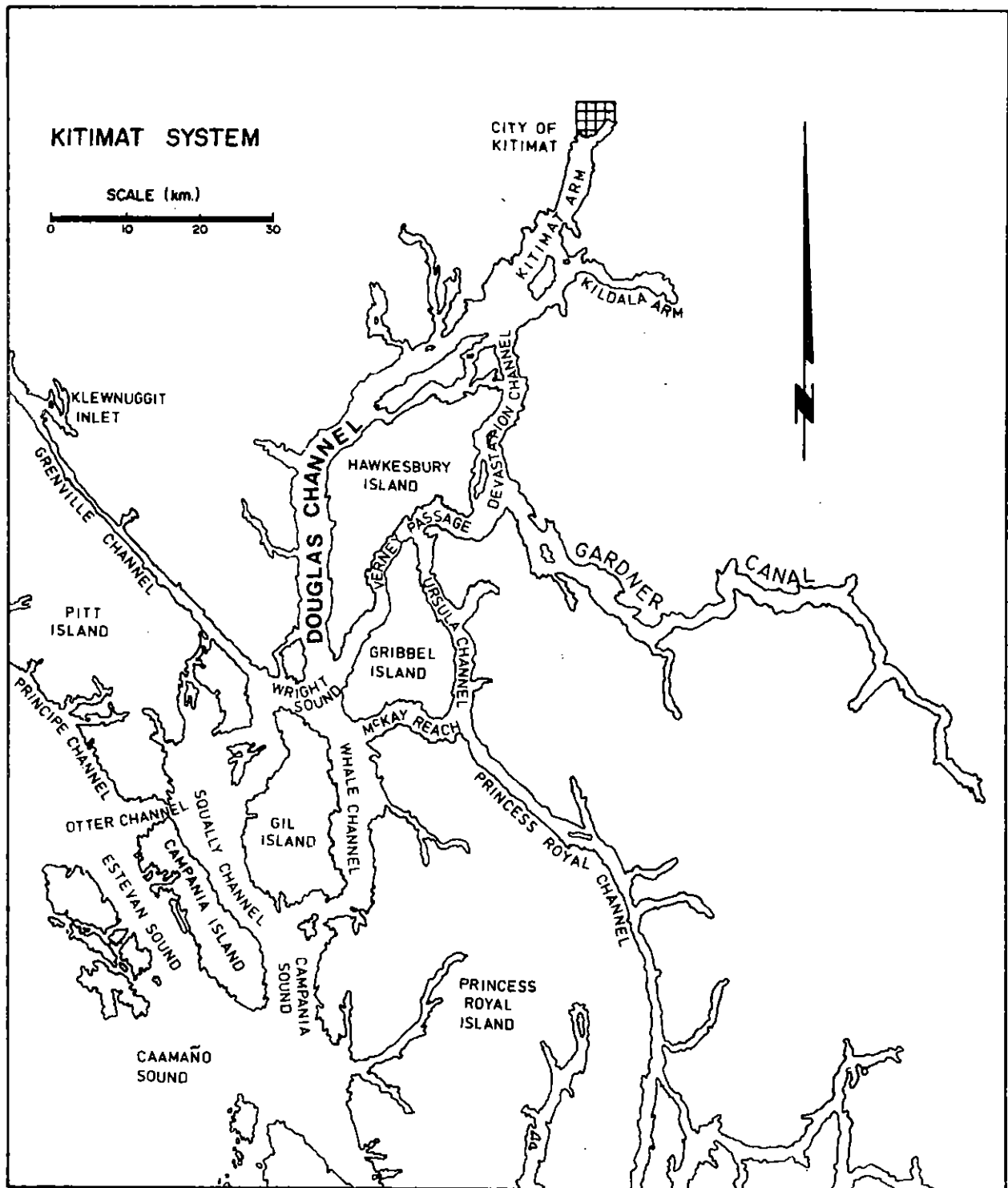


Figure 2. Map of Kitimat system.

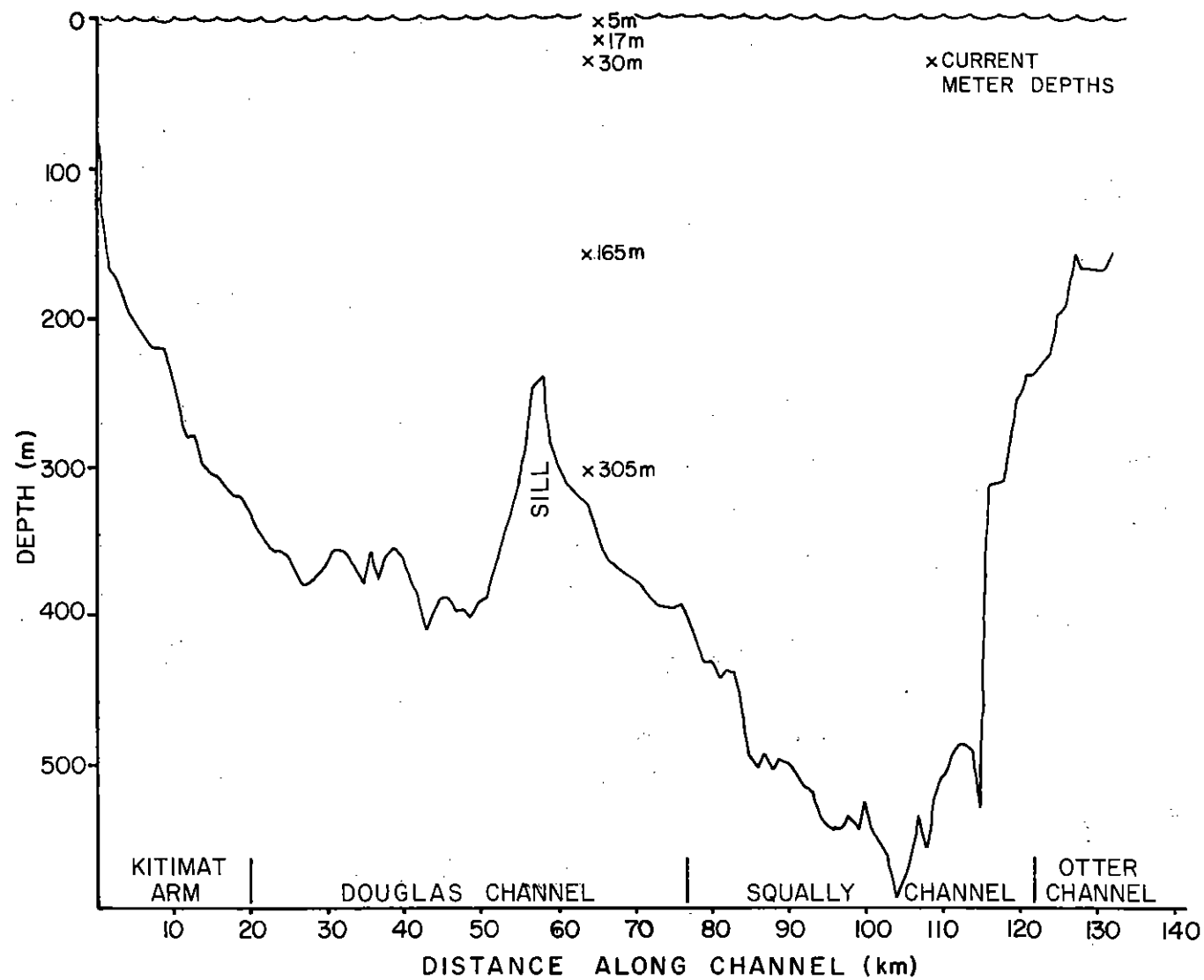


Figure 3. Bathymetry along Douglas Channel and adjoining channels.

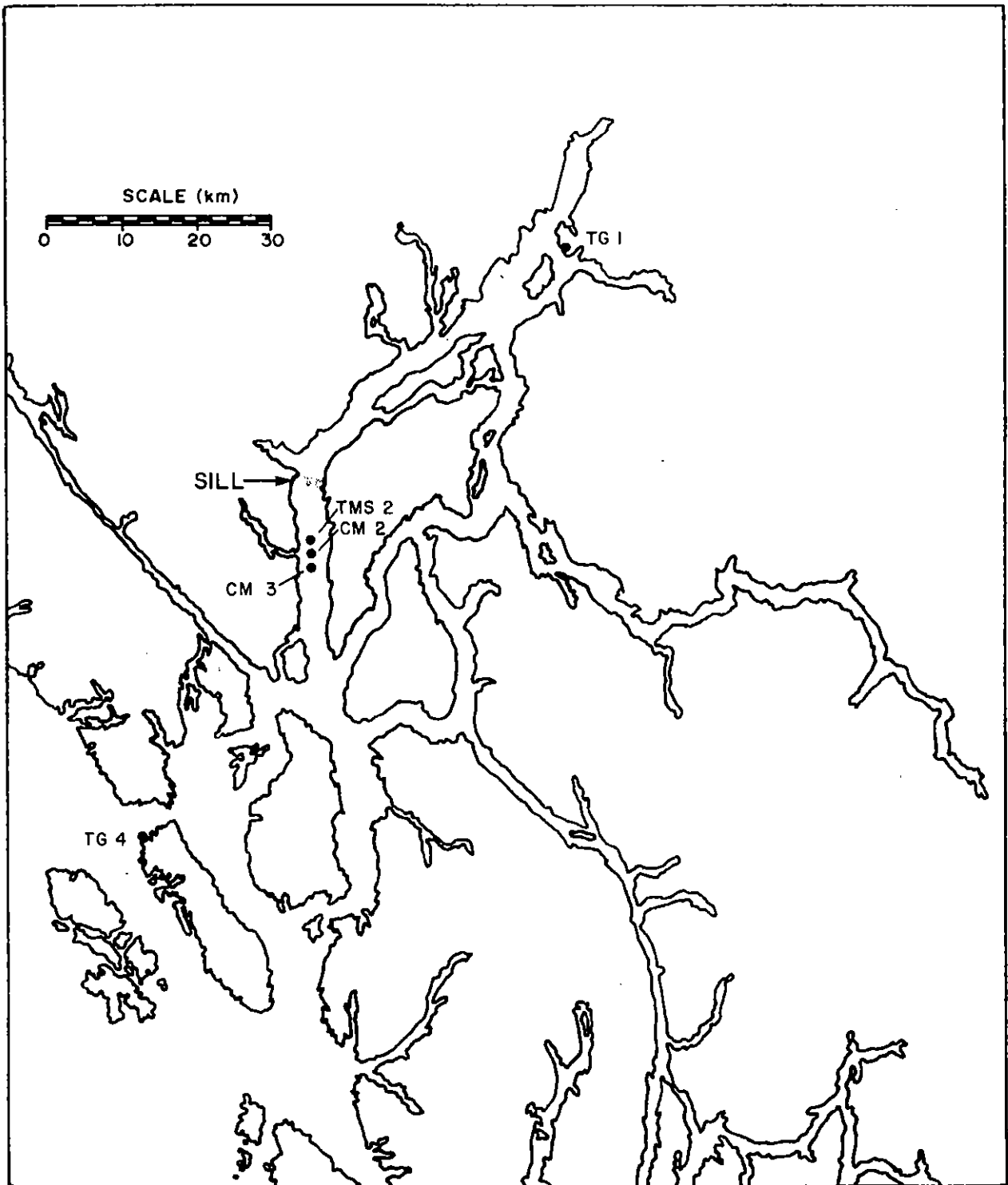


Figure 4. Locations of Instrumentation.

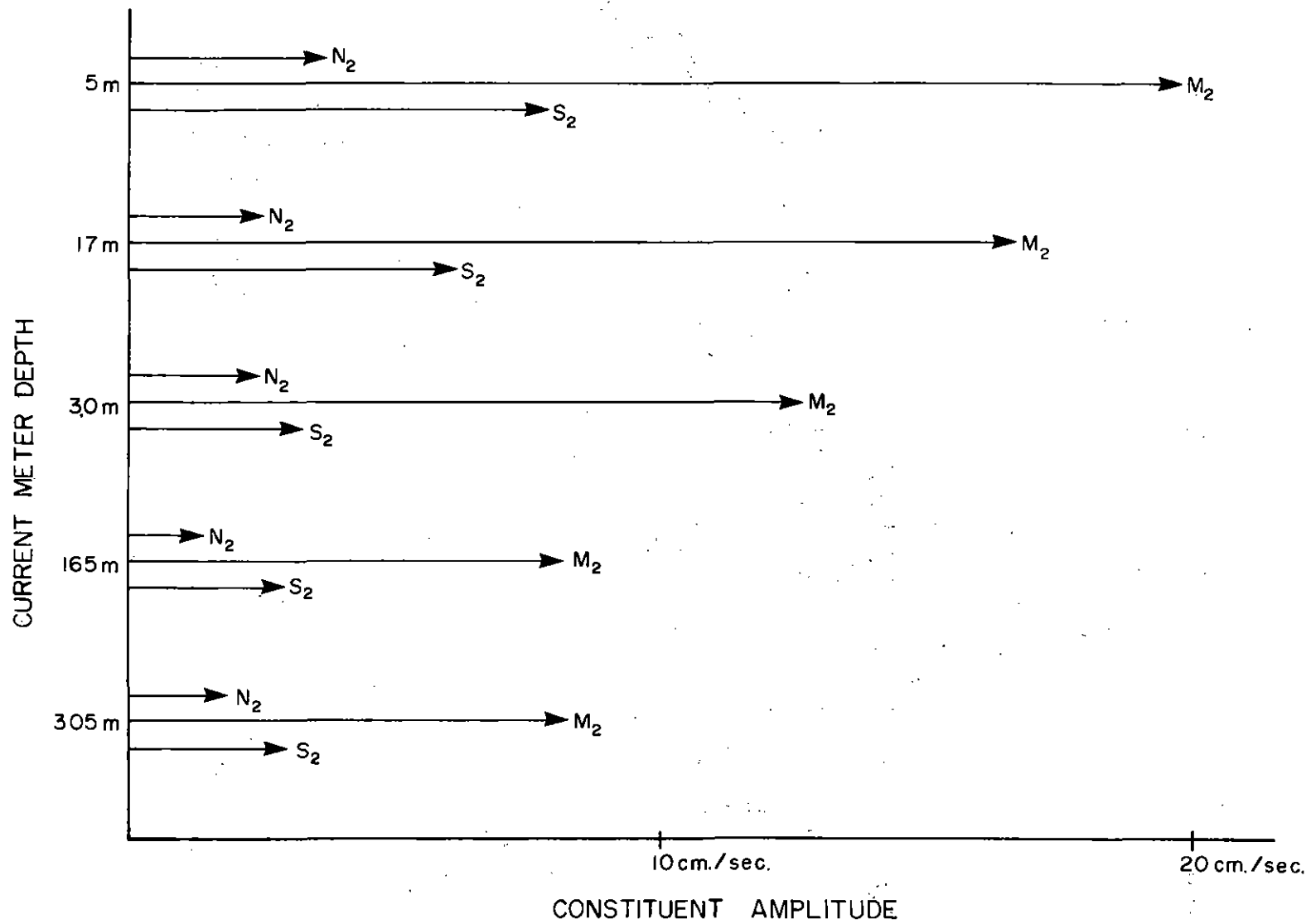


Figure 5. Amplitudes of semi-diurnal constituents of current velocity (July to September).

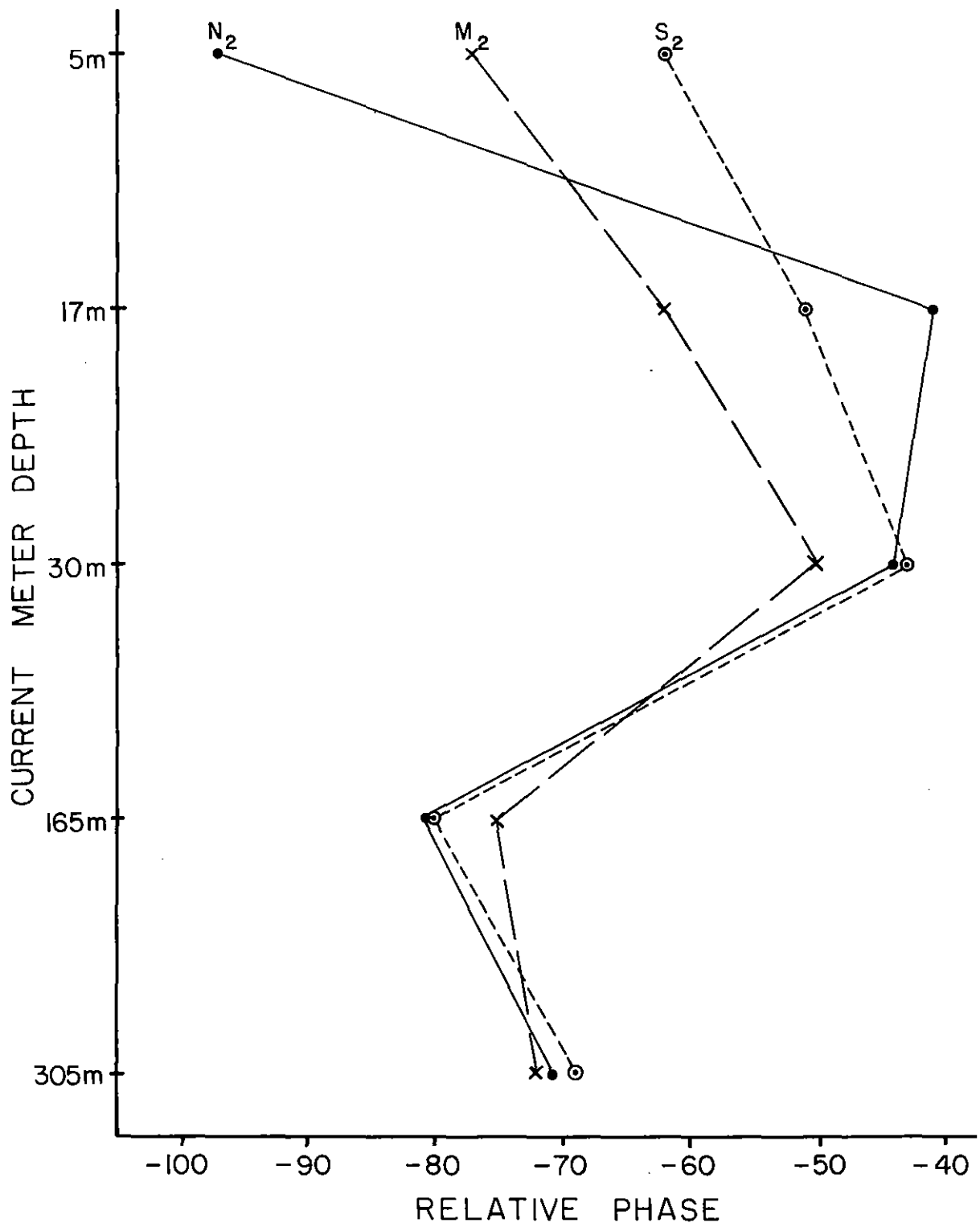


Figure 6. Phases of semi-diurnal constituents of current velocity relative to phases of tidal height constituents (July to September).

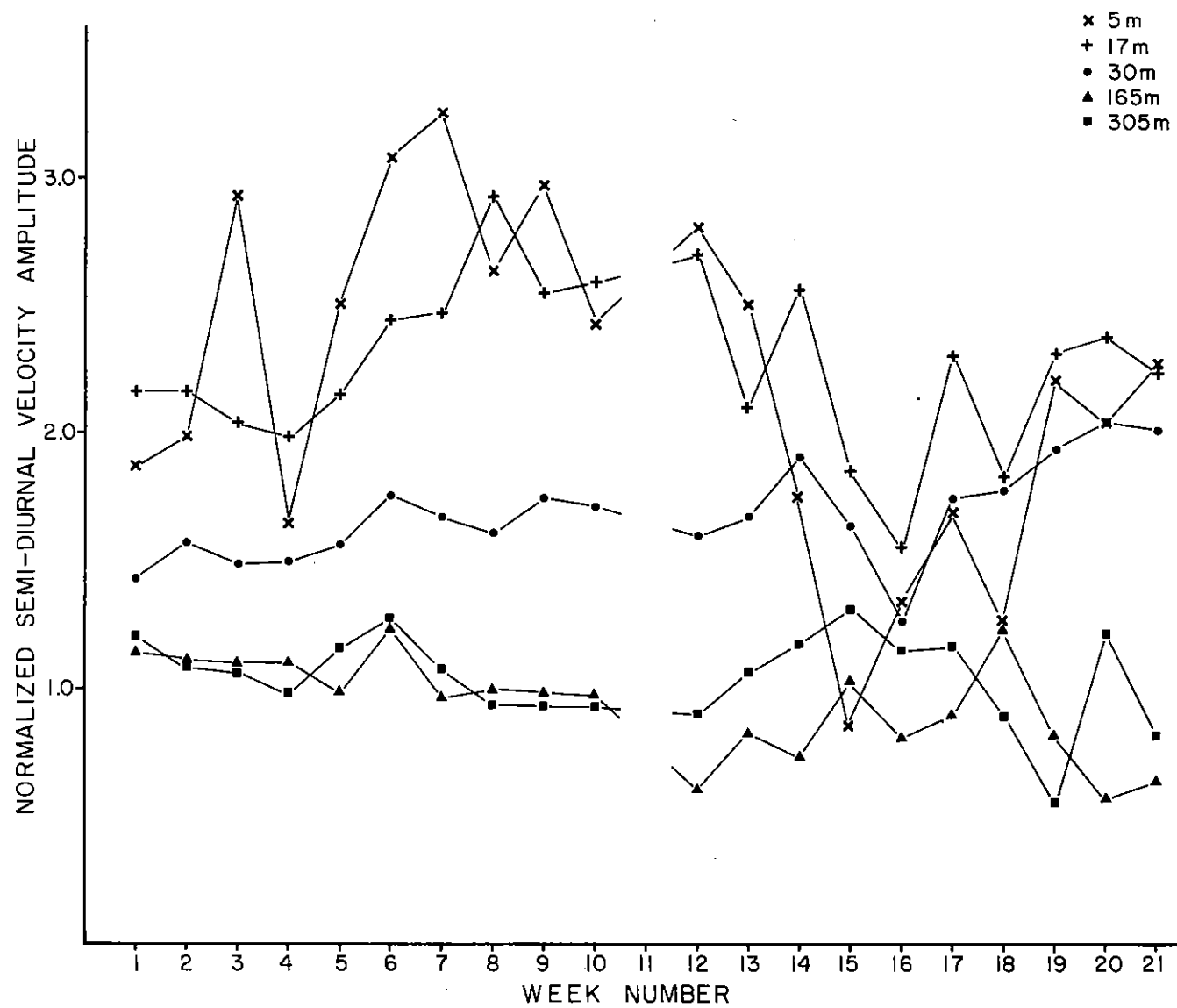


Figure 7. Observed weekly amplitudes of semi-diurnal velocity normalized with respect to the 'barotropic' velocity amplitudes.

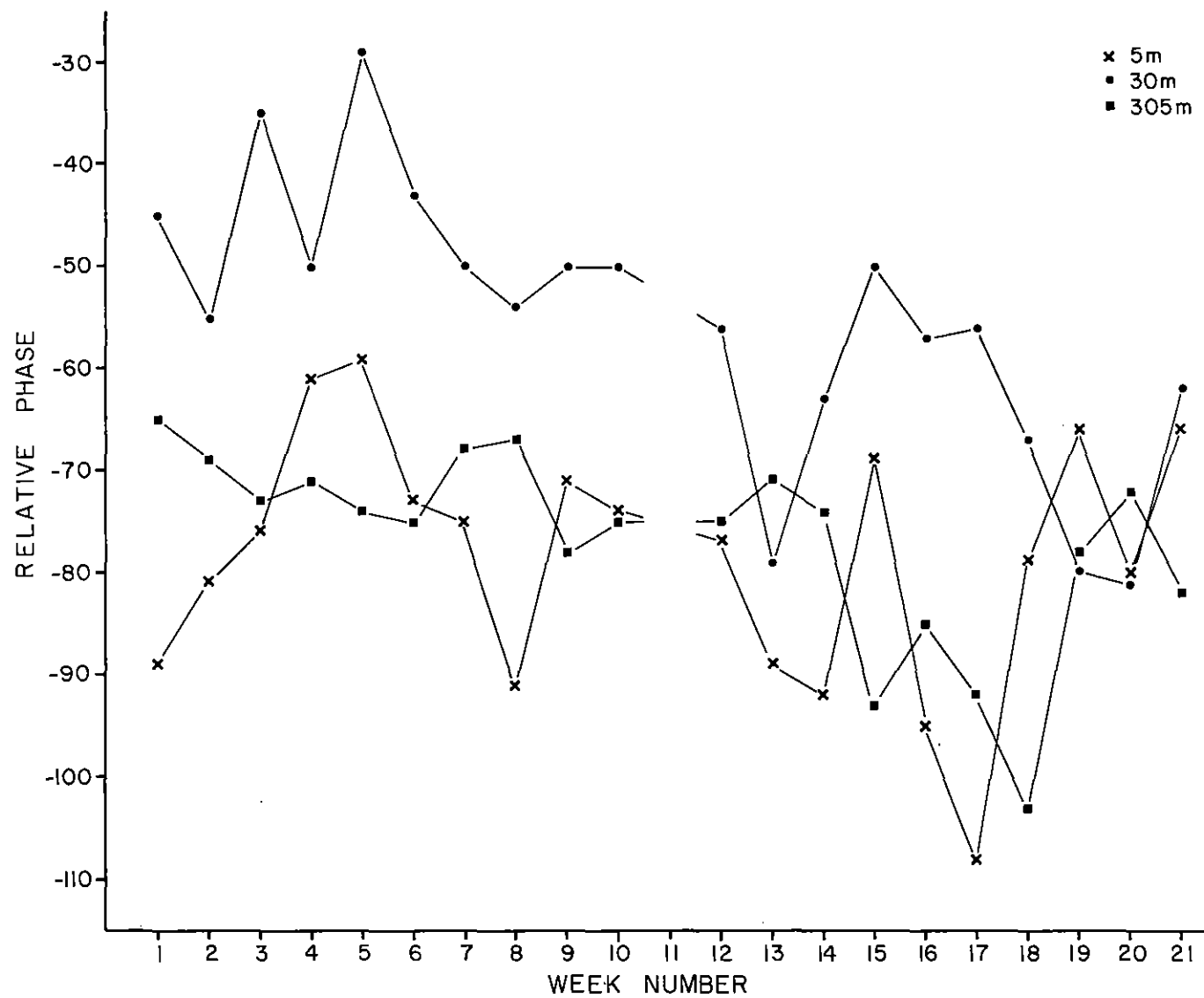


Figure 8. Observed weekly phases of semi-diurnal velocity relative to the phase of tidal height.

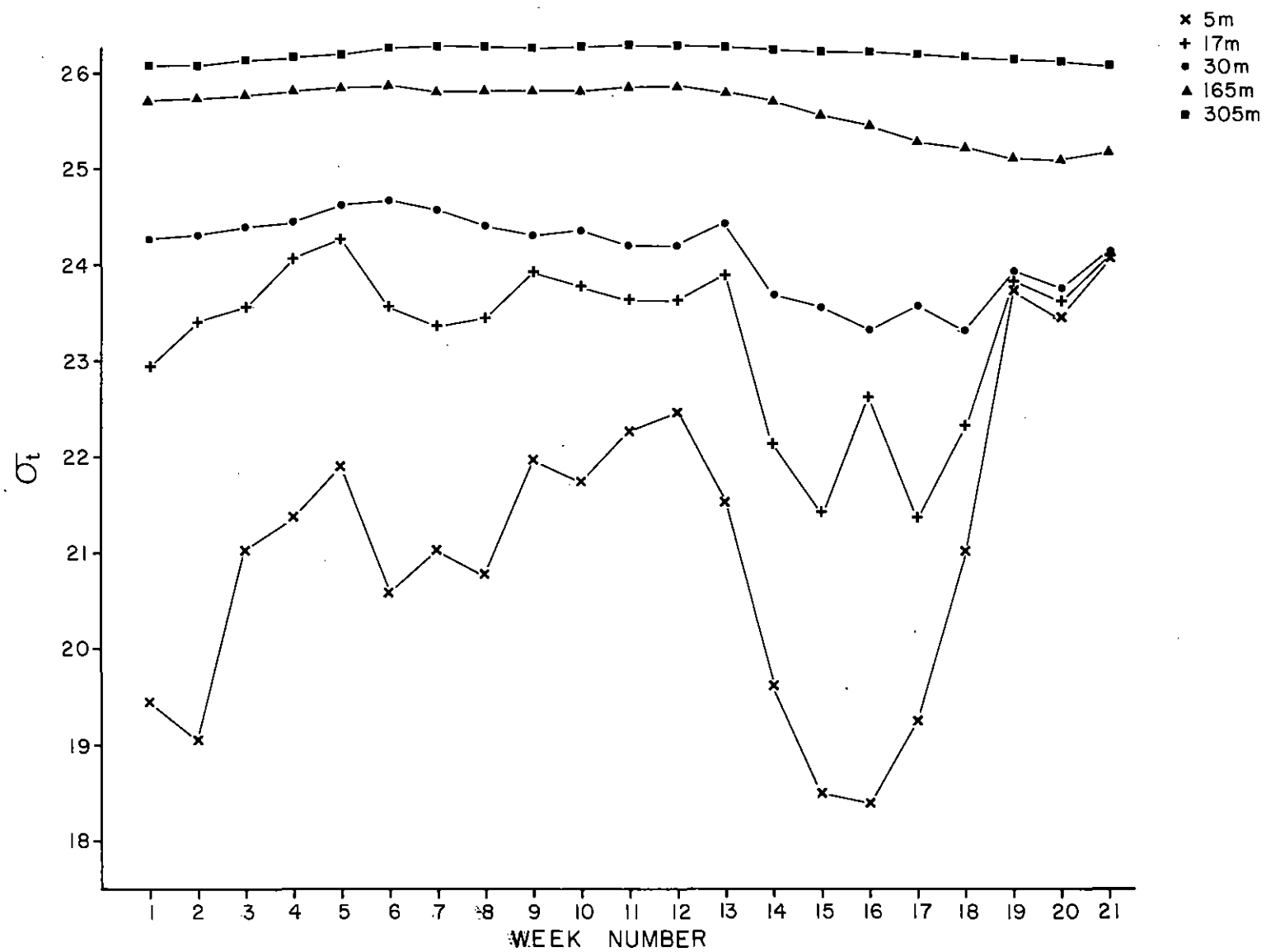


Figure 9. Weekly averaged densities (σ_T).

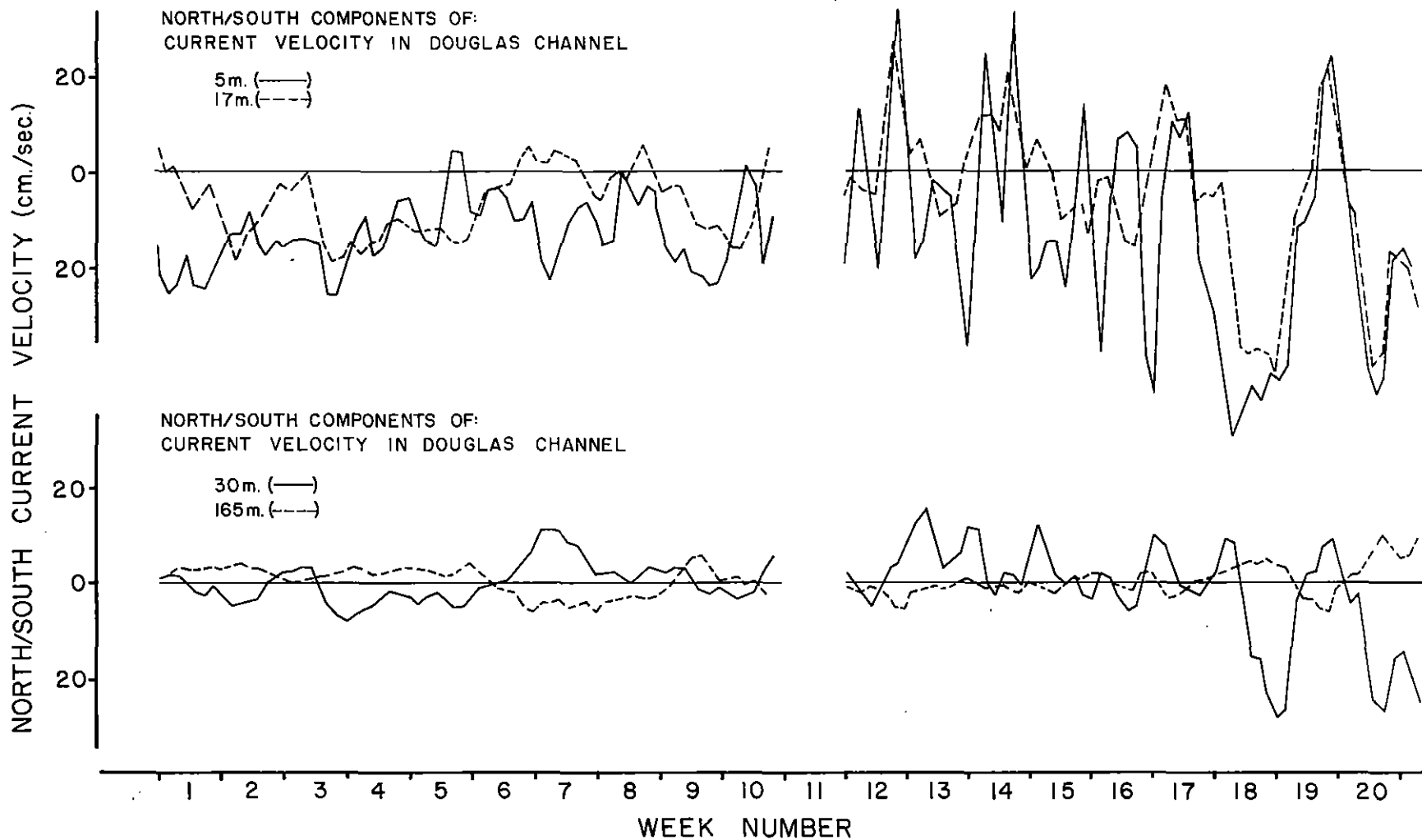


Figure 10. Low pass filtered components of current velocity in Douglas Channel.

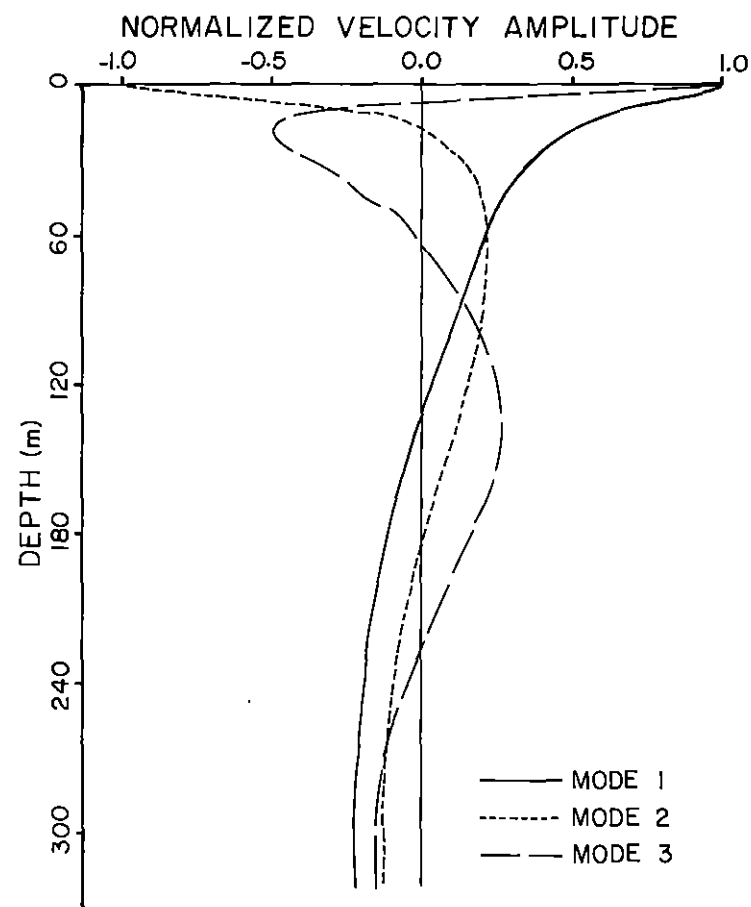
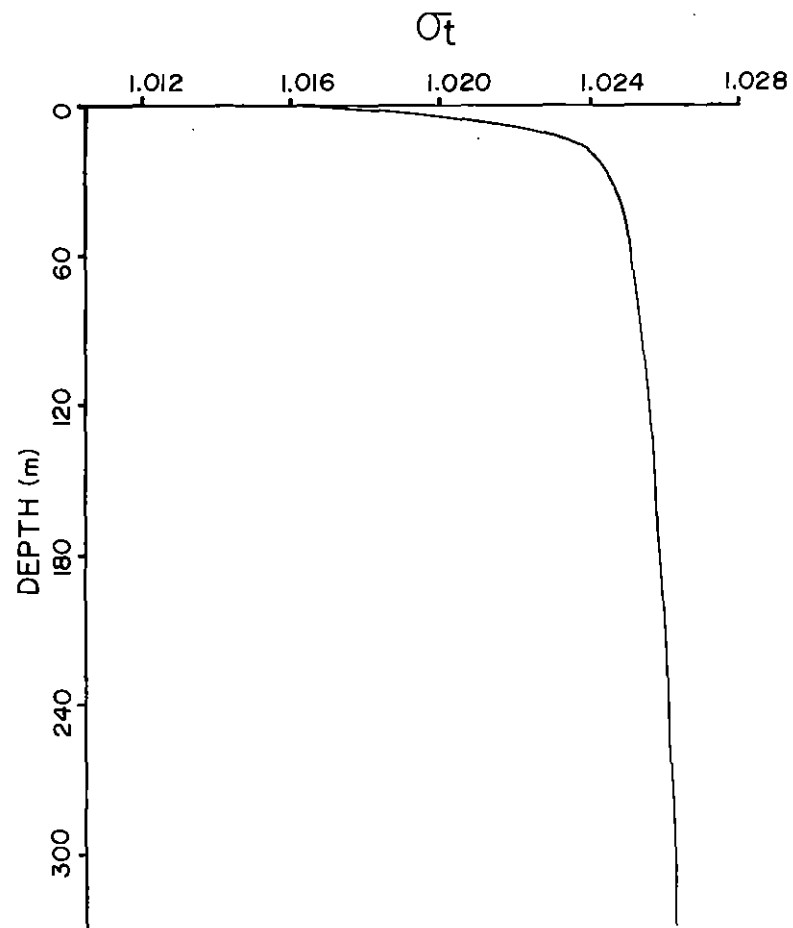


Figure 11a) Observed vertical density profile (week #3).
 b) Computed baroclinic horizontal velocity eigenfunctions (week #3).

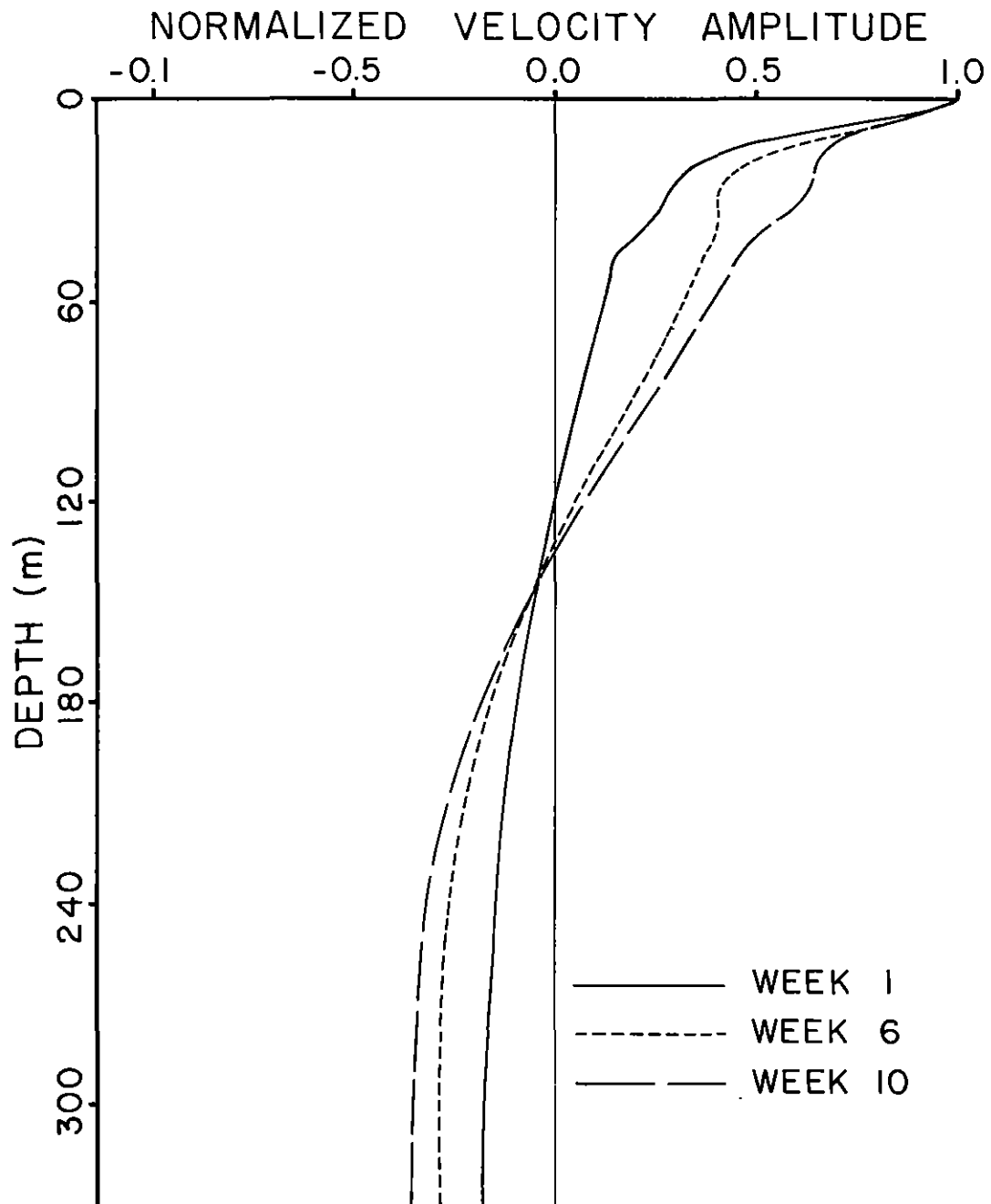


Figure 12. Computed first mode baroclinic horizontal velocity eigenfunctions (weeks #1, #6 and #10).

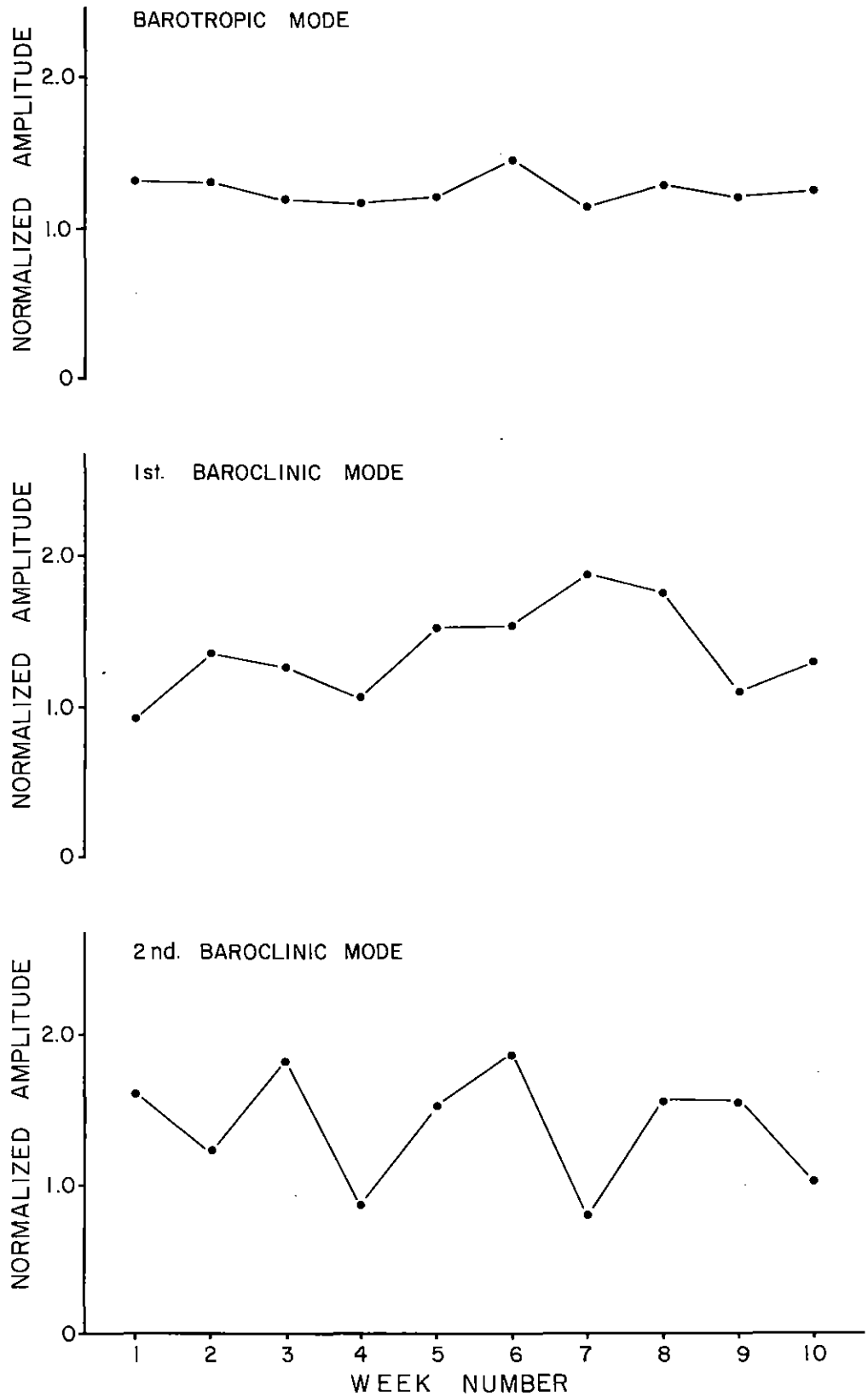


Figure 13. Fitted normalized velocity amplitudes for barotropic and first two baroclinic modes of velocity.

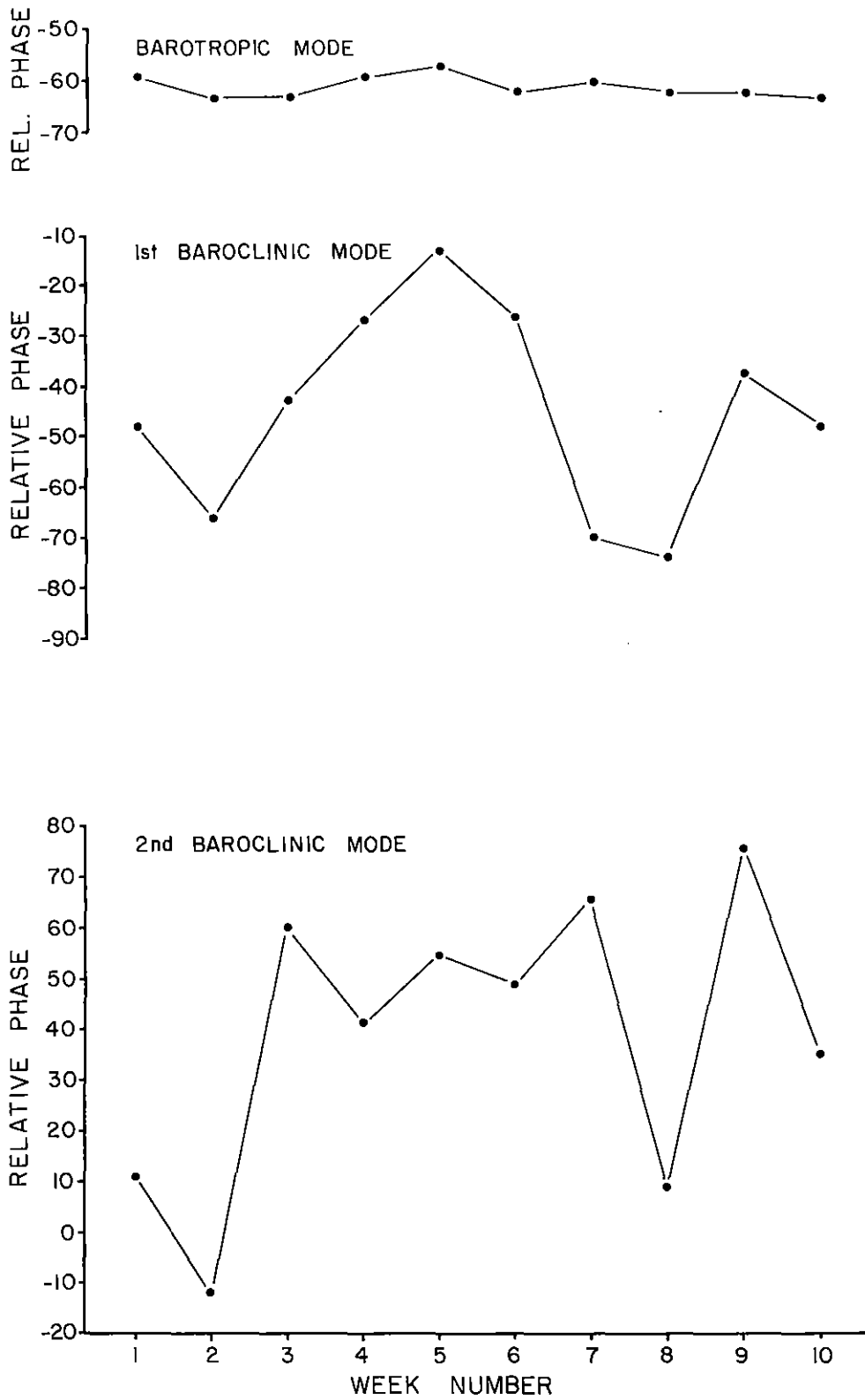


Figure 14. Fitted relative velocity phases for barotropic and first two baroclinic modes of velocity.

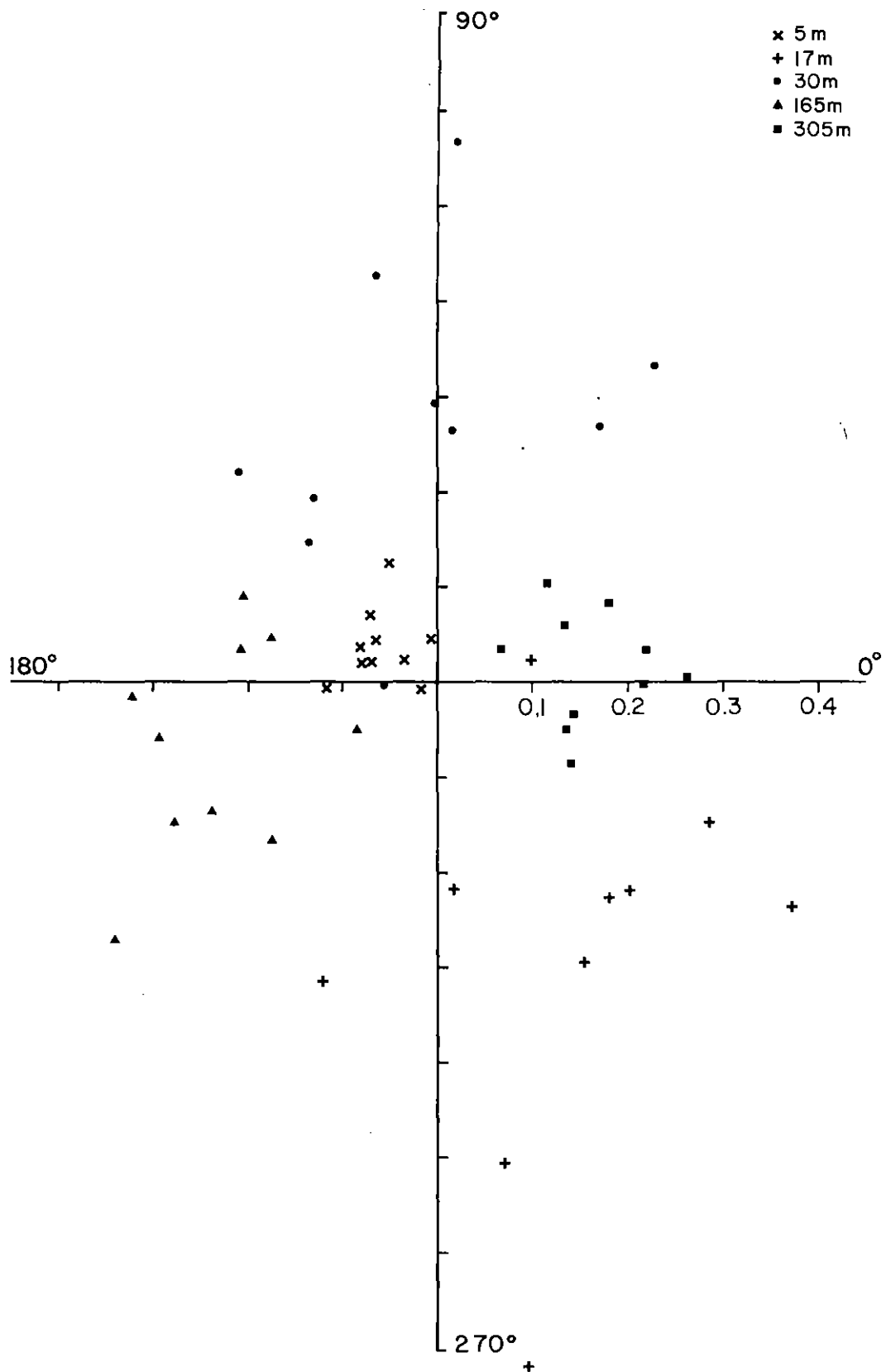


Figure 15. Normalized vector residuals ($\vec{R}_i$).

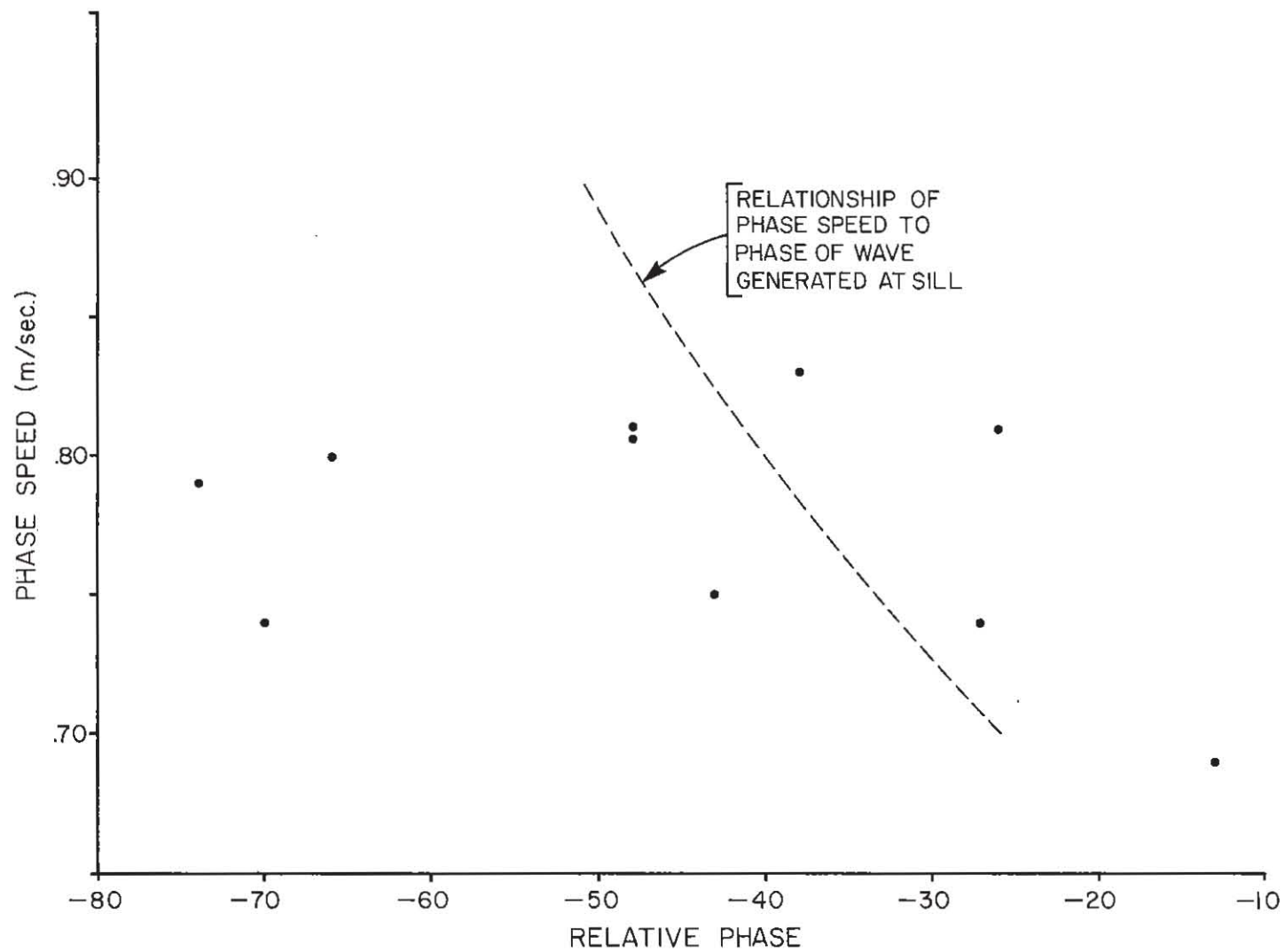


Figure 16. Phase speed versus fitted relative phase for the baroclinic mode.

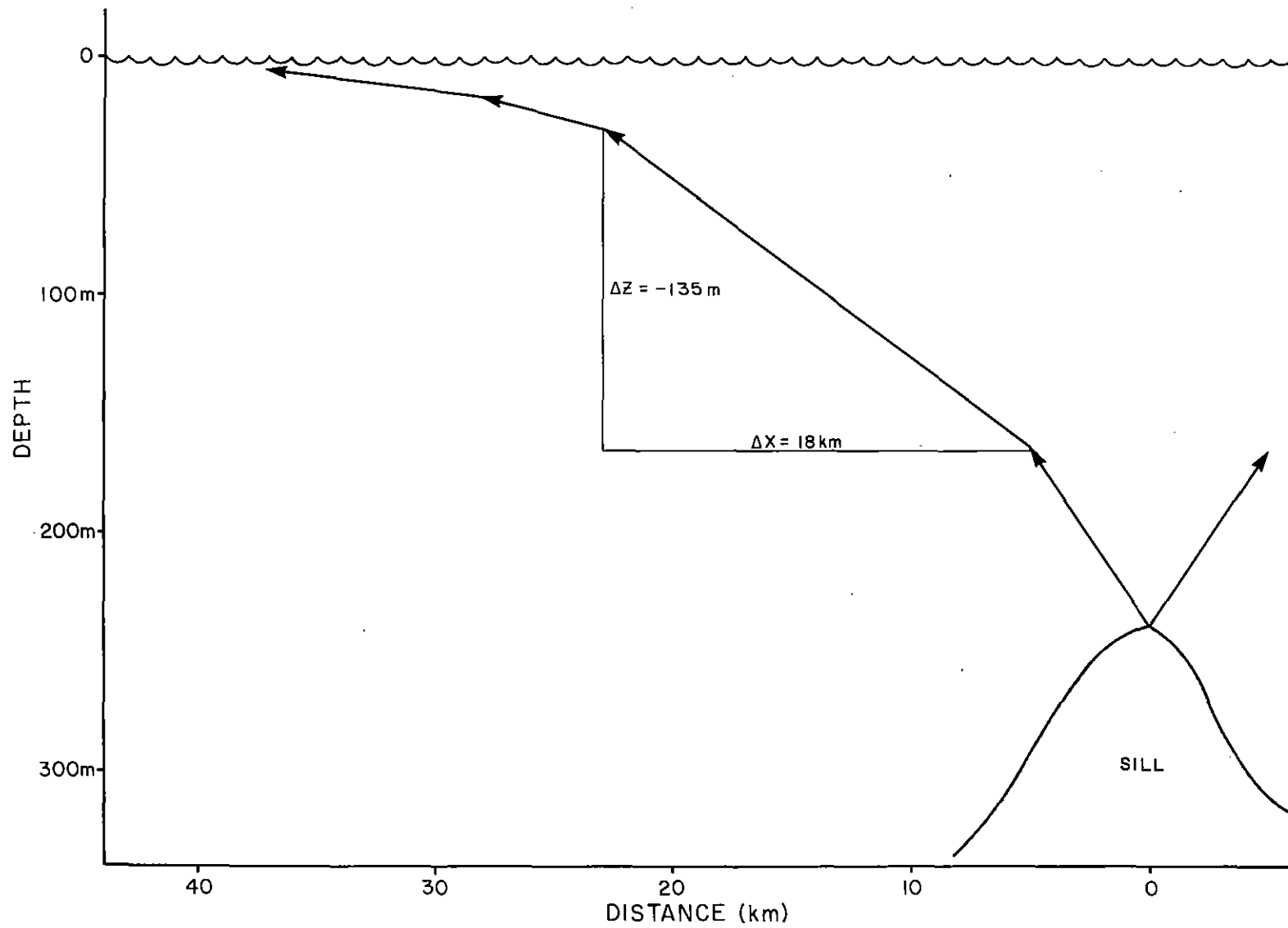


Figure 17. Schematic of baroclinic energy propagation from Douglas Channel sill (week #1).

